Abstract. A number of supernova remnants (SNRs) show nonthermal X-rays assumed to be synchrotron emission from shock accelerated TeV electrons. The existence of these TeV electrons strongly suggests that the shocks in SNRs are sources of galactic cosmic rays (CRs). In addition, there is convincing evidence from broad-band studies of individual SNRs and elsewhere that the particle acceleration process in SNRs can be efficient and non-linear. If SNR shocks are efficient particle accelerators, the production of CRs impacts the thermal properties of the shock heated, X-ray emitting gas and the SNR evolution. We report on a technique that couples nonlinear diffusive shock acceleration, including the backreaction of the accelerated particles on the structure of the forward and reverse shocks, with a hydrodynamic simulation of SNR evolution. Compared to models which ignore CRs, the most important hydrodynamical effects of placing a significant fraction of shock energy into CRs are larger shock compression ratios and lower temperatures in the shocked gas. We compare our results, which use an approximate description of the acceleration process, with a more complete model where the full CR transport equations are solved [?], and find excellent agreement for the CR spectrum summed over the SNR lifetime and the evolving shock compression ratio. The importance of the coupling between particle acceleration and SNR dynamics for the interpretation of broad-band continuum and thermal X-ray observations is discussed.

Key words: ISM: cosmic rays — acceleration of particles — shock waves — ISM: supernova remnants — X-rays: ISM
Hydrodynamic Simulation of Supernova Remnants Including Efficient Particle Acceleration

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1. Introduction

It is commonly believed that the shocks in supernova remnants (SNRs) produce the majority of galactic cosmic rays (CRs) with energies below \( \sim 10^{15} \) eV via diffusive shock acceleration (\( \gamma_{\text{eff}} \), see)\footnote{As of this writing, there is no unambiguous evidence for the production of TeV ions in SNRs (\( \gamma_{\text{eff}} \), see) for a discussion of SN1006 in this regard\cite{BKV2002}. The recent claim that TeV emission from SNR RX J1713.7-3946 (also called G347.3−0.5) observed by CANGAROO II is from pion-decay (\( \gamma_{\text{eff}} \), see)\footnote{The recent claim of TeV emission from SNR RX J1713.7-3946 (also called G347.3−0.5) observed by CANGAROO II is still under debate (\( \gamma_{\text{eff}} \), see, for example)\cite{RP2002,Butt2002}.
}Drury83,BE87. Convincing support for the production of TeV electrons in SNRs comes from the synchrotron interpretation of nonthermal X-ray emission (\( \gamma_{\text{eff}} \), see)\footnote{The recent claim of TeV emission from SNR RX J1713.7-3946 (also called G347.3−0.5) observed by CANGAROO II is still under debate (\( \gamma_{\text{eff}} \), see, for example)\cite{RP2002,Butt2002}.}R eynolds98 observed in an increasing number of young SNRs such as SN1006 (Koyama et al., 1995), Cas A (Allen et al., 1997), G347.3-0.5 (Slane et al., 1999), and RCW 86 (Borkowski et al., 2001).}

In addition to their putative role in accelerating cosmic rays, the shocks in SNRs heat the ambient interstellar medium and ejecta to X-ray emitting temperatures. The interpretation of these X-ray observations leads to inferences for important quantities such as the supernova (SN) explosion energy, ejecta mass and composition, ambient densities, shock speed, and rate of electron and proton equilibration. That CR production and thermal heating in SNRs may be coupled comes from the fact that diffusive shock acceleration is intrinsically efficient in high Mach number shocks if even a small fraction of the shock heated plasma is injected into the acceleration process (\( \gamma_{\text{eff}} \), see)\footnote{The recent claim of TeV emission from SNR RX J1713.7-3946 (also called G347.3−0.5) observed by CANGAROO II is still under debate (\( \gamma_{\text{eff}} \), see, for example)\cite{RP2002,Butt2002}.}

If strong coupling between particle acceleration and shock heating occurs, the modeling of efficient particle acceleration in SNRs offers the possibility of using the high-quality X-ray and gamma-ray data currently being collected by spacecraft (e.g., \textit{Chandra}, \textit{XMM-Newton}, INTEGRAL) to address fundamental questions concerning the SNR origin of CRs and the underlying physics of diffusive shock acceleration, particularly the injection of thermal particles into the acceleration process. Furthermore, strong coupling implies that the inferences made from X-ray observations may differ substantially between interpretations which include particle acceleration self-consistently and those that do not.

