Generation of polarization squeezed light in PPNC

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Abstract

Theoretical analysis is presented on quantum state evolution of polarization light waves at frequencies $\omega_0$ and $\omega_1$ in a periodically poled nonlinear crystal (PPNC). It is shown that the variances of all the four Stokes parameters can be squeezed.

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1 Introduction

During the last decade much attention has been paid to the realization of quantum information protocols [1] such as quantum teleportation, quantum cryptography. These protocols are based on the methods of nonlinear quantum optics. Nonlinear optical sources [2] play an important role in the generation of nonclassical states of light [3] and realization of optical quantum information protocols. The nonlinear optical processes such as degenerate parametric down conversion (type I and II processes) [2,3] and the Kerr effect [2,3] are used to create nonclassical states of light (squeezed states, polarization squeezed states and entangled states). The variance of one of the four Stokes parameters of polarization squeezed light is less than the corresponding value for the coherent state. Traditionally the degenerate parametric process (type II) and the Kerr effect are responsible for the generation of polarization squeezed states or polarization entangled states in ordinary nonlinear crystals. One can achieve suppression of variances of at least one of the Stokes parameters [5-9] ($S_0, S_1, S_2, S_3$) in the type II process by using ordinary nonlinear crystal. Most of the quantum information protocols are based on type II process [1] and Kerr effect, which are used to generate entangled states. The entangled states are used in the realization of quantum teleportation and quantum cryptography protocols. Experiments on quantum teleportation and quantum cryptography are performed by using ordinary nonlinear optical crystals with second and third order nonlinear susceptibilities. In the past few years, some experiments in the creation and realization of nonclassical and entangled states are performed by using PPNCs with second order nonlinear susceptibilities. The PPNCs [10,11], which have many interesting advantages as compared to ordinary nonlinear crystals were proposed by Bloembergen and co-authors in 1962. The main advantages of PPNCs against ordinary nonlinear crystals are: the quasi-phase-matching condition between the interacting waves; the highest nonlinear susceptibility coefficient can be used; multi-mode interaction of optical waves.

Recent experiments on quantum noise reduction [12,13] and the generation of entangled states [14] promising the applications of PPNCs in the realization of optical quantum information protocols. These experiments were based on type I and II processes. It should be noted that the parametric down conversion (type I) and frequency sum generation processes have been studied theoretically [see for instance, 4] and the references therein] and experimentally [12-15] very well. In a PPNC, the possibilities of type II process is much more complicated as compared to ordinary nonlinear crystals. The type II process in PPNC can be accompanied by three other nonlinear processes [4]. All these nonlinear processes can be quasi-phase-matched at certain coherent lengths [4]. Here we will study the generation of polarization squeezed states based on type II processes in PPNC with second order nonlinear susceptibility.

The main goal of this work is to show that PPNCs can suppress all the four variances of Stokes parameters below the standard quantum limit. The theoretical work that we present here is the first, to the best of our knowledge, to propose PPNCs for the generation of polarization squeezed light.

The structure of the paper is as follows. Section 2 describes the optical nonlinear processes and their Heisenberg equations of motions. Section 3 studies the behaviour of mean photon numbers of degenerate polarization (orthogonal) modes at frequencies $\omega_0$ and $\omega_1$. Section 4 analyzes the variances of Stokes parameters. The final section summarizes the results obtained in sections 3 and 4.
2 Equations of motions

We consider the five-frequency interaction of co-propagating light waves in a PPNC (see Fig.1). The four interaction processes of light waves at frequencies $\omega_0$, $2\omega_e$, $3\omega_e$, and $4\omega_e$ are [4]

$$\omega_0 + \omega_e = 2\omega_e,$$
$$\delta k_1 = k_{2e} - k_{10} - k_{1e} + m_1 G_1 = \Delta k_1 + m_1 G_1, \quad (1)$$
$$\omega_0 + 2\omega_e = 3\omega_e,$$
$$\delta k_2 = k_{3e} - k_{10} - k_{2e} + m_2 G_2 = \Delta k_2 + m_2 G_2, \quad (2)$$
$$\omega_0 + 3\omega_e = 4\omega_e,$$
$$\delta k_3 = k_{4e} - k_{1e} - k_{3e} + m_3 G_3 = \Delta k_3 + m_3 G_3, \quad (3)$$
$$\delta k_4 = k_{4e} - 2k_{2e} + m_4 G_4 = \Delta k_4 + m_4 G_4, \quad (4)$$

where $\Delta k_{1,2,3,4}$ is a phase mismatch for an ordinary nonlinear bulk crystal; $k_{jn}$ is a wave number of interacting modes ($n = 0$ stands for ordinary and $n = e$ for extraordinary); $G_j = 2\pi\Lambda_j^{-1}$ is the modulus of the vector of a reciprocal lattice with period $\Lambda_j$; $m_j = \pm 1, \pm 3, \ldots$, is the quasi-phase-matched order.

