This pedagogical overview will cover the current status of neutrino physics from an experimentalist’s point of view, focusing primarily on oscillation studies. The evidence for neutrino oscillations will be presented, along with the prospects for further refinement of observations in each of the indicated regions of two-flavor oscillation parameter space. The next steps in oscillation physics will then be covered (under the assumption of three-flavor mixing): the quest for $\theta_{13}$, mass hierarchy and, eventually, leptonic CP violation. Prospects for non-oscillation aspects of neutrino physics, such as kinematic tests for absolute neutrino mass and double beta decay searches, will also be discussed briefly.

1. Neutrinos and Weak Interactions

Neutrinos, the lightest of the fundamental fermions, are the neutral partners to the charged leptons. In the current picture, they come in three flavors ($e$, $\mu$, $\tau$)\(^{a}\) and interact only via the weak interaction. In the Standard Model of particle physics, neutrinos interact with matter in two ways: in a charged current interaction, the neutrino exchanges a charged $W^\pm$ boson with quarks (or leptons), producing a lepton of the same flavor as the interacting neutrino (assuming there is enough energy available to create the lepton.) In a neutral current interaction, a neutral $Z$ boson is exchanged; this type of interaction is flavor-blind, i.e. the rate does not depend on the flavor of neutrino (see Figure 1.)

2. Neutrino Mass and Oscillations

Neutrinos are known to be very much lighter than their charged lepton partners; direct measurements of neutrino mass yield only upper limits of

\(^{a}\)The $\tau$ neutrino has only recently been directly detected by the DONUT experiment\(^{b}\).
< 2 eV/c². However, the question of neutrino mass can be probed using the oscillatory behavior of free-propagating neutrinos, which is dependent on the existence of non-zero neutrino mass.

Neutrino oscillations arise from straightforward quantum mechanics. We assume that the $N$ neutrino flavor states $|\nu_f\rangle$, which participate in the weak interactions, are superpositions of the mass states $|\nu_i\rangle$, and are related by the Maki-Nakagawa-Sakata (MNS) unitary mixing matrix:

$$|\nu_f\rangle = \sum_{i=1}^N U_{fi} |\nu_i\rangle.$$  (1)

For the two-flavor case, assuming relativistic neutrinos, it can easily be shown that the probability for flavor transition is given by

$$P(\nu_f \rightarrow \nu_g) = 1 - |\langle \nu_f | \nu_g \rangle|^2 = \sin^2 2\theta \sin^2(1.27\Delta m^2 L/E),$$  (2)

for $\Delta m^2 \equiv m_2^2 - m_1^2$ (in eV²) and with $\theta$ the angle of rotation. $L$ (in km) is the distance traveled by the neutrino and $E$ (in GeV) is its energy.

Several comments are in order:

- Note that in this equation the parameters of nature that experimenters try to measure (and theorists try to derive) are $\sin^2 2\theta$ and $\Delta m^2$. $L$ and $E$ depend on the experimental situation.
- The neutrino oscillation probability depends on mass squared differences, not absolute masses.
- In the three-flavor picture, the transition probabilities can be computed in a straightforward way. The flavor states are related to the mass states according to

\[ |\nu_f\rangle = \sum_{i=1}^3 U_{fi} |\nu_i\rangle. \]
\[
\left( \begin{array}{c}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{array} \right) = \left( \begin{array}{ccc}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{array} \right)
\left( \begin{array}{c}
\nu_1 \\
\nu_2 \\
\nu_3
\end{array} \right)
\] (3)

and the transition probability is given by

\[
P(\nu_f \to \nu_g) = \delta_{fg} - 4 \sum_{j>i} \text{Re}(U_{fi}^* U_{gj} U_{gj}^* U_{fi}^*) \sin^2(1.27\Delta m_{ij}^2 L / E)
\]

\[
\pm 2 \sum_{j>i} \text{Im}(U_{fi}^* U_{gj} U_{gj}^* U_{fi}^*) \sin^2(2.54\Delta m_{ij}^2 L / E),
\]

again for \( L \) in km, \( E \) in GeV, and \( \Delta m^2 \) in eV\(^2\). The \( - \) refers to neutrinos and the \( + \) to antineutrinos.

