Progress in the physics of massive neutrinos

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Abstract

The current status of the physics of massive neutrinos is reviewed with a forward-looking emphasis. The article begins with the general phenomenology of neutrino oscillations in vacuum and matter and documents the experimental evidence for oscillations of solar, reactor, atmospheric and accelerator neutrinos. Both active and sterile oscillation possibilities are considered. The impact of cosmology (BBN, CMB, leptogenesis) and astrophysics (supernovae, highest energy cosmic rays) on neutrino observables and vice versa, is evaluated. The predictions of grand unified, radiative and other models of neutrino mass are discussed. Ways of determining the unknown parameters of three-neutrino oscillations are assessed, taking into account eight-fold degeneracies in parameters that yield the same oscillation probabilities, as well as ways to determine the absolute neutrino mass scale (from beta-decay, neutrinoless double-beta decay, large scale structure and Z-bursts). Critical unknowns at present are the amplitude of $\nu_\mu \rightarrow \nu_e$ oscillations and the hierarchy of the neutrino mass spectrum; the detection of $CP$ violation in the neutrino sector depends on these and on an unknown phase. The estimated neutrino parameter sensitivities at future facilities (reactors, superbeams, neutrino factories) are given. The overall agenda of a future neutrino physics program to construct a bottom-up understanding of the lepton sector is presented.
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1 Introduction

After decades of immense experimental and theoretical effort, major breakthroughs in our understanding of the properties of neutrinos have occurred recently. Since the time that the neutrino was first proposed by Pauli [1] and placed on a concrete theoretical foundation by Fermi [2], the question of whether neutrinos are massless or massive has persisted. The standard technique of probing neutrino masses through studies of the endpoints of decay spectra succeeded only in placing increasingly more restrictive upper limits on neutrino masses, presently down to about 2 eV in the case of neutrinos emitted in the beta decays of tritium [5, 6].

A more sensitive measure of small neutrino masses was known since 1968 from the proposal of Gribov and Pontecorvo that a neutrino of a given initial flavor could interchange its identity with other flavors [7], with a probability that is dependent on the distance from the location of the source [8]. Early searches for evidence of neutrino oscillations found only upper bounds on the oscillation probabilities. A long-standing deficit of solar neutrinos in the $^{37}\text{Cl}$ radiochemical experiment of Davis [9], compared to Standard Solar Model (SSM) expectations [10, 11], was attributed to oscillations, but this interpretation remained unproven until recently. The observation of an electron to muon ratio in the atmospheric neutrino events in the Kamiokande [12] and IMB [13] experiments of about a factor of 2 above expectations was interpreted as evidence for oscillations with neutrino mass-squared difference $\delta m^2 \sim 10^{-2}\text{eV}^2$ and near maximal neutrino mixing [14]. However, due to the prevailing theoretical prejudice that neutrino mixings would be small, based on the fact that quark mixings are small, this interpretation of the atmospheric neutrino data did not receive widespread acceptance.

The definitive evidence of atmospheric neutrino oscillations came ten years later from the Super-Kamiokande (SuperK) experiment [15]. With the ability to measure the zenith angular and energy distributions of both electron and muon events, the SuperK experiment convincingly established that the muon-neutrino flux has a deficit compared with the calculated flux that increased with zenith angle (or equivalently the path distance), while the electron-neutrino flux agreed with no-oscillation ex-

\footnote{For a historical perspective and discussions of early work on neutrinos, see Refs. [3, 4].}
pectations. The accumulation of high statistics data by SuperK eventually excluded interpretations other than $\nu_\mu$ to $\nu_\tau$ oscillations with nearly maximal mixing at a mass-squared-difference scale $\delta m^2 \sim 2.0 \times 10^{-3}$ eV$^2$. The energy and angular resolution of the SuperK experiment are too coarse to allow the first minimum in $\nu_\mu \rightarrow \nu_\mu$ to be resolved. That is the goal in the ongoing K2K [16] and the forthcoming MINOS [17] and CNGS [18, 19] accelerator experiments.

The evidence for solar neutrino oscillations continued to build as experiments with different detectors and energy sensitivity all found deficits of 1/3 to 1/2 in rates compared to the SSM. The $^{71}$Ga radiochemical experiments of SAGE [20], GALLEX [21] and GNO [22] found deficits in the flux of $pp$ neutrinos, which is the dominant product of the $pp$ reaction chain that powers the Sun$^2$. The SuperK water Cherenkov detector [24] accurately measured the electron energy spectrum from high-energy solar neutrinos ($\gtrsim 5$ MeV) originating from $^8$B decays in the Sun and found it to be flat [25, 26] with respect to the SSM prediction. A crucial theoretical aspect in the oscillations of solar neutrinos is coherent forward $\nu_e e \rightarrow \nu_e e$ scattering, first discussed by Wolfenstein [27], which affects electron neutrinos as they propagate through the dense solar core. Matter effects can produce large changes in the oscillation amplitude and wavelength compared to vacuum oscillations, as first shown by Barger, Whisnant, Pakvasa and Phillips [28]. A resonance enhancement$^3$ can be realized in matter for one sign of $\delta m^2$. Because of the prevailing prejudice that neutrino mixing would be small, there was a strong theoretical bias in favor of a resonant solar solution, which was the original solution to the solar neutrino problem proposed by Mikheyev and Smirnov (the so-called Mikheyev-Smirnov-Wolfenstein or MSW solution) [29]. In addition to this small mixing angle solution (known as SMA), other solutions with a large vacuum mixing angle were later identified that could account for the solar

$^2$The $pp$ neutrino flux is now determined experimentally to $\pm 2\%$ and is in agreement with the SSM predictions to $1\%$ (the theoretical uncertainty is also $1\%$) [23], thus confirming the essential ingredients of the SSM.

$^3$BWPP discovered this enhancement in a medium of constant density by varying the neutrino energy; the matter effect depends on the product of the neutrino energy and the electron density. Mikheyev and Smirnov applied the enhancement at a given neutrino energy to the propagation of solar neutrinos through the varying electron density in the Sun.
neutrino flux suppression [30].

The other solutions were named LMA (large mixing angle), LOW (low $\delta m^2$, low probability) [31], QVO (quasi-vacuum oscillations) [32] and VO (vacuum oscillations) [33]. These islands in the $(\delta m^2, \tan^2 \theta)$ in the solar neutrino oscillation parameter space are illustrated in Fig. 1. The flat energy spectrum relative to the SSM and the absence of a significant day/night difference (that can be caused by Earth-matter effects), measured by SuperK (see Fig. 2), led to a strong preference for the LMA solution. Subsequently, the Sudbury Neutrino Observatory (SNO) [35] measured the neutrino neutral currents as well as the charged currents, and confirmed the oscillations independent of the SSM normalization of the $^8$B flux. The recent SNO salt phase data [36] in conjunction with other solar neutrino data selects the LMA solution uniquely at more than the 3$\sigma$ level. The mass-squared difference indicated by the solar neutrino data is $\sim 6 \times 10^{-5}$ eV$^2$ and the mixing is large but not maximal, $\tan^2 \theta \sim 0.4$; see Section 4.

Figure 1: 90%, 95%, 99% and 3$\sigma$ C. L. allowed regions in the $(\delta m^2, \tan^2 \theta)$ oscillation parameter space, before any SNO data. From Ref. [34].
Figure 2: The energy spectrum and the day-night asymmetry (day rate – night rate divided by the average rate) as measured by SuperK. The solid lines are the predictions for $\delta m^2_s = 6.3 \times 10^{-5} \text{ eV}^2$ and $\tan^2 \theta_s = 0.55$. From Ref. [26].

The definitive confirmation of the LMA solar solution has come from the KamLAND experiment [37]. If CPT invariance is assumed, the probabilities of $\nu_e \rightarrow \nu_e$ and $\bar{\nu}_e \rightarrow \bar{\nu}_e$ oscillations should be equal at the same values of $L/E$. At the average distance $L \sim 180$ km of the reactors from the KamLAND detector and the typical energies of a few MeV of the reactor $\bar{\nu}_e$, the experiment has near optimal sensitivity to the $\delta m^2$ value of the LMA solar solution. The first year of data from KamLAND [38] shows the rate suppression of $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ expected from the solar LMA solution, vindicating the oscillation interpretation of the solar neutrino problem. The solar $\nu_e$ are oscillating to a combination of $\nu_\mu$ and $\nu_\tau$.

With three neutrinos ($\nu_e, \nu_\mu, \nu_\tau$) there are two distinct $\delta m^2$ that can account for the atmospheric and solar neutrino oscillations. However, the LSND accelerator experiment [39] found evidence at $3.3\sigma$ significance for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations at a higher $\delta m^2 \sim 0.2$ to 1 eV$^2$ with small mixing, $\sin^2 2\theta \sim 0.003$ to 0.04. Evidence for oscillations of $\nu_\mu \rightarrow \nu_e$ was also observed, although at lesser significance [40]. Such oscillations are constrained but not excluded by the KARMEN experiment [41]. A possible way to explain the LSND effect is to invoke oscillations involving a fourth, sterile neutrino with no Standard Model interactions [42]. Two possible scenarios are
considered, with $2 + 2$ and $3 + 1$ spectra of $\delta m^2$ [43, 44, 45, 46], where the large mass-squared difference of the LSND oscillations connects the upper and lower levels. However, there is a tension in the global fits between the solar and atmospheric data on one hand and the reactor and accelerator data on the other, with the conclusion that both schemes are highly disfavored [47]. The MiniBooNE [48] experiment, now in progress at Fermilab, is designed to test the $\nu_\mu \to \nu_e$ oscillation hypothesis for the LSND effect. The source of the antineutrino beam at LSND is muon decays, while at MiniBooNE the source is pion decays. An alternative nonoscillation interpretation of the LSND effect as a nonstandard muon decay [49] will not be tested by MiniBooNE. Since the LSND effect is fragile, this review emphasizes three neutrino oscillation phenomena.

The phenomenology of three neutrino mixing, like that of three quarks [50], involves three mixing angles ($\theta_{23} \equiv \theta_a$, $\theta_{12} \equiv \theta_s$, $\theta_{13} \equiv \theta_x$), two mass-squared differences ($\delta m^2_{31} \equiv \delta m^2_a$, $\delta m^2_{21} \equiv \delta m^2_s$), and one $CP$-violating phase ($\delta$) [7, 51, 52, 53, 54]. In order that $\delta$ be measurable, both $\delta m^2$ scales must contribute to the oscillation [52]. If neutrinos are Majorana [55], two further $CP$-violating phases ($\phi_2$, $\phi_3$) enter in the calculation of neutrinoless double-beta decay [56] but not oscillations [57]. We already have approximate knowledge of $\theta_a$, $\theta_s$, $\delta m^2_a$ (but not the sign of $\delta m^2_a$) and $\delta m^2_s$ (its sign is known from solar matter effects). The major challenge before us now is the measurement of $\theta_x$, for which we have only an upper bound, $\sin^2 2\theta_x \lesssim 0.2$ (for $\delta m^2_a = 2.0 \times 10^{-3} \text{eV}^2$) at the 95% C. L. from the CHOOZ [58] and Palo Verde [59] reactor experiments.

The amplitude of the oscillations $\nu_\mu \to \nu_e$ and $\nu_e \to \nu_\tau$ are governed by the size of $\theta_x$. Long-baseline and reactor experiments are planned with improved sensitivity to $\sin^2 2\theta_x$ [60]; see Section 10. The $CP$-violating phase enters oscillations via a factor $\sin \theta_x e^{-i\delta}$ and thus it is essential to establish a nonzero value of $\theta_x$ in order to pursue the measurement of $\delta$. The size of $CP$ violation in long-baseline experiments also depends on the value of $\delta m^2_s$. Oscillation parameter ambiguities exist [61] that must be resolved by the experiments.

The anticipated steps in the long-baseline program are off-axis beams [60, 62, 63], superbeams [64, 65], detectors with larger fiducial volumes and sophistication [60, 67].
66, 67], and neutrino factories [68, 69]. The ultimate sensitivities will be derived from neutrino factories, where the neutrino beams are obtained from the decays of muons that are stored in a ring with straight sections.

Although oscillations have allowed us to establish that neutrinos have mass, they do not probe the absolute neutrino mass scale. In particle and nuclear physics, the only avenues for this purpose are tritium beta decay and neutrinoless double-beta decay, and the latter only if neutrinos are Majorana particles. Experiments are beginning to probe the interesting region of $m_\nu$. Another route to the absolute mass is the power spectrum of galaxies, which gets modified on small scales when the sum of neutrino masses is nonzero [70]. The WMAP analysis [71] of the cosmic microwave background (CMB) and large-scale structure data have already given an upper limit on $\sum m_\nu$ of about 1 eV.

The field of neutrino physics is progressing at a rapid rate. The purpose of this review is to summarize the present status of the field and to discuss ways that progress can be made in experimentally answering the outstanding questions. For other recent reviews see Refs. [72, 73, 74, 75, 76]. We have provided an extensive bibliography, but due to the large number of papers on the subject, it may not be comprehensive.

## 2 Neutrino counting: $Z$-decays, CMB and BBN

Studies of $e^+e^-$ annihilation at the $Z$-resonance pole at the Large Electron Positron collider have determined the invisible width of the $Z$ boson. The experimental value $N_\nu = 2.984 \pm 0.008$ is close to the number expected from 3 active light neutrinos, though the value is $2\sigma$ low [54].

The cosmic microwave background (CMB) anisotropies and Big Bang Nucleosynthesis (BBN) probe the effective number of neutrinos ($N_\nu = 3 + \Delta N_\nu$) that were present in the early universe. The extra relativistic energy density due to sterile neutrinos, or other possible light particles, is normalized to that of an equivalent neutrino flavor as [77]

$$\rho_x \equiv \Delta N_\nu \rho_\nu = \frac{7}{8} \Delta N_\nu \rho_\gamma, \quad (1)$$

where $\rho_\nu$ is the energy density in photons. Sterile neutrinos would contribute to $\Delta N_\nu$. 


but so could other new physics sources.

The precise WMAP measurements [71] of the CMB have been analyzed to constrain $\Delta N_\nu$ [78, 79]. In Ref. [78], a flat universe with a cosmological constant $\Lambda$ as dark energy is assumed. The parameters varied are the reduced Hubble constant $h$, the baryon density $\omega_B = \Omega_B h^2$, the total matter density $\omega_M = \Omega_M h^2$ (comprised of baryons and cold dark matter), the optical depth $\tau$, the spectral index $n_s$ and amplitude $A_s$ of the primordial power spectrum ($P = A_s(k/k_*)^{n_s-1}$) and $\Delta N_\nu$. The HST measurement of $h = 0.72 \pm 0.08$ [80] is imposed as a top-hat distribution. This strong $h$ prior helps to break the degeneracy between $\omega_M$ and $\Delta N_\nu$. The universe is also required to be older than 11 Gyr, as inferred from globular clusters [81].

The effects of $N_\nu$ on the CMB are illustrated in Fig. 3, for a fixed normalization of the power spectrum. The resulting $\Delta N_\nu$ constraints are shown in Fig. 4, where $\eta_{10} \equiv 10^{10} n_B/n_\gamma = 274 \omega_B$. The best-fit is $\Delta N_\nu = -0.25$, but at 2$\sigma$, $\Delta N_\nu \leq 5.3$ is allowed; see Table 1 [78].

![Figure 3](image-url)

Figure 3: The power spectrum for the best-fit ($N_\nu = 2.75$) to the WMAP data [71] is the solid line. With all other parameters and the overall normalization of the primordial spectrum fixed, the spectra for $N_\nu = 1$, $N_\nu = 5$ and $N_\nu = 7$ are the dotted, dot-dashed and dashed lines, respectively. The data points represent the binned TT power spectrum from WMAP. From Ref. [78].
Figure 4: The 1σ and 2σ contours in the $\eta_{10} - \Delta N_\nu$ plane from an analysis [78] of WMAP data. The solid (dotted) lines correspond to $t_0 \geq 11$ (12) Gyr. The cross marks the best-fit at $\omega_B = 0.023$ and $\Delta N_\nu = -0.25$.

Table 1: The 2σ ranges (for 1 degree of freedom) of $N_\nu$ and $\eta_{10}$ from analyses [78] of WMAP data, deuterium and helium abundances and their combinations. The WMAP analysis involves the assumption of a flat universe, along with the strong HST prior on $h$ and the age constraint $t_0 \geq 11$ Gyr. For BBN the adopted primordial abundances are: $y_D \equiv 10^5(D/H) = 2.6 \pm 0.4$ [82], $Y = 0.238 \pm 0.005$ [83].

<table>
<thead>
<tr>
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<th>$N_\nu$ (2σ range)</th>
<th>$\eta_{10}$ (2σ range)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WMAP</td>
<td>0.9 – 8.3</td>
<td>5.58 – 7.26</td>
</tr>
<tr>
<td>$y_D + Y$</td>
<td>1.7 – 3.0</td>
<td>4.84 – 7.11</td>
</tr>
<tr>
<td>WMAP + $y_D + Y$</td>
<td>1.7 – 3.0</td>
<td>5.53 – 6.76</td>
</tr>
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BBN is a much better probe of $\Delta N_\nu$ than the CMB. The prediction of the primordial abundance of $^4$He depends sensitively on the early expansion rate, while the prediction of the D abundance is most sensitive to the baryon density $\eta_{10}$ [84]. The best-fit value of $\Delta N_\nu$ from BBN is $\Delta N_\nu = -0.7$ but at 2σ, $N_\nu = 3$ is allowed. The D-inferred baryon density is in excellent agreement with the baryon density determined from the CMB and Large Scale Structure.
A combined analysis of the BBN and the WMAP data yields the allowed regions in Fig. 5. As with the BBN analysis above, the best-fit value is $\Delta N_\nu = -0.7$ and $N_\nu = 3$ is allowed at 2$\sigma$.

![Figure 5](image)

Figure 5: The $1\sigma$, $2\sigma$ and $3\sigma$ contours in the $\eta_{10} - \Delta N_\nu$ plane from a combination of WMAP data and the adopted $D$ and $^4He$ abundances. From Ref. [78].

The BBN analysis is consistent with 3 neutrinos, but gives no support to the possible existence of extra neutrinos. Even one extra, fully thermalized neutrino ($\Delta N_\nu = 1$) is strongly disfavored [78]. We discuss the implications of this result for the sterile neutrino interpretation of the LSND experiment in Section 11.