The nonlinear process (1) describes the splitting of a photon of frequency $2\omega_e$ into two photons of orthogonal polarizations with degenerate frequencies $\omega_0$ and $\omega_e$. The second process (2) describes the sum frequency generation, i.e. photon of frequency $2\omega_e$ combines with the photon of frequency $\omega_0$, which gives rise to a photon of frequency $3\omega_e$. The third (3) and fourth (4) processes are responsible for the generation of the fourth harmonic with two different ways, i.e. a photon of frequency $\omega_0$ and a photon of frequency $3\omega_e$ combine, and a photon with frequency $4\omega_e$ appears as a result or the same can be achieved by the combination of two photons with frequencies $2\omega_e$.

It has been shown that the nonlinear processes (1-4) can be simultaneously quasi-phase-matched [4] in a single domain structure ($G_1 = G_2 = G_3 = G_4$) or at certain coherent lengths $L_{coh}^j$. The processes (1-4) under study can be simultaneously quasi-phase-matched with the following relationship

$$L_{coh}^{1,2} = 9L_{coh}^{3,4}. \quad (5)$$

The processes (1-4) can be described by the following interaction Hamiltonian

$$\hat{H}_I(z) = \hbar g(z)[\xi_1 \hat{a}_{10} \hat{a}_{1e} \hat{a}_{2e} e^{i\Delta k_1 z} + \xi_2 \hat{a}_{10} \hat{a}_{2e} \hat{a}_{3e} e^{i\Delta k_2 z} + \xi_3 \hat{a}_{10} \hat{a}_{3e} \hat{a}_{4e} e^{i\Delta k_3 z} + \xi_4 \hat{a}_{2e} \hat{a}_{4e} e^{i\Delta k_4 z} + HC], \quad (6)$$

where $\hbar$ is Planck’s constant, $\hat{a}_{jn}(\hat{a}^+_{jn})$ is the annihilation (creation) operator of photon of $j$th mode at frequency $j\omega_n$; $\xi_j$ is the nonlinear coupling coefficient; $g(z)$ is the periodic function equal to +1 or −1 at the domain thickness $l = \Lambda/2$; HC denotes Hermitian conjugate. The operators $\hat{a}_{jn}(\hat{a}^+_{jn})$ obey the following commutation rules

$$[\hat{a}_{jn}, \hat{a}^+_{pn}] = \delta_{jn, pn}, \quad \text{with} \quad j, p = 1, 2, 3, 4 \quad (7)$$

The interaction Hamiltonian (6) can be averaged over the period $\Lambda$, if the interaction length $z$ is much more than the period of modulation $\Lambda$, i.e. $z \gg \Lambda$. Then the interaction Hamiltonian (6) takes the form

$$\hat{H}_I = \hbar \gamma_j \alpha_{10} \alpha_{1e} e^{i\Delta k_j z} + \gamma_2 \alpha_{10} \alpha_{2e} \alpha_{3e} + \gamma_3 \alpha_{1e} \alpha_{3e} \alpha_{4e} + \gamma_4 \alpha_{2e} \alpha_{4e} + HC], \quad (8)$$

where

$$\gamma_j = \frac{\xi_j}{\Lambda} \int_{-\Lambda/2}^{+\Lambda/2} g(z) \exp (\pm i\Delta k_j z) = 2\xi_j/(\pi m_j).$$