- For three mass states, there are only two independent \( \Delta m_{ij}^2 \) values.
- If the mass states are not nearly degenerate, one is often in a “decoupled” regime where it is possible to describe the oscillation as effectively two-flavor, i.e. following an equation similar to 2, with effective mixing angles and mass squared differences. We will assume a two-flavor description of the mixing for most cases here.
- “Sterile” neutrinos, \( \nu_s \), with no normal weak interactions, are possible in many theoretical scenarios (for instance, as an isosinglet state in a GUT.)
- When neutrinos propagate in matter, the oscillation probability may be modified. This modification is known as the the “Mikheyev-Smirnov-Wolfenstein (MSW) effect” or simply the “matter effect”. Physically, neutrinos acquire effective masses via virtual exchange of W bosons with matter (virtual CC interactions.) For example, consider \( \nu_e \) propagating through solar matter: electron neutrinos can exchange W’s with electrons in the medium, inducing an effective potential \( V = \sqrt{2} G_F N_e \), where \( N_e \) is the electron density. Muon and \( \tau \)-flavor neutrinos, however, can exchange virtual Z bosons only with the matter (because there are no \( \mu \)'s and \( \tau \)'s present.) The probability of flavor transition may be either enhanced or suppressed in a way which depends on the density of matter traversed (and on the vacuum oscillation parameters.) A description of the phenomenology of neutrino matter effects may be found in e.g. References \(^1\)\(^,\)\(^,\)\(^2\). We will see below that matter ef-
fects become important for the solar neutrino oscillation case, and also for future long baseline experiments.

2.1. The Experimental Game

The basic experiment to search for neutrino oscillations can be described very simply.

(1) Start with some source of neutrinos, either natural or artificial.
(2) Calculate (or better yet, measure) the flavor composition and energy spectrum of neutrinos.
(3) Let the neutrinos propagate.
(4) Measure the flavor composition and energy spectrum after propagation. Have the flavors and energies changed? If so, is the change described by the oscillation equation 2? And if so, what are the allowed parameters?

The signature of neutrino oscillation manifests itself in one of two ways, either by disappearance or appearance. In “disappearance” experiments, neutrinos appear to be lost as they propagate, because they oscillate into some flavor with a lower interaction cross-section with matter. An example of disappearance is a solar neutrino experiment, for which $\nu_e$ transform into muon/tau flavor neutrinos, which are below CC interaction threshold at solar neutrino energies of a few MeV (solar $\nu_e$’s do not have enough energy to create $\mu$ or $\tau$ leptons.) In “appearance” experiments, one directly observes neutrinos of a flavor not present in the original source. For example, one might observe $\tau$’s from $\nu_\tau$ in a beam of multi-GeV $\nu_\mu$.

3. The Experimental Evidence

There are currently three experimental indications of neutrino oscillations. These indications are summarized in Table 1. We will now examine the current status of each of these observations.

3.1. Atmospheric Neutrinos

Atmospheric neutrinos are produced by collisions of cosmic rays (which are mostly protons) with the upper atmosphere. Neutrino energies range from about 0.1 GeV to 100 GeV. At neutrino energies $\gtrsim 1$ GeV, for which the geomagnetic field has very little effect on the primary cosmic rays, by geometry the neutrino flux should be up-down symmetric. Although
ν source | Experiments | Flavors | $E$ | $L$ | $\Delta m^2$ sensitivity (eV$^2$)
--- | --- | --- | --- | --- | ---
Sun | Chlorine Gallium | $\nu_e \rightarrow \nu_x$ | 5-15 MeV | $10^8$ km | $10^{-13} - 10^{-10}$ or $10^{-6} - 10^{-3}$
Reactor | Water Cherenkov Scintillator | $\nu_e \rightarrow \nu_x$ | 3-6 MeV | $\sim 180$ km | $10^{-5} - 10^{-3}$
Cosmic ray showers | Water Cherenkov Iron calorimeter Upward muons | $\nu_\mu \rightarrow \nu_x$ | 0.1-100 GeV | $10 - 10^5$ km | $10^{-2} - 10^{-3}$
Accelerator | LSND | $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ | 15-50 MeV | 30 m | 0.1-1

the absolute flux prediction has $\sim 15\%$ uncertainty; the flavor ratio (about two muon neutrinos for every electron neutrino) is known quite robustly, since it depends on the well-understood decay chain $\pi^\pm \rightarrow \mu^\pm \nu_\mu (\bar{\nu}_\mu) \rightarrow e^\pm \nu_e (\bar{\nu}_e) \bar{\nu}_\mu (\nu_\mu)$. The experimental strategy is to observe high energy interactions of atmospheric neutrinos, tagging the flavor of the incoming neutrino by the flavor of the outgoing lepton, which can be determined from the pattern of energy loss: muons yield clean tracks, whereas high-energy electrons shower. Furthermore, the direction of the produced lepton follows the direction of the incoming neutrino, so that the angular distribution reflects the neutrino pathlength distribution.

Super-Kamiokande\(^7\), a large water Cherenkov detector in Japan, has shown a highly significant deficit of $\nu_\mu$ events from below\(^7\), with an energy and pathlength dependence as expected from equation 2 (see Figure 2.) The most recent data constrain the two-flavor $\nu_\mu \rightarrow \nu_\tau$ oscillation parameters to a region as shown in Figure 3. The latest results from Soudan 2\(^2\) (an iron tracker) and from MACRO’s\(^7\) upward-going muon sample are consistent with the Super-K data.

