Abstract. Progress in determining the baryon spectrum using computer simulations of quarks and gluons in lattice QCD are summarized and some future plans are outlined.

Baryon spectroscopy on the lattice: recent results

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Baryon spectroscopy is plagued by numerous unresolved issues. The quark model predicts many more states [?,?] than are currently known. Experiments in Hall B at Jefferson Laboratory are currently mapping out of the spectrum of $N^*$ nucleon excitations, so the question of the so-called “missing resonances” should soon be resolved. A quark-diquark picture of baryons predicts a sparser spectrum [?]. Various bag and soliton models have also attempted to explain the baryon masses. The natures of the Roper resonance and the $\Lambda(1405)$ remain controversial. Experiment shows that the first excited positive-parity spin-$1/2$ baryon lies below the lowest-lying negative-parity spin-$1/2$ resonance, a fact which is difficult to reconcile in quark models. First principles studies of the baryon spectrum using lattice Monte Carlo methods are long overdue.

State of the art results for the low-lying hadron spectrum using the Iwasaki gauge action and a clover tadpole-improved fermion action are presented in Fig. 1. The quenched spectrum (upper left) deviates from experiment by under ten per cent. The inclusion of two flavors of light quark loops produces excellent agreement with experiment for the $K$ and $\phi$ mesons, but the baryon masses show significant deviations from their experimental values. The authors suggest that finite volume errors will explain these discrepancies.

During the last few years, a handful of lattice studies have begun at last to focus attention on the excited baryon spectrum. Other than some preliminary unquenched results, all estimates to date have utilized the quenched approximation, and most have used unphysically heavy quarks. Systematic errors due to discretization and finite volume are not yet under control. Although the current status of such calculations is still embryonic, a renewed interest of the lattice community in such calculations promises substantial progress in the near future. For example, the Lattice Hadron Physics Collaboration
FIGURE 1. State of the art results from the CP-PACS collaboration for the low-lying hadron spectrum. Quenched results (upper left) in the continuum limit are from Ref. [?] and are shown using solid symbols with the $K$ setting the scale, whereas the hollow symbols use the $\phi$ to set the scale. The differences between the two sets measure the systematic errors from quenching. Agreement with experiment is remarkable, indicating that quenching errors in these observables are not large. In the lower right and the two leftmost plots, solid symbols indicate results from Ref. [?] including two flavors of light quark loops, whereas open symbols are quenched results shown against the lattice spacing $a$. For the $K$ and $\phi$ mesons, one observes excellent agreement with experiment. However, the baryon results are problematic; the authors suggest that finite volume effects are to blame. Experimental results are indicated by the diamonds.

FIGURE 2. Mass estimates of the $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ $N$ and $\Delta$ baryons from Ref. [?] against the square of the pion mass $m_\pi$ in the quenched approximation. The results use an improved gauge field action and the fat-link irrelevant clover (FLIC) fermion action on a $16^3 \times 32$ lattice with spacing $a = 0.12$ fm, set by the string tension. Spin-projected results are compared with previous unprojected ones. Experimental values are shown near the vertical axis.

(LHPC) [?] recently formed and one of its major objectives is the computation of the $N^*$ spectrum. The formation of this collaboration was spearheaded by the late Nathan Isgur and is funded by the Department of Energy’s Scientific Discovery through Advanced Computing (SciDAC) initiative. I shall outline the plans of this collaboration later in this talk. But first, results from four selected recent baryon studies are presented.

Latest results in the quenched approximation from the CSSM Lattice collaboration using an improved gauge field action and a fat-link irrelevant clover (FLIC) fermion action are shown in Fig. 2. Rather heavy quark masses were used, but the level orderings are in qualitative agreement with those observed in experiments.

The first-excited state in the positive-parity spin-$1/2$ sector is found to be significantly higher [?,?,?,?] than the Roper mass when unphysically large quark masses are used. Recently, the use of overlap fermions has allowed quenched calculations [?] with realistically light quark masses, and this point appears to be crucial for identifying the Roper as a radial excitation of the nucleon. Fig. 3 shows dramatic changes in the quenched baryon masses as the pion mass drops below 300 MeV. An important note of caution concerning these findings is the use of empirical Bayesian constrained curve fitting to ex-

FIGURE 3. Two lowest-lying octet and decuplet baryon masses for both positive and negative parities in the quenched approximation from Ref. [?] at lattice spacing $a \approx 0.2$ fm against the square of the pion mass. Results were obtained on $16^3 \times 28$ lattices using an improved gauge action and overlap fermions for a large range of light quark masses. These figures emphasize the importance of simulating with sufficiently light quark masses. Experimental measurements are shown as bursts.