The Heisenberg operator equations corresponding to the interaction Hamiltonian (8) are given by

$$i\frac{d\hat{a}_{10}}{dz} = \gamma_2 \hat{a}_{2e} \hat{a}_{3e} + \gamma_1 \hat{a}_{1e} \hat{a}_{2e},$$
$$i\frac{d\hat{a}_{1e}}{dz} = \gamma_3 \hat{a}_{3e} \hat{a}_{4e} + \gamma_1 \hat{a}_{1e} \hat{a}_{2e},$$
$$i\frac{d\hat{a}_{2e}}{dz} = 2\gamma_4 \hat{a}_{4e} \hat{a}_{2e} + \gamma_2 \hat{a}_{1e} \hat{a}_{3e} + \gamma_1 \hat{a}_{1e} \hat{a}_{10},$$
$$i\frac{d\hat{a}_{3e}}{dz} = \gamma_2 \hat{a}_{1e} \hat{a}_{2e} + \gamma_3 \hat{a}_{1e} \hat{a}_{4e},$$
$$i\frac{d\hat{a}_{4e}}{dz} = \gamma_4 \hat{a}_{2e}^2 + \gamma_3 \hat{a}_{1e} \hat{a}_{3e}. \quad (9)$$
Assuming the pump modes at frequencies $\omega_{2e}$, $\omega_{4o}$ are classical and non-depleted at the input of a PPNC, i.e.

$$\hat{a}_{2e} = A_{2e}, \quad \hat{a}_{4o} = A_{4o},$$

(10)

(11)

where $A_{2e,4o} = e^{i\phi/2}$ are the complex amplitudes of the pump modes, we obtain the following linear system of equations, after the substitution of quantities (10) and (11) into (9):

$$\frac{d\hat{a}_{10}}{dz} = -\gamma_2 \hat{a}_{3e} + \gamma_1 \hat{a}_{1e}^+, \quad \frac{d\hat{a}_{1e}}{dz} = \gamma_3 \hat{a}_{3e}^+ + \gamma_1 \hat{a}_{1o},$$

$$\frac{d\hat{a}_{3e}}{dz} = \gamma_2 \hat{a}_{1o} + \gamma_3 \hat{a}_{1e}^+.$$  

(12)

For simplicity, we introduce a normalized interaction length parameter $\zeta$

$$\zeta = z \gamma_1.$$  

(13)

The quantity (13) is introduced into the set of equations (12), which reduce after straightforward algebra to the set of equations

$$\frac{d\hat{a}_{10}}{d\zeta} = -k_2 \hat{a}_{3e} + \hat{a}_{1e}^+, \quad \frac{d\hat{a}_{1e}}{d\zeta} = k_1 \hat{a}_{3e}^+ + \hat{a}_{1o},$$

$$\frac{d\hat{a}_{3e}}{d\zeta} = k_2 \hat{a}_{1o} + k_1 \hat{a}_{1e}^+.$$  

(14)

where $k_1 = \gamma_3 / \gamma_1$ and $k_2 = \gamma_2 / \gamma_1$. The set of linear equations (14) is solved by applying the Laplace transformation:

$$\hat{a}_{10}(\zeta) = \lambda_{11}(\zeta) \hat{a}_{10} + \lambda_{12}(\zeta) \hat{a}_{1e}^+ + \lambda_{13}(\zeta) \hat{a}_{3e},$$

$$\hat{a}_{1e}(\zeta) = \lambda_{21}(\zeta) \hat{a}_{10}^+ + \lambda_{22}(\zeta) \hat{a}_{1e} + \lambda_{23}(\zeta) \hat{a}_{3e}^+, \quad \hat{a}_{3e}(\zeta) = \lambda_{31}(\zeta) \hat{a}_{10} + \lambda_{32}(\zeta) \hat{a}_{1e}^+ + \lambda_{33}(\zeta) \hat{a}_{3e}.$$  

(15)