3 Neutrino mixing and oscillations

3.1 Vacuum oscillations

The dramatic increase in our knowledge of neutrino properties has come from observational evidence of neutrino oscillations. These neutrino flavor changes require that the neutrino flavor states, $\nu_\alpha$ are not the same as the neutrino mass eigenstates, $\nu_i$. The eigenstates are related by a unitary matrix $V$ [7],

$$\nu_\alpha = \sum V_{\alpha i}^* \nu_i . \quad (2)$$
$V$ is often denoted as $V_{MNS}$, where MNS represents the authors of Ref. [7]. For 3 neutrinos, the mixing matrix $V$ is specified by three rotation angles $\theta_a$, $\theta_x$, $\theta_s$ ($0 \leq \theta_i \leq \pi/2$) and three $CP$-violating phases $\delta$, $\phi_2$ and $\phi_3$ ($0 \leq \delta, \phi_i \leq 2\pi$). $V$ can be conveniently written as the matrix product

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_a & s_a \\ 0 & -s_a & c_a \end{bmatrix} \begin{bmatrix} c_x & 0 & s_x e^{-i\delta} \\ 0 & 1 & 0 \\ -s_x e^{i\delta} & 0 & c_x \end{bmatrix} \begin{bmatrix} c_s & s_s & 0 \\ -s_s & c_s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{1}{2}\phi_2} & 0 \\ 0 & 0 & e^{i\frac{1}{2}\phi_3+\delta} \end{bmatrix}$$

(3)

where $c_i$ denotes $\cos \theta_i$ and $s_i$ denotes $\sin \theta_i$. The angle $\theta_a$, customarily denoted as $\theta_{23}$, governs the oscillations of atmospheric neutrinos, the angle $\theta_s$ ($\theta_{12}$) describes solar neutrino oscillations, and the angle $\theta_x$ ($\theta_{13}$) is an unknown angle that is bounded by reactor neutrino experiments at short distances ($L \sim 1\text{ km}$). The oscillation probabilities are independent of the Majorana phases $\phi_2$ and $\phi_3$. Vacuum neutrino oscillations are given by

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_j V_{\beta j} e^{-i\frac{\Delta m^2_{jj}L}{2E_\nu}} V^*_{\alpha j} \right|^2,$$

(4)

where the $m_j$ are the neutrino eigenmasses. The oscillation probabilities depend only on differences of squared neutrino masses. The oscillation arguments for the atmospheric and solar phenomena are

$$\Delta_a \equiv \frac{\delta m^2_{aa}L}{4E_\nu}, \quad \Delta_s \equiv \frac{\delta m^2_{ss}L}{4E_\nu},$$

(5)

respectively, where

$$\delta m^2_a = m_3^2 - m_1^2, \quad \delta m^2_s = m_2^2 - m_1^2.$$

(6)

The vacuum oscillation probabilities are

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_x \sin^2 \Delta_a - (c_x^4 \sin^2 2\theta_s + s_x^2 \sin^2 2\theta_x) \sin^2 \Delta_s + s_x^2 \sin^2 2\theta_x \left( \frac{1}{2} \sin 2\Delta_s \sin 2\Delta_a + 2 \sin^2 \Delta_a \sin^2 \Delta_s \right),$$

(7)

$$P(\nu_e \to \nu_\mu) = s_x^2 \sin^2 2\theta_x \sin^2 \Delta_a + 4J(\sin 2\Delta_s \sin^2 \Delta_a - \sin 2\Delta_a \sin^2 \Delta_s)$$

where $J$ is a function of the $CP$-violating phases and $\Delta_a$ and $\Delta_s$. Sometimes it is denoted as $V_{PMNS}$ or $V_{MNSP}$ to acknowledge the contributions of Pontecorvo.

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\[-(s_a^2s_s^2\sin^22\theta_x - 4K)[\frac{1}{2}\sin 2\Delta_s \sin 2\Delta_a + 2\sin^2\Delta_s \sin^2\Delta_a] + [c_x(c_\alpha^2 - s_x^2s_s^2)\sin^22\theta_a + s_x^2s_s^2\sin^22\theta_x - 8Ks_a^2]\sin^2\Delta_s, \quad (8)\]

\[
P(\nu_\mu \to \nu_\mu) = 1 - (c_x^4\sin^22\theta_a + s_a^2\sin^22\theta_x)\sin^2\Delta_a + [c_x(c_\alpha^2 - s_x^2s_s^2)\sin^22\theta_a + s_x^2s_s^2\sin^22\theta_x - 8Ks_a^2] \times [\frac{1}{2}\sin 2\Delta_s \sin 2\Delta_a + 2\sin^2\Delta_s \sin^2\Delta_a] - [\sin^22\theta_a(c_\alpha^2 - s_x^2s_s^2)^2 + s_x^2\sin^22\theta_a(1 - c_\alpha^2\sin^22\theta_a) + 2s_x\sin 2\theta_a\cos 2\theta_a\sin 2\theta_a\cos 2\theta_a - 16Ks_a^2s_s^2 + \sin^22\theta_a c_x^2(c_\alpha^2 - s_x^2s_s^2) + s_x^2s_s^2\sin^22\theta_x] \sin^2\Delta_s, \quad (9)\]

\[
P(\nu_\mu \to \nu_\tau) = c_x^4\sin^22\theta_a\sin^2\Delta_a + 4J(\sin 2\Delta_s \sin^2\Delta_a - \sin 2\Delta_a \sin^2\Delta_s) - [c_x^2\sin^22\theta_a(c_\alpha^2 - s_x^2s_s^2)^2 + 4K\cos 2\theta_a] \times [\frac{1}{2}\sin 2\Delta_s \sin 2\Delta_a + 2\sin^2\Delta_s \sin^2\Delta_a] + [\sin^22\theta_a(c_\alpha^2 - s_x^2s_s^2)^2 + s_x^2\sin^22\theta_a(1 - \sin^22\theta_a\delta) + 4K\cos 2\theta_a + s_x\sin 2\theta_a\cos 2\theta_a\sin 2\theta_a\cos 2\theta_a(1 + s_x^2\delta)] \sin^2\Delta_s, \quad (10)\]

where the CP-violating quantity \(J\) [85] is

\[
J = \frac{1}{8}c_x \sin 2\theta_x \sin 2\theta_a \sin 2\theta_a s_\delta, \quad (11)\]

and for convenience we have defined

\[
K = \frac{1}{8}c_x \sin 2\theta_x \sin 2\theta_a \sin 2\theta_a c_\delta. \quad (12)\]

Oscillation probabilities for other neutrino channels may be obtained by probability conservation, \textit{i.e.}, \(\sum_\alpha P(\nu_\alpha \to \nu_\beta) = \sum_\beta P(\nu_\alpha \to \nu_\beta) = 1\). Probabilities for antineutrino channels are obtained by replacing \(\delta\) by \(-\delta\) in the corresponding neutrino formulae. Also, since CPT is conserved for ordinary neutrino oscillations, the antineutrino probabilities are given by \(P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = P(\nu_\beta \to \nu_\alpha)\), and \(P(\nu_\alpha \to \nu_\beta)\) is obtained from \(P(\nu_\beta \to \nu_\alpha)\) by replacing \(\delta\) by \(-\delta\).

For oscillations of atmospheric and long-baseline neutrinos, the oscillation argument \(\Delta_a\) is dominant, and the vacuum oscillation probabilities in the leading oscillation approximation (where only the \(\delta m^2_a\) oscillations are appreciable) are

\[
P(\nu_e \to \nu_e) \simeq 1 - \sin^22\theta_x \sin^2\Delta_a \quad (13)\]

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\[ P(\nu_e \rightarrow \nu_\mu) \approx \sin^22\theta_x \sin^2\Delta_a \]  \hspace{1cm} (14)

\[ P(\nu_\mu \rightarrow \nu_\mu) \approx 1 - (c_x^4 \sin^22\theta_a + s_a^2 \sin^22\theta_x) \sin^2\Delta_a. \]  \hspace{1cm} (15)

\[ P(\nu_\mu \rightarrow \nu_\tau) \approx c_x^4 \sin^22\theta_a \sin^2\Delta_a. \]  \hspace{1cm} (16)

For vacuum oscillations of solar neutrinos, where \(|\Delta_a| \gg 1\), the terms involving \(\Delta_a\) approach their average values over a complete cycle and

\[ P(\nu_e \rightarrow \nu_e) \approx 1 - \frac{1}{2} \sin^22\theta_x - c_x^4 \sin^22\theta_a \sin^2\Delta_a. \]  \hspace{1cm} (17)

Note that in the limit \(\theta_x \rightarrow 0\), \(\nu_\mu \rightarrow \nu_\tau\) oscillations of atmospheric neutrinos and \(\nu_e \rightarrow \nu_e\) oscillations of solar neutrinos completely decouple, \(i.e.,\) they are determined by independent parameters, and each has the form of two-neutrino oscillations.

### 3.2 Matter effects on oscillations

The scattering of \(\nu_e\) on electrons in matter can modify the vacuum oscillation probabilities [27, 28, 86]. For the two-neutrino case, with mixing angle \(\theta\) and mass-squared difference \(\delta m^2\), the oscillation probability amplitude \(\sin^2\theta_m\) in matter is

\[ \sin^2\theta_m = \frac{\sin^22\theta}{\left(\frac{A}{\delta m^2} - \cos 2\theta\right)^2 + \sin^22\theta}, \]  \hspace{1cm} (18)

where

\[ A = 2\sqrt{2}G_F N_e E_\nu = 1.54 \times 10^{-4} \text{ eV}^2 Y_e \rho (\text{g/cm}^3) E_\nu (\text{GeV}), \]  \hspace{1cm} (19)

and \(N_e\) is the electron number density, which is the product of the electron fraction \(Y_e\) and matter density \(\rho\). The oscillation amplitude in matter is enhanced if \(\delta m^2 > 0\), and a resonance occurs \(i.e.,\) the amplitude reaches its maximal value of unity) at the critical density \(N_e^c = \delta m^2 \cos 2\theta / (2\sqrt{2}G_F E_\nu)\). For antineutrinos, \(A \rightarrow -A\) in Eq. (18), and the oscillation amplitude in matter is enhanced if \(\delta m^2 < 0\). For \(N_e\) much larger than the critical density, the oscillation amplitude in matter is strongly suppressed for both neutrinos and antineutrinos. The effective value of \(\delta m^2\), and hence the oscillation wavelength, is also changed in matter:

\[ \delta m^2_m = \delta m^2 \sqrt{\left(\frac{A}{\delta m^2} - \cos 2\theta\right)^2 + \sin^22\theta}. \]  \hspace{1cm} (20)
The angle $\theta_m$ represents the mixing between the flavor eigenstates $\nu_\alpha$ and the instantaneous eigenstates in matter $\nu_{im}$:

$$
\begin{bmatrix}
\nu_{1m} \\
\nu_{2m}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta_m & -\sin \theta_m \\
\sin \theta_m & \cos \theta_m
\end{bmatrix}
\begin{bmatrix}
\nu_e \\
\nu_\mu
\end{bmatrix}.
$$

(21)

If the electron density is $N_e^0$ when the neutrino is created, the initial value of $\theta_m$ is given by

$$
\cos 2\theta_0^m = -\frac{A^0}{\delta m^2} - \cos 2\theta
\sqrt{(A^0/\delta m^2 - \cos 2\theta)^2 + \sin^2 2\theta},
$$

(22)

where $A^0 = 2\sqrt{2}G_F N_e^0 E_\nu$. A neutrino originally created as a $\nu_e$ can be expressed in terms of the lower and upper eigenstates as $\nu_e = \cos \theta^0_m \nu_{1m} + \sin \theta^0_m \nu_{2m}$.

### 3.3 Solar neutrino oscillations

For matter of varying density, such as for neutrinos propagating through the Sun, the instantaneous eigenstates change as the neutrinos propagate. Once a solar neutrino reaches the vacuum, the lower eigenstate is $\nu_1 = \cos \theta \nu_e - \sin \theta \nu_\mu$ and the upper eigenstate is $\nu_2 = \sin \theta \nu_e + \cos \theta \nu_\mu$.

A $\nu_e$ created far above the resonance density is predominantly in the upper eigenstate. Then if the neutrino propagation is adiabatic, the neutrino will remain in the upper eigenstate, and if $\theta$ is small it will be predominantly $\nu_\mu$ once it reaches the vacuum [87, 88] (see Fig. 6). For nonadiabatic propagation, if the probability of jumping from one eigenstate to another is $P_x$, then averaging over the oscillations [89]

$$
\langle P(\nu_e \rightarrow \nu_e)\rangle = \frac{1}{2} \left[ 1 + (1 - 2P_x) \cos 2\theta \cos 2\theta_0^m \right],
$$

(23)

where [90]

$$
P_x = \frac{\exp(-\frac{\pi}{2} \gamma F) - \exp(-\frac{\pi}{2} \gamma \frac{F}{\sin^2 \theta})}{1 - \exp(-\frac{\pi}{2} \gamma \frac{F}{\sin^2 \theta})},
$$

(24)

is the transition probability with $F = 1 - \tan^2 \theta$ for the exponentially varying matter density in the Sun, and [88, 89]

$$
\gamma = \frac{(\delta m^2)^2 \sin^2 2\theta}{4\sqrt{2}G_F E_\nu^2 |dN_e/dL|_e},
$$

(25)
measures the degree of adiabaticity of the transition. In Eq. (25), $|dN_e/dL|_c$ is the density gradient at the critical density. This process is analogous to level crossings in atoms [91]. For an electron neutrino that is created well above the resonance density ($\theta_m^0 \approx \pi/2$) and which undergoes a perfectly adiabatic transition ($\gamma \to \infty, P_x \to 0$), the oscillation probability is $P(\nu_e \to \nu_e) = \sin^2 \theta$. Thus a very large depletion of solar $\nu_e$’s is possible even for small vacuum mixing angles. This is known as the MSW effect, and was first studied numerically in Ref. [29]. In the extreme nonadiabatic limit ($\gamma \to 0$) [92], $P_x \to \cos^2 \theta$ and the oscillation probability approaches $1 - \frac{1}{2} \sin^2 2\theta$, the expected value for two-neutrino vacuum oscillations averaged over the oscillations.

The range over which $P(\nu_e \to \nu_e) < \frac{1}{2}$ is

$$\frac{\delta m^2 \cos 2\theta}{2\sqrt{2}G_F N_0^e} < E_\nu < \delta m^2 \sin 2\theta \sqrt{\frac{\pi}{8\sqrt{2}\ln 2G_F|dN_e/dL|_c}}. \quad (26)$$

For neutrino energies below this range, the initial density is below the critical density and the neutrino starts with a large fraction in the lower eigenstate. For neutrino energies above this range, the transition becomes very nonadiabatic and the neutrino has a high probability of hopping from the upper eigenstate to the lower eigenstate. In either case, a large component of the neutrino ends up in the lower eigenstate, in which case the survival probability is greater than $\frac{1}{2}$. For $N_0^e = 100N_A/cm^3$ (the approximate number density at the center of the Sun), neutrinos with energies in the range $2\text{ MeV} \lesssim E_\nu \lesssim 20\text{ GeV}$ will have survival probabilities smaller than $\frac{1}{2}$, assuming the best-fit oscillation parameters of the LMA solution ($\delta m_s^2 = 7 \times 10^{-5} \text{ eV}^2$.
and $\sin^2 2\theta_s = 0.83$).

Exact formulae that include cases when the neutrino is created near resonance are presented in Ref. [93]. More discussions of exact formulae for the transition probability are given in Ref. [94]. A semi-classical treatment for an arbitrary density profile is given in Ref. [95]. Formulae for the MSW effect in a three-neutrino model are presented in Ref. [96].

### 3.4 Long-baseline oscillations through the Earth

Oscillations of long-baseline neutrinos are affected by electrons in the Earth if the path length is an appreciable fraction of the Earth’s diameter. The full propagation equations for three neutrinos in matter are

$$i\frac{d\nu_{\alpha}}{dL} = \frac{1}{2E_{\nu}} \sum_{\beta} \left( A_{\alpha e} \delta_{\beta e} + \sum_i V_{\beta i}^* \delta m_i^2 V_{\alpha i} \right) \nu_{\beta}.$$  \hspace{1cm} (27)

A constant density approximation often provides a good representation of the neutrino propagation over long baselines through the Earth. Since $\delta m_s^2 \ll |\delta m_a^2|$ and $\theta_x$ is small, the probabilities can be expanded to second order in terms of the small parameters $\theta_x$ and $|\delta m_s^2/\delta m_a^2|$ [97, 98]. The following useful approximations for $\delta m_a^2 > 0$ are obtained [61]:

$$P(\nu_\mu \rightarrow \nu_e) = x^2 f^2 + 2xy fg (\cos \delta \cos \Delta - \sin \delta \sin \Delta) + y^2 g^2,$$  \hspace{1cm} (28)

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = x^2 \bar{f}^2 + 2xy \bar{f} g (\cos \delta \cos \Delta + \sin \delta \sin \Delta) + y^2 g^2,$$  \hspace{1cm} (29)

where

$$x = \sin \theta_a \sin 2\theta_x,$$  \hspace{1cm} (30)

$$y = \alpha \cos \theta_a \sin 2\theta_s,$$  \hspace{1cm} (31)

$$f, \bar{f} = \sin \left[ (1 \mp \hat{A}) \Delta \right] / (1 \mp \hat{A}),$$  \hspace{1cm} (32)

$$g = \sin(\hat{A} \Delta) / \hat{A},$$  \hspace{1cm} (33)

and

$$\Delta = |\Delta_a|, \quad \hat{A} = |A/\delta m_a^2|, \quad \alpha = |\delta m_s^2/\delta m_a^2|.$$  \hspace{1cm} (34)
For $\delta m_a^2 < 0$, the corresponding formulae are

\[ P(\nu_\mu \to \nu_e) = x^2 f^2 - 2xyf g(\cos \delta \cos \Delta + \sin \delta \sin \Delta) + y^2 g^2, \]  
\[ P(\bar{\nu}_\mu \to \bar{\nu}_e) = x^2 f^2 - 2xyf g(\cos \delta \cos \Delta - \sin \delta \sin \Delta) + y^2 g^2. \] 

(35)

(36)

Oscillation probabilities for an initial $\nu_e$ and final $\nu_\mu$ can be found by changing the sign of the $\sin \delta$ term in Eqs. (28, 29, 35), and (36). These expansions are nearly exact for distances less than 4000 km when $E_\nu \gtrsim 0.5$ GeV \cite{61}. For more accurate results at longer distances, Eq. (27) may be integrated numerically over the density profile of the neutrino path.

Other approximate solutions can also be useful in certain situations. At relatively short distances where the matter effect is not as large, an expansion can be made in $\alpha$ and $A/\delta m_a^2$ for the constant density solution \cite{99}. Relationships between the vacuum and matter oscillation parameters for three-neutrino oscillations are given in Ref. \cite{100}. Exact results for the three-neutrino case with constant density are given in Refs. \cite{28, 101}. Several properties of the general three neutrino solution with a nonconstant density profile are discussed in Ref. \cite{102}. Consequences of random density fluctuations are discussed in Ref. \cite{103}; they are not expected to play an important role in most situations.

The evolution in Eq. (27) is modified if a sterile neutrino $\nu_s$, is involved \cite{104}:

\[ A\delta_{\alpha e}\delta_{\beta e} \to 2\sqrt{2} G_F E_\nu \left[ N_e \delta_{\alpha e} \delta_{\beta e} - \frac{1}{2} N_n (\delta_{\alpha \beta} - \delta_{\alpha s} \delta_{\beta s}) \right], \]

(37)

where $N_n$ is the neutron number density. In two-neutrino oscillations between $\nu_e$ and a sterile neutrino, the electron number density $N_e$ is changed to an effective number density $N_{e, eff} = N_e - \frac{1}{2} N_n$, which changes the critical density for a resonance in oscillations of solar neutrinos. Also, in two-neutrino oscillations between a $\nu_\mu$ or $\nu_\tau$ and a sterile neutrino, $N_{e, eff} = -\frac{1}{2} N_n$; consequently there can be substantial matter effects in $\nu_\mu \to \nu_s$ oscillations of atmospheric neutrinos.

4 The solution to the solar neutrino problem

Decades of study of neutrinos from the Sun \cite{9, 20, 21, 22, 25, 26, 35, 36} have convincingly established that neutrino oscillations are the cause of the deficits of 1/3 to
1/2 in the measured electron-neutrino flux relative to the Standard Solar Model expectations [11]. The water Cherenkov experiments of SuperK and SNO measure the high energy neutrinos ($E \gtrsim 5$ MeV) from the $^8$B chain, the Chlorine experiment also detects the intermediate energy neutrinos from $^7$Be and $pep$, and the GALLEX, GNO and SAGE experiments have dominant contributions from the $pp$ neutrinos [10]; see Fig. 7 [11]. Until recently, the interpretation of the deficits depended on comparisons with SSM predictions of the flux. With the SNO experiment, which directly measures the total active neutrino flux via neutral currents, the evidence for flavor conversion becomes robust.