Super-K has also been able to shed some light on the flavors involved in the atmospheric $\nu_\mu$ disappearance. Assuming a two-flavor oscillation, the missing $\nu_\mu$’s could have oscillated into either $\nu_\mu$, $\nu_\tau$ or $\nu_e$. The oscillation cannot be pure $\nu_\mu \rightarrow \nu_e$, because there is no significant excess of $\nu_e$ from below. In addition, the CHOOZ\(^2\) and Palo Verde\(^7\) experiments have ruled out disappearance of reactor $\bar{\nu}_e$; only small mixing to $\nu_e$ is allowed\(^7\). (see Section 6.1.)\(^b\)

The $\nu_\mu \rightarrow \nu_\tau$ hypothesis is difficult to test directly. Super-K expects relatively few charged current (CC) $\nu_\tau$ interactions, and the products of such interactions in the detector are nearly indistinguishable from other atmo-

\(^b\)In fact, a potential small $\nu_\mu \rightarrow \nu_e$ mixing is extremely interesting, as we will see in Section 6.
spheric neutrino events. However, recently Super-K has employed several strategies to distinguish $\nu_{\mu} \rightarrow \nu_{\tau}$ from $\nu_{\mu} \rightarrow \nu_{s}$\textsuperscript{7}. First, one can look for an angular distortion of high-energy neutrinos due to matter effects of sterile neutrinos propagating in the Earth: unlike $\nu_{\tau}$’s, sterile neutrinos do not exchange $Z^0$’s with matter in the Earth, resulting in a matter effect that effectively suppresses oscillation. The effect is more pronounced at higher energies. Such distortion of the high-energy event angular distribution is not observed. Second, one can look at neutral current (NC) events in the detector: if oscillation is to a sterile neutrino, the neutrinos “really disappear” and do not interact via NC. A NC-enriched sample of multiple-ring Super-K events shows no deficit of up-going NC events. Together, these measurements exclude two-flavor $\nu_{\mu} \rightarrow \nu_{s}$ at 99% C. L., for all parameters allowed by the Super-K fully-contained events\textsuperscript{7}. The maximum allowed admixture of sterile neutrinos is about 20%\textsuperscript{7}.

There is one more piece of evidence from Super-K suggesting that $\nu_{\mu} \rightarrow \nu_{\tau}$ oscillations are primarily responsible for the observed disappearance\textsuperscript{1,7}. Because the energy threshold for tau production is about 3.5 GeV and only a small fraction of the atmospheric neutrino flux exceeds this energy, only about 90 $\nu_{\tau}$-induced $\tau$ leptons are expected in Super-K’s 1489 day sample, given the measured oscillation parameters. Tau leptons decay with a very short lifetime into a variety of modes, and can be observed as ener-
getic multi-ring events; such events are very difficult to disentangle from a large background of multi-ring CC and NC events. Nevertheless, three independent Super-K analyses which select “τ-like” events have determined excesses of up-going ντ events consistent with τ appearance at about the 2σ level.

3.1.1. Long Baseline Experiments

The next experiments to explore atmospheric neutrino parameter space are the “long-baseline” experiments, which aim to test the atmospheric neutrino oscillation hypothesis directly with an artificial beam of neutrinos. In order to achieve sensitivity to the oscillation parameters indicated by Super-K, L/E must be such that for ~1 GeV neutrinos, baselines are hundreds of kilometers. A beam is created by accelerating protons and bombarding a target to produce pions and other hadrons; pions are then focused forward with a high-current magnetic “horn” and allowed to decay in a long pipe. The neutrino flavor composition and spectrum can be measured in a near
The first long-baseline experiment is the K2K (KEK to Kamioka) experiment, which started in March 1999, and which saw the first artificial long-distance neutrinos in June 1999. K2K sends a beam of $\langle E_\nu \rangle \sim 1$ GeV $\nu_\mu$ 250 km across Japan to the Super-K experiment. K2K can look for $\nu_\mu$ disappearance (the beam energy is not high enough to make significant numbers of $\tau$’s.) Preliminary K2K results do show a deficit of observed neutrinos: $80.1^{+6.2}_{-5.4}$ beam events in the fiducial volume are expected, based on beam-modeling and near detector measurements; however only 56 single-ring $\nu_\mu$ events were seen at Super-K. The far spectrum was also measured. The best fit oscillation parameters using both spectrum and suppression information are entirely consistent with the atmospheric results. See Figure 4. Somewhat more than half of K2K data has now been taken. The beam resumed in early 2003 after repair of Super-K. The next generation long baseline experiments will be discussed in Section 5.3.