where $\hat{a}_{jn} = \hat{a}_{jn}(0)$ and

$$q = \sqrt{1 + k_1^2 - k_2^2},$$

$$\begin{align*}
\lambda_{11}(\zeta) &= \frac{1}{q^2}(-k_1^2 \cosh q \zeta + q \cosh q \zeta + k_2^2), \\
\lambda_{12}(\zeta) &= \frac{1}{q^2}(-k_1 k_2 \cosh q \zeta + k_1 k_2 + q \sinh q \zeta), \\
\lambda_{13}(\zeta) &= \frac{1}{q^2}(-q k_2 \sinh q \zeta + k_1 \cosh q \zeta - k_1), \\
\lambda_{21}(\zeta) &= \frac{1}{q^2}(-q \sinh q \zeta - k_1 k_2 + k_1 k_2 \cosh q \zeta), \\
\lambda_{22}(\zeta) &= \frac{1}{q^2}(k_2^2 \cosh q \zeta + q^2 \cosh q \zeta - k_2^2), \\
\lambda_{23}(\zeta) &= \frac{1}{q^2}(q k_1 \sinh q \zeta - k_2 \cosh q \zeta + k_2), \\
\lambda_{31}(\zeta) &= \frac{1}{q^2}(q k_2 \sinh q \zeta - k_1 + k_1 \cosh q \zeta), \\
\lambda_{32}(\zeta) &= \frac{1}{q^2}(q k_1 \sinh q \zeta + k_2 \cosh q \zeta - k_2), \\
\lambda_{33}(\zeta) &= \frac{1}{q^2}(1 + q^2 \cosh q \zeta - \cosh q \zeta).
\end{align*}$$  

(16)

If $k_1 = k_2 = 0$, then the solution (15) corresponds to the conventional degenerate parametric down conversion process (type II) [3]. Using (15) we can calculate the statistical properties of interacting modes with frequencies $\omega_o$, $\omega_{2e}$, $3\omega_o$ at normalized interaction length $\zeta$.  

3
3 Evolution of mean photon numbers

The evolution of mean photon number of degenerate polarization modes at frequencies $\omega_o$ and $\omega_e$ in a PPNC are calculated by using (15) for the following initial conditions at the input of the PPNC: the two orthogonal modes are in polarized coherent states $|\alpha_{1o}\rangle$, $|\alpha_{1e}\rangle$, $|\alpha_{1o,e}\rangle = |\alpha_{1o,e}|e^{i\omega_{1o,e}}$, where $|\alpha_{1o,e}| = \sqrt{\langle N_{1o,e}(0) \rangle}$ and the wave at frequency $3\omega_e$ is in the vacuum state $|0\rangle$:

$$<N_{1o,e}(\zeta)> = <\alpha_{1o}|<\alpha_{1e}|<0|\hat{a}_{1o,e}^+(\zeta)\hat{a}_{1o,e}(\zeta)|0\rangle |\alpha_{1e}> |\alpha_{1o}>.$$  \hfill (17)

The expressions for mean photons (17) are:

$$<N_{1o}(\zeta)> = \lambda_{12}^2(\zeta) + \lambda_{12}^1(\zeta)|\alpha_{1o}|^2 + 2\lambda_{11}(\zeta)\lambda_{12}(\zeta)\cos(\phi_{1o} + \phi_{1e}) + \lambda_{11}^2(\zeta)|\alpha_{1o}|^2,$$  \hfill (18)

$$<N_{1e}(\zeta)> = \lambda_{12}^1(\zeta) + \lambda_{12}^2(\zeta) + \lambda_{22}^1(\zeta)|\alpha_{1e}|^2 + 2\lambda_{21}(\zeta)\lambda_{22}(\zeta)\cos(\phi_{1o} + \phi_{1e}) + \lambda_{21}^2(\zeta)|\alpha_{1o}|^2.$$  \hfill (19)

It is well known that the parametric down conversion process (type II) depends upon the initial phases $\phi_{1o,e}$ of the polarized (orthogonal) coherent states. The values of mean photon numbers (18) and (19) depend upon the sum of initial phases, i.e. $\phi_{1o} + \phi_{1e}$. Under the condition $\phi_{1o} + \phi_{1e} = 2\pi s$, the mean photon numbers (18) and (19) start increasing rapidly with the growth of interaction length $\zeta$, under the condition $\phi_{1o} + \phi_{1e} = \pi s$, they start decreasing and then monotonically increasing (see Fig. 2) and $s = \pm 1, \pm 2, \ldots$

Fig. 2 demonstrates the dependence of mean photon numbers $<N_{1o}>$ and $<N_{1e}>$ on the normalized interaction length under the condition $\phi_{1o} + \phi_{1e} = \pi$. It is seen that the behaviour of mean photon numbers at frequencies $\omega_o$ (curve 1) and $\omega_e$ (curve 2) are quite different from the evolution of the mean photon number for the case of parametric down conversion (type II) in an ordinary nonlinear crystal. The mean photon number (curve 3) for the case of an ordinary nonlinear crystal is calculated by putting $k_1 = k_2 = 0$ into the expressions (18) and (19) under the same initial conditions.