![Figure 7: The neutrino flux predictions of the Standard Solar Model [11]. From Ref. [54].](image)

The SNO experiment utilizes a heavy water target and measures the following processes [35, 36]:

\[
\text{Charged-Current (CC): } \nu_e + d \rightarrow e^- + p + p
\]
\[
\text{Neutral-Current (NC): } \nu_x + d \rightarrow \nu_x + n + p
\]
\[
\text{Elastic-Scattering (ES): } \nu_x + e^- \rightarrow \nu_x + e^-
\]

The CC/NC ratio establishes the oscillations of $\nu_e$ to $\nu_\mu$ and $\nu_\tau$ flavors,

\[
\frac{CC}{NC} = \frac{\text{flux}(\nu_e)}{\text{flux}(\nu_e + \nu_\mu + \nu_\tau)}.
\]
Only $\nu_e$ are produced in the Sun; the $\nu_\mu$ and $\nu_\tau$ fluxes are a consequence of oscillations. The charged-current signal was found to be suppressed by 7.6$\sigma$ from the neutral-current signal; see Fig. 8.

![Neutral Current (NC) Elastic Scattering (ES) Charged Current (CC)](image)

Figure 8: Evidence for neutrino flavor change seen by SNO. The open (filled) circles represent the 2003 SNO flux results, relative to the SSM, under the assumption of an undistorted (unconstrained) $^8$B neutrino energy spectrum.

The day and night energy spectra of charged-current events are potentially sensitive to matter effects on oscillations that occur when the neutrinos travel through the Earth [105], though a significant day-night asymmetry is yet to be established. SuperK (SNO) measured the day minus night $\nu_e$ rate to be $-2.1\% \pm 2.0\%^{+1.3\%}_{-1.2\%}$ ($-7.0\% \pm 4.9\%^{+1.2\%}_{-1.3\%}$) of the average rate [25, 26] ([35]); see Fig. 9. From a global fit to neutrino data, regions of the solar oscillation parameters have been determined, as shown in Fig. 10 [36]. The Large Mixing Angle (LMA) solution is preferred at more than 3$\sigma$. The best fit to the solar data is $\delta m^2_{\odot} = 6.5 \times 10^{-5}$ eV$^2$ and $\tan^2 \theta_s = 0.40$ [36]. The survival probability versus neutrino energy for LMA parameters is shown in Fig. 11.

The KamLAND experiment [37] measures the electron antineutrino flux at the Kamiokande site from surrounding reactors. The dominant reactor is at $L = 160$ km and the average distance from the sources is $L \sim 180$ km. The measured reaction is $\bar{\nu}_e + p \rightarrow e^+ + n$. If $CPT$ invariance holds, which is expected in quantum field theory, then $P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P(\nu_e \rightarrow \nu_e)$. Therefore, for the LMA solar solution, reactor antineutrinos should also disappear due to oscillations. For any other solar oscilla-
Figure 9: The SuperK day-night asymmetry as a function of $\delta m^2_s$, with the $1\sigma$ band shaded. The solid line is the prediction for $\tan^2 \theta_s = 0.55$, and the cross-hatched bands are the ranges of $\delta m^2_s$ allowed by KamLAND. From Ref. [26].

Figure 10: 90%, 95%, 99% and $3\sigma$ C.L. allowed regions from a fit to the Homestake, GALLEX+GNO and SAGE rates, and the SuperK and SNO spectra (with NC sensitivity enhanced by salt). The ellipse is a projection of the $3\sigma$ region from three years of KamLAND data assuming the best-fit LMA parameters ($\delta m^2_s = 6.5 \times 10^{-5}$ eV$^2$, $\tan^2 \theta_s = 0.40$). Adapted from Ref. [36] and Ref. [106].

In the oscillation solution, no disappearance would be observed at KamLAND. The KamLAND expectation [106, 108] for 3 years data, assuming the LMA oscillation parameters, is shown in Fig. 10 by the narrow ellipse superimposed on the LMA region from solar data. With sufficient data, the KamLAND experiment should “see” the oscillations in the positron energy spectra, as illustrated in Fig. 12 [106].
Figure 11: The flux-weighted survival probability for an LMA solution. Adapted from Ref. [107].

Figure 12: KamLAND is expected to detect the spectral distortion resulting from oscillations. Adapted from Ref [106].
The first KamLAND results [38] are based on 145 days of operation. The data give spectacular confirmation of the solar oscillation analysis predictions; see Fig. 13. The fractional number of events \( \frac{N(\text{observed}) - N(\text{bkg})}{N(\text{expected})} = 0.611 \pm 0.085(\text{stat}) \pm 0.041(\text{syst}) \) excludes no oscillations at 99.95% C. L. and eliminates all solar solutions but LMA [38]. A combined analysis of solar and KamLAND data select the LMA solution uniquely at the \( 4 - 5 \sigma \) C. L [110]. Maximal mixing is excluded at the \( 5.4 \sigma \) C. L. and \( \delta m^2_s < 10^{-4} \text{ eV}^2 \) at greater than the 99% C. L. [36]; see Fig. 14.

Figure 13: The ratio of the measured to expected \( \bar{\nu}_e \) flux from reactor experiments. The shaded region encompasses the fluxes predicted for oscillation parameters in the 95% C. L. region from an analysis of pre-SNO salt phase data [109]. The dotted curve corresponds to \( \delta m^2_s = 5.5 \times 10^{-5} \text{ eV}^2 \) and \( \tan^2 \theta_s = 0.42 \). From Ref. [38].

KamLAND has not only eliminated all oscillation solutions other than LMA, but has relegated nonoscillation solutions to the solar neutrino problem to be at most subleading effects. Non-standard neutrino interactions (NSNI) [111], which lead to energy-independent conversion probabilities, were consistent with the flat energy spectra seen by SuperK and SNO [112]. With KamLAND data, NSNI are generically rejected as the leading cause of \( \bar{\nu}_e \)-disappearance at about the 3\( \sigma \) C. L. Also, the resonant and nonresonant spin-flavor precession solutions [113] are allowed only at the 99.86% and 99.88% C. L., respectively [114]. Yet another excluded alternative invokes the violation of the equivalence principle to induce oscillations even for mass-
Figure 14: 90%, 95%, 99% and 3σ C. L. allowed regions from a combined fit to KamLAND and solar neutrino data. The best-fit point is at $\delta m^2_s = 7.1 \times 10^{-5}$ eV$^2$ and $\tan^2 \theta_s = 0.41$. From Ref. [36].

less neutrinos [115]. These three solutions fail because the KamLAND baseline is too short for any significant disappearance to occur.

The continuation of the KamLAND reactor experiment will provide a measurement of $\delta m^2_s$ that will be precise to about 10%. Data from solar neutrinos require that the sign of $\delta m^2_s$ is positive and that the mixing angle $\theta_s$ is nonmaximal. A more precise determination of $\theta_s$ is important to test models of neutrino mass. Future SNO data (from the $^3$He proportional counter phase) should reduce the presently allowed range of $\theta_s$ by measuring the CC/NC ratio and the day-night asymmetry more precisely. As far as the parameters responsible for the mixing of solar neutrinos are concerned, the goal of any new solar or reactor experiment should be to pin down $\theta_s$ more accurately than a combination of future SNO and KamLAND data. Borexino [116] or any other $^7$Be solar neutrino experiment will not improve on the accuracy with which three years of KamLAND data and solar data will determine $\theta_s$ [23]. A future $pp$ solar neutrino experiment with better than 3% precision can lead to significant improvement. Proposals for measuring $pp$ neutrinos include LENS [117], MOON [118], SIREN [119], XMASS [120], CLEAN [121], HERON [122] and GENIUS [123]. Another avenue for
an improved $\theta_s$ measurement is a lithium-based radiochemical detector that detects neutrinos from the CNO cycle [124]. A reactor neutrino experiment with baseline such that the measured survival probability is a minimum can lead to a more precise measurement of $\theta_s$ [125]. Since the reactor neutrino spectrum is accurately known and KamLAND will determine $\delta m^2_s$ precisely, such an experiment is conceivable. For $\delta m^2_s = 7 \times 10^{-5}$ eV$^2$, the required baseline is 70 km.

5 Atmospheric neutrinos

The first compelling evidence for neutrino oscillations came from the measurement of atmospheric neutrinos. Interactions of cosmic rays with the atmosphere produce pions and kaons that decay to muon neutrinos, electron neutrinos, and their antineutrinos:

$$\pi^+, K^+ \rightarrow \nu_\mu \mu^+ \rightarrow \nu_\mu e^+ \nu_e \bar{\nu}_\mu,$$  \hspace{1cm} (42)

$$\pi^-, K^- \rightarrow \bar{\nu}_\mu \mu^- \rightarrow \bar{\nu}_\mu e^- \nu_e \nu_\mu.$$  \hspace{1cm} (43)

On average there are twice as many muon neutrinos as electron neutrinos at energies of about 1 GeV, although the electron neutrinos tend to be at somewhat lower energies since they are produced only in a secondary decay. The atmospheric neutrino flux is well understood: the normalizations are known to 20% or better and ratios of fluxes are known to 5% [126]. The flux falls off rapidly with neutrino energy for $E_\nu \gtrsim 1$ GeV.

Neutrinos observed at different zenith angles have path distances that vary from $L \sim 10 - 30$ km for downward neutrinos to $L \sim 10^4$ km for upward neutrinos, as illustrated in Fig. 15. The ratio of observed to expected neutrino events provides a sensitive measure of neutrino oscillations, especially since different values of neutrino baselines and energies can be studied. Initial evidence for atmospheric neutrino oscillations was an overall depletion of muon neutrinos [12, 13] compared to the theoretical expectation. The SuperK experiment [15] has studied CC events in four categories: fully contained ($E_\nu \sim 1$ GeV), partially contained ($E_\nu \sim 10$ GeV), upward-going stopped ($E_\nu \sim 10$ GeV) and through-going ($E_\nu \sim 100$ GeV). The contained events have the highest statistics and give the more precise measurement, but all of the data samples are fully consistent with the same oscillation parameters. Zenith angle distributions for the $e$-like and $\mu$-like contained events are shown in Fig. 16 along with
Figure 15: A schematic view (not to scale) of the different zenith angles of atmospheric neutrinos and distances they travel before detection.

The no oscillation expectation and the best fit assuming oscillations. Atmospheric neutrinos are primarily sensitive to the leading oscillation which involves $\theta_a$ and $\theta_x$. The angle $\theta_x$ is constrained to be smaller than $13^\circ$ (for $\delta m_a^2 = 2.0 \times 10^{-3} \text{ eV}^2$) at the 95% C. L. Assuming $\theta_x = 0$ in Eqs. (9) and (10) (i.e., atmospheric muon neutrinos oscillate exclusively to tau neutrinos), the latest (preliminary) analysis by the SuperK collaboration of their data yields best-fit values $\sin^2 2\theta_a = 1.00$ (maximal mixing) and $\delta m_a^2 = 2.0 \times 10^{-3} \text{ eV}^2$ [127]. The 90% C. L. ranges for these oscillation parameters are $\sin^2 2\theta_a \gtrsim 0.90$ and $\delta m_a^2 \simeq (1.3 - 3.0) \times 10^{-3} \text{ eV}^2$. Both the Soudan-2 [128] and MACRO [129] experiments have also measured atmospheric neutrinos and find allowed regions consistent with the SuperK result (see Fig. 17).
Figure 16: Zenith angle distributions for $e$-like and $\mu$-like contained atmospheric neutrino events in SuperK [127]; $\cos \Theta = 1$ corresponds to downward events with $L \sim 15$ km and $\cos \Theta = -1$ corresponds to upward events with $L \sim 13000$ km. The lines show the best fits with and without oscillations; the best-fit is $\delta m^2_a = 2.0 \times 10^{-3}$ eV$^2$ and $\sin^2 2\theta_a = 1.00$.

Figure 17: 90% C. L. allowed regions for $\nu_\mu \rightarrow \nu_\tau$ oscillations of atmospheric neutrinos for Kamiokande, SuperK, Soudan-2 and MACRO. From Ref. [130].
If \( \theta_x \neq 0 \), then \( \nu_e \) would participate in the oscillations of atmospheric neutrinos with amplitude \( \sin^2 2\theta_x \). In the SuperK data the number of observed electron neutrinos is consistent with \( \theta_x = 0 \) (see Fig. 16), whereas an enhancement would be expected if there were \( \nu_\mu \leftrightarrow \nu_e \) oscillations (due to the 2:1 ratio of \( \nu_\mu \) to \( \nu_e \) in the flux). Furthermore, the zenith angle distribution of the SuperK muon sample is inconsistent with oscillations involving \( \nu_e \). Figure 18 shows the SuperK allowed region in \( \sin^2 \theta_x - \delta m^2_a \) plane. There is also slightly greater than 2\( \sigma \) evidence in the SuperK data of hadronic showers from \( \tau \) decays [130, 132], consistent with the hypothesis that the primary oscillation of atmospheric neutrinos is \( \nu_\mu \rightarrow \nu_\tau \).

Another consequence of \( \theta_x \neq 0 \) is that reactor \( \bar{\nu}_e \) fluxes should exhibit disappearance due to the leading oscillation when \( L/E_\nu \geq 40 \) m/MeV. Data from the CHOOZ reactor experiment [58] (\( L \sim 1000 \) m, \( E_\nu \sim 3 \) MeV) place upper limits on \( \theta_x \) for the values of \( \delta m^2_a \) indicated by the atmospheric neutrino data; similar limits have been obtained from the Palo Verde reactor experiment [59] (see Fig. 18). For the \( \delta m^2_a \) values obtained from the SuperK collaboration’s latest two-neutrino analysis (slightly lower than from previous analyses), bounds on \( \theta_x \) are quite sensitive to

Figure 18: Allowed regions in \( \sin^2 \theta_x - \delta m^2_a \) plane for three-neutrino oscillations from SuperK atmospheric data. From Ref. [131].
While for $\delta m^2_a = 2.0 \times 10^{-3}$ eV$^2$, the 95% C. L. upper bound on $\sin^2 2\theta_x$ is 0.2, for $\delta m^2_a = 1.3 \times 10^{-3}$ eV$^2$, the corresponding bound is 0.36. Thus, it has become necessary to specify the $\delta m^2_a$ for which a bound on $\theta_x$ is quoted.

The K2K experiment, in which $\nu_\mu$ with energies typically $\sim 1.4$ GeV are directed from KEK to SuperK ($L = 250$ km), has measured a $\nu_\mu$ survival probability consistent with the atmospheric neutrino results, $P(\nu_\mu \rightarrow \nu_\mu) = 0.70^{+0.11}_{-0.10}$ [16]. The K2K allowed region, from the number of events and the spectrum shape combined, is consistent with the allowed region from the atmospheric neutrino data (see Fig. 19). An approximate two-neutrino analysis that adopted the SuperK allowed regions in Fig. 19 in combination with K2K data found $\delta m^2_a = (2.0^{+0.8}_{-0.6}) \times 10^{-3}$ eV$^2$ at the 95% C. L. [133]. Imposing this $\delta m^2_a$ range as a prior in a three-neutrino analysis of CHOOZ, KamLAND and pre-SNO salt phase solar data yielded $\sin^2 2\theta_x \leq 0.17$ at the 95% C. L. after marginalizing over $\delta m^2_a$ [133].

Figure 19: Allowed regions in $\sin^2 \theta_a - \delta m^2_a$ plane from K2K, compared with the allowed regions from the SuperK atmospheric data [130].

The MINOS experiment [17] expects to detect atmospheric neutrinos via the $\nu_\mu$ and $\bar{\nu}_\mu$ charged-current reactions and to determine the signs of the resulting charged leptons with a magnetic field, thereby separately testing oscillations of $\nu_\mu \rightarrow \nu_\mu$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$.  

30
Since upward-going atmospheric neutrinos traverse a large fraction of the Earth’s diameter, matter effects could be relevant. The dominant oscillation of atmospheric neutrinos appears to be $\nu_\mu \rightarrow \nu_\tau$; for two-neutrino $\nu_\mu \rightarrow \nu_\tau$ oscillations there would be no matter effects. However, probability conservation in three-neutrino oscillations involves large matter effects which change the $\nu_\mu \rightarrow \nu_\tau$ oscillation probability to a small extent.

Specific relationships between the changes in the oscillation phase and the matter density can lead to an enhancement of the oscillation probability, analogous to the classical phenomenon of parametric resonance [134]. Also, constructive quantum mechanical interference between the probability amplitudes for different density layers can give total neutrino flavor conversion [135]. Such enhancement phenomena generally require passage of the neutrino through the Earth’s core. Conditions for observing matter effects in the Earth’s mantle and core in atmospheric neutrino experiments are discussed in Ref. [136].

6 Absolute neutrino mass

Neutrino oscillations tell us nothing about the absolute scale of neutrino masses. The standard technique for probing the absolute mass is to study the end-point region of the electron spectrum in tritium beta-decay. The effect of a nonzero neutrino mass is to suppress and cut off the electron distribution at the highest energies. The effective neutrino mass that could be determined in beta-decay is [137]

$$m^2_\beta = \sum |V_{ei}|^2 m^2_i.$$  \hspace{1cm} (44)

The present limit from the Troitsk [5] and Mainz [6] experiments is $m_\beta \leq 2.2$ eV at 2\sigma. Future sensitivity down to $m_\beta = 0.35$ eV is expected in the KATRIN experiment [138], which will begin in 2007.

The sum of neutrino masses $\Sigma \equiv \sum m_\nu$, can be probed in cosmology by measuring $\omega_\nu = \Sigma/(94 \text{ eV})$ using the large scale structure (LSS) of the universe [70]. Relativistic neutrinos do not cluster on scales smaller than they can travel in a Hubble time. Thus, the power spectrum is suppressed on scales smaller than the horizon when the neutrinos become nonrelativistic. (For eV neutrinos, this is the horizon at matter-radiation
equality). The effect is subtle. Lighter neutrinos freestream out of larger scales and cause the power spectrum suppression to begin at smaller wavenumbers [139]:

$$k_{nr} \approx 0.026 \left( \frac{m_\nu \omega_M}{1 \text{ eV}} \right)^{1/2} \text{Mpc}^{-1} ,$$

(45)

assuming almost degenerate neutrinos. On the other hand, heavier neutrinos constitute a larger fraction of the matter budget and suppress power on smaller scales more strongly than lighter neutrinos [70]:

$$\frac{\Delta P}{P} \approx -8 \frac{\Omega_\nu}{\Omega_M} \approx -0.8 \left( \frac{\Sigma}{1 \text{ eV}} \right) \left( \frac{0.1}{\omega_M} \right).$$

(46)

The galaxy power spectrum is influenced by the sum of neutrino masses, even down to 0.1 eV [70]. Analysis of the 2dF Galaxy Redshift Survey (2dFGRS) data found a limit of $\Sigma \leq 2.2$ eV on the masses of degenerate neutrinos, or about 0.7 eV for each neutrino [140]. An improved limit of $\Sigma \leq 0.71$ eV was obtained by the WMAP collaboration in an analysis of CMB data in conjunction with the 2dFGRS and the Lyman alpha forest power spectrum data [71]. Since there are questions about the treatment of the uncertainties [141] in the Lyman alpha forest data, the WMAP collaboration performed an analysis without this data and find the bound is strengthened to $\Sigma \leq 0.63$ eV [71]. With only CMB and 2dFGRS data, other analyses found the limit on the summed neutrino masses to be about 1 eV [142]. In connection with the above limits, it is interesting that an argument relying on anthropic selection concluded that $\Sigma \sim 1$ eV so that neutrinos cause a small but nonnegligible suppression of galaxy formation [143]. In the future, lensing measurements of galaxies and the CMB by large scale structure may also provide a sensitive probe of $\Sigma$ [144].

