Acceleration parameter of the Universe expansion and cosmological equation of state

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Abstract. We analyze the astronomical data on the explosions of supernovae Ia as well as the precision measurement of cosmic microwave background in order to get a model-independent determination of the acceleration parameter for the Universe evolution. The effective parameter of state in the cosmological equations, which gives the ratio of pressure to the energy density, as shown, can naturally get the values less than -1, if one introduces a nonzero mass of the graviton in the framework of relativistic theory of gravitation (RTG). We demonstrate the model equivalence of evolution equations for the flat homogeneous isotropic universe in the general relativity to those of RTG. Some constraints on the graviton mass value are obtained.

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1. Introduction

Recently, a progress in the quality of actual astronomical observations resulted in the essential great advance in the development of cosmological investigations. From the point of view dealing with the study of Universe evolution, one should emphasize, first of all, the modern experimental works on the measurement of brightness-red shift dependence for the supernovae Ia [1], which can be used as “standard candles of Universe” due to a specific critical mechanism of their explosions. Such the investigations allow us to determine functional parameters for the dependence of distances between the objects on the red shifts, i.e., actually, to study the character of Universe expansion. Second, the precision measurements of the cosmic microwave temperature and its angular anisotropy [2] (see Figs. 1 and 2) open the opportunity to look at the epoch of early Universe, so that the Fourier-density of multipole moments for the temperature anisotropy gives the possibility to extract some cosmological parameters such as the normalized total density of energy and that of baryonic matter, the contribution by the curvature of 3-dimensional isotropic homogeneous space in the Einstein equations for the evolution of universe and some other characteristics, which determine the conditions for the generation of the anisotropy related with the production of large scale structure of the Universe.

The principal conclusions, which can be drawn from the above mentioned astronomical observations with a high degree of reliability, could be summarized for our purposes by the following statements:
(i) The density of nonrelativistic matter composes the fraction equal to $\Omega_M \approx 0.27$ from the total density of the energy.

(ii) The Universe is flat in practice.

(iii) The fraction of the baryonic matter is small $\Omega_b \approx 0.04$.

(iv) The age of the Universe is close to 13.7 billion years.

(v) The Universe is expanding with an acceleration.

Thus, the cosmological evolution is determined by two factors. The first is the dark nonrelativistic matter dominating over the baryonic matter. The second is the dark energy with a negative pressure, called "a quintessence" [4], which covers the boundary case of cosmological constant. Usually the results of data processing by the experiments are represented versus two variables such as the fractions of densities for the dark matter and dark energy or the dark matter fraction and the dark energy state parameter equal to the ratio of pressure to the energy density $w = p/\rho$. In this respect, two questions are to the moment. First, what is a model-independent estimate for the acceleration parameter $\hat{q}$ in the Universe evolution? It is important to make the analysis with no additional suggestions on the matter species giving the contribution into the cosmological Einstein equations, particularly taking into account the necessary correlations between the parameters describing the evolution, which makes the deduction of constraints more complex, if one tries to extract the limits on $\hat{q}$ from the known constraints on the model parameters such as the densities of energy-momentum tensor for the dark components. The second question is caused by the admissible region of state parameter values $w_Q$ for the quintessence, which can penetrate to $w_Q < -1$ as follows from the data handling (see the text and figures below) [5, 6]. In the case of fundamental character of the above constraint on the state parameter of quintessence, the evolution of Universe could reach the situation of so-called "big rip" [7], since at $w_Q < -1$ the horizon of future events, i.e. the distance, at which an observer remains causally-connected with the events by means of physical signals propagating with the speed of light, will decrease with the evolution up to the sizes of atoms, which looks unreasonable. Thus, it is worth to consider the problem with the following respect: at which circumstances can the state parameter of dark energy effectively take the values less than $-1$? We have to note a possibility to get the effects of $w_Q < -1$ beyond the framework of the general relativity (GR), if one introduces the graviton mass in the relativistic theory of gravitation (RTG) [8]‡.

In the present paper we analyze the parameters of the Universe expansion and get the constraints on the graviton mass. Early such the constraints were posed at the level of

$$m_{gr} \lesssim 10^{-66} \text{ g,}$$

(deduced from, first, the observation of gravitationally connected galaxies at rather large distances between them, and, second, the age of Universe [8, 12], which results in