The difference between the evolutions of mean photon numbers (18) (curve 1) and (19) (curve 2) illustrated in Fig. 2 is related with the interaction process (2), which is responsible for the sum harmonic generation, i.e., photon of frequency $\omega_e$ combines with the photon of pump frequency $2\omega_o$, which gives rise to photon with frequency $3\omega_e$. This can be easily seen by evaluating the mean photon numbers $<N_{3e}(\zeta)>$ using (15) under the same initial conditions:

$$<N_{3e}(\zeta)> = \lambda_{23}^2(\zeta) + \lambda_{23}^1(\zeta)|\alpha_{1e}|^2 + 2\lambda_{31}(\zeta)\lambda_{23}(\zeta)\cos(\phi_{1o} + \phi_{1e}) + \lambda_{31}^2(\zeta)|\alpha_{1o}|^2.$$  \hfill (20)

The curve (4) of Fig. 2 illustrates the growth of the mean photon number $<N_{3e}>$ as the normalized interaction length increases.

At earlier stages (normalized interaction length $\zeta \approx 0 \div 0.4$), the mean photon number of frequency $\omega_e$ decreases (see Fig. 2, (curve 2)) and later starts increasing monotonically. The decreasing of mean photon number $<N_{1e}>$ is connected with the nonlinear process (3), which is responsible for the sum harmonic generation at frequency $4\omega_o$ and is realized by the combination of photons with frequencies $\omega_e$ and $3\omega_e$. The fourth harmonic generation process (3) is complicated as compared to (1) and (2). So, the interaction process (3) starts acting at later stages of interaction as compared to the interaction process (2).

The calculations of photon variances $\langle \Delta N_{1o,e}(\zeta) \rangle$ of polarization modes with frequencies $\omega_o$ and $\omega_e$ have shown that they are super-Poissonian [3]. The same can be seen in the ordinary parametric down conversion process (type II) in ordinary nonlinear crystals by putting the nonlinear coupling coefficients $k_1$, $k_2$ equal to 0 in the expressions (16) and substituting $\hat{a}_3(0) = 0$ into the solution (15).

4 Stokes parameters

The polarization properties of orthogonal modes at frequencies $\omega_o$ and $\omega_e$ can be analyzed by the Stokes parameters [8]

$$\hat{S}_0(\zeta) = \hat{a}_{1o}^+(\zeta)\hat{a}_{1o}(\zeta) + \hat{a}_{1e}^+(\zeta)\hat{a}_{1e}(\zeta),$$

$$\hat{S}_1(\zeta) = \hat{a}_{1o}^+(\zeta)\hat{a}_{1o}(\zeta) - \hat{a}_{1e}^+(\zeta)\hat{a}_{1e}(\zeta),$$

$$\hat{S}_2(\zeta) = \hat{a}_{1e}^+(\zeta)\hat{a}_{1o}(\zeta) + \hat{a}_{1o}^+(\zeta)\hat{a}_{1e}(\zeta),$$

$$\hat{S}_3(\zeta) = i[\hat{a}_{1o}^+(\zeta)\hat{a}_{1o}(\zeta) - \hat{a}_{1e}^+(\zeta)\hat{a}_{1e}(\zeta)].$$  \hfill (21)

The Stokes parameters (21) obey the commutation relations of the SU(2) algebra [8]

$$[\hat{S}_0(\zeta), \hat{S}_{1,2,3}(\zeta)] = 0; \quad [\hat{S}_1(\zeta), \hat{S}_2(\zeta)] = 2i\hat{S}_3(\zeta);$$