The $Z$-burst mechanism [145] provides another astrophysical probe of the absolute scale of neutrino masses [146]. The Greisen-Zatsepin-Kuzmin (GZK) cutoff energy ($\sim 5 \times 10^{19} \text{ eV}$) [147] expected in the cosmic ray spectrum is absent in data from the AGASA experiment [148], although the GZK cutoff may be respected by the HiRes data [149]. A possible explanation for ultra high energy cosmic ray events above the GZK cutoff is resonant annihilation of ultra high energy neutrinos on the cosmic neutrino background to produce $Z$ bosons which decay in a burst of about 20 photons and 2 super-GZK nucleons. The average energy of the secondary nucleon arriving at
The neutrino mass scale constrained by atmospheric data from below and by cosmology from above is suitable to initiate $\sim 10^{20}$ eV air-showers. Higher $m_\nu$ would lower $E$, thus precluding $Z$-bursts as an explanation of the super-GZK events. Lower $m_\nu$ would necessitate an unrealistically large neutrino flux at the resonant energy. The speculative assumption of the $Z$-burst mechanism is that a substantial cosmic neutrino flux exists at $\sim 10^{22}$ eV. If the existence of this flux is confirmed at teraton neutrino detectors [151], $Z$-bursts must occur.

All neutrino masses are linked to the lightest mass by the values of $\delta m^2_a$ and $\delta m^2_s$ determined by the neutrino oscillation studies [152]. If the scale of the lightest mass is small, then the heaviest mass is approximately $\sqrt{\delta m^2_a} \approx 0.05$ eV and a neutrino mass hierarchy exists. Since the sign of $\delta m^2_a$ is unknown, there are two possible hierarchies, as illustrated in Fig. 20. The mass hierarchy is an important discriminant of neutrino mass models. If the scale of the lightest mass is larger than 0.05 eV, then the neutrino masses are approximately degenerate.

\begin{align*}
\text{normal} & \quad m_3 \quad m_2 \quad m_1 \\
\delta m^2_a > 0 & \quad \delta m^2_a < 0 \\
\text{inverted} & \quad m_2 \quad m_1 \quad m_3
\end{align*}

Figure 20: The patterns of relative mass differences in normal (left) and inverted (right) neutrino mass hierarchies.

In the Standard Model with massive neutrinos and no other new physics, neutrinoless double-beta decay ($0\nu\beta\beta$) probes the absolute mass, provided that neutrinos are Majorana particles; see Fig. 21. Numerous theoretical analyses have been made of what can be learned about the neutrino sector from $0\nu\beta\beta$ [153, 154, 155, 156]. The decay rate depends on the $\nu_e - \nu_e$ element of the mass matrix [56]:

$$M_{ee} = \left| \sum V^2_{ei} m_i \right|. \quad (48)$$

33
The prediction is insensitive to $\theta_x$ and $\delta m_s^2$ because they are small. Setting $\theta_x = 0 = \delta m_s^2$, the following relation between $M_{ee}$ and $\Sigma$ is obtained for both hierarchies [154]:

$$M_{ee} = \left( 2\Sigma - \sqrt{\Sigma^2 + 3 \delta m_s^2 a} \right) \left| c_s^2 + s_s^2 e^{i\phi} \right| / 3,$$

where $\phi$ is a Majorana phase. For a given measured value of $M_{ee}$ both upper (since $\theta_s \neq \pi/4$) and lower bounds are implied for $\Sigma$. These bounds are displayed in Fig. 22. The present upper limit on $M_{ee}$ is 0.35 eV at the 90% C. L. [157], with an overall factor of 3 uncertainty associated with the $0\nu\beta\beta$ nuclear matrix elements [158]. A detection of neutrinoless double beta decay, corresponding to $M_{ee} = 0.39$ eV, has been reported [159], but this experimental result is highly controversial [160].
An extensive review of past and proposed $0\nu\beta\beta$ experiments has been made in Ref. [158]. Future experiments include CUORE (130Te) [161], EXO (136Xe) [162], XMASS (136Xe) [120], GENIUS (76Ge) [123], Majorana (76Ge) [163] and MOON (100Mo) [118]. The upcoming experiments are expected to have sensitivity better than 50 meV, which is the critical mass scale of $\sqrt{|\delta m^2|}$.

There has been speculation about detecting $CP$ violation using $0\nu\beta\beta$ [155]. However, it has been shown in Ref. [156] that this is impossible in the foreseeable future since there is no reliable method for estimating the uncertainty in the nuclear matrix elements. Moreover, the further the solar amplitude is constrained away from unity, the more stringent will be the precision requirement on the matrix elements for such a detection to be made even in principle [156]. Even under extremely optimistic assumptions, at best it may be possible to determine whether $\phi$ is closer to 0 or to $\pi$, corresponding to $CP$ conservation.

7 Supernova neutrinos

Stars with masses above eight solar masses undergo collapse. Once the core of the star becomes constituted primarily of iron, further compression of the core does not ignite nuclear fusion and the star is unable to thermodynamically support its outer envelope. As the surrounding matter falls inward under gravity, the temperature of the core rises and iron dissociates into $\alpha$ particles and nucleons. Electron capture on protons becomes heavily favored and electron neutrinos are produced as the core gets neutronized (a process known as neutronization). When the core reaches densities above $10^{12}$ g/cm$^3$, neutrinos become trapped (in the so-called neutrinosphere). The collapse continues until $3 - 4$ times nuclear density is reached, after which the inner core rebounds, sending a shock-wave across the outer core and into the mantle; see Fig. 23. This shock-wave loses energy as it heats the matter it traverses and incites further electron capture on the free protons left in the wake of the shock. During the few milliseconds in which the shock-wave travels from the inner core to the neutrinosphere, electron neutrinos are released in a pulse. This neutronization burst carries away approximately $10^{51}$ ergs of energy. However, 99% of the binding energy $E_b$, of
the protoneutron star is released in the following $\sim 10$ seconds primarily via $\beta$-decay (providing a source of electron antineutrinos), $\nu_e\bar{\nu}_e$ [165] and $e^+e^-$ annihilation and nucleon bremsstrahlung [166] (sources for all flavors of neutrinos including $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_\tau$ and $\bar{\nu}_\tau$), in addition to electron capture. The neutrinos following the neutronization burst are the ones of interest in the following discussion.

Figure 23: Schematic illustration of a supernova explosion. The dense Fe core collapses in a fraction of a second and gets neutronized (lower-left). The inner core rebounds and gives rise to a shock-wave (lower-right). The protoneutron star cools by the emission of neutrinos. From Ref. [164].

We focus on charged current $\nu_e$ and $\bar{\nu}_e$ interactions. We therefore cannot distinguish between the different nonelectron species, and denote them collectively as $\nu_x$ ($x = \mu, \tau, \bar{\mu}, \bar{\tau}$). The various cooling processes result in a state of approximate (within a factor of two or so) equipartition of energy with the luminosity of the electron neutrinos $L_{\nu_e}$ being up to 10% larger than the that of the electron antineutrinos.

5However, note that weak magnetism can cause the luminosities and temperatures of the $\nu_\mu,\tau$ and $\nu_e$ to differ by about 10% [167]. Practically speaking, this effect is too small to be detectable.
and 50 – 100% larger than that of $\nu_x$ [165]. The degree to which equipartition is violated can be quantified through constants $\beta_{\bar{\nu}_e}$ and $\beta_{\nu_x}$ which are defined by

$$L_{\nu_e} = \beta_{\bar{\nu}_e} L_{\bar{\nu}_e} = \beta_{\nu_x} L_{\nu_x},$$

(50)

where $1 \leq \beta_{\bar{\nu}_e} \lesssim 1.1$ and $1 \leq \beta_{\nu_x} \lesssim 2$. Perfect equipartition corresponds to $\beta_{\bar{\nu}_e} = \beta_{\nu_x} = 1$.

Since the protoneutron star is opaque to neutrinos, it takes a few tens of seconds for them to diffuse out. The $\nu_e$ and $\bar{\nu}_e$ interact with nuclear matter via both charged and neutral current reactions (with a smaller cross-section for $\bar{\nu}_e$), while the $\nu_x$ experience only neutral current scattering. Consequently, the different species have neutrinospheres such that their radii obey $R_{\nu_e} > R_{\bar{\nu}_e} > R_{\nu_x}$. Each neutrino species decouples at a temperature characterized by the temperature at the surface of its neutrinosphere. Based on this simple argument, which relies only on the well-known interaction strengths of neutrinos with matter, the hierarchy of average energies is

$$\langle E_{\nu_e} \rangle < \langle E_{\bar{\nu}_e} \rangle < \langle E_{\nu_x} \rangle.$$

(51)

Early supernova (SN) models typically predicted [168]

$$\langle E_{\nu_e} \rangle = 10 - 12 \text{ MeV},$$

$$\langle E_{\bar{\nu}_e} \rangle = 14 - 17 \text{ MeV},$$

$$\langle E_{\nu_x} \rangle = 24 - 27 \text{ MeV},$$

$$E_b = 1.5 - 4.5 \times 10^{53} \text{ ergs}.$$ (52)

The inclusion of additional energy transfer processes in modern SN codes indicates that the hierarchy of average energies is likely smaller than originally expected. There is evidence that nuclear recoils can lower $\langle E_{\nu_x} \rangle$ by as much as 20% [169]. Also, nucleon bremsstrahlung softened the spectra in the simulations of Ref. [170]. The cumulative effect is that $\langle E_{\nu_x} \rangle / \langle E_{\bar{\nu}_e} \rangle$ is typically expected to be 1.1, and unlikely to be greater than 1.2 [171].

The energy spectra of neutrinos can be modeled by pinched Fermi-Dirac distributions. The unoscillated differential flux (flux per unit energy) at a distance $D$ can be written as

$$F_{\alpha} = \frac{L_{\alpha}}{24\pi D^2 T_{\alpha}^4 |Li_{4}(-e^{\eta_{\alpha}})|} \frac{E^2}{e^{E/T_{\alpha}-\eta_{\alpha}} + 1},$$

(53)
where $\alpha = \nu_e, \bar{\nu}_e, \nu_x$, $Li_n(z)$ is the polylogarithm function and $\eta_\alpha$ is the degeneracy parameter. The temperature of the neutrinos, $T_\alpha$, is related to $\langle E_\alpha \rangle$ via $\langle E_\alpha \rangle = 3\frac{Li_4(-e^{\eta_\alpha})}{Li_3(-e^{\eta_\alpha})}T_\alpha$.

Because of the extremely high density of the matter in the neutrino production region of a SN, all flavors of neutrinos start out in pure mass eigenstates. As the neutrinos stream out from the production region, they pass through a density profile that is well-represented by $V_0(R_\odot/r)^3$ [172], where $R_\odot$ is the solar radius and $V_0$ is a constant. Due to the wider range of densities that the neutrinos encounter, both the solar and atmospheric scales contribute to the oscillation dynamics. The hierarchical nature of the two scales ($|\delta m^2_s| \ll |\delta m^2_a|$) and the smallness of the mixing parameter $\sin^2 2\theta_x$ imply that the dynamics can be approximately factored so that oscillations are governed by $\delta m^2_a$ and $\sin^2 2\theta_x$ at high densities ($10^3 - 10^4$ g/cm$^3$), and by $\delta m^2_s$ and $\sin^2 2\theta_s$ at low densities ($\sim 20$ g/cm$^3$ for the LMA solution) [173]; see Fig. 24. Transitions in the latter region are adiabatic. In the high density region, neutrinos (antineutrinos) pass through a resonance if $\delta m^2_a > 0$ ($\delta m^2_a < 0$). The jumping probability is the same for both neutrinos and antineutrinos [174] and is of the form $P_H \sim e^{-\sin^2 \theta_x (|\delta m^2_a|/E_\nu)^{2/3}V_0^{1/3}}$ [175]. Note the exponential dependence of $P_H$ on $\sin^2 \theta_x$. That SN neutrinos provide a handle on the sign of $\delta m^2_a$ can be seen as follows.

![Schematic level-crossing diagram for neutrinos emitted by a SN in the case of a normal mass hierarchy. $\nu_{\mu'}$ and $\nu_{\tau'}$ are basis states which diagonalize the $(\nu_{\mu}, \nu_{\tau})$ submatrix of the Hamiltonian governing the neutrino evolution.](image-url)
For the normal and inverted hierarchies, the survival probability of electron antineutrinos is given by [173]

$$\bar{p} = \bar{P}_{1e},$$

and

$$\bar{p} = P_H \bar{P}_{1e} + (1 - P_H) \sin^2 \theta_x,$$

respectively. Here, \( \bar{P}_{1e} = \bar{P}_{1e}(E_\nu, \delta m_s^2, \sin^2 2\theta_s) \) is the probability that an antineutrino reaching the Earth in the \( \nu_1 \) mass eigenstate will interact with the detector as a \( \bar{\nu}_e \). If \( \sin^2 2\theta_x \ll 10^{-3} \), then \( P_H \approx 1 \) and the survival probabilities for the two hierarchies are the same. Thus, the normal and inverted mass hierarchies are indistinguishable for \( \sin^2 2\theta_x \ll 10^{-3} \). If \( \sin^2 2\theta_x > \sim 10^{-3} \), for the inverted hierarchy \( \bar{p} \approx \sin^2 \theta_x \ll 0.05 \) and the original electron antineutrinos have all been swapped for the more energetic \( \mu \) and \( \tau \) antineutrinos by the time they exit the supernova envelope, resulting in a harder incident spectrum. Thus, the initial \( \bar{\nu}_e \) spectrum would have to be softer for the inverted hierarchy than for the normal hierarchy.

The detection of neutrinos from SN 1987A was momentous. The 11 events at Kamiokande II [176] and 8 events at the Irvine Michigan Brookhaven [177] detectors have lent strong support to the generic model of core collapse supernovae [178]. The significance of these few events provides a tantalizing glimpse into the physics potential offered by a future galactic SN event. Despite the fact that only a few galactic SN are expected per century, the potential payoff is so huge that experiments dedicated to SN neutrino detection have been proposed [66].

Attempts have been made to extract neutrino oscillation parameters from the 19 SN 1987A events (see Refs. [179, 180, 181] for recent analyses). However, conclusions drawn from these analyses depend crucially on the assumed neutrino temperatures and spectra. For example, it was claimed that the data favor the normal hierarchy over the inverted hierarchy provided \( \sin^2 \theta_x \gtrsim 10^{-4} \) [180], but this conclusion was contradicted [181]. Table 2 shows the results of two-parameter fits in \( E_b \), and \( T_{\bar{\nu}_e} \), for the normal hierarchy and the inverted hierarchy (with \( \sin^2 2\theta_x = 0.01 \)). In all cases the likelihoods are comparable, so the data do not favor one neutrino mass hierarchy over the other. While SN 1987A was of great astrophysical significance, it did not allow firm deductions about neutrino mixing.
Table 2: Best fit values for $E_b$ and $T_{\bar{\nu}_e}$ from two-parameter fits to all the KII and IMB data. Results are for the cases of no oscillations, the inverted hierarchy with $\sin^2 2\theta_x = 0.01$ and the normal hierarchy. From Ref. [181].

<table>
<thead>
<tr>
<th></th>
<th>$E_b$ (10$^{53}$ ergs)</th>
<th>$T_{\bar{\nu}_e}$ (MeV)</th>
<th>$\ln(L_{\text{max}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>no oscillations</td>
<td>3.2</td>
<td>3.6</td>
<td>-42.0</td>
</tr>
<tr>
<td>$\langle E_{\nu_x} \rangle / \langle E_{\bar{\nu}_e} \rangle = 1.25, \sin^2 2\theta_x = 0.01$</td>
<td>3.1</td>
<td>2.9</td>
<td>-42.0</td>
</tr>
<tr>
<td>$\langle E_{\nu_x} \rangle / \langle E_{\bar{\nu}_e} \rangle = 1.25, \text{ normal}$</td>
<td>3.2</td>
<td>3.4</td>
<td>-41.9</td>
</tr>
<tr>
<td>$\langle E_{\nu_x} \rangle / \langle E_{\bar{\nu}_e} \rangle = 1.4, \sin^2 2\theta_x = 0.01$</td>
<td>3.1</td>
<td>2.6</td>
<td>-42.0</td>
</tr>
<tr>
<td>$\langle E_{\nu_x} \rangle / \langle E_{\bar{\nu}_e} \rangle = 1.4, \text{ normal}$</td>
<td>3.4</td>
<td>3.2</td>
<td>-41.6</td>
</tr>
<tr>
<td>$\langle E_{\nu_x} \rangle / \langle E_{\bar{\nu}_e} \rangle = 1.7, \sin^2 2\theta_x = 0.01$</td>
<td>3.2</td>
<td>2.1</td>
<td>-42.0</td>
</tr>
<tr>
<td>$\langle E_{\nu_x} \rangle / \langle E_{\bar{\nu}_e} \rangle = 1.7, \text{ normal}$</td>
<td>4.2</td>
<td>2.7</td>
<td>-41.2</td>
</tr>
<tr>
<td>$\langle E_{\nu_x} \rangle / \langle E_{\bar{\nu}_e} \rangle = 2.0, \sin^2 2\theta_x = 0.01$</td>
<td>3.2</td>
<td>1.8</td>
<td>-42.0</td>
</tr>
<tr>
<td>$\langle E_{\nu_x} \rangle / \langle E_{\bar{\nu}_e} \rangle = 2.0, \text{ normal}$</td>
<td>5.8</td>
<td>2.2</td>
<td>-40.6</td>
</tr>
</tbody>
</table>

Neutrinos from a galactic SN incident on a large water or heavy water detector could in principle provide much information on neutrino oscillations. A determination of $\theta_x$ and the neutrino mass hierarchy from SN neutrinos is special in that degeneracies arising from the unknown $CP$ phase $\delta$ and whether $\theta_a$ is above or below $\pi/4$ do not contaminate it, i.e., the eight-fold parameter degeneracies that are inherent in long baseline experiments [61] are absent. This cleanness results because (i) nonelectron fluxes do not depend on the $CP$ phase $\delta$ [182], and so SN neutrinos directly probe $\theta_x$, and (ii) whether $\theta_a$ is above or below $\pi/4$ is immaterial since this parameter does not affect the oscillation dynamics.

Investigations of the effect of neutrino oscillations on SN neutrinos have been made in Refs. [173, 183, 184]. Whether or not the mass hierarchy can be determined and $\theta_x$ be constrained depends strongly on how much $\langle E_{\nu_x} \rangle / \langle E_{\bar{\nu}_e} \rangle$ is greater than unity [173]. The higher the value of $\langle E_{\nu_x} \rangle / \langle E_{\bar{\nu}_e} \rangle$, the better the possible determinations. As noted earlier, $\langle E_{\nu_x} \rangle / \langle E_{\bar{\nu}_e} \rangle$ is expected to be about 1.1, and no larger than 1.2 [171]. Then, assuming that the $\eta_\alpha$ values predicted by SN models are accurate, a safe deduction is that observations of a galactic SN at SuperK or Hyper-Kamiokande (HyperK)
could place either a lower or upper bound on $\theta_x$ if the neutrino mass hierarchy is inverted [184]. The hierarchy can be determined if it is inverted and $\sin^2 2\theta_x \gtrsim 10^{-3}$; for $\sin^2 2\theta_x \lesssim 10^{-4}$, the survival probabilities of the electron antineutrinos are similar for both hierarchies rendering them indistinguishable even in principle. On the other hand, if the hierarchy is normal, neither can $\theta_x$ be constrained nor can the hierarchy be determined [184]. Nonetheless, the importance of the detection of a galactic SN should not be understated because of its major impact on the understanding of the SN explosion mechanism.