Figure 4. Left: Expected beam neutrino spectrum for no oscillations (dashed line), data (points), expected spectrum with systematic error normalized to the number of observed events (boxes) and best fit to the oscillation hypothesis (solid) for the K2K 1999-2001 data sample. Right: allowed region in oscillation parameter space corresponding to K2K 1999-2001 data sample, using both suppression and spectrum.

### 3.2. Solar Neutrinos

The deficit of solar neutrinos was the first experimental hint of neutrino oscillations. The solar neutrino energy spectrum is well-predicted, and depends primarily on weak physics, being rather insensitive to solar physics.
The three “classic” solar neutrino detectors (chlorine, gallium and water Cherenkov), with sensitivity at three different energy thresholds, together observe an energy-dependent suppression which cannot be explained by any solar model (standard or non-standard). The observed suppression in all three experiments can be explained by neutrino oscillation at certain values of $\Delta m^2$ and mixing angle: see Figure 5. The “classic” allowed regions at higher values of $\Delta m^2$ (“small mixing angle”, “large mixing angle” and “low”) are those for which matter effects in the Sun come into play. There are also solutions at very small $\Delta m^2$ values for which matter effects in the Sun are not involved: these are known as “vacuum” oscillation or “just-so” solutions. Figure 5 shows the mixing angle axis plotted as $\tan^2 \theta$, rather than as the more conventional $\sin^2 2\theta$, to make evident the difference between $0 < \theta < \pi/4$ and $\pi/4 < \theta < \pi/2$: these regions are not equivalent when one considers matter effects.

Before 2000, the most precise real-time solar neutrino data came from Super-K via the elastic scattering reaction $\nu_{e,x} + e^- \rightarrow \nu_{e,x} + e^-$, which proceeds via both CC and NC channels, with a cross-section ratio of about 1:6. In this reaction, the Cherenkov light of the scattered electron is measured. The scattered electrons point away from the direction of the sun.

Possible “smoking guns” for neutrino oscillations include a distortion from the expected shape that would be hard to explain by other than non-standard weak physics. The latest Super-K solar neutrino spectrum shows no evidence for distortion. Another “smoking gun” solar neutrino measurement is the day/night asymmetry: electron neutrinos may be regenerated in the Earth from their oscillated state for certain oscillation parameters. The latest measured Super-K day/night asymmetry is $\frac{(\text{day} - \text{night})}{(\text{day} + \text{night})/2} = -0.021 \pm 0.020^{+0.013}_{-0.012} \text{ (syst)}$: regeneration is therefore a relatively small effect, if it is present at all. Together, the energy spectrum and day/night observations place strong constraints on solar neutrino parameters. In particular, Figure 5 shows the Super-K results overlaid on the global flux fit parameters: large mixing angles are favored, and the small mixing angle and vacuum solutions from the global flux fit are disfavored at 95% C.L.. Global flux fit $\nu_e \rightarrow \nu_x$ solutions are also disfavored.

The information from Super-K served primarily to constrain parame-
Figure 5. Solar neutrino parameter space: the shaded areas show the “classic” global flux fit solutions from chlorine, gallium and water Cherenkov experiments (from Reference 7).

The true “smoking gun” for solar neutrino oscillations recently came from the Sudbury Neutrino Observatory, a detector comprising 1 kton of D$_2$O in Sudbury, Canada, with the unique capability to detect neutral current reactions from the breakup of deuterium, $\nu_x + d \rightarrow \nu_x + p + n$: since this reaction is flavor-blind, it measures the total active neutrino flux from the sun. Neutrons from this reaction can be detected via various methods: capture on d itself, capture on Cl ions from dissolved salt, and neutron detectors. In addition, the charged current reaction $\nu_e + d \rightarrow \nu_e + p + e^-$ specifically tags the $\nu_e$ component of the solar flux. SNO also observes the same neutrino-electron elastic scattering (ES) interaction as Super-K, which proceeds via both CC and NC channels.

SNO’s recent results are summarized in Figure 7, which shows the mea-
Figure 6. Solar neutrino parameter space: on the left, the light grey areas show the “classic” global flux fit solutions from chlorine, gallium and the SNO experiment’s CC measurement. The darker grey shaded regions indicate Super-K’s excluded regions from spectral and day/night information (and the darkest grey regions indicate the overlap.)