$$[\hat{S}_2(\zeta), \hat{S}_3(\zeta)] = 2i\hat{S}_1(\zeta); \quad [\hat{S}_3(\zeta), \hat{S}_1(\zeta)] = 2i\hat{S}_2(\zeta).$$  \hfill (22)
The Heisenberg uncertainty relation for the Stokes parameters (21) is given by [8]
\[
< \Delta \hat{S}_i^2(\zeta) > = | < \Delta \hat{S}_k > |^2, (i, j, k = 1, 2, 3; i \neq j \neq k).
\]
\[
\hat{S}_{0,1}(\zeta) = p_{0\pm}(\zeta) + p_{1\pm}(\zeta) \hat{a}_{3e}^* \hat{a}_{3e} + p_{2\pm}(\zeta) \{ \hat{a}_{1e} \hat{a}_{3e} + \hat{a}_{1e}^* \hat{a}_{3e} \} + p_{3\pm}(\zeta) \{ \hat{a}_{1e}^* \hat{a}_{1e} + p_{4\pm}(\zeta) \{ \hat{a}_{1e} \hat{a}_{3e} + \hat{a}_{1e}^* \hat{a}_{3e} \}
+ p_{5\pm}(\zeta) \{ \hat{a}_{1e} \hat{a}_{1e} + \hat{a}_{1e}^* \hat{a}_{1e} \} + p_{6\pm}(\zeta) \hat{a}_{1e} \hat{a}_{1e},
\]
\[
\hat{S}_{2,3}(\zeta) = i^{0,1} \{ q_{0}(\zeta) \{ \hat{a}_{3e}^2 + \hat{a}_{3e}^* \} + q_{1}(\zeta) \{ \hat{a}_{1e}^2 + \hat{a}_{1e}^* \} + q_{2}(\zeta) \{ \hat{a}_{1e} \hat{a}_{3e} \hat{a}_{1e}^* \} + q_{3}(\zeta) \{ \hat{a}_{1e} \hat{a}_{3e} \hat{a}_{1e}^* \}
+ q_{4}(\zeta) \{ \hat{a}_{1e} \hat{a}_{1e}^* \hat{a}_{1e} \} + q_{5}(\zeta) \{ \hat{a}_{1e}^2 + \hat{a}_{1e}^* \} \}
\]
\]
where \( i^0 = 1, i^1 = i \) stand for \( \hat{S}_2(\zeta), \hat{S}_3(\zeta) \) and
\[
p_{0\pm}(\zeta) = \lambda_{12}(\zeta) \pm \lambda_{21}(\zeta) \pm \lambda_{22}(\zeta),
\]
\[
p_{1\pm}(\zeta) = \lambda_{12}(\zeta) \pm \lambda_{21}(\zeta) \pm \lambda_{22}(\zeta),
\]
\[
p_{2\pm}(\zeta) = \lambda_{11}(\zeta) \lambda_{12}(\zeta) \pm \lambda_{21}(\zeta) \lambda_{22}(\zeta),
\]
\[
p_{3\pm}(\zeta) = \lambda_{11}(\zeta) \lambda_{12}(\zeta) \pm \lambda_{21}(\zeta) \lambda_{22}(\zeta),
\]
\[
p_{4\pm}(\zeta) = \lambda_{11}(\zeta) \lambda_{22}(\zeta) \pm \lambda_{12}(\zeta) \lambda_{22}(\zeta),
\]
\[
p_{5\pm}(\zeta) = \lambda_{11}(\zeta) \lambda_{32}(\zeta) \pm \lambda_{12}(\zeta) \lambda_{32}(\zeta),
\]
\[
p_{6\pm}(\zeta) = \lambda_{11}(\zeta) \lambda_{22}(\zeta) \pm \lambda_{12}(\zeta) \lambda_{22}(\zeta),
\]
\[
q_{0}(\zeta) = \lambda_{13}(\zeta) \lambda_{23}(\zeta),
\]
\[
q_{1}(\zeta) = \lambda_{13}(\zeta) \lambda_{22}(\zeta) + \lambda_{12}(\zeta) \lambda_{23}(\zeta),
\]
\[
q_{2}(\zeta) = \lambda_{12}(\zeta) \lambda_{22}(\zeta),
\]
\[
q_{3}(\zeta) = \lambda_{13}(\zeta) \lambda_{21}(\zeta) + \lambda_{11}(\zeta) \lambda_{23}(\zeta),
\]
\[
q_{4}(\zeta) = \lambda_{12}(\zeta) \lambda_{21}(\zeta) + \lambda_{11}(\zeta) \lambda_{22}(\zeta),
\]
\[
q_{5}(\zeta) = \lambda_{11}(\zeta) \lambda_{21}(\zeta).
\]
Further for simplicity and clearness, we will write \( p_j(\zeta) (j=0,1,2,3,4,5,6) \) as \( p_j \) and \( q_j(\zeta) (j=0,1,2,3,4,5) \) as \( q_j \). The expressions for the variances of the Stokes parameters (23) and (24) become very lengthy. So, we write down the expressions of variances for the Stokes parameter \( \hat{S}_j(\zeta) \) under the same initial conditions applied for expressions (17)-(20)
\[
< \Delta \hat{S}_j(\zeta) > \leq \hat{S}_j(\zeta) > - \hat{S}_j(\zeta) >^2, (j = 0, 1, 2, 3)