FIGURE 4. The low-lying nucleon masses in the quenched approximation from Ref. [?] (left) and Ref. [?] (right). Results on the left use domain wall fermions in a small volume with spatial extent $L_a \approx 1.5$ fm. On the right, results in three different volumes $L_a \approx 2.2 - 3.0$ fm were extrapolated using $1/L^3$ to the infinite volume limit. The Wilson gauge and Wilson fermion actions with $\beta = 6.0$ and spacing $a \sim 0.1$ fm were used with quark masses yielding pion masses in the range $m_\pi = 0.6 - 1.0$ GeV. Maximum entropy methods were employed. Experimental values are shown as bursts.

tract the excited state mass from a single correlation function. Although likely reliable, an analysis using several operators in a correlation matrix would be much preferred. Also, at such light quark masses, one must very carefully check finite volume errors (as evidenced in the next study described below). In a more recent paper [?], these authors have also addressed the issue of pollution of the first-excited state observed in Fig. 3 by unphysical $\eta' N$ quenched artifacts. Such ghost contributions were distinguished by obtaining results in two volumes $16^3 \times 28$ and $12^3 \times 28$, corresponding to lattice extents 3.2 and 2.4 fm, respectively. The conclusion, within the quenched approximation, that the Roper is a radial excitation of the nucleon with three valence quarks was confirmed.

A large sensitivity of the Roper resonance to finite volume errors in the quenched approximation has recently been reported in Ref. [?]. In Fig. 4, one sees that for $m_\pi^2 \approx 0.5$ GeV$^2$, the infinite volume results for the $N'$ Roper are degenerate with the negative parity $N^*$, in disagreement with the results shown in Fig. 3. The lattice spacings and actions differ, so this discrepancy could simply be a discretization artifact. Also, maximum entropy methods are being employed, which further muddies the issue. Nevertheless, an important message seems clear: large volumes and small quark masses are especially important for reliable results in baryon spectroscopy.

Doubly charmed baryons, also of current experimental interest, have also been studied recently [?] in the quenched approximation using an improved gauge action on anisotropic lattices with the D234 action for the light quarks...
and a nonrelativistic (NRQCD) action for the heavy quarks. Two lattice spacings $a \sim 0.15, 0.22$ fm and four light quark masses were used. These authors found that mass splittings between $J = \frac{1}{2}$ and $\frac{3}{2}$ baryons from color hyperfine interactions were not suppressed, unlike the meson sector. Many charmed and bottom baryons were studied. No finite volume checks were done, and radiative corrections to the couplings in the NRQCD action were ignored. Results using a clover fermion action at $\beta = 6.2$ have also been presented \cite{?} recently.

The Lattice Hadron Physics Collaboration is currently using large sets of extended operators with correlation matrix techniques to capture a significant portion of the baryon spectrum. Excited states will be extracted without resorting to maximum entropy methods. We believe that the construction of good operators is crucial: the operators have been designed with one eye towards maximizing overlaps with the low-lying states of interest, and the other eye towards minimizing the number of source needed in computing the required quark propagators. For example, the three-quark operators we plan to use are expressed in terms of smeared quark fields $\tilde{\psi}$, the covariant three-dimensional Laplacian $\tilde{\Delta}$, and the $p$-link covariant displacement $\tilde{D}_j^{(p)}$ by

\begin{align}
(1): & \phi_{ABC}^F \varepsilon_{abc} \Gamma_{\alpha \beta \gamma} (\tilde{\Delta}^{n1} \tilde{\psi})_{Aa} (\tilde{\Delta}^{n2} \tilde{\psi})_{Bb} (\tilde{\Delta}^{n3} \tilde{\psi})_{Cc} \\
(2): & \phi_{ABC}^{F} \varepsilon_{abc} \Gamma_{\alpha \beta \gamma} (\tilde{\Delta}^{n1} \tilde{\psi})_{Aa} (\tilde{\Delta}^{n2} \tilde{\psi})_{Bb} (\tilde{D}_j^{(p)} \tilde{\Delta}^{n3} \tilde{\psi})_{Cc} \\
(3): & \phi_{ABC}^{F} \varepsilon_{abc} \Gamma_{\alpha \beta \gamma} (\tilde{\Delta}^{n1} \tilde{\psi})_{Aa} (\tilde{D}_j^{(p1)} \tilde{\Delta}^{n2} \tilde{\psi})_{Bb} (\tilde{D}_k^{(p2)} \tilde{\Delta}^{n3} \tilde{\psi})_{Cc},
\end{align}

where $n_1, n_2, n_3, p, p_1, p_2$ are positive integers, $j, k = \pm 1, \pm 2, \pm 3$ are spatial directions, $\alpha, \beta, \gamma$ are Dirac spin indices, $A, B, C$ are quark flavors, and $a, b, c$ indicate colors. Different powers of the spatial Laplacian $\tilde{\Delta}$ are utilized to build up radial structure, and the displacement operator $\tilde{D}_j$ is used to incorporate orbital structure. The group theoretical projections necessary to obtain operators transforming irreducibly under the symmetries of the lattice have been carried out using software written in Maple. Degeneracy patterns among the different irreducible representations of the cubic group must be exploited to identify angular momentum $J$ eigenstates in the continuum limit. We hope to present our first results in the near future. Note that our operator construction approach can be easily adapted for mesons, pentaquark systems, and so on.

Regardless of how the operators are constructed, extracting the baryon spectrum in lattice simulations remains a challenge. It is especially important for baryons that the Monte Carlo calculations be done in large volumes with realistically light quark masses and without the quenched approximation. Furthermore, all baryon studies must ultimately confront the thorny issue of treating unstable resonances. The techniques for doing this are well known \cite{?}, but are untested in QCD. However, computing speeds continue to increase and large computer clusters dedicated to hadron physics are coming on-line. Given the renewed interest in baryon spectroscopy, substantial progress is inevitable.
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