‡ Note that the criticism of RTG by its enemies in the papers by Zel'dovich and Grischchuk [9] is reduced, in fact, to the statement on the absolute perfection of GR and the negation of a necessity for any modification of GR, that was recently recognized due to a clear conceptual delimitation of GR from some different consistent theories of gravitation and demonstrated by one of the RTG opponents: Grischchuk and a co-author published the paper on the theory of massive graviton [10] in the spirit of RTG with a single difference, that the tensor gravitational field is splitted in mass with respect to the graviscalar, while in RTG these fields are degenerated in mass. In [11] a possibility of negative value for the masses squared has been considered with an application to the cosmology with the acceleration of Universe.
more strict constraints, in fact, equivalent to the condition that the Compton length of the graviton expressed in units with the speed of light equal to 1 should be greater than the age of the Universe. We confirm this conclusion and point to a principal feature of theories with the massive graviton, which necessary requires the introduction of two metric tensors and, therefore, a distinct separation of notions such as the rest mass of the graviton, the projection of the rest energy onto the Minkowskian space-time and the fading length of the gravitational field for the static massive body in the Riemannian space-time, since two latter quantities get the red shifts in the evolution of the Universe. We offer a new method, which allows us to enforce the constraints on the Minkowskian projection of the rest energy, at least, by 12 orders of magnitude. This advantage is caused by the fact that the age of Universe significantly less sensitive to the graviton mass in comparison with the marking the observed objects with high red shifts. Indeed, the projection of the graviton rest energy essentially restricts the limits of the scale factor variation, $a$, since it removes the cosmological singularity, i.e., it gives $a_{\text{min}} > 0$. The observed temperature of cosmic microwave background (CMB), the fact of the nucleosynthesis and formation of nuclei from the quarks [13] provide us with the minimally admissible limits of red shift, that superimposes strict constraints on the projection of the graviton rest energy.
2. Evolution parameters

The scale factor \( a(t) \) determines the metric interval for the homogeneous expanding universe

\[
ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - \kappa r^2} + r^2 d\omega^2 \right],
\]

where the expression for the differential over the angles \( d\omega^2 \) depends on the spatial curvature \( \kappa \). With a sufficient accuracy for the description of data on the supernovae, we can introduce a model-independent parametrization of the scale factor in the form

\[
a(t) / a_0 = 1 - \tau + \frac{\dot{q}}{2} \tau^2 - \frac{\beta}{6} \tau^3, \quad \tau = -H_0 t,
\]

where \( H_0 \) is a Hubble constant, \( \dot{q} \) is the acceleration parameter in the Universe evolution, \( \beta \) is a coefficient of expansion. A physical meaning of introduced parameters becomes clear after the substitution of (3) into the Einstein equations:

\[
\frac{\dot{a}}{a} = -\frac{1}{3} [\rho_M + (1 + 3w_Q)\rho_Q],
\]

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{2}{3}(\rho_M + \rho_Q) - \frac{\kappa}{a^2},
\]

where \( \dot{a} = da/dt \). By definition, \( H = \dot{a}/a \) and \( H^2 \dot{q}(t) = \ddot{a}/a \). To the current moment of time \( t_0 \) we put \( H(t_0) = H_0 = h \cdot 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \), \( \dot{q} = \ddot{q}(t_0) \). Introducing the energy-momentum tensor of matter as a sum of contributions by the nonrelativistic matter (the pressure \( p_M = 0 \)) and the quintessence with the state parameter\(^\S\) \( w_Q \):

\(^\S\) The boundary value of \( w_Q = -1 \) reproduces the case of cosmological constant.
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\[ p_Q = w_Q \rho_Q, \]  
we can easily find the law for the evolution of energy density

\[ \rho_Q = \rho_Q^0 \left( \frac{a_0}{a(t)} \right)^{3(1+w_Q)}, \]  
and express the normalized densities in terms of evolution parameters in (3)

\[ \Omega_{\text{tot}} = - \frac{2}{1+3w_Q} (\beta + 2\dot{q}) - 2\ddot{q}, \]  
\[ \Omega_M = - \frac{2}{3w_Q} (\beta + 2\dot{q} + 3w_Q\dot{q}), \]  
where \( \Omega_X = 2\rho_X / 3H_0^2 \). Note the linear character of dependence for the densities \( \Omega_X \) on the evolution parameters \( \dot{q} \) and \( \beta \).

At \( w_Q = -1 \) we get

\[ \Omega_{\text{tot}} = \beta, \]  
\[ \Omega_M = \frac{2}{3} (\Omega_{\text{tot}} - \dot{q}), \]  
wherefrom we see that in the case of cosmological constant the coefficient \( \beta \) coincides with the normalized density of total energy \( \Omega_{\text{tot}} \).

The parameters \( \dot{q} \) and \( \beta \) can be directly extracted from the data on the dependence of the supernovae brightness \( m \) on the red shift \( z = a_0 / a - 1 \)

\[ m(z) = M - 5 \log D_L(z), \]  
where \( M \) is a constant, and

\[ D_L(z) = (1+z) a_0 \int \frac{dt}{a(t)}. \]  
The characteristic description of supernovae data is presented in Fig. 3.

The supernovae data determine the region of admissible values for the parameters \( \dot{q} \) and \( \beta \). This region has nonempty intersections with the parameter regions, which follow the constraints extracted from other astronomical observations. The most effective restrictions are coming from the measurements of total energy and matter densities.

