A detailed description of how black holes grow in full, non-linear general relativity is presented. The starting point is the notion of dynamical horizons. Expressions of fluxes of energy and angular momentum carried by gravitational waves across these horizons are obtained. Fluxes are local and the energy flux is positive. Change in the horizon area is related to these fluxes. A notion of angular momentum and energy is associated with cross-sections of the horizon and balance equations, analogous to those obtained by Bondi and Sachs at null infinity, are derived. These in turn lead to generalizations of the first and second laws of black hole mechanics. The relation between dynamical horizons and their asymptotic states —the isolated horizons— is discussed briefly. The framework has potential applications to numerical, mathematical, astrophysical and quantum general relativity.