Despite the expected efficiency of diffusive shock acceleration, X-ray spectra from SNRs have generally been modeled and interpreted assuming that the shocks place an insignificant fraction of their energy in cosmic rays. Exceptions to this include the early works of Chevalier (1983), Heavens (1984), and Boulares & Cox (1988). Chevalier (1983) investigated the effects of cosmic-ray pressure on SNR dynamics using a two-fluid, self-similar solution with an arbitrary fraction of thermal gas (adiabatic index \( \gamma = 5/3 \)) and relativistic gas (\( \gamma = 4/3 \)). More recent work has been done by Dorfi & Böhringer (1993) and Dorfi (1994). In our preliminary work (\( \gamma_{\text{eff}} \), see)\footnote{The recent claim of TeV emission from SNR RX J1713.7-3946 (also called G347.3−0.5) observed by CANGAROO II is still under debate (\( \gamma_{\text{eff}} \), see, for example)\cite{RP2002,Butt2002}.}DEB2000, we developed a model which coupled the approximate nonlinear (NL) acceleration calculation of Berezhko & Ellison (1999) with an analytic, self-similar description of the SNR hydrodynamics (\( \gamma_{\text{eff}} \), see)\footnote{The recent claim of TeV emission from SNR RX J1713.7-3946 (also called G347.3−0.5) observed by CANGAROO II is still under debate (\( \gamma_{\text{eff}} \), see, for example)\cite{RP2002,Butt2002}.}

The effects of efficient particle acceleration on SNR hydrodynamics where calculated in a hydro computer simulation of SNRs by Blondin & Ellison (2001). This was done by globally changing the effective ratio of specific heats, \( \gamma_{\text{eff}} \), from 5/3 to values approaching 1, but did not include coupling between the acceleration and the hydro. As \( \gamma_{\text{eff}} \) was decreased, the shocked gas became more compressible, the shock compression ratio increased, and the interaction region between the forward and reverse shocks narrowed. Blondin & Ellison (2001) were able to show in two and three-dimensional simulations that if the interaction region was narrow enough, convective instabilities
produced Rayleigh-Taylor fingers of dense ejecta material which were able to reach and perturb the forward shock.

Here, we introduce and describe in detail a CR-Hydro model which uses the same NL acceleration calculation of Berezhko & Ellison (1999), but replaces the self-similar description used in Decourchelle et al. (2000) with a 1-D hydro simulation such as that used by Blondin & Ellison (2001). The simulation is more general than the analytic approach used by Decourchelle et al. (2000) since it is not restricted to self-similar evolution and allows for a continuous change in the acceleration efficiency as the SNR evolves. We show, however, that when acceleration efficiency is nearly constant during the self-similar phase, the two models closely correspond, providing an important check on the validity of both models. While we do not calculate X-ray thermal spectra in this paper, we show, with various examples, how the efficient production of CR protons influences SNR evolution and discuss the implications this has on the interpretation of X-ray observations.

2. CR-Hydro Model

Our CR-Hydro model couples a spherically symmetric hydrodynamic simulation with a calculation of nonlinear diffusive shock acceleration. The particle acceleration calculation determines how energy is divided between the thermal gas and relativistic particles, and provides the particle distribution over all energies behind the shock, as well as the effective ratio of specific heats, \( \gamma_{\text{eff}} \), defined below. In future work, we will use the electron and ion distribution functions to calculate the broad-band continuum photon emission from radio to TeV energies (\cite{DEG2001}, e.g.,) and use the self-consistent thermal properties in a non-equilibrium calculation of X-ray lines (\cite{DEB2000}, e.g.,)\cite{DEB2000}.

For now, we restrict ourselves to calculating CR proton spectra integrated over the SNR lifetime.

2.1. Hydrodynamic Simulation

We use a standard hydrodynamic simulation in one dimension to model the effects of a supernova explosion in the interstellar medium (ISM) (\cite{EE84}, see) and references therein\cite{BE2001}. We are free to choose arbitrary ejecta and ISM mass density profiles, but to facilitate our comparisons discussed below, we adopt the parameters determined for SN1006 by Berezhko et al. (2002), i.e., we assume a constant density, constant temperature ISM, and take the initial ejecta density profile to be \( \rho \propto r^{-n} \), with \( n = 7 \) and a constant density plateau at small radii (\cite{Chev82a}, e.g.).