For the matter density we use the data

\[ \Omega_M \cdot h^2 = 0.135^{+0.008}_{-0.009}, \quad h = 0.71 \pm 0.04, \]  
which results in

\[ \Omega_M = 0.27 \pm 0.04. \]  
The total energy density measured by WMAP is equal to

\[ \Omega_{\text{tot}} = 1.02 \pm 0.02. \]  
Then, in the model with the cosmological constant we get Fig. 4 and find the acceleration parameter

\[ \dot{q} = 0.61 \pm 0.05. \]  
However, the introduction of arbitrary value for the state parameter different from \( w_Q = -1 \) leads to an additional dependence of fitting results for the values of \( \dot{q} \) and \( \beta \) as shown in Fig. 5.
We have to stress that the case of cosmological constant or $w_Q$ close to $-1$ looks preferable because of the visual coincidence for the intersection region of three domains, but from the statistical point of view the situations shown in Fig. 5 have rather high probabilities, and they are significant, too. Therefore, making some substitutions in terms of parameters $\{w_Q, \hat{q}\}$, we get the result shown in Fig. 6 taking into account the parametric dependence on $\Omega_{\text{tot}}$, which is not strong as seen.

This result is in agreement with (15) in the case of cosmological constant. With the accuracy up to one standard deviation we find

$$-1.25 < w_Q < -0.87,$$

which agrees with the standard representation in the plain of variables $\{\text{the matter density, the state parameter of quintessence}\}$ as shown in Fig. 7. For the arbitrary admissible value of the state parameter for the quintessence we get the one-sigma constraint on the acceleration parameter in the Universe expansion

$$0.52 < \hat{q} < 0.83.$$

Note, that the minimal value of quadratic deviation in the description of data takes place at

$$w_Q \approx -1, \quad \hat{q} \approx 0.62, \quad \Omega_M \approx 0.27.$$
The age of the Universe $t_0$ is determined by the equation

$$H_0 t_0 = \int_0^1 \frac{da \sqrt{a}}{\sqrt{\Omega_M + (\Omega_{\text{tot}} - \Omega_M)a^{-3wQ + (1 - \Omega_{\text{tot}})a}}}.$$ (18)

Curves of constant ages are shown in Fig. 8 in comparison with the region of admissible values for the state parameter of quintessence and normalized density of nonrelativistic matter at fixed value of the Hubble constant. Note, that in this way the accuracy of the obtained value for the Universe age is basically determined by an uncertainty in the measurement of the Hubble constant, while the uncertainty caused by the variation of parameters describing the evolution in this representation: the state parameter of quintessence and the matter density, is approximately twice less than the former one. The variation of the total energy density weakly influences the age of the Universe. Thus, we get the estimate

$$t_0 = 13.78 \pm 0.88 \text{ Gyrs.}$$ (19)

The uncertainty in this procedure for the extraction of the Universe age is 4 times greater than in the WMAP experiment, since the latter uses a detail information on the CMB, *viz.*, a red shift for a final rescattering of CMB, a size of region for the CMB formation as well as the position of main peak in the multipole expansion of its anisotropy (see Fig. 2).
Thus, we have shown that our analysis is in agreement with other similar investigations, and it allows us to extract some definite constraints on the value of acceleration in the Universe evolution.

3. Equivalence of the evolution equations in GR and RTG

The equations of RTG for the evolution of flat homogeneous Universe can be written down in the following form:

\[
\frac{\ddot{a}}{a} = -\frac{1}{3} \left[ \rho_M + (1 + 3w_Q)\rho_Q \right] - \frac{1}{6} m_{gr}^2 \left[ 1 - \frac{1}{a^6} \right],
\]

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{2}{3} \left( \rho_M + \rho_Q \right) - \frac{1}{6} m_{gr}^2 \left[ 1 - \frac{3}{2a^2} + \frac{1}{2a^6} \right],
\]

We are working in the unit system, where the gravitational constant, the speed of light and the Planck constant are normalized by the condition $4\pi G = c = \hbar = 1$. 

\[Figure 5.\] The evolution of fitting results with the change of state parameter of quintessence $w_Q$. The notations are the same as in Fig. 4.
where \( m_{\text{gr}} \) is the graviton mass, and \( \kappa \) determines the coefficient in the Riemannian metric of the interval
\[
ds^2 = dt^2 - \kappa^2 a^2(t) [dr^2 + r^2 d\omega^2],
\]
consistent with the Minkowskian metric
\[
d_s^2 = \frac{1}{a^6(t)} dt^2 - [dr^2 + r^2 d\omega^2],
\]
so that the condition following from the principle of causality: the trajectories of null cone in the effective Riemannian space-time \( ds^2 = 0 \) lie in the causally-connected region of Minkowskian space-time \( ds^2_\gamma \geq 0 \), leads to
\[
\frac{\kappa^2}{a^4} - 1 \geq 0,
\]
which makes \( \kappa^2 \geq a^4_{\text{max}} \).

By the geometrization of RTG the liquid equation, i.e. the conservation of energy-momentum tensor, takes the same form as in GR
\[
\dot{\rho} + 3H(\rho + p) = 0,
\]
so that in the case of linear relation between the pressure and density \( p_X = w_X \rho_X \), we get the linear equations for the energy densities with the solutions pointed in (6).