\]
The variances of Stokes parameters (26) are normalized by dividing them by the variance of the Stokes parameter \( \hat{S}_0(\zeta) \), i.e.
\[
V_j(\zeta) = \frac{< \Delta \hat{S}_j^2(\zeta) >}{< \Delta \hat{S}_0^2(\zeta) >},
\]
where \(< \Delta \hat{S}_0^2(\zeta) > = | \alpha_{1e} |^2 + | \alpha_{1e} |^2. \) For the case of coherent light, the relative variances (27) take values 1. If at least one of the relative variances (27) takes value less than 1, the light is said polarization squeezed. The relative variances \( V_j(\zeta) \) read
\[
V_{0,1}(\zeta) = \frac{1}{<\Delta \hat{S}_0^2(\zeta)>} \left[ p_{0\pm}^2 + p_{2\pm}^2 + p_{5\pm}^2 + (p_{2\pm}^2 + 2(p_{0\pm} + p_{3\pm} + p_{5\pm}) |\alpha_{1e}|^2 \right].
\]
\[
V_{2,3}(\zeta) = \frac{1}{<\Delta \hat{S}_0^2(\zeta)>} \left[ q_{0}^2 + q_{2}^2 + q_{5}^2 + (2q_{2}^2 + 2q_{5}^2 + 2q_{0}^2) |\alpha_{1e}|^4 \cos (4\phi_{1e}) \right] \left[ q_{2}^2 + q_{4}^2 + q_{5}^2 \right] |\alpha_{1e}|^2
+ 2(2q_{0}q_{2} + 2q_{2}q_{4}) |\alpha_{1e}|^3 \cos (\phi_{1e} + \phi_{1e}) + 4q_{2} |\alpha_{1e}|^3 \cos (\phi_{1e} + \phi_{1e}) \left[ 2(2q_{0} - 3\phi_{1e}) + 4q_{2} |\alpha_{1e}|^3 \cos (\phi_{1e} + \phi_{1e}) \right] \left[ 2(2q_{0} - 3\phi_{1e}) + 4q_{2} |\alpha_{1e}|^3 \cos (\phi_{1e} + \phi_{1e}) \right]
\]
\[
\vdots
\]
\[
- \left( i^{0,1} | \alpha_{1e} |^2 \{ e^{i2\phi_{1e}} - e^{-i2\phi_{1e}} \} + q_{0} |\alpha_{1e}|^2 \left[ e^{i\phi_{1e} - i\phi_{1e}} \pm e^{-i\phi_{1e} + i\phi_{1e}} \right] + q_{0} |\alpha_{1e}|^2 \left[ e^{i2\phi_{1e}} \pm e^{-i2\phi_{1e}} \right] \right].
\]
Under the conditions $\phi_{10} + \phi_{1c} = \pi$ and $k_1 < k_2$ expressions (28) and (29) become more favorable for the generation of polarization squeezed light and are evaluated for different initial mean photon numbers in polarized coherent states. The Fig. 3-5 demonstrate the evolution of expressions (28) and (29) as functions of the normalized interaction length $\zeta$. Fig. 3 and Fig. 4 illustrate the relative variances of Stokes parameters $S_{0,2}$ and $S_1$ and simulate the spontaneous parametric down conversion process, i.e. when the photons with frequencies $\omega_0$ and $\omega_c$ at the input of the PPNC are each having mean single photons $|\alpha_{10,c}|^2 = 1$ in their orthogonal coherent states. Fig 3. shows that the variances of the Stokes parameters $S_{0,2}$ start with the sub-Poissonian [3] statistics and later $\zeta > 0.45$ become super-Poissonian. The evolution of the variances of the Stokes parameters $S_2$ and $S_3$ are almost the same. So, that is why we are not demonstrating the variance of $S_3$. Moreover, the variances of the Stokes parameters $S_{0,2,3}$ do not differ from the same variances of the Stokes parameters for an ordinary nonlinear crystal. The latter one can be seen by putting the values of nonlinear coupling coefficients $k_1$, $k_2$ equal to 0 in the expressions (16) and substituting $\hat{a}_{3c}(0) = 0$ into (15).