Broadly speaking, the binding energy of the star and the temperatures of the different flavors of neutrinos and antineutrinos are determinable; see Fig. 25 [184].

Figure 25: Determination of the binding energy $E_b$, the supernova neutrino mean energies (temperatures) and $\sin^2 2\theta_x$ for the normal mass hierarchy. The left-hand and right-hand panels correspond to data simulated at $\sin^2 2\theta_x = 10^{-5}$ and $\sin^2 2\theta_x = 10^{-2}$, respectively. The cross-hairs mark the theoretical inputs, and the 90% and 99% C. L. regions are light and dark shadings, respectively. $\langle E_{\nu_e} \rangle$ is the mean energy of the nonelectron neutrinos. From Ref. [184].
Precise measurements of the temperatures can provide an unique window into the dominant microphysics of neutrino transport in addition to confirming the expectation that $T_{\bar{\nu}_e} < T_{\nu_x}$. With very large detectors it may even be possible to estimate how equipartitioned the energy is among the neutrino flavors. This knowledge would help refine SN codes that predict different degrees to which equipartitioning is violated. For example, in Ref. [185] an almost perfect equipartitioning is obtained while according to Refs. [165, 186], equipartitioning holds only to within a factor of 2.

Since it is difficult to determine $\theta_x$ and the mass hierarchy simultaneously from galactic SN data, an interesting prospect is to consider what can be learned from a future SN if $\sin^2 2\theta_x$ is already known to be larger than 0.01 from a future reactor or accelerator neutrino experiment. With this information, SuperK would easily discriminate between the two hierarchies if the hierarchy is inverted. If the hierarchy is normal, Earth-matter effects on the SN neutrino flux could be used to verify that this is the case, with either a high-statistics detector like the proposed half-megaton HyperK [60] or at a high-resolution scintillation detector [187].

8 Model building

8.1 Patterns of neutrino masses and mixings

One of the important challenges in particle physics is to understand the spectrum of fermion masses. The mixing matrix in the quark sector, $V_{CKM}$, is given by the product $V_u^\dagger V_d$, where $V_u$ and $V_d$ are the unitary transformations applied to the left-handed up and down quarks to diagonalize the up and down quark mass matrices. Similarly, the mixing matrix that enters into neutrino oscillations is $V_{MNS} = V_L^\dagger V_\nu$, where $V_L$ and $V_\nu$ are the unitary transformations applied to the left-handed charged leptons and neutrinos to diagonalize the charged lepton and neutrino mass matrices. In the quark sector, all mixing angles in $V_{CKM}$ are small and there is a mass hierarchy among the generations, whereas in the lepton sector (although not necessarily in the neutrino sector) a mass hierarchy exists with two large mixing angles and one small mixing angle in $V_{MNS}$. A remarkable property of neutrino masses is that they are so much lighter than the charged leptons. Any theory of fermion mass must
reconcile the extreme differences between quark and lepton masses and mixings. A plethora of papers has addressed the problem of neutrino mass over the years in the context of different models, which we are unable to exhaustively cover here; for recent comprehensive reviews and more references, see Refs. [73, 74, 75].

Since absolute neutrino masses are not yet known, there are three possible mass patterns for neutrinos: (i) normal hierarchy \( m_1 \ll m_2 \ll m_3 \), (ii) inverted hierarchy \( m_2 \gtrsim m_1 \gg m_3 \), and (iii) quasi-degenerate \( m_1 \simeq m_2 \simeq m_3 \). Because \( V_{MNS} \) is a product of the mixing matrices for the charged leptons and neutrinos, the observed mixing in neutrino oscillations can originate from \( V_L \), \( V_\nu \), or a combination of the two. Viable models exist with different combinations of mass pattern and origins of the mixing angles.

In models where the charged lepton mixing matrix is approximately diagonal, \( V_{MNS} \) derives directly from \( V_\nu \). If there are three Majorana neutrinos, then there are nine independent parameters in the mass matrix: three absolute masses and six mixing matrix parameters (see Eq. 3). Six of these may be measured in neutrino oscillations (three mixing angles, the Dirac phase, and two mass-squared differences). The absolute mass scale may be determined by measuring tritium beta-decay, \( 0\nu\beta\beta \) decay or the suppression of the matter power spectrum. The magnitude of the \( \nu_e - \nu_e \) mass matrix element (which depends on the three mixing angles, absolute masses, and two Majorana phases) may be determined from \( 0\nu\beta\beta \) experiments (although the value of the associated nuclear matrix elements makes this measurement less than precise). Therefore the complete \( 3 \times 3 \) mass matrix for Majorana neutrinos cannot be fully determined by experiment in the near future, and in practice neither of the Majorana phases will be well-measured. In models where the charged lepton mass matrix is nondiagonal, there are even more independent parameters. However, symmetry arguments can help to reduce the number of parameters.

Soon after the initial SuperK discovery that atmospheric neutrinos oscillate with maximal or nearly maximal amplitude, it was noted that the neutrino sector might exhibit bimaximal mixing [188], \( i.e., \) maximal or nearly maximal mixing of solar and atmospheric neutrinos. Then the mixing matrix has the unique form (up to state
redefinitions)

\[
V_{\text{MNS}} = \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
-\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}}
\end{pmatrix}.
\]

Perturbations on this basic form can yield mixing that is not quite maximal, and can make \( V_{e3} = s_x e^{-i\delta} \) nonzero. More recently, solar neutrino and KamLAND data disfavor maximal mixing in the solar sector (\( \sin^2 2\theta_s \leq 0.95 \) at the 3\( \sigma \) level), and the emphasis is now on finding models that can give bi-large mixing.

### 8.2 The seesaw mechanism

A popular model for understanding the smallness of neutrino masses is the seesaw mechanism [189], which is particularly well-motivated in a grand unified theory (GUT). In the one-generation version, the neutrino mass matrix in the \( \nu_L - \nu_R \) basis is

\[
M_\nu = \begin{pmatrix}
0 & m_D \\
\frac{1}{m_D} & m_R
\end{pmatrix},
\]

where \( m_D \) is a Dirac mass and \( m_R \) a right-handed Majorana mass. The eigenmasses are then approximately \(-m_D^2/m_R\) and \( m_R \) (the negative value of the lighter state can be made positive by a redefinition of the phases of the neutrino fields). If the heaviest light neutrino has mass of order \( \sqrt{\left| \delta m^2 \right|} \simeq 0.05 \) eV, and the Dirac mass is the \( \tau \) lepton mass then \( m_R \sim 10^{11} \) GeV. Other interesting possibilities for \( m_D \) are the electroweak vacuum expectation value or the top quark mass which would imply \( m_R = 10^{15} \) GeV (close to the GUT scale). Since heavy right-handed neutrinos exist in most grand unified models, a GUT/seesaw model is very attractive.

In practice, there are three generations of neutrinos and the neutrino mass matrix for the light neutrinos is \(-M_D^T M_R^{-1} M_D\), where \( M_D \) is the \( 3 \times 3 \) matrix describing the Dirac neutrino masses (presumably related by symmetries to the charged lepton or quark masses) and \( M_R \) is the \( 3 \times 3 \) matrix describing the right-handed Majorana neutrino masses. If \( M_D \) has a hierarchical form (similar to the charged lepton mass spectrum), then the neutrino mixing angles in \( V_{\text{MNS}} \) tend to be small unless there is an unnatural conspiracy between \( M_D \) and \( M_R \) [190, 191]. The choice of particular
forms for $M_D$ and $M_R$ can avoid this problem [191, 192]. A more detailed discussion of such models and relevant references can be found in Ref. [73].

### 8.3 GUT models

Since GUT models relate quarks and leptons, the fact that quark mixing angles are large and lepton mixing angles are small is potentially a problem for these models. In fact, many GUT models predicted small solar neutrino mixing, now ruled out by data from SNO and KamLAND. However, there is a way around this difficulty, due to the fact that GUT theories relate leptons and quarks of opposite chiralities, e.g., right-handed down quarks are in the same fermion multiplets as left-handed charged leptons, and vice versa. Therefore the mixing we observe in the lepton sector is connected to the right-handed quark mixing, which is unknown and not constrained. So-called lopsided models [193] take advantage of this fact. The three-generation Dirac mass matrices for the charged leptons and down quarks have the forms

$$
M_L \propto \begin{pmatrix}
  x & x & x \\
  x & 0 & \epsilon \\
  x & \sigma & 1
\end{pmatrix},
M_d \propto \begin{pmatrix}
  x & x & x \\
  x & 0 & \sigma \\
  x & \epsilon & 1
\end{pmatrix},
$$

where $\epsilon \ll \sigma \sim 1$. The entries involving only the second and third generations come from specific Higgs Yukawa interactions, with no contribution to the middle diagonal. The entries with an “$x$” involve the first generation and are very small, and in many (although not all) models are generated by Froggatt-Nielsen diagrams mediated by exotic vector-like matter fields [194]; see Fig. 26. The up-quark and neutrino mass matrices are approximately diagonal. Then the quark mixing element $V_{bu}$ is small, but the leptonic mixing element $V_{\mu 3}$, which relates to the mixing of atmospheric neutrinos, is large. In lopsided models, the large atmospheric neutrino mixing comes from the diagonalization of the charged lepton mass matrix; the solar neutrino mixing angle arises from the structure of the three-generation right-handed Majorana neutrino mass matrix, also determined in some models by Froggatt-Nielsen diagrams. Most lopsided models are embedded in a GUT [195], but some are not [196]. In many cases, horizontal (i.e., family) symmetries determine the textures of the mass
matrices. A Monte Carlo study suggests that lopsided textures are favored by the data [197]. In any GUT framework, proper comparison with data can only be made after allowing for the renormalization group running of the neutrino mass terms in the Lagrangian [198]; for recent discussions, see Ref. [199]. Of particular importance are how zeroes in the mass matrices behave under renormalization and the stability of the large mixing angles [200]. For a more complete discussion and further references on GUT models, see Refs. [73, 201].

![Froggatt-Nielsen diagrams](image)

Figure 26: Froggatt-Nielsen diagrams. Here \( a \) and \( b \) are family-indices and \((\chi, \bar{\chi})\) are vector-like fields of mass \( M \) and \( \langle \theta \rangle \) is the vacuum expectation value of the flavor Higgses or flavons. The tree-level diagram (a) generates the mass of the third family and the lighter masses are obtained by \( \mathcal{O}(\langle \theta \rangle/M)^n \) suppressions from diagrams (b) and (c). From Ref. [76].

Lopsided models can yield either large or small solar neutrino mixing [202]; for a discussion of which models can naturally yield the LMA solution, see Ref. [203]. The known SO(10) models that satisfy all experimental data favor a normal hierarchy [201], i.e., \( m_3 > m_1, m_2 \) in the neutrino sector, and therefore \( \delta m^2_\alpha > 0 \). Predictions for \( V_{e3} \) vary [73].

Another interesting possibility is to assume that the unification group is replicated at the Planck scale, i.e., there is one copy for each generation. This leads to family-dependent \( U(1) \) symmetries as the theory breaks down to become the Standard Model at low energies. Some consequences of such models are discussed in Ref. [204]. Models that utilize family symmetries can have either (i) both large mixing angles
in the neutrino sector deriving from the neutrino mass matrix [205], or (ii) the large atmospheric neutrino mixing angle deriving from the charged lepton mass matrix and the large solar neutrino mixing angle deriving from the neutrino mass matrix [206]. SUSY GUT models with \( R \)-parity violation are discussed in Ref. [207].

### 8.4 Non-GUT models

There are many alternatives to explicit GUT models (although it is often assumed that they could emerge from an unspecified grand unified theory). Some popular possibilities include: (i) the Zee model, (ii) models with low-energy new physics in which neutrino masses are generated as loops, such as supersymmetry with \( R \)-parity violation, (iii) so-called “democratic” models, (iv) models with triplet Higgs bosons, (v) models with specific textures for the neutrino mass matrix or special relationships involving the entries in the mass matrix, (vi) models with dynamical electroweak symmetry breaking, and (vii) models with large extra dimensions. In the non-GUT models, \( M_\nu \) is unrelated to the charged fermion mass matrices, and hence \textit{a priori} there are few constraints on its structure. In many cases, horizontal (family) symmetries can be used to provide constraints and produce a phenomenologically compelling model.

#### Zee model

In the Zee model [208], which invokes radiative neutrino masses via a charged \( SU(2)_L \) singlet and a Higgs field in the loop (see Fig. 27), the neutrino mass matrix has the approximate form

\[
M_\nu = \begin{pmatrix}
0 & A & B \\
A & 0 & 0 \\
B & 0 & 0
\end{pmatrix},
\]

where \( A \sim B \). In Zee-type models, the diagonal elements of \( M_\nu \) are zero, and the remaining off-diagonal elements may be nonzero (but small compared to \( A \) and \( B \)). Such a texture can also result from an approximate \( L_e - L_\mu - L_\tau \) symmetry [209]; for some other possibilities see Ref. [210]. The mass matrix in Eq. (59) yields large mixing for both solar and atmospheric neutrinos (although some specific models yield
vacuum solar neutrino oscillations that are now excluded). The mass hierarchy for the Zee-type mass matrix is inverted, \( i.e., m_3 \ll m_1, m_2 \) and \( \delta m_a^2 < 0 \). The prediction of the Zee model that the solar neutrino mixing is nearly maximal mixing is now excluded by data.

\[
\begin{align*}
\ & v_L \\
\ & l_R \\
\ & \phi \\
\ & h \\
\ & l_L \\
\ & v_L
\end{align*}
\]

Figure 27: Neutrino mass is generated at one loop in the Zee model.

**New physics at low energy**

In models with low-energy new physics, neutrinos couple to a heavy fermion in the theory. Mass terms for the light neutrinos are generated by loop diagrams involving the neutrino and the heavy fermion. If the heavy fermion coupling to the second and third generation neutrinos is larger than to the first generation, a normal mass hierarchy and large mixing for atmospheric neutrinos results. In the minimal supersymmetric extension of the Standard Model (MSSM) radiative neutrino mass generation is a direct consequence of R-parity violation [211] (see Fig. 28). For specific realizations to explain the neutrino anomalies see Ref. [212]. R-parity violating SUSY models that reproduce the neutrino mass and mixing parameters can have specific signatures in future collider experiments, such as lepton-number violating final states [213], neutralino decay within the detector [214, 215], neutralino decay branching ratios [215, 216], multi-\( b \)-jet events with an isolated charged lepton [217],

\[
\begin{align*}
\ & v_L \\
\ & b_L \\
\ & \lambda' \\
\ & \widetilde{q}_R \\
\ & \widetilde{q}_L \\
\ & v_L
\end{align*}
\]

Figure 28: The dominant one-loop diagram which generates Majorana neutrino masses for left-handed neutrinos in \( R \)-parity violating models. The coupling \( \lambda' \) violates lepton number as well as \( R \)-parity.
and multi-lepton events [218]. It is also possible that neutrino masses are generated at the two-loop level [219].

**Flavor democracy**

In models with flavor democracy [220], the quark and charged lepton mass matrices have the form

\[ M \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{60} \]

while the neutrino mass matrix is approximately diagonal. This scenario also leads to large mixing for solar and atmospheric neutrinos and a normal mass hierarchy. The democratic model (and many other non-seesaw models) predicts that the solar neutrino mixing angle is maximal and the atmospheric neutrino mixing angle is large but not necessarily maximal, whereas the data indicate the opposite.

**Triplet Higgs bosons**

In models with triplet Higgs bosons, horizontal symmetries are used to constrain the texture of the neutrino mass matrix [221]. An example that uses an \( S_2 \times S_2 \) permutation symmetry in a four-neutrino theory (with one neutrino becoming heavy) is given in Ref. [222].

**Textures or special relationships**

Since it unlikely that all nine parameters of the neutrino mass matrix can be determined, many studies have examined simpler structures with fewer independent parameters (with the assumption that the charged lepton mass matrix is diagonal). Some recent examples:

(i) It has been shown that \( 3 \times 3 \) Majorana mass matrices with three or more independent zero entries are excluded by current neutrino data but there are seven distinct textures with exactly two independent zeroes that are acceptable [223]. Note that since a Majorana mass matrix is symmetric, a reflected off-diagonal
zero is not counted as independent. Two of these textures lead to a normal mass hierarchy and the other five to a quasi-degenerate mass spectrum. In fact, it is possible to fully determine the neutrino mass spectra corresponding to these textures [224]. Several aspects of these seven matrices have been studied in Ref. [225]. Some of these textures can be realized in a seesaw model with or without extra $U(1)$ flavor symmetries [226]. These textures can also be obtained in models with three Higgs triplets and a sufficiently massive triplet Majoron [221].

(ii) The weak-basis independent condition $\det(M_\nu = 0)$ (which would be approximately true if the lightest neutrino was nearly massless) can also lead to a complete determination of the neutrino mass matrix [227].

(iii) Another possible condition that the neutrino mass matrix might obey is form invariance, $UMU^T = M$, where $U$ is a specific unitary matrix such that $U^N$ represents a well-defined discrete symmetry in the neutrino flavor basis [228]. This condition leads to a variety of possible mass matrices, including all three of the allowed mass patterns [229]. If the discrete symmetry is the non-Abelian group $A_4$, the mass pattern is quasi-degenerate [230].

(iv) If the sum of the neutrino masses is zero [231], which can occur in models whose neutrino mass matrix can be expressed as the commutator of two matrices, only the inverted hierarchy and quasi-degenerate mass patterns are allowed by current neutrino data [232].

If the charged lepton mass matrix is not assumed to be diagonal, then there are more independent parameters. A simplifying ansatz may be used to reduce the number of parameters. For example, the Fritzsch ansatz [233] assumes $M_{11} = M_{22} = M_{13} = M_{31} = 0$ for both the charged leptons and neutrinos, which can lead to acceptable phenomenology. The large mixing of atmospheric neutrinos can come from $V_L$ [234].

50
Dynamical electroweak symmetry breaking

The seesaw mechanism can be realized in models with dynamical electroweak symmetry breaking (extended technicolor, or ETC, models). By suppressing the Dirac mass terms $m_R$ in Eq. (57), the heavy Majorana scale need not be so high. (typically the $m_D$ terms can be much smaller than the ETC scale) [235]. Dynamical electroweak symmetry breaking due to a neutrino condensate has also been considered [236].

Extra dimensions

Theories with large extra dimensions [237] have been postulated to avoid the hierarchy problem. In such theories, there is no very large scale (e.g., the GUT or Planck scale), and so the smallness of the neutrino masses cannot be obtained from a conventional seesaw mechanism; instead, it is a consequence of the suppressed coupling between the active neutrinos on the brane (the usual four-dimensional world) and sterile neutrinos in the bulk (Kaluza-Klein modes) or on other branes, associated with the small overlap of their wavefunctions. According to the particular model and coupling mechanisms, the neutrino masses can be either Dirac or Majorana. Some examples of models of neutrino mass in extra dimensions are given in Refs. [238]. However, no evidence of Kaluza-Klein modes, whose effects are like those of sterile neutrinos, has been found in the oscillation data. For a more complete discussion of theories with large extra dimensions, see Ref. [75].