Then equations (20)-(21) are reduced to the form
\[
\frac{\ddot{a}}{a} = -\frac{1}{3} \left[ \rho_M + \sum_j (1 + 3w_j) \rho_j \right],
\]
\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{2}{3} (\rho_M + \sum_j \rho_j),
\]
where the index \( j \) runs the values \( \{Q, 1, -\frac{1}{3}, -1\} \), whereas the state parameters are equal to \( w_j = \{w_Q, 1, -\frac{1}{3}, -1\} \), and the corresponding densities of energy are determined by the graviton mass

\[
\rho_{-1} = -\frac{1}{4} m^2_{\gamma r} \frac{1}{4\pi G},
\]

\[
\rho_{-1/3} = \frac{3}{8} m^2_{\gamma r} \frac{1}{4\pi G} \frac{1}{a^2},
\]

\[
\rho_1 = -\frac{1}{8} m^2_{\gamma r} \frac{1}{4\pi G} \frac{1}{a^6},
\]

where we have explicitly shown the energy dimension for the density of energy by the introduction of gravitational constant.

Therefore, we have shown that the equations of RTG (20)-(21) are deduced to the equations of GR (4)-(5) in the flat space (\( \kappa = 0 \)) as expressed in the form of (26)-(27) with the specific set of contributions to the energy-momentum tensor of matter. Definitely, the graviton mass leads to the terms analogous to the cosmological constant (\( w = -1 \)), the stiff matter (\( w = 1 \)), and the quintessence with \( w = -1/3 \) imitating the term of constant spatial curvature.

In other words, if we do not concern for the origin of matter state equations one could, in an appropriate way, introduce some additional terms of quintessence and stiff matter in RTG, so that they could compensate the contributions by the graviton mass and lead to the equations for the evolution of flat homogeneous isotropic Universe in GR. The inverse statement is true, too. Thus, in this context, the difference between
the RTG and GR is reduced to a model motivation for choosing the components in the energy-momentum tensor of matter.

A separate item is the constraints following from the causality principle according to (24). A contradiction does not appear, if only whether the expansion of Universe is restricted by a maximal value of the scale factor or the corresponding parameter tends to infinity, \( \kappa \to \infty \). It is important to note that in the evolution equations the quantity \( \varpi \) can be generally replaced by an arbitrary effective parameter \( \hat{\varpi} \), since the introduction of arbitrary density for the quintessence with the state parameter \( w = -1/3 \) does not change the parametrization of Riemannian metric, but it “renormalizes” the value of \( 1/a^2 \)-term in the evolution equations. The special role of that term imitating the contribution of spatial curvature is revealed also by the equation on the normalized density of energy

\[
\Omega_{\text{tot}} + \frac{m_{\text{gr}}^2}{4\hat{\varpi}^2 a_0^2 H_0^2} = 1.
\]

(31)

In RTG if there is no contribution by the quintessence with \( w = -1/3 \) except the term due to the graviton mass we get

\[ \Omega_{\text{tot}} < 1. \]

In a generic case the value of parameter \( \hat{\varpi}^2 \) can get the negative sign, too.

4. Parameters of RTG

For the definiteness of analysis of the Universe evolution parameters in RTG we will accept that at the current moment the scale factor is far away from its maximal value, so that we can take the limit of \( \varpi \to \infty \) and neglect the term of (29) imitating the
curvature of homogeneous isotropic space in equations (20)-(21). Thus, in RTG we will put
\[ \Omega_{\text{tot}} = 1. \] (32)

Further, the constant term (28) independent of the scale enters as a part of general contribution by the cosmological constant or, probably, due to a contribution caused by dynamical fields, so that for our purposes we can neglect a dependence of their state parameters on the scale factor and put \( w = -1 \) with a sufficient accuracy. Therefore, the only term leading to the difference between the evolution equations for the flat Universe in presence of cosmological constant in GR and RTG is the component of stiff matter (30) with the state parameter \( w = 1 \). Such the additional term has the clear motivation in RTG due to the graviton mass.

Of course, a role of such the exotic term like (30) cannot be extremely significant at the current moment, so that we can expect
\[ \Omega_{\text{gr}} = \frac{1}{12} \frac{m_{\text{gr}}^2}{H_0^2} \frac{1}{a_0^6} \ll 1. \] (33)

Note, that for the sum of cosmological term and the contribution by the graviton mass, the effective parameter of state takes the form
\[ w_{\text{eff}} = -\frac{\Omega_{\Lambda} + \Omega_{\text{gr}}}{\Omega_{\Lambda} - \Omega_{\text{gr}}} \approx -1 - \frac{2\Omega_{\text{gr}}}{\Omega_{\Lambda}}, \] (34)
which imitates \( w < -1 \). The comparison of (34) with the region of \( w \) values shown in Fig. 8 allows one to extract the estimate of \( \Omega_{\text{gr}} \sim 0.02 - 0.03 \), which is of the same order of magnitude as the contribution by the baryonic matter.