On the right, the light shaded areas indicate allowed regions from Super-K data alone and SSM $^8$B neutrino flux.

sured fluxes $\phi_{\mu\tau}$ vs $\phi_e$.\(^d\) The CC measurement, which tags $\nu_e$ flux $\phi_e$, is represented by a vertical bar on this plot. Since the neutral current flux is flavor-blind and therefore represents a measurement of the sum of $\phi_{\mu\tau}$ and $\phi_e$, i.e. $\phi_{NC} = \phi_{\mu\tau} + \phi_e$, the NC measurement corresponds to a straight line with slope $-1$ on this plot. The intersection with the vertical CC line indicates the composition of the solar neutrino flux: it is approximately $1/3 \, \nu_e$ and $2/3 \, \nu_{\mu,\tau}$. The ES reaction measures both $\nu_e$ and $\nu_{\mu,\tau}$ with a known ratio, $\phi_{ES} = 0.154\phi_{\mu\tau} + \phi_e$ for SNO, so that the ES measurement corresponds to a line on the plot with slope $-1/0.154$; it provides a consistency check. The conclusion from SNO is that solar neutrinos really are oscillating (into active neutrinos.) The solar neutrino problem is solved!

The detailed measurements from SNO incorporating observed day/night asymmetry and energy spectra shrink the allowed parameters down to small

\(^d\)Note that one cannot distinguish between $\nu_\mu$ and $\nu_\tau$ at low energy since the NC interaction does not distinguish between them.
regions, shown in Figure 7. At 99% C.L., only the LMA region is left.

So far, SNO’s NC measurement comes from capture of neutrons on $d$; SNO continues to run, and will provide cross-checked NC measurements using salt and helium neutron counters.

There is one more recent chapter in the solar neutrino story. KamLAND, a 1 kton scintillator detector at the Kamioka mine in Japan, has investigated solar neutrino oscillation parameters using reactor neutrinos rather than solar neutrinos directly. Reactors produce $\bar{\nu}_e$ of few-MeV energies abundantly; assuming vacuum oscillations, the baseline required to observe oscillations with LMA parameters is about $\sim 100$ km. Note that no significant matter effects are expected at this baseline. KamLAND observes the sum of the fluxes of neutrinos from reactors in Japan and Korea, with roughly a 180 km average baseline, via the inverse beta decay reaction $\bar{\nu}_e + p \rightarrow e^+ + n$; $\bar{\nu}_e$’s are tagged using the coincidence between the positron and the 2.2 MeV $\gamma$-ray from the captured neutron. In December 2002, the KamLAND experiment announced an observed suppression of reaction $\bar{\nu}_e$ consistent with LMA parameters: see Figure 8. Solar neutrino oscillations are therefore now confirmed using a completely independent source of neutrinos and experimental technique. In addition, the LMA solution is strongly indicated.

3.3. LSND

The third oscillation hint is the only “appearance” observation: the Liquid Scintillator Neutrino Detector (LSND) experiment at Los Alamos has ob-
13

1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0

N obs / N exp

10
10
10
10
10
10

Distance to Reactor (m)

ILL
Savannah River
Bugey
Rovno
Goesgen
Krasnoyarsk
Palo Verde
Chooz
KamLAND

θ

22sin

0.0
0.2
0.4
0.6
0.8
1

0

(\text{eV}^2 m^{-1})

10^{-6}
10^{-5}
10^{-4}
10^{-3}

Rate excluded
Rate+Shape allowed

LMA
Palo Verde excluded
Chooz excluded

An experiment at Rutherford-Appleton Laboratories in the U. K. called KARMEN, which has roughly similar neutrino oscillation sensitivity as does LSND (although with a shorter 17.5 m baseline), does not however confirm the LSND result\textsuperscript{7}. This detector expects fewer signal events than does LSND, but has a stronger background rejection due to the pulsed nature of the ISIS neutrino source. However, due to somewhat different sensitivity, KARMEN’s lack of observation of $\bar{\nu}_e$ appearance cannot rule out all of the parameter space indicated by LSND: see Figure 9.

4. Where Do We Stand?

Now we can step back and view the big picture. Where do we stand? The current experimental picture for the three oscillation signal indications can be summarized:

Figure 8. Left: Ratio of measured to expected $\bar{\nu}_e$ for various experiments as a function of baseline; the point on the far right is the KamLAND result. The shaded area represents the expectation from the solar LMA solution, and the dotted line is the best fit to the oscillation hypothesis. Right: Allowed and excluded regions in oscillation parameter space for various experiments (as indicated in the legend.)

served an excess of $\bar{\nu}_e$ events\textsuperscript{7} from a beam which should contain only $\bar{\nu}_\mu$, $\nu_e$, and $\nu_\mu$ from positive pion and muon decay at rest. The result is interpreted as $\sim$20-50 MeV $\bar{\nu}_\mu$’s oscillating over a 30 m baseline. See Figure 9 for the corresponding allowed region in parameter space, which is at large $\Delta m^2$ and small mixing. (The large mixing angle part of this range is ruled out by reactor experiments.)
Figure 9. The shaded region shows the LSND allowed regions at 90% and 99% C.L.; the region to the right of the KARMEN2 line is excluded by KARMEN. Also shown are exclusions by the reactor experiments Bugey and Chooz, the NuTeV and Nomad excluded regions, and the reach of the mini-BooNE experiment (see Section 5.1.)