Alternatively, extra dimensions can be generated dynamically at low energies from a theory which is four-dimensional and renormalizable at high energies (a process called dimensional deconstruction [239]). Acceptable neutrino phenomenology appears to be possible in such a scenario [240].

Neutrino anarchy

Finally, even though it is aesthetically pleasing to think that symmetries in one form or another account for the structure seen in the neutrino masses and mixings, it could be that an essentially random three-neutrino mass matrix can give the appropriate phenomenology [241]. In models with neutrino anarchy, it was originally thought
that large mixing angles are quite natural, and the value of $\theta_x$ could lie just below the current experimental bound. However, a recent study suggests that large mixing angles are not preferred if the mass matrix elements are truly random in a basis-independent way [242]. Statistical analyses of nonrandom structures have also been performed [197, 243].

9 Leptogenesis

The range of neutrino masses that the data suggest lends credibility to the seesaw mechanism. A direct consequence of the seesaw mechanism is leptogenesis [244]. This process generates a net lepton asymmetry $Y_L \equiv (n_L - n_{\bar{L}})/s$ because all of Sakharov’s conditions [245] are met: (i) the heavy right-handed neutrinos $N_i$ decay into a lepton-Higgs pair ($lH$) and into the $CP$ conjugate pair with different partial widths, thereby violating lepton number; (ii) $CP$ violation results from phases in the Yukawa couplings and neutrino mass matrices; (iii) the cosmological expansion yields the departure from thermal equilibrium.

As the universe cools and the $N_i$ drop out of equilibrium, their decays lead to a $CP$ asymmetry [246],

$$\epsilon_i = \frac{\Gamma(N_i \rightarrow lH) - \Gamma(N_i \rightarrow \bar{l}H^*)}{\Gamma(N_i \rightarrow lH) + \Gamma(N_i \rightarrow \bar{l}H^*)}. \quad (61)$$

The lepton asymmetry generated will be $Y_L \sim \sum \epsilon_i/g_\ast$, where $g_\ast$ is the number of degrees of freedom.

Since sphaleron [247] interactions preserve $B - L$ but violate $B$ and $L$ [248], the lepton asymmetry is partially converted to a baryon asymmetry $Y_B$. In terms of the initial $B - L$ [249],

$$Y_B = aY_{B-L} = \frac{a}{a-1}Y_L, \quad (62)$$

where $a$ depends on the processes in equilibrium. In the seesaw extended SM (MSSM), $a = 28/79$ ($a = 8/23$). Note that Eq. (62) is valid only for temperatures far above the weak scale (for a review see Ref. [250]).

An interesting aspect of leptogenesis is that it requires the low energy neutrino masses to be sufficiently light that the baryon asymmetry is not washed out by
neutrino-mediated $L$-violating scatterings [251]$^6$. This has been further explored in Refs. [253, 254]. Reference [254] finds that for the mass-squared differences relevant to solar and atmospheric neutrino oscillations, leptogenesis is the unique source of the baryon asymmetry provided the lightest of the heavy neutrinos is $O(10^5)$ times lighter than the other heavy neutrinos.

Although a very appealing idea, leptogenesis is difficult to test [255]. The $\epsilon_i$ can be expressed independently of the neutrino mixing matrix $V$ of Eq. (3) [256]. Any connection between the $CP$ phase $\delta$ and the $CP$ violation required for leptogenesis requires assumption about the texture of the Yukawa matrix, and is therefore model-dependent [257].

The only way to test leptogenesis directly is to constrain the Yukawa matrix and the masses of right-handed neutrinos. This can be achieved by searching for lepton flavor violating decays and the electric dipole moments of the charged leptons to which orders of magnitude improved sensitivity are expected in the near future [258]: $\mu \rightarrow e \gamma$, $\tau \rightarrow \mu \gamma$, $\tau \rightarrow e \gamma$ and $\mu \rightarrow 3e$. Even so, any determination of the Yukawa matrix will depend on how precisely the Higgs sector is known and on assumptions about the GUT model.

10 Future long-baseline experiments

A summary of present knowledge of neutrino parameters is given in Table 3, along with the near future projects that will improve this knowledge. The mixing angle $\theta_x$, the Dirac $CP$ phase $\delta$, and the sign of $\delta m^2_\delta$ are as yet undetermined; their measurement will be the main goal of future long-baseline neutrino experiments. In this section, we discuss the the next-generation long-baseline experiments that are being considered after MINOS, ICARUS, and OPERA.

$^6$If leptons couple to a heavy SU(2)$_L$ triplet in addition to $N_i$, neutrino masses do not necessarily induce asymmetry washout effects and an upper bound on the neutrino masses cannot be placed [252].
Table 3: Present knowledge of neutrino parameters and future ways of improving this knowledge.

<table>
<thead>
<tr>
<th>3-neutrino observables</th>
<th>Present knowledge (≈ 95% C. L.)</th>
<th>Near future</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_a$</td>
<td>$45^\circ \pm 10^\circ$</td>
<td>$P(\nu_\mu \to \nu_\mu)$ MINOS, CNGS</td>
</tr>
<tr>
<td>$\theta_s$</td>
<td>$32.5^\circ \pm 3.6^\circ$</td>
<td>SNO NC, KamLAND</td>
</tr>
<tr>
<td>$\theta_x$</td>
<td>$\leq 13^\circ$ (for $</td>
<td>\delta m^2_a</td>
</tr>
<tr>
<td>$</td>
<td>\delta m^2_a</td>
<td>$</td>
</tr>
<tr>
<td>$\text{sgn}(\delta m^2_a)$</td>
<td>unknown</td>
<td>$P(\nu_\mu \to \nu_\mu), P(\bar{\nu}_\mu \to \bar{\nu}_e)$ LBL</td>
</tr>
<tr>
<td>$</td>
<td>\delta m^2_s</td>
<td>$</td>
</tr>
<tr>
<td>$\text{sgn}(\delta m^2_s)$</td>
<td>+ (MSW)</td>
<td>done</td>
</tr>
<tr>
<td>$\delta$</td>
<td>unknown</td>
<td>$P(\nu_\mu \to \nu_\mu), P(\bar{\nu}_\mu \to \bar{\nu}_e)$ LBL</td>
</tr>
<tr>
<td>Majorana</td>
<td>unknown</td>
<td>$0\nu\beta\beta$</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>unknown</td>
<td>$0\nu\beta\beta$ (if $\simeq 0, \pi$)</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>unknown</td>
<td>hopeless</td>
</tr>
<tr>
<td>$m_\nu$</td>
<td>$\sum m_\nu &lt; 1$ eV</td>
<td>LSS, $0\nu\beta\beta, \beta$-decay</td>
</tr>
</tbody>
</table>

10.1 Conventional neutrino beams and superbeams

The near term agenda is to confirm atmospheric neutrino oscillations in accelerator experiments and improve the accuracy with which $|\delta m^2_a|$ and $\sin^2 2\theta_a$ are determined. Experiments that measure $\nu_\mu$ disappearance will establish the first oscillation minimum in $P(\nu_\mu \to \nu_\mu)$. The K2K experiment from KEK to SuperK [16], a distance of $L = 250$ km, has begun taking data again following the restoration of the SuperK detector. The MINOS experiment from Fermilab to the Soudan mine [17], at a distance of $L = 730$ km, will begin in 2005. It is expected to obtain 10% precision on $|\delta m^2_a|$ and $\sin^2 2\theta_a$ in 3 years running. The CERN to Gran Sasso (CNGS) experiments, ICARUS [18] and OPERA [19], also at a distance $L = 730$ km but with higher neutrino energy, are expected to begin in 2007. The appearance of $\nu_\tau$ should be observed in the CNGS experiments, which would confirm that the primary oscillation of atmospheric neutrinos is $\nu_\mu \to \nu_\tau$.

The three parameters that are not determined by solar and atmospheric neutrino
experiments are $\theta_x$, $\text{sgn}(\delta m_a^2)$, which fixes the hierarchy of neutrino masses, and the $CP$-violating phase $\delta$. The appearance of $\nu_e$ in $\nu_\mu \rightarrow \nu_e$ oscillations is the most critical measurement, since the probability is proportional to $\sin^2 2\theta_x$ in the leading oscillation, for which there is currently only an upper bound ($0.2$ for $\delta m_a^2 = 2.0 \times 10^{-3}$ eV$^2$ at the 95% C. L., from the CHOOZ reactor experiment [58]). By combining ICARUS/MINOS/OPERA data, it may be possible to establish whether $\sin^2 2\theta_x > 0.01$ at 95% C. L. [259].

The study of $\nu_\mu \rightarrow \nu_e$ oscillations also allows one to test for $CP$ violation in the lepton sector [53]. Intrinsic $CP$ violation in the Standard Model requires both $\delta \neq 0$ or $\pi$ and $\theta_x \neq 0$. In vacuum, the $CP$ asymmetry in the $\nu_\mu \rightarrow \nu_e$ channel, to leading order in the $\delta m^2$’s, is

$$
\frac{P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)}{P(\nu_\mu \rightarrow \nu_e) + P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)} \approx \left( \frac{\sin 2\theta_s \sin 2\theta_a}{2 \sin^2 \theta_a} \right) \left( \frac{\sin 2\Delta_s}{\sin 2\theta_x} \right) \sin \delta.
$$

For large-angle solar and atmospheric neutrino mixing the first factor on the right-hand side of Eq. (63) is of order unity. The existence of $CP$ violation requires that the contribution of the sub-leading scale, $\Delta_s$, is nonnegligible, so large $L/E_\nu$ values are essential. In practice, the $CP$ conserving and $CP$ violating contributions may have similar size [260], depending on the values of $L/E_\nu$ and $\theta_x$. Furthermore, Earth-matter effects can induce fake $CP$ violation, which must be folded into any deduction of $\delta$ (on the other hand, matter effects are essential in determining the sign of $\delta m_a^2$).

The standard proposed method for measuring $CP$ violation is to compare event rates in two charge conjugate oscillations channels, such as $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$. However, there are three two-fold parameter degeneracies that are present when two such measurements are made, which may result in an overall eight-fold degeneracy (a parameter degeneracy occurs when two or more parameter sets are consistent with the same data):

(i) The $(\delta, \theta_x)$ ambiguity [61, 97, 261, 262, 263], in which two different parameter pairs, $(\delta, \theta_x)$ and $(\delta', \theta'_x)$, lead to the same values for $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$.

(ii) The $\text{sgn}(\delta m_a^2)$ ambiguity [61, 261, 264, 265], where $(\delta, \theta_x)$ for one sign of $\delta m_a^2$ gives the same values for the oscillation probabilities as $(\delta', \theta'_x)$ with the opposite sign of $\delta m_a^2$. 

55
(iii) The \((\theta_a, \pi/2 - \theta_a)\) ambiguity \([61, 266]\), where \((\theta_a, \delta, \theta_x)\) gives the same values for the oscillation probabilities as \((\pi/2 - \theta_a, \delta', \theta'_x)\). This ambiguity exists because the channel used to determine \(\theta_a\), \(\nu_\mu\) survival, only measures \(\sin^2 2\theta_a\). The ambiguity vanishes at the experimentally preferred value for \(\theta_a\) (= \(\pi/4\)).

For each of these degeneracies, a duplicity in inferred values of \(\delta\) and \(\theta_x\) is possible; thus each of these degeneracies can confuse CP-violating parameter sets with CP-conserving ones, and vice versa. In many cases these degeneracies persist for all experimentally allowed values of \(\theta_x\). An overview of these parameter degeneracies can be found in Refs. \([61, 267]\).

There are two “magic” baselines that are valuable to resolve some of these parameter degeneracies:

(i) The detector is located at a distance that corresponds to the first peak of the leading oscillation \((\Delta_a = \pi/2)\):

\[
L \simeq 620 \text{ km} \left(\frac{E}{1 \text{ GeV}}\right) \left(\frac{2.0 \times 10^{-3} \text{ eV}^2}{\delta m^2_a}\right) .
\]

Then the \(\nu_\mu \rightarrow \nu_e\) probability depends only on \(\sin \delta\) and not \(\cos \delta\) (see Eq. 28), the \((\delta, \theta_x)\) degeneracy is broken, and \(\theta_x\) is uniquely determined for a given \(\text{sgn}(\delta m^2_a)\) and \(\theta_a\) \([61, 261]\). There is a residual \((\delta, \pi - \delta)\) degeneracy, but this degeneracy does not mix CP violating and CP conserving solutions. Figure 29 shows the remaining degeneracies when \(L/E_\nu\) is chosen to be at the first peak of the oscillation. Furthermore, if \(L\) is taken to be very long, large matter effects will break the \(\text{sgn}(\delta m^2_a)\) ambiguity. The minimum distance needed depends on the size of \(\theta_x\) and \(\delta m^2_s\), but generally \(L \geq 1000 \text{ km}\) is required.

(ii) The detector is located at a distance such that \(\hat{A}\Delta_a = G_F N_e L/\sqrt{2} \simeq \pi\), which for the Earth’s density profile implies \(L \simeq 7600 \text{ km}\). Then only the leading oscillation term survives in Eqs. \(28, 29, 35\), and \(36\), and the oscillation probabilities for \(\nu_e\) appearance are independent of \(\delta\) and \(\delta m^2_s\) \([61]\). This allows an unambiguous measurement of \(\theta_x\) \([268, 269]\) (modulo the \((\theta_a, \pi/2 - \theta_a)\) ambiguity).
For each of these magic baselines, additional measurements at different $L$ and/or $E_{\nu}$ values would be necessary to break the remaining degeneracies and determine the precise values of $\delta$ and $\theta_a$.

Figure 29: Remaining degeneracies when $\Delta_a = \frac{\pi}{2}$ for (a) the $(\delta, \theta_x)$ ambiguity, (b) the $\text{sgn}(\delta m_a^2)$ ambiguity, and (c) the $(\theta_a, \frac{\pi}{2} - \theta_a)$ ambiguity. In (a), each value of $\sin^2 2\theta_x$ describes a distinct line in probability space. In (b), the ambiguity in $\sin^2 2\theta_x$ is small, but in the overlap region there is still an ambiguity in $\text{sgn}(\delta m_a^2)$ and a corresponding large uncertainty in $\delta$. In (c), the ambiguity in $\delta$ is small, but there may be a large uncertainty in $\sin^2 2\theta_x$ when $\theta_a \neq \frac{\pi}{4}$. In all cases there remains a $(\delta, \pi - \delta)$ ambiguity since only $\sin \delta$ is being measured. Adapted from Ref. [61].
Precision measurements of $\delta$ and $\theta_x$ can be made at future off-axis [62] neutrino beam experiments proposed for the Main Injector at Fermilab (NuMI) [63, 270, 271, 272] and the Japan Hadron Facility (JHF), also called the Japan Proton Accelerator Research Complex (J-PARC) [60]. Compared to on-axis beams, off-axis beams have a much narrower energy spectrum, smaller beam contamination, and a suppression of the high-energy tail that results in lower backgrounds to $\nu_e$ events in the detector. They also are well-suited for multiple detectors (a detector cluster [273]) that allows more than one measurement to be made simultaneously.

Strategies for the future include superbeams [65, 274], with upgrades of the neutrino flux by a factor of 4 for SuperNuMI (S NutzungMI) [275], a factor of 5 for SuperJHF (SJHF) [60], and a factor of 5 for a low-energy option for CNGS [276], which will allow smaller values of $\theta_x$ to be probed. A high-intensity neutrino beam at Brookhaven is also being considered [277]. Many different superbeam scenarios have been considered:

(i) Having detectors at different off-axis angles allows one to modify both the baseline and neutrino energy. Multiple detectors utilizing an off-axis beam such as the one at Fermilab can shift degeneracies to $\sin^2 2\theta_x \leq 0.01 - 0.001$ [273]. Alternatively, having detectors at two different on-axis distances from the same superbeam can provide multiple measurements that help to remove parameter degeneracies. One such possibility is to combine a moderate distance such as JHF to SuperK ($L = 295\text{ km}$), with a much longer distance such as JHF to a detector near Beijing ($L \simeq 2100\text{ km}$) [278]. One specific proposal is for a SJHF to HyperK experiment to run 2 years with neutrinos and 5 years with antineutrinos from, and then run with 5 years with neutrinos from SJHF to a very large Water Cherenkov detector near Beijing. This very ambitious scenario would require a separate beamline for the 2100 km measurement, but could resolve the $(\delta, \theta_x)$ ambiguity down to $\sin^2 2\theta_x = 0.005$ [279]. Finally, one can vary the beam energy at the same baseline [273, 280].

(ii) Two superbeam experiments do significantly better than one in parameter determinations [281, 282]. For example, the combination of data from a SJHF to SuperK experiment ($\theta_{\text{off-axis}} = 2\text{ deg}$, $E_\nu \simeq 0.6\text{ GeV}$, $L = 295\text{ km}$, 22.5 kt
water Cherenkov) and a SNuMI to southwestern Ontario experiment (1 deg, $E_\nu = 1.8 \text{ GeV}$, $L \approx 900 \text{ km}$, 20 kt low-Z calorimeter), both with 2 years $\nu_\mu$ running and 6 years $\bar{\nu}_\mu$ running, would be sensitive to the sign of $\delta m^2_a$ and to $CP$ violation for $\sin^2 2\theta_x \gtrsim 0.03$ [281]. Running with only $\nu_\mu$ at SNuMI and SJHF may allow one to determine $\text{sgn}(\delta m^2_a)$ if $\theta_x$ is not too small [283], although $\theta_x$ and $\delta$ will not be well-measured without $\bar{\nu}_\mu$ data.

(iii) Another approach is to use a wide-band superbeam, for example from Brookhaven to a National Underground Science Laboratory [277]. The measurement of quasi-elastic events allows a determination of the neutrino energy with reasonable precision. The lower-energy events are more sensitive to the $\delta$ terms in the oscillation probability, while the higher-energy events are more sensitive to the sign of $\delta m^2_a$. Binning the quasi-elastic events is roughly equivalent to running many narrow-band beams simultaneously, which can help to resolve neutrino parameter degeneracies. In principle only a neutrino beam is required. However, running with an antineutrino beam would provide essential confirmation (especially of $CP$ violation) and probe lower in $\theta_x$ (if $\delta m^2_a < 0$).

In any long-baseline neutrino experiment there is a trade-off between detector size and the ratio of the $\nu_e$ CC signal events to background: generally speaking, detector technologies that allow a bigger reduction in background cannot be built as large [65]. Four types of detectors that have been studied for $\nu_\mu \rightarrow \nu_e$ detection are (i) water Cherenkov (backgrounds of order $10^{-2}$ of the number of unoscillated CC events, maximum fiducial volume of order 500 kt) [60, 66], (ii) iron scintillator ($3 \times 10^{-3}$, 50 kt) [271, 284, 285], (iii) liquid argon ($3 \times 10^{-3}$, 50 kt) [67] and (iv) low-Z calorimetric [272]. There is also an additional background of order $3 \times 10^{-3}$ due to $\nu_e$ contamination in the beam. The larger water Cherenkov detectors generally do better when the neutrino flux is less (such as for a conventional beam before superbeam upgrade or for baselines $\geq 4000 \text{ km}$ where there is a large $1/R^2$ fall-off of the flux), whereas the smaller detectors that can measure $e^\pm$ positions on a finer scale generally do better when there is more flux (such as with superbeams or for baselines below 4000 km).
Another option being considered is an intense $\nu_e$ beam (with negligible contamination) from radioactive ion decay leading to a so-called beta-beam [286] in a proposed experiment from CERN to Frejus with a baseline of 130 km.