The variance of the Stoke’s parameter $S_1$ is more interesting due to its sub-Poissonian statistics, which is shown in Fig. 4. It should be noted that the variance of Stok’s parameter $S_3$ for parametric down conversion process (type II) in an ordinary nonlinear crystal is super-Poissonian. The latter can be checked by putting the values of $k_1 = k_2 = 0$ in the expressions (16) and substituting $\hat{a}_{3c}(0) = 0$ into (15). So, the PPNC can suppress variances of the Stokes parameters $S_{0,1,2}$, under certain initial values of phases $(\phi_{10} + \phi_{1c} = \pi)$ of polarized modes and nonlinear coupling coefficients ($k_1 < k_2$).

The expressions (28) and (29) are calculated under the same initial conditions but with different mean photons $|\alpha_{10,c}|^2 = 10^3$ in each polarized mode. The Fig. 5 shows the evolution of variances of all the four Stokes parameters $S_{0,1,2,3}$. It is seen that all the variances of Stokes parameters have sub-Poissonian statistics. After analyzing the Fig. 3-5, it is clear that the larger the mean photon numbers in degenerate polarized modes, the more the squeezing in the variances of all the Stokes parameters. Moreover, the variances of Stokes parameters $S_{2,3}$ are $\approx 0$, i.e. the generation of almost Fock states.

5 Conclusion

In this paper we have investigated the problem of the generation of polarization squeezed light in PPNC with second order nonlinear susceptibility. In the case of classical and non-depleted modes at frequencies $2\omega_0$ and $4\omega_0$, the exact solution of the Heisenberg equations of motions is obtained. The solution is exact quantum solution showing new possibilities for the generation of polarized squeezed states of light. We have used this solution to calculate the mean photon numbers of interacting modes and variances of Stokes parameters. It is found that the evolution of mean photon numbers of interacting modes with frequencies $\omega_0$ and $\omega_c$ differ with the evolution of mean photon numbers of the same interacting modes in ordinary nonlinear crystals (degenerate parametric process (type II)). The reasons are found, which explain the peculiarities of degenerate parametric down conversion (type II) in PPNCs.

Also, it is shown that all the four variances of the Stokes parameters can be squeezed (variances of the Stokes parameters are smaller than the value of variance of coherent state) simultaneously. Optimal initial conditions are found under which the squeezing of variances of the Stokes parameters can be obtained.

So, PPNCs can be good candidates for the generation of polarization squeezed states of light, which can be used for the realization of quantum information protocols.

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References


Fig. 1 Sketch of generation of polarization squeezed light in PPNC with second order nonlinear susceptibility. The light waves involved are described by their frequencies $\omega_0$, $\omega_c$, $2\omega_c$, $3\omega_c$, and $4\omega_c$.

Fig. 2 Mean photon numbers $< N_{1\omega}(\zeta) >$ (1), $< N_{1c}(\zeta) >$ (2), $< N_{3c}(\zeta) >$ (4) with frequencies $\omega_0$, $\omega_c$, and $3\omega_c$, respectively and $< N_{1\omega,c}(\zeta) > (k_1 = k_2 = 0)$ (3), as functions of the normalized interaction length $\zeta$. The curves (1-4) are calculated corresponding to the initial photon numbers $< N_{1\omega,c}(0) >= 1$ and $< N_{3c}(0) >= 0$. 
Fig. 3 Normalized variances $V_0(\zeta)$ (0) and $V_2(\zeta)$ (2) of the Stokes parameters $\hat{S}_0(\zeta)$ and $\hat{S}_2(\zeta)$. Curves (0 and 2) are calculated corresponding to the initial photon numbers $<N_{10,e}(0)> = 1$ and $<N_{3e}(0)> = 0$

Fig. 4 Normalized variance $V_1(\zeta)$ of the Stoke's parameter $\hat{S}_1(\zeta)$. Curve is calculated corresponding to the initial photon number $<N_{10,e}(0)> = 1$ and $<N_{3e}(0)> = 0$
Fig. 5 Normalized variances $V_0(\zeta)$ (0) and $V_1(\zeta)$ (1), and $V_2(\zeta)$ (2), of the Stokes parameters $\hat{S}_0(\zeta)$, $\hat{S}_1(\zeta)$ and $\hat{S}_2(\zeta)$. Curves (0-2) are calculated corresponding to the initial photon numbers $<N_{10e}(0)> = 10^3$ and $<N_{3e}(0)> = 0$. 