The investigation of evolution equations leads to the relations connecting the introduced parameters
\[ 1 = \Omega_M + \Omega_{\Lambda} - \Omega_{\text{gr}}, \] (35)
\[ \hat{q} = 1 - \frac{3}{2} \Omega_M + 3 \Omega_{\text{gr}}, \] (36)
\[ \beta = 1 - 9 \Omega_{\text{gr}}, \] (37)
which allows us to use the procedures described in the previous sections in order to extract the parameters values, so that the result is presented in Fig. 9. In this way, it is necessary to take into account the fact that the graviton mass removes the cosmological singularity, since the beginning of evolution takes place at a nonzero minimal value of the scale factor
\[ \left( \frac{a_{\min}}{a_0} \right)^3 = \frac{1}{2\Omega_{\Lambda}} \left( \Omega_M^2 + 4\Omega_{\Lambda}\Omega_{\text{gr}} - \Omega_M \right). \] (38)

The interpretation of obtained constraints in the framework of RTG suggests, first of all, the validity of the inequality providing the stability of gravitational field
\[ m_{\text{gr}}^2 \geq 0, \]
which, in accordance with both the small value of \( \Omega_{\text{gr}} \) and (34), leads to the strict constraint on the effective value for the state parameter of "quintessence" in the presence of cosmological constant
\[ w_{\text{eff}} \leq -1, \] (39)
which implies the cutting off the down region in Fig. 9, where $\Omega_{gr} < 0$. As for the physical meaning of the quantity $\Omega_{gr}$, in the Minkowskian metric (23) we can define the invariant

$$\mathcal{P} = \gamma_{\mu\nu} \mathcal{G}^{\mu\alpha} v^\beta p_\alpha p_\beta,$$

where $p_\mu$ is a four-momentum of graviton, which effectively moves in the Riemannian space-time, so that the equation of its mass shell is given by

$$g_{\mu\nu} p_\mu p_\nu = m_{gr}^2.$$

Then, in the case of rest energy $p_\mu = (m_{gr}, 0)$ the invariant $\mathcal{P} = \mathcal{E}^2$ depends on the value of red shift $z$,

$$\mathcal{E}(z) = \mathcal{E}_0 (1 + z)^3, \quad \mathcal{E}_0 = \frac{m_{gr}}{a_0^3}, \quad (40)$$

i.e. there is a cubic red shifting because the currently observed projection of the graviton rest energy into the Minkowskian space-time is less its value in the past. In other words, the quantity $g^{\mu\nu} p_\nu = m_{gr} u^\mu$ defines the 4-velocity of free graviton $u^\mu$, which is equal to $u_0^\mu = (1, 0)$ in the rest frame, while its projection determine the quantity $\mathcal{E}^2 = m_{gr}^2 \gamma_{00} u_0^0 u_0^0$. Therefore, the obtained constraints can be represented in the form

$$0 \leq \mathcal{E}_0 \leq 2 \cdot 10^{-66} \text{ g}. \quad (41)$$

The minimum of $\chi^2$ takes place at the value of projection

$$\mathcal{E}_0 = 0.67 \cdot 10^{-66} \text{ g}.$$
As seen in Fig. 9, the estimate of the Universe age in RTG agrees with the calculations in GR. The Compton length of graviton $\lambda_{gr} = 1/E_0$ in units $c = 1$ practically exceeds the age of Universe $t_0$

$$\lambda_{gr} \geq 0.8 \cdot c \cdot t_0.$$  

In addition, equation (38) makes the constraint on the maximal value of red shift admissible

$$\frac{1}{(1 + z_{max})^3} = \frac{1}{2\Omega_\Lambda} \left( \sqrt{\Omega_M^2 + 4\Omega_\Lambda \Omega_{gr} - \Omega_M} \right) \approx \frac{\Omega_{gr}}{\Omega_M},$$  

so that the direct astronomical observation of objects with high red shifts can give essentially more strict constraint on the projection of graviton rest energy than, for instance, the data on the Universe age, since the expansion can be rather long, while the variation of scale factor remains small due to the quadratic dependence of the factor on the time. Indeed, from the analysis performed above we deduce

$$\frac{\Omega_{gr}}{\Omega_M} \leq 0.1, \quad z_{max} \geq 1.2,$$

and the observation of quasars¶ with $z \sim 6$ leads to the reduction of upper limit by 6 times for the projection of the graviton rest energy $E_0$. Note that in accordance with (34), the enforcement of constraint on $E_0$ results in a decrease of deviation of state parameter from $-1$ in the case of cosmological constant in RTG.

It is extremely important to stress that the red shift and, hence, the variation of the scale factor are directly related with the temperature $T \sim 1/a$, so that the fact caused by a high temperature of a process in the past guarantees a corresponding variation of the scale factor, too.