10.2 Future reactor experiments

A different approach to determining $\theta_x$ without the complication of parameter degeneracies is to measure $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ at a reactor experiment using two larger versions of the CHOOZ detector, one near and one more distant ($\lesssim 10$ km) from the reactor [287, 288, 289]. Table 4 lists the detector distances for three proposals. Ignoring terms cubic or higher in the small parameters $\theta_x$ and $\Delta_s$, the oscillation probability is given approximately by

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta_x \sin^2 \Delta_a - c_x^4 \sin^2 2\theta_s \sin^2 \Delta_s.$$ (65)

It is independent of $\delta$, $\theta_a$, and $\text{sgn}(\delta m^2_a)$. Discovery down to $\sin^2 2\theta_x = 0.01$ at 90% C. L. may be possible [287, 288, 289, 290] for a reactor experiment with exposure of about 400 t-GW-yr, where the tonnage refers to the detector size and GW to the reactor power; an actual measurement of $\sin^2 2\theta_x$ is possible for $\sin^2 2\theta_x \simeq 0.05$. A comparison of the sensitivity to $\sin^2 2\theta_x$ in superbeam and reactor experiments is shown in Fig. 30. When a reactor measurement is combined with results from long-baseline experiments, it may also be possible to determine $\delta$ and $\theta_a$.

Table 4: Proposed reactor neutrino experiments for measuring $\theta_x$ using two detectors.

<table>
<thead>
<tr>
<th>Site</th>
<th>$L_1$ (km)</th>
<th>$L_2$ (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Krasnoyarsk [287]</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>Kashiwazaki [288]</td>
<td>0.3</td>
<td>1.7</td>
</tr>
<tr>
<td>Diablo Canyon [289]</td>
<td>0.15</td>
<td>1.2</td>
</tr>
</tbody>
</table>

10.3 Neutrino factories

A neutrino factory (NuFact) [68] is the ultimate technology for neutrino oscillation studies. Muons would be stored in a flat oval ring. Their decays will give neutrino
Figure 30: Sensitivity to $\sin^2 2\theta_x$ at 90% C.L. in reactor experiments with 400 t-GW-yr (Reactor-I) and 8000 t-GW-yr (Reactor-II), and for a JHF-SuperK experiment assuming $\delta m^2_s = 7 \times 10^{-5}$ eV$^2$ (LMA-I) and $\delta m^2_s = 1.4 \times 10^{-4}$ eV$^2$ (LMA-II). The left edges of the bars show the sensitivities assuming statistical uncertainties only, while the right edges of the bars show the sensitivities as the uncertainties from systematics, parameter correlations, and parameter degeneracies are progressively included. From Ref. [290].

beams in the directions of the straight sections of the storage ring. Stored muons of energies 20 GeV and above are needed; energies as high as 50 GeV have been considered in design studies. A decaying $\mu^+$ in the ring yields both $\bar{\nu}_\mu$ and $\nu_e$; detection of the charge of the final state lepton in a charge-current event allows one to determine the initial neutrino flavor. Table 5 shows the six oscillation channels possible with stored $\mu^+$; the six charge-conjugate channels can be tested using $\mu^-$ decays.

The neutrino spectrum from a NuFact ranges from zero to the stored muon energy, $E_\mu$, with a broad peak at $0.7 E_\mu$ ($0.6 E_\mu$) for muon (electron) neutrinos when the muons are not polarized. The neutrino flux in the forward direction is approximately $n_0 \gamma^2/(\pi L^2)$, where $n_0$ is the number of decaying muons in the straight section of the ring, $\gamma = E_\mu/m_\mu$, and $L$ is the baseline. An entry-level NuFact produces a time integrated $n_0 \sim 10^{20}$ and a high-performance NuFact has $n_0 \sim 10^{21}$. Early studies of the capabilities of a NuFact can be found in Refs. [69, 291]. See also more recent study group reports in Ref. [292] and the reviews in Ref. [293].
Table 5: Signals for oscillation channels assuming a decaying $\mu^-$ in a neutrino factory.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Detect</th>
<th>Nomenclature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_\mu \to \nu_\mu$</td>
<td>$\mu^-$</td>
<td>right–sign $\mu$ survival</td>
</tr>
<tr>
<td>$\nu_\mu \to \nu_e$</td>
<td>$e^-$</td>
<td>right–sign $e$ appearance</td>
</tr>
<tr>
<td>$\nu_\mu \to \nu_\tau$</td>
<td>$\tau^-$</td>
<td>right–sign $\tau$ appearance</td>
</tr>
<tr>
<td>$\bar{\nu}_e \to \bar{\nu}_e$</td>
<td>$e^+$</td>
<td>wrong–sign $e$ survival</td>
</tr>
<tr>
<td>$\bar{\nu}<em>e \to \bar{\nu}</em>\mu$</td>
<td>$\mu^+$</td>
<td>wrong–sign $\mu$ appearance</td>
</tr>
<tr>
<td>$\bar{\nu}<em>e \to \bar{\nu}</em>\tau$</td>
<td>$\tau^+$</td>
<td>wrong–sign $\tau$ appearance</td>
</tr>
</tbody>
</table>

Golden channel: $\nu_e \to \nu_\mu$

Since the sign of the detected lepton is critical for determining the oscillation channel being observed, most studies have focused on the $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$ oscillation channels with final state muon detection. Employing a magnetized iron detector allows one to determine the sign of the detected charged lepton, and the backgrounds are quite small, of order $3 \times 10^{-5}$. This small background compared to $\nu_e$ detection in a superbeam, plus the fact that NuFact neutrino fluxes can be one or two orders of magnitude greater than that of a superbeam, are the two main reasons a NuFact is clearly superior.

There have been many many studies of the physics capabilities of a NuFact using muon appearance [97, 265, 294, 295, 296, 297] (for a discussion of the physics that can be done with electron appearance, see Ref. [298]). By comparing $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$ event rates, factoring in Earth-matter effects, very precise determinations of the oscillation parameters $\theta_x$, $\delta$, and $\text{sgn}(\delta m^2_a)$ can be made. Figure 31 shows the ratio of antineutrino to neutrino muon appearance events versus baseline for $\delta m^2_a > 0$ and $\delta m^2_a < 0$ for $\sin^2 2\theta_x = 0.004$ and several values of $\delta$. The different signs of $\delta m^2_a$ are clearly distinguishable when $L \geq 2000$ km, and these measurements are especially sensitive to the amount of $CP$ violation when $L \sim 3000$ km. In practice, if a NuFact is run with $\mu^-$ about twice as long as with $\mu^+$, then the total number of CC events in the neutrino and antineutrino channels will be about the same since $\bar{\nu}_e$ cross section is about half that of the $\nu_e$ cross section.
Figure 31: Ratio of antineutrino to neutrino appearance events versus baseline in a neutrino factory for $\sin^2 2\theta_x = 0.004$ and several values of $\delta$. Both $\delta m_a^2 > 0$ and $\delta m_a^2 < 0$ cases are shown. From Ref. [295].

One obstacle to making precise measurements of $\delta$ and $\theta_x$ and determining the sign of $\delta m_a^2$ is that the other neutrino mass and mixing parameters, i.e., $\theta_a$, $\theta_s$, $\delta m_s^2$, and $\delta m_a^2$, may not be precisely known. Measurements of $\nu_\mu$ survival in a superbeam or NuFact will reduce current uncertainties in $\theta_s$ and $\delta m_s^2$, while future KamLAND and SNO measurements will reduce the uncertainties in $\theta_s$ and especially $\delta m_s^2$.

As with superbeams, there is the possibility of having an eight-fold parameter degeneracy using neutrino and antineutrino event rates in a NuFact. The $\text{sgn}(\delta m_a^2)$ ambiguity can be resolved by choosing a baseline $\geq 2000$ km. The ambiguity is easier to resolve for large $\theta_x$ (due to the larger matter effect) and small $\delta m_s^2$ (due to the smaller size of the $CP$ violating term in the oscillation probability). There have been a number of different proposals for resolving the $(\delta, \theta_x)$ ambiguity in the golden channel:

(i) Since a NuFact has a broad spectrum of neutrino energies, measuring the energy
of the detected muon gives information about the modulation of the oscillation probability with $E_{\nu}$. A 10% muon energy resolution is sufficient to remove the $(\delta, \theta_x)$ ambiguity [296]. A combined fit with the $\nu_\mu$ survival channel can also improve the measurement of $\delta$, $\theta_x$, and $\text{sgn}(\delta m^2_{31})$. An understanding of parameter correlations and degeneracies is essential for extracting meaningful constraints from the data [263, 296].

(ii) Another idea is to combine $\nu_e \rightarrow \nu_\mu$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ measurements from neutrino factory experiments at two baselines. Having one detector at $L \simeq 3000$ km and another at $L \simeq 7300$ km would provide good discrimination between degenerate solutions for a wide range of $\delta$ and $\theta_x$ [97] (see Fig. 32). Measurements at 7300 km, near the magic baseline, where the $\delta$ and $\delta m^2_s$ dependence is minimal [61], would provide an unambiguous measurement of the leading oscillation amplitude $\sin^2 \theta_a \sin^2 2\theta_x$ [268]. With an oval design this scenario would require two separate runs for both stored $\mu^+$ and $\mu^-$, whereas a triangular configuration could allow data to be taken simultaneously at two baselines.

(iii) Since a superbeam will most certainly be a precursor to a NuFact, it is quite natural to combine data from a superbeam and a NuFact to help resolve the $(\delta, \theta_x)$ ambiguity. Studies show that it is possible to remove this ambiguity for $\sin^2 2\theta_x \geq 0.0005$ [299]. In this sense, superbeams and neutrino factories are complementary. Some comparisons of superbeam versus NuFact performance are given in Refs. [65, 263]. It should be noted that even if $\theta_x = 0$, subleading terms in the oscillation probability associated with $\delta m^2_s$ can lead to observable effects in appearance experiments [295].

A summary of typical capabilities of future long-baseline experiments is given in Table 6. A neutrino factory would have sensitivity down to $5 \times 10^{-4}$ and possibly lower in $\sin^2 2\theta_x$ for both $\text{sgn}(\delta m^2_{31})$ and $CP$-violation determinations. A 1% determination of $\delta m^2_s$ should be possible at a NuFact. Thus, it appears that precision reconstruction of the neutrino mixing matrix would be possible with a neutrino factory even for very small values of $\theta_x$. 

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Figure 32: Fits to $\delta$ and $\theta_x \equiv \theta_{13}$ at a neutrino factory using hypothetical results from two baselines, 2810 km and 7332 km, for several input values of $\delta$ and $\theta_x$ (and $\theta_a = \pi/4$). The three curves in each case represent the 90%, 95%, and 99% C. L. ranges of allowed parameters. Expected uncertainties in the oscillation parameters have been included, in addition to a 1% uncertainty in the matter density. From Ref. [97], in whose notation $\Delta m^2_{23} > 0$ implies $\delta m^2_a > 0$.

Table 6: Approximate $3\sigma$ reaches in $\sin^2 2\theta_x$ of future neutrino oscillation experiments.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Reach in $\sin^2 2\theta_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Discovery</td>
</tr>
<tr>
<td>Reactor</td>
<td>0.01</td>
</tr>
<tr>
<td>Conventional $\nu$ beam</td>
<td>0.01</td>
</tr>
<tr>
<td>Superbeam</td>
<td>0.003</td>
</tr>
<tr>
<td>Entry-level NuFact</td>
<td>0.0005</td>
</tr>
<tr>
<td>High-performance NuFact</td>
<td>0.00005</td>
</tr>
</tbody>
</table>

**Silver channel:** $\nu_e \rightarrow \nu_\tau$

Another advantage of a NuFact over a superbeam is that it has available the oscillation channel $\nu_e \rightarrow \nu_\tau$. The oscillation probabilities for $\nu_e \rightarrow \nu_\tau$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$ can be obtained from those for $\nu_e \rightarrow \nu_\mu$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$ by the transformations $\sin \theta_a \leftrightarrow \cos \theta_a$.  

65
and $\delta \rightarrow -\delta$. Comparing the muon and tau channels in a NuFact therefore allows one to (i) help resolve the $(\theta_a, \frac{\pi}{2} - \theta_a)$ ambiguity since the leading term in the oscillation probability is proportional to $\sin^2 \theta_a$ for $\nu_e \rightarrow \nu_\mu$ and $\cos^2 \theta_a$ for $\nu_e \rightarrow \nu_\tau$ [61], and (ii) help resolve the $(\delta, \theta_x)$ ambiguity since the $\delta$ dependence is different in the two channels [300].

10.4 $T$ and $CPT$ symmetries

If $CPT$ is conserved, then $CP$ violation implies $T$ violation, which can be measured, e.g., by comparing $\nu_e \rightarrow \nu_\mu$ to $\nu_\mu \rightarrow \nu_e$. There have been many phenomenological studies of $T$ violation in the literature [267, 297, 301]. Unlike $CP$ violation, matter does not induce $T$ violation in a long-baseline neutrino oscillation experiment, due to the symmetric matter distribution (i.e., the matter distribution is the same from the detector to the source as for the source to the detector), although matter can modify the amount of $T$ violation.

If $CPT$ is not conserved, then $P(\nu_\alpha \rightarrow \nu_\beta)$ is not necessarily equal to $P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha)$ in a vacuum; matter can induce fake $CPT$ violation (for a study of matter-induced $CPT$ violation see Ref. [302]). In a NuFact, $CPT$ violation can be tested down to very low levels by comparing the survival channels $\nu_\mu \rightarrow \nu_\mu$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu$. There have been a number of studies of possible tests of $CPT$ violation in the neutrino sector [303].

11 The outlier: LSND

The focus of this review so far has been on three-neutrino phenomenology. However, the results of the LSND experiment may cast some doubt on the three-neutrino picture. The LSND experiment found evidence for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations at $3.3\sigma$ significance (oscillation probability $(2.64 \pm 0.67 \pm 0.45) \times 10^{-3}$ [39] in data on $\mu^-$ decays at rest taken from 1993-1998. Evidence for $\nu_\mu \rightarrow \nu_e$ oscillations was found at lesser significance from $\pi^+$ decay in flight, with oscillation probabilities $(2.6 \pm 1.0 \pm 0.5) \times 10^{-3}$ in the 1993-1995 data [40] and $(1.0 \pm 1.6 \pm 0.4) \times 10^{-3}$ in the 1996-1998 data (see the last paper of Ref. [39]). There was a significant difference in the analysis of the decay in
flight data in the two time periods due to changes in the neutrino production target, so the two $\nu_\mu \rightarrow \nu_e$ samples were not combined. The KARMEN experiment [41] also searched for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations with a null result; the KARMEN data rule out a large fraction of the LSND allowed region, but still allow a limited region of oscillation parameters [304]. The Bugey reactor experiment [305], which tests the oscillation channel $\bar{\nu}_e \rightarrow \bar{\nu}_e$, excludes the part of the LSND region with $\sin^22\theta_L \gtrsim 0.04$. In a two-neutrino parameter space, the indicated oscillation parameters from a combination of LSND and KARMEN data that are consistent with the constraint from Bugey are $\delta m^2_L \sim 0.2 - 1 \text{ eV}^2$, $\sin^22\theta_L \sim 0.003 - 0.04$ and $\delta m^2_L \sim 7 \text{ eV}^2$, $\sin^22\theta_L \sim 0.004$ at the 90% C. L. (see Fig. 33). The bulk of the allowed region is a narrow band in $(\sin^22\theta_L, \delta m^2_L)$ plane lying along the line described approximately by $\sin^22\theta_L(\delta m^2_L)^{1.64} = 0.0025$, between $\delta m^2_L = 0.2$ and 1 eV$^2$. The MiniBooNE experiment [48] will search for $\nu_\mu \rightarrow \nu_e$ oscillations over the entire parameter space allowed by the LSND results.

Figure 33: The 90% C. L. allowed region (shaded) from a combined fit to $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ data from LSND [39] and KARMEN [41]. The 90% C. L. allowed region from LSND alone is unshaded. The 90% C. L. exclusion regions from KARMEN, Bugey ($\bar{\nu}_e \rightarrow \bar{\nu}_e$) [305], CCFR ($\nu_\mu \rightarrow \nu_e$) [306] and NOMAD ($\nu_\mu \rightarrow \nu_e$) [307] and the expected 90% C. L. sensitivity of the MiniBooNE experiment [48] are also shown. From Ref. [304].
11.1 Four neutrinos?

The LSND parameters are very different from the oscillation parameters that explain the solar and atmospheric neutrino data; in particular, $|\delta m^2_L| \gg |\delta m^2_\alpha|, |\delta m^2_s|$. Since a theory with three neutrinos has at most two independent mass-squared differences, a third $\delta m^2$ scale suggests that there may be a fourth light neutrino participating in neutrino oscillations [42]. Because neutrino counting experiments indicate that there are only three light active neutrinos [54], a fourth light neutrino must be sterile, $i.e.$, it does not participate in the weak interactions [308].

In Section 2, we saw that an extra fully thermalized neutrino is strongly disfavored by BBN [78]. The only way the LSND sterile neutrino can be reconciled with BBN is if its mixing with the active neutrinos occurs only after the active neutrinos have decoupled from the $e^\pm - \gamma$ plasma. Then, the LSND neutrino is not thermalized and the effective number of neutrinos remains equal to 3. An electron neutrino asymmetry,

$$L_e \equiv \frac{n_{\nu_e} - n_{\bar{\nu}_e}}{n_{\gamma}} \approx 0.01 - 0.1,$$

accomplishes this [309] and simultaneously improves the agreement between the BBN prediction for the primordial $^4$He abundance and the observationally inferred value [310]. Also note that since the production of the LSND neutrino is suppressed, cosmological bounds on $\sum m_\nu$ pertain only to the active neutrinos. A confirmation of the LSND signal by MiniBooNE could be interpreted as a hint for a large neutrino asymmetry in the universe.

There are two types of mass spectra possible in four-neutrino models [43, 44]. In $3 + 1$ models, one mass eigenstate is separated from a nearly degenerate triplet of mass eigenstates by $\delta m^2_L$; the triplet has a mass ordering like that of a three-neutrino model. The well-separated mass eigenstate can be either lighter or heavier than the other three, and the triplet can have a normal or inverted hierarchy, so there are four possible variations of $3 + 1$ models. In $2 + 2$ models there is one pair of closely-spaced mass eigenstates separated from another closely-spaced pair by $\delta m^2_L$; one pair has a mass-squared difference of $\delta m^2_s$ and the other $\delta m^2_\alpha$, and the solar $\delta m^2$ can be in either the upper or lower pair. Figure 34 shows the six possible mass spectra with four neutrinos.
11.2 Four neutrino models

The 3 + 1 models are a straightforward extension of a three-neutrino model: the three active neutrinos have mass-squared differences and mixings similar to those in a three-neutrino model, and the sterile neutrino state has only small mixing with active neutrinos. However, 3 + 1 models have trouble accounting for the LSND results and simultaneously obeying the constraints of earlier accelerator and reactor experiments [43, 44]. This can be demonstrated as follows.