- For atmospheric neutrino parameter space: evidence from Super-K, Soudan 2 and MACRO is very strong for \( \nu_\mu \to \nu_x \). Furthermore, Super-K’s data favor the \( \nu_\mu \to \nu_\tau \) hypothesis over the \( \nu_\mu \to \nu_s \) one. These oscillation parameters have been independently confirmed using the K2K beam of \( \sim 1 \text{ GeV} \) \( \nu_\mu \)'s to Super-K.

- For solar neutrino parameter space (\( \nu_e \to \nu_x \)): The solar neutrino problem is now solved. While Super-K data favored large mixing via day/night and spectral measurements, SNO’s D\(_2\)O-based NC and CC measurements have confirmed that solar neutrinos are oscillating, and have shrunk down the allowed parameter space to the LMA region using day/night and spectral measurements. Better yet, the KamLAND experiment has independently confirmed the LMA solution using reactor \( \bar{\nu}_e \)'s. Oscillation to sterile neutrinos is disfavored.

- The LSND indication of \( \bar{\nu}_\mu \to \bar{\nu}_e \) still stands; KARMEN does not rule out all of LSND’s allowed parameters.

What do these data mean? There is an obvious problem. Under the assumption of three generations of massive neutrinos, there are only two
Figure 10. Oscillation parameter space showing all three indications of oscillation, in the two-flavor mixing approximation. At high $\Delta m^2$, the parameters allowed by LSND are shown by dotted lines, and the part not excluded by Karmen is shown as a solid region. Allowed atmospheric neutrino parameters are shown at large mixing and $\Delta m^2$ of about $2.5 \times 10^{-3}$ eV$^2$. Also shown by dotted lines are the “classic” solar neutrino solutions at small $\Delta m^2$: “small mixing angle” (SMA), “large mixing angle” (LMA), and “low”, which all involve matter effects in the sun, and the vacuum solutions at very small $\Delta m^2$. With new information from SNO and KamLAND, only the LMA solution is now allowed, as indicated by the solid region at about $4.5 \times 10^{-4}$ eV$^2$.

independent values of $\Delta m^2_{ij}$: we must have $\Delta m^2_{13} = \Delta m^2_{12} + \Delta m^2_{23}$. However, we have three measurements which give $\Delta m^2_{ij}$ values of three different orders of magnitude. So, if each hint represents two-flavor mixing, then something must be wrong. All data cannot be satisfactorily fit assuming three-flavor oscillations. One way to wriggle out of this difficulty is to introduce another degree of freedom in the form of a sterile neutrino (or neutrinos) or else invoke some exotic solution (e.g. CPT violation). (We cannot introduce another active neutrino, due to the $Z^0$ width mea-
surements from LEP, which constrain the number of light active neutrinos to be three\(^2\): any new light neutrino must be sterile.) Although pure mixing into \(\nu_s\) is now disfavored by solar and atmospheric neutrino results, a sterile neutrino is still barely viable as part of some four-flavor mixing\(^7\). Of course, it is also possible that some of the data are wrong or misinterpreted. Clearly, we need more experiments to clarify the situation.

5. What’s Next for Two-flavor Oscillations?
So what’s next? First, let’s consider the next experiments for each of the interesting regions of two-flavor parameter space.

5.1. LSND Neutrino Parameter Space
The next experiment to investigate the LSND parameter space will be BooNE (Booster Neutrino Experiment.) This will look at \(\sim 1\) GeV neutrinos from the 8 GeV booster at Fermilab, at a baseline of about 500 m (with a second experiment planned at longer baseline if an oscillation signal is seen.) This experiment is primarily designed to test \(\nu_\mu \rightarrow \nu_e\) at about the same \(L/E\) as LSND. Since the neutrino energy is higher, and the backgrounds are different, systematics will presumably be different from those at LSND. BooNE, which started in 2002, expects to cover all of LSND parameter space\(^7\) (see Figure 9.) If a signal is found, the BooNE collaboration plans to build another detector at a longer baseline to further test the oscillation hypothesis.

5.2. Solar Neutrino Parameter Space
Now that the latest results from SNO and KamLAND have squeezed the allowed solar mixing parameters down to the LMA region, solar neutrino physics is entering a precision measurement era. Over the next few years, we expect to have cross-checks of NC measurements from SNO, using different neutron detection techniques (salt, NCDs.) From KamLAND we expect better precision from improved statistics and systematics; KamLAND will also attempt to measure the solar neutrino flux directly. Borexino, a planned 300 ton scintillator experiment at the Gran Sasso Laboratory in Italy\(^7\) with very low radioactive background, hopes to measure the solar \(^7\)Be line at 0.86 MeV.