The observation of cosmic microwave background and its temperature reliably gives $z_{max} > 10^3$, so that the upper limit of the projection for the graviton rest energy decreases by 4 orders of magnitude

$$0 \leq E_0 \leq 2 \cdot 10^{-70} \text{ g. (43)}$$

The requirement of temperatures sufficient for the usual nucleosynthesis or the inflational expansion of the Universe evidently can give more strict constraints. The temperature for forming the CMB is $T_{CMB} \sim 1 \text{ eV}$, while the temperature necessary for the nucleosynthesis is of the order of $T_{BBN} \sim 1 \text{ MeV}$, which decreases the limit by several orders of magnitude. The temperature for the quark transformation to nuclei is $T_B \sim 100 \text{ MeV}$, which gives $z_{max} > 10^{13}$. We have to take into account the fact that at the red shift $z_{eq} \approx 3 \cdot 10^3$ the energy density of nonrelativistic matter becomes equal to that of radiation, which scale dependence is determined by the factor of $1/a^4$. Therefore at the red shift $z > z_{eq}$ we have to use the straightforward generalization of (42)

$$\frac{\Omega_{gr}}{\Omega_M} \approx \frac{1}{(1 + z_{max})^2} \frac{1}{1 + z_{eq}}.$$  

Thus, the requirement of existing the temperatures sufficient for the forming of nuclei from the quarks leads to

$$0 \leq E_0 \leq 10^{-78} \text{ g. (45)}$$

¶ The red shift of spectral lines for the quasar emission, in general, could depend on some ‘local’ parameters of the objects, not only on the global scale factor of evolution.
While the effects of cubic red shift for the projection of the graviton rest energy is a strict fact of RTG in the consideration of evolution for the homogeneous isotropic Universe, the scale factor $a_0$ at the moment $t_0$ could be, by the first sight, a subject of choosing the units in a local-Galilean reference frame, where one usually puts the speed of light $c$ equal to unit, i.e., $a_0 \equiv 1$, and the constraints on the value of projection given above, in fact, would coincide with the constraints on the absolute value of the graviton rest mass.

Thus, in the study of the Universe evolution in the framework of RTG with the presence of effective cosmological constant, probably, of a dynamical nature, there is a possibility to imitate a “quintessence” with an effective state parameter $w_{\text{eff}} < -1$ due to the graviton mass. The constraints, which can be obtained for the graviton mass in the procedure of describing the evolution parameters, in a sense, are reduced to the requirement of the fact that the Compton length of graviton should be greater than the Universe age multiplied by the speed of light. However, the requirement of sufficiently high temperatures in the evolution, allowing for the production of CMB with the measured temperature spectrum as well as the nucleosynthesis, and, hence, the existence of the epoch with high red shifts, is more strict, which decreases the upper limit on the projection of the graviton rest energy by 12 orders of magnitude.

Finally, let us mention the possibility of aesthetically clumsy and conceptually unnatural transition to an unstable gravitational field by the substitution $m_{\text{gr}} \rightarrow i/(2\tau_{\text{gr}})$, where $\tau_{\text{gr}}$ is a “lifetime of graviton”. This procedure allows one to associate the observed cosmological constant with the contribution by the graviton, if the lifetime is close to the Universe age $\tau_{\text{gr}} \approx t_0/4$, so that the gravitons with the wavelength $\lambda_{\text{gr}} \leq \tau_{\text{gr}}$ remain stable, but in this way, the observed lifetime $^* \tau_{\text{gr}} a_0^3 \geq t_0$, and again we cannot avoid the problem with the introduction of dark nonbaryonic matter as well as the initial singularity.

5. Interpretation of results in RTG

The procedure of fixing the scale $a_0 = 1$ is logically meaningful, but it is not completely correct operation, since the scale factor at the current moment, in fact, is the additional degree of freedom for the parameters of the Universe evolution. We can easily show that at such the normalization and given mass of the graviton, equation (44) determines the maximal admissible value for the red shift, and under the condition of $w_{\text{Q}} > -1$ the presence of cosmological term in equations (20), (21) due to the graviton mass gives the red shift of current moment $z_f$ with respect to the maximal value of scale factor, when the expansion is changed by the contraction, so that

$$\frac{\Omega_{\text{Q}}}{(1 + z_f)^{3(1 + w_{\text{Q}})}} \approx \frac{m_{\text{gr}}^2}{6H_0^2},$$

where $1 + z_f = a_{\text{max}}/a_0$ and, for definiteness, $a_{\text{max}}^2 = \kappa$. Combining (44) and (46), we get

$$a_{\text{max}}^6 \approx \frac{\Omega_{\text{Q}}}{2\Omega_M}(1 + z_{\text{max}})^2(1 + z_{\text{eq}})(1 + z_f)^3(1 - w_{\text{Q}}),$$

$^*$ More accurately, we are talking on the maximal speed of physical signal in the Minkowskian space-time. The photons moves in the effective Riemannian space-time with the speed of $c/(\kappa a_0)$.