Assume a neutrino mass spectrum such that the nearly degenerate triplet of mass eigenstate is lighter than the remaining state and exhibits a normal hierarchy (the first spectrum shown in Fig. 34); similar conclusions can be drawn for the other three 3 + 1 spectra. Then $\delta m^2_{43} \simeq \delta m^2_{42} \simeq \delta m^2_{41} \gg \delta m^2_{32} \approx \delta m^2_{21} \approx \delta m^2_a$, and the oscillation probabilities for the leading oscillation, due to $\delta m^2_L$, are

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \simeq 4 |V_{\mu 4}|^2 |V_{e 4}|^2 \sin^2 \Delta_L,$$

$$P(\nu_\mu \rightarrow \nu_\mu) \simeq 1 - 4 |V_{\mu 4}|^2 (1 - |V_{\mu 4}|^2) \sin^2 \Delta_L,$$

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \simeq 1 - 4 |V_{e 4}|^2 (1 - |V_{e 4}|^2) \sin^2 \Delta_L,$$

where $\Delta_L \equiv \delta m^2_L L/(4E_\nu)$, analogous to Eq. (5). At $L/E_\nu$ values appropriate for atmospheric neutrinos,

$$P(\nu_\mu \rightarrow \nu_\mu) \simeq 1 - 4 |V_{\mu 3}|^2 (1 - |V_{\mu 3}|^2 - |V_{\mu 4}|^2) \sin^2 \Delta_a,$$

$$P(\nu_\mu \rightarrow \nu_\mu) \simeq 1 - 4 |V_{\mu 3}|^2 (1 - |V_{\mu 3}|^2 - |V_{\mu 4}|^2) \sin^2 \Delta_a,$$

Figure 34: The six possible mass spectra in four-neutrino models.
and for solar neutrinos

\[ P(\nu_e \rightarrow \nu_e) \simeq 1 - 4|V_{e1}|^2|V_{e2}|^2 \sin^2 \Delta_s. \]  

(71)

There are very stringent limits on \( \nu_\mu \) disappearance from the CCFR [306], NOMAD [307] and CDHS [311] accelerator experiments that constrain \( |V_{\mu 4}| \) to be very small or very close to unity via Eq. (68). However, if \( |V_{\mu 4}| \) is close to unity then the amplitude of atmospheric neutrino oscillations cannot be as large as required by observation (see Eq. 70); therefore, \( |V_{\mu 4}|^2 \ll 1 \). Similarly, the Bugey reactor experiment [305] puts severe limits on \( \bar{\nu}_e \) disappearance that constrain \( |V_{e4}| \) to be very small or very close to unity, and the observation of large angle mixing in solar neutrino oscillations therefore implies \( |V_{e4}| \) cannot be close to unity (see Eq. 71), so \( |V_{e4}|^2 \ll 1 \).

Since the oscillation amplitude for the LSND experiment is \( 4|V_{\mu 4}|^2|V_{e4}|^2 \), it has an upper limit of approximately one-fourth of the product of the CDHS and Bugey oscillation amplitude bounds (see Eqs. 67-69). In practice, the limits on \( |V_{\mu 4}| \) and \( |V_{e4}| \) depend on \( \delta m^2_{L} \), as does the allowed oscillation amplitude from LSND, so a comparison must be made for each value of \( \delta m^2_{L} \). Early analyses [43, 44] concluded that the 3 + 1 model could not consistently explain the LSND, accelerator, and reactor data. Upon the release of the final data analysis by the LSND collaboration (the last paper of Ref. [39], in which the central value of the average oscillation probability decreased from 0.0031 to 0.0026), it was thought that perhaps the 3 + 1 models would be revived [45, 46]. Subsequently, a Bayesian analysis of all relevant data was made [312]. Figure 35 shows the incompatibility of the combined accelerator and reactor upper limit on the oscillation amplitude in 3 + 1 models and the region allowed by LSND and KARMEN. An analysis of all relevant data yielded the best overall goodness of fit to be \( 5.6 \times 10^{-3} \) [47]. Thus, the viability of 3 + 1 models is tenuous.

The 2 + 2 models are not a simple extension of a three-neutrino model, since the removal of the sterile neutrino does not leave the standard three-neutrino mass spectrum. If the lower pair of neutrino mass eigenstates are primarily responsible for solar neutrino oscillations, then we have \( \delta m^2_{42} \simeq \delta m^2_{32} \simeq \delta m^2_{31} \simeq \delta m^2_{41} = \delta m^2_{L} \gg \delta m^2_{13} = \delta m^2_{a} \gg \delta m^2_{21} = \delta m^2_s \) (the case where the lower pair of mass eigenstates are primarily responsible for atmospheric neutrino oscillations leads to similar conclusions). Then
the oscillation probabilities for the leading oscillation are

\[
P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \simeq 4|V_{\mu 3}V_{\mu 4}^*|^2 \sin^2 \Delta L,
\]

\[
P(\nu_\mu \rightarrow \nu_\mu) \simeq 1 - 4(|V_{\mu 3}|^2 + |V_{\mu 4}|^2)(1 - |V_{\mu 3}|^2 - |V_{\mu 4}|^2) \sin^2 \Delta L,
\]

\[
P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \simeq 1 - 4(|V_{e 3}|^2 + |V_{e 4}|^2)(1 - |V_{e 3}|^2 - |V_{e 4}|^2) \sin^2 \Delta L,
\]

At an \(L/E_\nu\) appropriate for atmospheric neutrinos,

\[
P(\nu_\mu \rightarrow \nu_\mu) \simeq 1 - 4|V_{\mu 3}|^2|V_{\mu 4}|^2 \sin^2 \Delta_a,
\]

and the oscillation probability for solar neutrinos in the 2 + 2 case is the same as in the 3 + 1 case (Eq. 71).

If \(\nu_e\) is primarily connected to \(\nu_1\) and \(\nu_2\), and \(\nu_\mu\) is primarily connected to \(\nu_3\) and \(\nu_4\), i.e., \(|V_{e 1}|^2 + |V_{e 2}|^2, |V_{\mu 3}|^2 + |V_{\mu 4}|^2 \simeq 1 \gg |V_{e 3}|^2, |V_{e 4}|^2, |V_{\mu 1}|^2, |V_{\mu 2}|^2\), then it is possible to simultaneously fit the solar, atmospheric, and LSND data in 2 + 2 models [43, 44].
For example, if $|V_{\mu 3}|^2 + |V_{\mu 4}|^2 = 1$ and $|V_{e 1}|^2 + |V_{e 2}|^2 \simeq 1$, then it is easy to have large mixings of solar and atmospheric neutrinos while suppressing oscillations in the sensitivity regions of CDHS and Bugey. Since $P(\bar{\nu}_\mu \to \bar{\nu}_e)$ and $P(\bar{\nu}_e \to \bar{\nu}_e)$ are both proportional to the second power of small parameters ($V_{e 3}$ and $V_{e 4}$), the LSND oscillation amplitude is approximately constrained by the Bugey bound in the 2 + 2 case (in contrast to one-fourth of the product of the CDHS and Bugey bounds in the 3 + 1 case). This less severe constraint is readily satisfied for a wide range of $\delta m^2_4$; in fact, if either $V_{e 3}$ or $V_{e 4}$ is zero, then the constraint exactly reduces to the simple two-neutrino Bugey constraint shown in Fig. 33. The situation with the roles of $\nu_e$ and $\nu_\mu$ reversed, i.e., $\nu_e$ primarily connected to $\nu_3$ and $\nu_4$, and $\nu_\mu$ primarily connected to $\nu_1$ and $\nu_2$, gives similar results. Some examples of explicit 2 + 2 models are given in Refs. [44, 313].

The preceding arguments relied upon the assumption that large mixing solutions for solar or atmospheric neutrinos are satisfactory for oscillations to sterile neutrinos. In 2 + 2 scenarios, solar neutrinos oscillate to a linear combination of $\nu_\tau$ and $\nu_s$, and atmospheric neutrinos oscillate to the orthogonal combination [44]:

$$
\nu_e \to -\sin \alpha \nu_\tau + \cos \alpha \nu_s , \quad (76)
$$

$$
\nu_\mu \to \cos \alpha \nu_\tau + \sin \alpha \nu_s . \quad (77)
$$

The oscillation probabilities in SuperK and SNO are [314]

$$
R_{SK} = \beta P + r \beta \sin^2 \alpha (1 - P), \quad (78)
$$

$$
R_{SNO}^{CC} = \beta P , \quad (79)
$$

$$
R_{SNO}^{NC} = \beta P + \beta \sin^2 \alpha (1 - P) , \quad (80)
$$

where $R$ is the ratio of observed to expected rates in a given experiment, $\beta$ is the ratio of the actual $^8$B neutrino flux to the SSM prediction, $P$ is the average oscillation probability for $^8$B neutrinos, and $r = \sigma_{\nu_\mu,\nu_\tau}/\sigma_{\nu_e} \simeq 1/6.48$ is the ratio of $\nu_\mu,\nu_\tau$ to $\nu_e$ elastic scattering cross sections on electrons in the energy range of the SuperK experiment.\(^7\) The ratio of the SNO NC to CC data is then

$$
\frac{R_{SNO}^{NC}}{R_{SNO}^{CC}} = 1 + \sin^2 \alpha \left( \frac{1}{P} - 1 \right) . \quad (81)
$$

\(^7\)SNO can also measure $\nu e$ scattering, which is equivalent to $R_{SK}$.
If the $^8$B neutrino flux is assumed to be known from the SSM (i.e., $\beta \equiv 1$), then $P$ is determined from SNO CC data and the sterile fraction can be deduced from the SNO NC/CC measurement. However, if $\beta$ is allowed to be free, $\sin^2 \alpha$ cannot be determined since $R_{SNO}^{CC}$ measures only the combination $\beta P$ and not $P$ itself [314].

Early analyses of the solar neutrino data showed that solar solutions with pure $\nu_e \rightarrow \nu_s$ ($\alpha = 0$) did not provide as good a fit to the solar neutrino data (see the second paper of Ref. [30]). The main difficulty with pure sterile neutrino solutions is that they give similar predictions for the Chlorine and SuperK experiments, whereas active neutrino solutions give a larger value for SuperK due to neutral current $\nu_\mu e$ and/or $\nu_\tau e$ interactions in the detector, which is in better agreement with the experimental data. Oscillations to sterile neutrinos have different matter effects since the value of $\delta m^2_s$ that gives resonant oscillations is $\delta m^2_s = 2\sqrt{2}G_F E_\nu(N_e - \frac{1}{2}N_n)/\cos 2\theta_s$ instead of the value $2\sqrt{2}G_F E_\nu N_e/\cos 2\theta_s$ for oscillations to active neutrinos (see Eq. 37).

Imposing the SSM uncertainties on $\beta$, the pure sterile solution is excluded at the 7.6$\sigma$ level, although a large sterile fraction could still be allowed even after the SNO CC and NC data is included [107, 314, 315]. The SuperK data (or, equivalently, the SNO $\nu e$ scattering data) do not give an independent constraint; in fact, there is a sum rule [314, 316]

$$R_{SNO}^{NC} = [R_{SK} - (1 - r)R_{SNO}^{CC}] / r ,$$

(82)

that must be obeyed for any value of $\sin^2 \alpha$. A future test of $\sin^2 \alpha$ that is independent of the SSM $^8$B neutrino flux predictions would be a neutrino-nucleon NC measurement of intermediate energy solar neutrinos [314], or the independent measurement of $P$, such as in the KamLAND reactor experiment [315].

The opposite extreme is to have pure sterile solutions to the atmospheric neutrino data ($\alpha = \frac{\pi}{2}$). Here there are strong matter effects due to coherent forward scattering in the Earth that is present for $\nu_\mu$ but not $\nu_s$ (see Eq. 37), with $N_{eff} = -\frac{1}{2}N_n$; for pure $\nu_\mu \rightarrow \nu_\tau$ oscillations, matter effects are small. The SuperK atmospheric data rule out pure $\nu_\mu \rightarrow \nu_s$ oscillations at the 99% C. L. [132, 317], and require $\sin^2 \alpha$ to be smaller than 0.2 at the 90% C. L. [127, 131]. Other analyses also found that a substantial sterile component is allowed by the atmospheric neutrino data [318].

Since pure $\nu_e \rightarrow \nu_s$ solar and pure $\nu_\mu \rightarrow \nu_s$ atmospheric neutrino oscillations are
ruled out, but partial sterile solutions are allowed in each case, the only remaining option in the $2+2$ scenario is to have an active-sterile mixture for both solar and atmospheric neutrinos ($0 < \alpha < \pi/2$). The analysis of Ref. [47] indicates $\sin^2 \alpha \geq 0.55$ from pre-SNO salt phase solar neutrino data and $\sin^2 \alpha \leq 0.35$ from atmospheric neutrino data (both at 99% C. L.), and that the overall goodness of fit of $2+2$ models is only $1.6 \times 10^{-6}$, much worse than that of the $3+1$ models.

To summarize the situation for four neutrinos, the most recent analysis [47] shows that $3+1$ scenarios provide a better fit than $2+2$ scenarios, but neither scenario gives a good description of the combined solar, atmospheric, accelerator, and reactor data. However, the inclusion of additional sterile neutrinos can enhance the size of the LSND amplitude allowed by CDHS and Bugey in the $3+1$ scenario [46], and a recent study found that a $3+2$ model does significantly better than the $3+1$ models in fitting the data [319]. Also, studies using a more complete set of four-neutrino parameters indicate there may still be room for the $2+2$ models [320]. Therefore, although the positive appearance result in LSND remains puzzling, a consistent four-neutrino explanation may still be possible.

### 11.3 Three-neutrino models with CPT violation

It has been suggested that if CPT were not conserved, then oscillations of three active neutrinos could describe the solar, atmospheric and LSND data simultaneously [321]. In this proposal, the mass matrices (and hence mass-squared differences) for neutrinos and antineutrinos are different, which violates CPT (while preserving Lorentz invariance) $^8$.

In the original versions of CPT-violating models, the neutrino sector had the usual three-neutrino mass spectrum that can account for the oscillation of solar and atmospheric neutrinos, while in the antineutrino sector the mass-squared differences account for the oscillation of antineutrinos in the atmospheric and LSND experiments (the weak indication for $\nu_\mu \to \nu_e$ oscillations in LSND must be ignored). KamLAND data, consistent with oscillations of $\bar{\nu}_e$ at the $\delta m^2$ scale, forced a modification of $^8$Whether such a model can be constructed using nonlocality of the interactions is still a matter of debate [322].
the antineutrino spectrum, so that it describes the oscillation of antineutrinos in LSND and KamLAND (but not in the atmosphere) [323]. Since the atmospheric data does not distinguish between neutrinos and antineutrinos, this latter scenario was not in obvious contradiction to the data. However, one analysis of atmospheric data indicates that \textit{CPT}-violating scenarios are not in good agreement with the atmospheric data [324]. Furthermore, global analyses of all data including KamLAND excludes these \textit{CPT}-violating scenarios at the \(3\sigma\) level [324, 325].

We await the MiniBooNE results on \(\nu_{\mu} \rightarrow \nu_{e}\) oscillations to confirm or reject the LSND effect. A positive signal will rule out current \textit{CPT}-violating models, and force a reconsideration of the disfavored four-neutrino models, possibly by extending them to include more than one sterile state. A negative result will rule out the standard four-neutrino scenario (which predicts the same oscillations for neutrinos and antineutrinos in LSND), but does not completely extinguish the LSND flame since it does not test the primary LSND \(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_{e}\) channel. In either case, it will be difficult to exclude the very speculative possibility of the LSND anomaly arising from \textit{CPT}-violation in four-neutrino models [326].

12 Summary and outlook

Great advances have been made over the past five years in our understanding of the neutrino sector. The field is now poised for further breakthroughs. In this article we have strived to summarize the present state of the field and the ongoing experimental, phenomenological, and theoretical efforts towards a bottom-up reconstruction of the fundamental properties and theory of neutrinos. In this concluding section we very briefly recapitulate the highlights of these recent accomplishments and the major planned directions for future progress.

The standard formalism of three-neutrino mixing and oscillation probabilities was recounted. The modifications from \(\nu_{e}\) scattering in matter were quantified as appropriate for solar neutrinos and for long-baseline accelerator neutrino experiments. The experimental resolution of the solar neutrino problem, particularly by the SNO experiment, as the large mixing angle oscillation solution, was discussed along with its
validation by the KamLAND reactor and other neutrino experiments. The evidence for $\nu_\mu \to \nu_\tau$ oscillations, from the dependence on the $\nu_\mu \to \nu_\mu$ oscillation probability versus zenith angle in the Super-Kamiokande atmospheric neutrino experiment, was summarized.

In the framework of three-neutrino mixing, the major physics goal is to determine the six neutrino oscillation parameters: two mass-squared differences ($\delta m^2_s, \delta m^2_a$), three mixing angles ($\theta_s$, $\theta_a$, $\theta_x$), and a $CP$ phase $\delta$. From present data the approximate values are $\delta m^2_s \approx 7 \times 10^{-5} \text{eV}^2$, $|\delta m^2_a| \approx 2 \times 10^{-3} \text{eV}^2$, $\theta_s \approx 33^\circ$, $\theta_a \approx 45^\circ$. The unknown parameter $\theta_x$ (with present limit $\theta_x \leq 13^\circ$ for $\delta m^2_a = 2.0 \times 10^{-3} \text{eV}^2$ at the 95% C. L.) is critical to further understanding; it can be probed in $\nu_\mu \to \nu_e$ or $\nu_e \to \nu_\mu$ appearance channels. Both off-axis and broad-band beams are being considered for its measurement. More intense neutrino beams (superbeams or neutrino factories) and longer baselines ($\gtrsim 1000 \text{ km}$) are essential to determine the sign of $\delta m^2_a$ from matter effects and have sensitivity to $CP$-violation. Expected sensitivities of proposed future oscillation experiments to $\theta_x$, $\text{sgn}(\delta m^2_a)$, and $\delta$ are compared; see Table 6. Reactor experiments at baselines $\lesssim 10 \text{ km}$ could also determine $\theta_x$ if $\sin^2 2\theta_x \gtrsim 0.05$, but only at about the 90% C. L.

The evidence for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations at a higher $\delta m^2$ ($\approx 0.1 - 1 \text{ eV}^2$) may imply the existence of a sterile neutrino if $CP$ is conserved. Global fits to data now reject this possibility at more than the 99% C. L. The ongoing MiniBooNE experiment is designed to confirm or exclude the LSND effect. Big Bang Nucleosynthesis, combined with cosmic microwave background measurements by WMAP, strongly disfavors more than three thermalized neutrinos, but a large $\nu_e$-asymmetry allows an escape from this constraint.

The search for the absolute scale of neutrino mass in beta decay, neutrinoless double-beta decay, and large scale structure in the universe was discussed. The best present constraint comes from cosmology and bounds the sum of neutrino masses to be less than about 1 eV.

Galactic supernova explosions produce high fluxes of neutrinos and antineutrinos that can determine the neutrino mass hierarchy (normal or inverted) if the mixing angle $\theta_x$ is known from accelerator experiments.
Models of neutrino mass are largely of two generic types, Grand Unification with a seesaw mechanism and radiative mass generation. Unified models can accommodate all present knowledge of quark and lepton masses and mixing, with different models giving different predictions for the unknown neutrino parameters. Successful Grand Unified models predict $\delta m_a^2 > 0$, for which there would be one heavier neutrino mass and two lighter masses. Also in unified models, the neutrino masses are hierarchical: $m_3 \gg m_2 \gg m_1$. Radiative mass generation involves lepton number violating interactions that have testable implications for collider physics experiments.

Unified models offer a possible explanation of the baryon asymmetry of the universe in terms of a neutrino-antineutrino asymmetry. In such models the $CP$-violating phase in heavy Majorana neutrino decays that generate leptogenesis may be related to the $CP$-violating phase of neutrino oscillations at low energy.

The ways by which the unknown neutrino oscillation parameters can be determined by an ambitious future experimental program with baselines at long distances from intense sources was detailed. Conventional neutrino beam experiments are now under construction. Later these may be upgraded with more intense superbeams. Eventually neutrino factories will provide the ultimate sensitivity and precision in determining neutrino mixing and mass-squared difference parameters.

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