The true frontier for solar neutrino experiments is the real-time, spectral measurement of the flux of neutrinos below 0.4 MeV produced by pp
reactions in the sun, which are responsible for most of the solar energy generation. The pp flux is precisely known, which will aid in precision measurements of mixing parameters; in addition if the total pp flux is well-known, measurement of the active component will help constrain a possible sterile admixture. The pp flux is also a new window on solar energy generation. Because the pp flux is very large, one can build relatively small (tens to hundreds of tons) detectors and still expect a reasonable rate of neutrino interactions. The challenge is to achieve low background at low energy threshold. There are a number of innovative new solar neutrino experiments aiming to look at the very low energy pp solar flux, among them LENS, Heron, solar-TPC and CLEAN.

5.3. Atmospheric Neutrino Parameter Space

Two-flavor oscillation studies at atmospheric neutrino parameters has also entered a precision measurement era.

The K2K experiment will continue, now that Super-K has been refurbished to 47% of its original number of inner detector phototubes after the accident of November 2001. The results published so far represent about half of the total number of protons on target for the neutrino beam; the next few years will see both systematic and statistical precision improvements in mixing parameter measurements.

The next set of long baseline experiments to explore atmospheric oscillation parameter space have $\sim 730$ km baselines and will start in a few years. The NuMi beamline\(^7\) will send a $\nu_\mu$ beam from Fermilab to Soudan, with a beam energy of $3 - 8$ GeV, and a baseline of 735 km. The far detector, MINOS\(^7\) is a magnetic iron tracker. A primary goal is to attain 10% precision on $2-3$ mixing parameters $\Delta m^2_{23}$ and $\sin^2 2\theta_{23}$.

CNGS (Cern Neutrinos to Gran Sasso)\(^7\) is a $\sim 20$ GeV $\nu_\mu$ beam from CERN to the Gran Sasso 730 km away. The two planned CNGS detectors, OPERA\(^7\) and Icarus\(^7\), are focused on an explicit $\nu_\tau$ appearance search. Because when $\tau$'s decay they make tracks only about 1 mm long, both detectors are fine-grained imagers. Icarus is a liquid argon time projection chamber, and OPERA is a hybrid emulsion/scintillator detector. Both experiments expect a few dozen $\tau$ events over several years of running.

6. Beyond Two-Flavor Oscillations

The previous section discussed the future of neutrino oscillation studies in the context of two-flavor oscillations. As noted in section 2, however, this
is an approximation valid for well-separated mass states, which appears to be the case. However a full description requires three flavors.

In the following, we will assume that a “standard” three-flavor picture is valid. If mini-BooNE confirms the LSND effect, we will have to rethink our picture, and our goals.

In this “standard” picture, neutrino mixing can be described by six parameters: two independent $\Delta m^2_{ij}$ ($\Delta m^2_{12}$, $\Delta m^2_{23}$), three mixing angles ($\theta_{12}$, $\theta_{23}$, $\theta_{13}$), and a CP violating phase $\delta$. The mixing matrix $U$ of equation 2 can be written as a product of three Euler-like rotations, each described by one of the mixing angles:

$$
U = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix} \begin{pmatrix}
c_{13} & 0 & s_{13}e^{i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta} & 0 & c_{13}
\end{pmatrix} \begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
$$

where “$s$” represents sine of the mixing angle and “$c$” represents cosine.

The “1-2” matrix describes solar mixing; the “2-3” matrix describes atmospheric neutrino mixing. The “1-3” or “e3” mixing is known to be small; $\theta_{13}$ may be zero. The mass-squared difference $\Delta m^2_{23} \sim 2 \times 10^{-3}$ eV$^2$ describes the atmospheric mixing, and $\Delta m^2_{12} \sim 4.5 \times 10^{-5}$ eV$^2$ describes solar mixing. Neutrino oscillation experiments tell us only about mass-squared differences; the absolute mass scale is known only to be less than about 2 eV. It is also as yet unknown whether the mass hierarchy is “normal”, i.e. the solar mixing is described by two lighter states, or “inverted”, i.e. the solar mixing is described by two heavier states (see Figure 11.)

The remaining questions can be addressed by neutrino oscillation experiments are:

- Is $U_{e3}$ non-zero?
- Is the hierarchy normal or inverted?
- Is 2-3 mixing maximal, or just large?
- Is the CP-violating phase non-zero?