While the modeling of particular, young SNRs depends critically on the ejecta and ISM densities (\cite{DEG2001}, e.g.,)\cite{DEB2000}, the general character of the results we present here are insensitive to these details. Specifically, we begin at some time \( t_{\text{start}} < 0.1 t_{\text{ch}} \)

with undisturbed ejecta and ISM separated by a contact discontinuity. Here, \( t_{\text{ch}} = R_{\text{ch}}/V_{\text{ch}} \) is the characteristic age with \( R_{\text{ch}} = [3M_{\text{ej}}/(4\pi\rho_0)]^{1/3} \), \( V_{\text{ch}} = \sqrt{2E_{\text{ej}}/M_{\text{ej}}} \), and \( \rho_0 = (1 + 4f_{\text{He}})m_pn_p \), where \( E_{\text{ej}} \) is the explosion energy, \( M_{\text{ej}} \) is the ejecta mass, \( n_p \) is the ISM proton number density, \( f_{\text{He}} \) is the helium to proton number ratio, and \( m_p \) is the proton mass.\footnote{At the start of the simulation, the ejecta temperature is assumed to be low enough to be insignificant.}

The magnetic field, \( B \), is ignored in our hydrodynamic model (we implicitly assume that \( B^2/8\pi \) is small compared to the thermal pressure), but it is an important parameter for the particle acceleration discussed next.

2.2. Nonlinear Diffusive Shock Acceleration

The full details of the nonlinear acceleration model used here are given in Berezhko & Ellison (1999) and Ellison et al. (2000). This is an approximate, algebraic model of diffusive shock acceleration containing the essential physics of NL acceleration, but which parameterizes important properties of the process such as the injection efficiency and the maximum energy particles achieve. While more complete models of nonlinear shock acceleration exist (\cite{EE96}, e.g.,)\cite{JE91,BEK96,MD2001}, our algebraic approximation is easier to include in global models of SNRs. It is computationally fast making it far less time consuming to do parameter searches and to compare model results with observations. In Sect. ?? we show by direct comparison that it gives similar results to the more physically complete model of Berezhko et al. (2002).

Briefly, the nonlinear effects in diffusive shock acceleration are: (i) the self-generation of magnetic turbulence by counter-streaming energetic particles. Backstreaming particles produce turbulence in the magnetic field which leads to stronger scattering of the particles and hence to more acceleration, quickly leading to saturated turbulence levels near \( \delta B/B \sim 1 \) in strong shocks; (ii) the modification (i.e., smoothing) of the shock precursor by the backpressure of energetic particles. The precursor influences the subshock compression, \( r_{\text{sub}} \), the injection and acceleration efficiencies, and the shape of the accelerated spectrum. Since particle diffusion lengths are generally increasing functions of momentum (\cite{EE96}, e.g.,)\cite{BE87,GBSE93}, high momentum particles sample a broader portion of the flow velocity profile, and hence experience larger effective total compression ratios, \( r_{\text{tot}} \), than low momentum particles. Consequently, higher momentum particles have a flatter power-law index than those at lower momenta and can dominate the pressure in a NL fashion. The resultant superthermal distribution has a characteristic concave upward curvature until the spectrum turns over at the highest energies from losses (\cite{EE84,Blasi2002,MDJ2002}; and (iii) the increase in \( r_{\text{tot}} \) from relativistic particle pressure and particle es-

\footnote{Throughout this paper the subscript 0 (2) implies values upstream (downstream) from the shock.}
As relativistic particles are produced and contribute significantly to the total pressure, their softer equation of state makes the shocked plasma more compressible ($\gamma \to 4/3$). Even more important, as the highest energy particles escape from strong shocks they drain away energy flux which must be compensated for by ramping up the overall compression ratio to conserve the fluxes. Just as in radiative shocks, this is equivalent to $\gamma \to 1$ and $r_{\text{tot}}$ can become arbitrarily large (?), e.g.,) KE86, BE99, Malkov97. As the overall compression increases ($r_{\text{tot}} > 4$), the subshock compression ratio, $r_{\text{sub}}$, which is responsible for heating the gas, must become less than the test-particle (TP) value ($r_{\text{sub}} < 4$), causing the temperature of the shocked gas to drop below TP values. These changes in shock compression occur simultaneously with changes in the shape of the accelerated particle spectrum, thus linking X-ray heating to cosmic-ray production. For reviews on diffusive shock acceleration see Drury (1983); Blandford & Eichler (1987); Berezhko & Krymsky (1988); Jones & Ellison (1991); and Malkov & Drury (2001).