$^*$ In other words, if we accept the normalization $a_0 = 1$ then the constraint on the term of stiff matter leads to $\tau_{\text{gr}} \geq t_0$, and the observed cosmological constant is 20 times greater than the contribution caused by the instability of graviton.
which at \(z_f > 0\) gives rather strict constraint on \(a_{\text{max}}\), i.e., the parameter \(\kappa\). Under the condition \(a_0 = 1\) the equations shown above result in a definite value of \(z_f\) and, hence, the maximal value of scale factor, too. Indeed, at such the normalization we get \(a_{\text{max}} = 1 + z_f\), and equation (47) can be resolved with respect to \(z_f\). Nevertheless, the equality \(a_0 = 1\) simply gives a “synchronization” for the relation between the mass and energy in the Riemannian and Minkowskian space-times.

Note that the equations of Universe evolution in RTG determine the constraint on the upper limit of contribution imitating the spatial curvature

\[
0 < \Omega_k < 3\Omega_{\text{gr}}.
\]

Probably, it is worth to emphasize that the physically clear choice of normalization condition for the scale factor could be the equality of speed of light to unit in the Riemannian space-time, when the physical distances at the moment coincide with the distances in the Minkowskian space-time, i.e. with the distances in the co-moving coordinates,

\[
\kappa a_0 = 1, \quad (48)
\]

whereas the inequality \(\kappa > 1\) should be valid, since \(a^2(t) \leq \kappa\), which in the constraint on the projection of the graviton rest energy \(\mathcal{E}_0\), leads to the inequality

\[
0 \leq m_{\text{gr}} \kappa^3 \leq 10^{-78} \text{ g}, \quad (49)
\]

and the upper limit for the graviton mass would be, in general, more strict, if we take into account the fact that the Universe expansion takes place with the acceleration and there is no experimental evidence for a transition to a deceleration and the further contraction, which would indicate reaching the maximal value of the scale factor, i.e., \(\kappa \gg 1\). However, the substitution of (48) into equations (44) and (46) leads to the contradiction because of

\[
(1 + z_f)^{3w_Q^{-1}} = \frac{\Omega_Q}{2\Omega_M} (1 + z_{\text{max}})^2 (1 + z_{eq}),
\]

wherefrom we deduce \(z_f < 0\). Thus, we draw the conclusion that the choice of \(a_0 = 1\) could be both physically meaningful and consistent. Nevertheless, one should clearly understand that in RTG, in principle, the scale factor does not allow an arbitrary multiplicative renormalization as was explicitly shown in the examples with different choices of \(a_0\). Indeed, for the complete description of evolution for the homogeneous isotropic Universe it is necessary to unambiguously determine the gravitational field expressed in terms of \(a\), i.e., to introduce the following observable quantities, which physical meaning is defined with no ambiguity:

- the invariant mass of graviton \(m_{\text{gr}}\), which determines the contribution to the cosmological constant and corresponding fraction of energy

\[
\Omega_{\Lambda}^{\text{fr}} = -\frac{m_{\text{gr}}^2}{6H_0^2};
\]

- the projection of the graviton rest energy

\[
\mathcal{E} = \frac{m_{\text{gr}}}{a^3},
\]

which has got the cubic red shift and gives the contribution considered above, \(\Omega_{\text{gr}}\):
• the inverse fading length of the gravitational field for the static massive object at large distances:

$$r_{\text{gr}}^{-1} = \frac{m_{\text{gr}}}{a_{\text{max}}^2 a},$$

which has got the red shift and determines the contribution imitating the spatial curvature

$$\Omega_k^\text{gr} (a) = \frac{m_{\text{gr}}^2}{4 \kappa^2 a^2 H_0^2}.$$  

Thus, the question, how many times the temporal and spatial scales are sprained with respect to the Minkowskian space-time to the current moment, should be solved experimentally, and it does not permit an arbitrary prescription for the scale factor. The feature pointed out is an integral attribute of the theory with two metrics, where one can introduce the notion of mass for the graviton, only.

In this paper we have put the upper limit for the projection of the graviton rest energy, which has also determined the limit for the minimal admissible values of fading length. However, due to possible high red shift in the projection the obtained constraint is not valid for the graviton mass itself. The corresponding constraint on the graviton mass follows from the condition that the negative contribution to the cosmological constant as caused by the graviton mass should not exceed unit $\Omega_k^\text{gr} < 1$, since it gives the minimal admissible density of energy, which cannot be greater than the energy density observed to the moment. Therefore, we get

$$m_{\text{gr}} \leq 3.8 \cdot 10^{-66} \text{ g}.$$  

It is worth to pay an attention to the following item: the signal moving with the maximal admissible speed in the Minkowskian space-time, $ds^2 = 0$, should propagate in the space-like region of Riemannian space-time, $ds^2 < 0$, therefore, with the naive interpretation of free waves it would have a “super-light speed” $v^2 = \kappa/a^4 = a_{\text{max}}^4/a^4 \geq 1$. However, in accordance with the geometrization of RTG all matter fields move in the effective Riemannian space-time, so that the physical signals cannot propagate with a super-light speed. The only field, which is sensitive to the geometry of Minkowskian space-time is the massive graviton field. For the case of evolution of the homogeneous isotropic Universe under consideration the gravitational field has a large virtuality and it cannot be approximated by a free wave on the mass shell. Indeed, the dependence of scale factor on the time implies that the gravitational field has zero wave length with the variation of its frequency. In addition, the variation of the field with the time takes place immediately in the whole space, i.e., it is instant, which witnesses for the field virtuality as well as for the inconsistency of free field description. Thus, the considered gravitational field depending on the time gives the time-dependent intervals, only, i.e., there is no contradiction with the description of signal propagation and its causality.