### 6.1. The Next Step: e3 Mixing

The next question which can be approached experimentally is that of e3 mixing. A consequence of a non-zero $U_{e3}$ matrix element will be a small

---

*Majorana phases, which cannot be measured in oscillation experiments and in general are very difficult to observe*, will not be considered here.
appearance of $\nu_e$ in beam of $\nu_\mu$: for $\Delta m_{23}^2 >> \Delta m_{12}^2$ (as is the case), and for $E_\nu \sim L\Delta m_{23}^2$, ignoring matter effects we find

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 (1.27\Delta m_{23}^2 L/E). \quad (5)$$

This expression illustrates that $\theta_{13}$ manifests itself in the amplitude of an oscillation with 2-3-like parameters. Since $\nu_e$ appearance has never been observed at these parameters, this amplitude (and hence $\theta_{13}$) must be small.

The best limits so far, shown in Figure 12, come from a reactor experiment, CHOOZ, which observed no disappearance of reactor $\bar{\nu}_e$. The on-axis long baseline experiments mentioned in section 3.1.1 can likely improve this limit by a factor of approximately five. To do better than this is a difficult job: since the modulation may be parts per thousand or smaller, one needs both good statistics and low background data. The primary sources of background for a long baseline experiment are: intrinsic beam $\nu_e$ contamination, misidentified NC resonant $\pi^0$ production (since $\pi^0$ decay to $\gamma$-rays which make electron-like showers), and other misidentified particles.

### 6.1.1. Off-Axis Beams

A promising next step for measurement of $\theta_{13}$ (assuming it is large enough to be measured) is an off-axis detector at a long baseline neutrino beam. An

---

1In the literature one finds limits and sensitivities to this mixing angle variously expressed in terms of $\theta_{13}$ (in radians or degrees), $\sin \theta_{13}$, $\sin^2 \theta_{13}$, $\sin^2 2\theta_{13}$, $\sin^2 2\theta_{13}\equiv |U_{e3}|^2$; no convention has yet emerged. For $\Delta m_{12}^2 << \Delta m_{23}^2$ $\sim \Delta m_{13}^2$, $\sin^2 2\theta_{13} \equiv \sin^2 2\theta_{13}\sin^2 \theta_{23}$ is the measured $\nu_e$ appearance amplitude. $|U_{e3}| = \sin \theta_{13}$, and for $\theta_{23} \sim \pi/4$ and $\theta_{13}$ small, it follows that $\sin^2 2\theta_{13}\sim 4 \sin^2 2\theta_{13} \sim 2 \sin^2 \theta_{13}$. 

---

Figure 11. Normal and inverted hierarchies in the three-flavor picture, with mass-squared values of the three states indicated vertically, and possible flavor composition of the mass states indicated by the horizontal divisions.
Figure 12. Expected sensitivity to $\sin^2 2\theta_{13}$ with J-PARC$\nu$, for various beam configurations (a 2° off-axis configuration, labeled OAB 2deg on the plot, has been selected.) The CHOOZ excluded region for $\bar{\nu}_e$ disappearance is shown for comparison. NuMi off-axis and new reactor experiments have comparable sensitivity.

experiment placed a few degrees off axis has some kinematic advantages. Because two-body pion decay kinematics imply that neutrino energy becomes relatively independent of pion energy off-axis, the neutrino spectrum becomes more sharply peaked. Therefore an off-axis siting is favorable for background reduction and oscillation fits, in spite of the reduction in $\nu$ flux.

There are two major long-baseline off-axis detector projects currently under consideration. The first of these is J-PARC$\nu^7$, comprising a high power ($0.77$ MW) beam from the J-PARC facility in Japan (currently under construction.) The far detector, 2° off-axis, will be a fully refurbished Super-K at 295 km. The second off-axis proposal$^7$, for the U.S., is to exploit the NuMi beam which will exist for MINOS, and build a new detector off-axis at a distance of 700-900 km. Various detector technologies and sites are under discussion. Sensitivity to $\theta_{13}$ for both possibilities will be roughly a factor of 10-20 better than the CHOOZ limit.
6.1.2. Reactor Experiments

Another possibility currently under consideration by groups in Russia, Japan and the U.S. is an upgraded reactor experiment employing the same strategy as CHOOZ, i.e. search for disappearance of $\bar{\nu}_e$. The challenge for this type of experiment is reduction of systematics to the level where a few percent or smaller modulation is evident. Multiple detectors may help to achieve this.

6.2. Leptonic CP Violation and Mass Hierarchy

The long-term goal of long baseline oscillation experiments in the observation of CP violation in the lepton sector. The basic idea is to measure a difference between $\nu_\mu \rightarrow \nu_e$ and $\nu_\mu \rightarrow \bar{\nu}_e$ transition probabilities. However, it is not simple to extract a CP violating phase from the measurements: transition rates depend on all MNS matrix parameters, and in addition are affected by the presence of matter.

Following the analysis of Reference 7: the approximate transition probabilities for neutrinos and antineutrinos (assuming $\theta_{13}$, $\Delta_{12}$, $L$ and $\Delta_{12}/\Delta_{13}$