The most important parameters associated with nonlinear shock acceleration are the Mach numbers (i.e., the shock speed, $u_0$, pre-shock hydrogen number density, $n_{H0}$, and preshock magnetic field, $B_0$), the injection efficiency, $\eta_{\text{inj}}$ (i.e., the fraction of total protons which end up with superthermal energies), and the maximum proton energy produced, $E_{\text{max}}$. As described in Berezhko & Ellison (1999), our model includes Alfvén heating in the precursor which reduces the efficiency compared to adiabatic heating and makes the magnetic field strength an important parameter. For given sonic and Alfvén Mach numbers (i.e., given $M_S = \sqrt{n_{H0}u_0^2/(\gamma p_0)$ and $M_A = \sqrt{4\pi n_{H0}u_0^2}/B_0$), and a given shock size and age, $\eta_{\text{inj}}$ sets the overall acceleration efficiency and determines the importance of NL effects. With other parameters fixed, Alfvén wave heating causes the acceleration efficiency to decrease with increasing $B_0$. In a complete model of diffusive shock acceleration, the injection efficiency would be determined from first principles. However, no current model of diffusive shock acceleration can do this and injection remains dependent on approximations of poorly understood wave-particle interactions. Here, we investigate the effects of efficient acceleration by varying $\eta_{\text{inj}}$. A principle aim of future work is to constrain $\eta_{\text{inj}}$ from models using X-ray and broad-band observations of particular SNRs.

In order to compare our results directly to those of Berezhko et al. (2002), we assume as they do that the magnetic field is turbulent, adopting the Bohm limit for strong particle scattering, and somewhat arbitrarily take the field downstream from the shock to be the compressed upstream magnetic field i.e., $B_2 = r_{\text{tot}}B_0$, where $B_0(B_2)$ is the upstream (downstream) magnetic field strength. We do not expressly consider shock obliquity, i.e., the angle between the local shock normal and $B_0$, even though this may be an important factor for understanding emission around the rims of some SNRs (see) for a discussion of the effects of shock obliquity in a test-particle description of particle acceleration in SNRs|Reynolds98. As a crude approximation, we could model the asymmetry seen in many SNRs, including SN 1006, by combining results for different quadrants of the remnant where values of the magnetic field and injection parameter were varied.

In our examples presented here, we take the unshocked ISM field to be $B_0 = 20 \mu G$ to match the value determined by Berezhko et al. (2002) for SN 1006. For simplicity, unless explicitly stated we use the same constant value for the field in the unshocked supernova ejecta even though, in reality, this field is likely to weaken considerably with time because of flux conservation [in the discussions associated with Figs. ?? ( Sect. ??) and ?? (Sect. ??), we show some effects of a weak ejecta field].

The maximum energy cosmic rays obtain depends, in part, on the scattering mean free path, $\lambda$, which is assumed to be,

$$\lambda = \eta_{\text{mfp}} r_g ,$$

where $\eta_{\text{mfp}} \gtrsim 1$ is taken to be a constant and $r_g = p/(qB)$ is the gyroradius in SI units. Small values of $\eta_{\text{mfp}}$ imply strong scattering and allow higher maximum proton energies in a given system. The Bohm limit implies $\eta_{\text{mfp}} \sim 1$.

The phase-space momentum distributions for protons, $f(p)$, are calculated as in Ellison et al. (2000) and consist of a thermal component, a three-component power law at superthermal energies, and a turnover at the highest energies given by

$$\exp \left[ -\frac{1}{\alpha} \left( \frac{p}{p_{\text{max}}} \right)^\alpha \right].$$

Here, $\alpha$ is a constant and $p_{\text{max}} = E_{\text{max}}/c$ is determined by setting the acceleration time equal to the SNR age, $t_{\text{snr}}$, or by setting the diffusion length of the highest energy particles equal to some fraction, $f_{sk}$, of the shock radius, $R_{sk}$, whichever gives the lowest $p_{\text{max}}$ (see) BaringEtal99. With these assumptions, $p_{\text{max}}$ is proportional to the largest magnetic field for a significant population of superthermal particles to be produced. If, in addition, electrons are to be investigated, the simulations must use the short electron time-step yet run for many proton time-scales increasing computation time considerably.

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4 While in principle plasma simulations, where particles move in response to Newton’s and Maxwell’s equations (particle-in-cell), can fully describe wave-particle interactions and injection, in practice, they have not yet done so because of computational limits. The limits have been insurmountable for two main reasons: (i) The simulations must be performed fully in 3D because 1- or 2-D simulations unphysically prevent cross-field diffusion (Jokipii et al., 1993; Jones et al., 1998). In all cases except strictly parallel shocks (where the upstream magnetic field is parallel to the shock normal), cross-field diffusion will be an essential part of the injection and acceleration process; and (ii) In order for NL effects to become apparent or field amplification to occur on large scales, the simulations must be run long enough in a large enough box with enough resolution by varying $\eta_{\text{inj}}$. A principle aim of future work is to constrain $\eta_{\text{inj}}$ from models using X-ray and broad-band observations of particular SNRs.