6. Conclusion

The investigations have shown that with a high confidence level we can claim that, first, the expansion of the Universe to the current moment of time takes place with

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\(^\ddagger\) See the Appendix.
the acceleration, and the value of acceleration parameter is numerically determined in the limits

\[ 0.52 < \dot{q} < 0.83. \]

Second, the answer on the question whether the graviton is massive or not, requires a distinct separation of notions such as the mass, the projection of the rest energy with the metric tensor of Minkowskian space-time as well as the fading length. We have shown that the probability of nonzero projection for the graviton is extremely low, at least, it cannot essentially influences the Universe evolution from the times and, basically, temperatures of nucleosynthesis up to a far future with a condition that the contribution of dark energy, i.e., the quintessence, will fade. Moreover, one could involve the only known mechanism for the solution of problem why the value of fluctuations in the angle anisotropy of CMB is small, which is related also with the spectrum of primary fluctuations in the matter density in agreement with the observed large-scale structure of the Universe. This mechanism is the inflation of Universe at early stages. Such the way can, probably, give more strict constraints on the value of graviton energy projection. We can state that the introduction of somehow essential value for the projection of graviton energy leads to the model of cold expanding Universe, which is in contradiction with the observed physical phenomena witnessing for the scenario of hot Universe. The constraints on the invariant mass of graviton is less strong.

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Appendix

Consider a spherically symmetric gravitational field of massive body at large distances in the case of quasi-static expansion in the framework of RTG. The quasi-static approximation implies that we neglect the dependence of scale factor on the time, i.e., we will suggest this dependence adiabatic and use the following linear approximation in \( \phi(r) \to 0 \) for the interval of Riemannian metric \( g_{\mu\nu} \)

\[
ds^2 \approx \kappa^2 a^2 \left\{ 1 + \phi(r) \right\} d\tau^2 - \kappa^2 a^2 \left\{ 1 - \phi(r) \right\} \left[ dr^2 + r^2 d\omega^2 \right],
\]

which is consistent with the Minkowskian metric according to the equation

\[
D_\mu (\sqrt{-g}g^{\mu\nu}) = 0,
\]

where \( D_\mu \) is the covariant derivative with the Monkowskian metric connection, so that

\[
ds^2_\gamma \approx \kappa^2 a^2 \frac{d\tau^2}{a^2} - \left[ dr^2 + r^2 d\omega^2 \right],
\]

whereas the change of time normalization inessential for the static consideration,

\[
dt = \kappa a \, d\tau
\]

\[\dagger\dagger\]The RTG authors state the cyclic character of the Universe evolution.
leads to the asymptotic limit of adiabatically expanding Universe described in the
text, at $\phi \to 0$. It is important that if we neglect the expansion then the quasi-flat
limit of Riemannian metric implies that the influence of external matter, which density
of energy only adiabatically depends on the time, is reduced to the introduction of
an effective cosmological constant. In this way, the flat asymptotics guarantees the
compensation between the contribution into the cosmological term by the graviton
mass and that of the matter. In this case the equations for the gravitational field $\phi(r)$
in RTG are deduced to the only equation

$$\phi'' + \frac{2}{r} \phi' - m^2_{gr} \phi = 0,$$

(A.3)

where $\phi' = d\phi/dr$, and the right hand side of equation is equal to zero because of the
requirements to get zero for the total effective adiabatic cosmological constant. The
solution of (A.3) is

$$\phi(r) = -\frac{2M}{r} \exp[-m_{gr} r],$$

(A.4)

where $M$ is the mass of body, wherefrom we see that the fading length of gravitational
field at large distances in the coordinates of Minkowskian space-time is given by
the inverse graviton mass, while in the Riemannian space-time, where the physical
distances are measured by paths of light, the fading length has got the quasi-static
conformal re-scaling, and

$$r^{-1}_{gr} = \frac{m_{gr}}{\kappa a},$$

(A.5)

and the red shift does take place.

References

Garnavich P M et al. 1998 Astrophys. J. 509 74
(Garnavich P M et al. 1998 Preprint astro-ph/9806396)


(Chiba T 1999 Preprint gr-qc/9903094)

Preprint astro-ph/0305392


(Caldwell R R 2002 Preprint astro-ph/9908168)
Gibbons G W 2003 Preprint hep-th/0302199
[8] Loginov A A 2001 The theory of gravity (Moscow: Nauka)
(Sami M 2002 Preprint hep-th/0210258)
(Gershtein S S, Logunov A A and Mestvirishvili M A 1997 Preprint hep-th/9711147)
Gershtein S S, Logunov A A, Mestvirishvili M A and Tkachenko N P 2003 Preprint astro-
ph/0305125