Neutrino-induced fission of neutron-rich nuclei

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We calculate neutrino-induced fission cross sections for selected nuclei with $Z = 84 - 92$. We show that these reactions populate the daughter nucleus at excitation energies where shell effects are significantly washed out, effectively reducing the fission barrier. If the r-process occurs in the presence of a strong neutrino fluence, and electron neutrino average energies are sufficiently high, perhaps as a result of matter-enhanced neutrino flavor transformation, then neutrino-induced fission could lead to significant alteration in the r-process flow in slow outflow scenarios.

In this letter we calculate neutrino capture-induced fission cross sections for heavy nuclei associated with the r-process. Matter-enhanced neutrino flavor transformation could enhance this process in the supernova/compact object environments commonly invoked as r-process sites. Recent observations of r-process abundances in low-metallicity, old galactical halo stars \cite{1} show patterns which agree with the solar r-process distribution for nuclides with mass numbers $A > 130$, but do not reproduce the solar r-process pattern for the lighter r-process elements. In particular, these observed abundances show a peak around mass number $A \sim 195$, which follows the solar r-process distribution, and enhanced structures at around $A \sim 90$ and $\sim 132$ which do not follow the solar pattern.

It was recognized some time ago \cite{2} that $\nu_e$ capture on heavy nuclei in the post-collapse supernova environment would leave the daughter nucleus at the high excitation energies characteristic of Gamow-Teller resonances. This leaves these nuclei vulnerable to fission. Recently, Qian has demonstrated \cite{3} that fission, induced by charged-current neutrino reactions within this neutrino-driven wind scenario \cite{4}, can account for the observed abundance patterns. In this model it is proposed that neutrino-induced fission occurs after the r-process freezes out (i.e. all initial neutrons are exhausted) and the progenitor nuclei decay to stability. It is further proposed \cite{3} that no fission cycling occurs during the r-process, i.e. neutrino-induced reactions are unimportant during the r-process.

Neutrino capture-induced fission cross sections have not been calculated before. Two aspects of nuclear physics conspire to make this process potentially important in dense environments with large neutrino fluxes: (1) the weak strength distribution in the charged current (neutrino capture) channel shows that the post-capture daughter nucleus will be left in a highly excited state; and (2) fission barriers are lower at higher excitation energy.

It is expected that charged-current reactions on r-process nuclides will have larger partial fission cross sections than neutral-current reactions, despite the fact that the latter can be induced by $\nu_{\mu,\tau}$ neutrinos and their antiparticles, which, in a core bounce supernova explosion, might have larger average energies ($\langle E_{\nu} \rangle \sim 20 - 25$ MeV) than the $\nu_e$ neutrinos have ($\langle E_{\nu} \rangle \sim 10$ MeV). For the neutron-rich nuclei along the r-process path neutrino capture cross sections are quite large, as both allowed channels (Fermi and Gamow-Teller (GT)) are governed by sum rules which scale with the neutron excess $N - Z$ and the $\nu_e$ neutrino energy is large enough to excite the centroids of these allowed responses. Furthermore, the isobaric analogue state (IAS) and the GT centroid are located at energies in the daughter nucleus ($E \sim 20 - 30$ MeV) which are significantly above the fission barriers in these nuclei. Hence, fission can represent an important, even dominant decay mode following neutrino-induced reactions on neutron-rich nuclei. Obviously the principal competing decay mode is neutron emission, as neutron thresholds are also quite low in r-process nuclei.

Our calculations of neutrino-induced reactions proceed through two steps. First we calculate the neutrino cross sections as functions of excitation energy in the final nucleus and then determine the decay mode of the final nuclear state using a statistical approach. The neutrino cross sections are calculated with the random phase approximation (RPA), considering multipoles up to $J = 4$ and both parities. (See refs. \cite{5}.) Our RPA scheme treats proton and neutron degrees of freedom separately and employs a partial occupancy formalism for non-closed shell nuclei. We adopt a zero-range Migdal force as a residual interaction. We note that the RPA satisfies the Fermi and Ikeda sum rules, which fix the total strength for the allowed transitions.

In the second step we calculate for each final state with well-defined energy, angular momentum, and parity the branching ratios into the various decay channels using the stastical model code SMOKER \cite{6}, considering proton, neutron, $\alpha$ and $\gamma$ emission as well as fission. The fission barriers employed here were taken from the compilation of Howard and Möller \cite{7} and the neutron separation energies from the mass table of Hilf \textit{et al.} \cite{8}. The final states in the residual nucleus were taken from the experimentally known levels supplemented at higher energies by an appropriate level density formula \cite{6}.
Assuming a typical supernova $\nu_e$ neutrino spectrum, i.e., a Fermi-Dirac spectrum with temperature $T_\nu = 4$ MeV and zero chemical potential, we have calculated the total $(\nu_e, e^-)$ cross section and the neutrino-induced fission cross section for selected even-even nuclei with charge numbers $Z = 84 – 92$, covering the range from stability all the way to unstable r-process nuclides (Fig. 1). As the total cross sections are dominated by allowed contributions (Fermi, GT), the cross sections increase linearly with neutron excess within the various isotope chains, simply reflecting the sum rules governing these two multipole transitions. Fission is an important decay mode, in particular for the Ra, Th, and U isotopes (Fig. 1). The differences between the total and the fission cross sections, are mainly accounted for by the partial $(\nu_e, e^- n)$ cross sections, although for the lighter Po and Rn isotopes the decay into the gamma channel can compete with the fission decay branch. Due to the relatively high thresholds (Coulomb barriers), branchings into the proton and $\alpha$ channels are negligible. The competition between the two dominant decay modes, neutron emission and fission, are shown in Fig. 2 for selected Th and U isotopes. In our calculation, fission dominates the decay, except for the most neutron-rich nuclides shown. We note that this calculation only considers the decay branchings in the daughter nucleus, and does not follow multiple decays; i.e., it represents the “first-chance” fission cross sections [9].

The competition between the dominant decay modes (fission, neutron emission) in a neutrino capture-excited daughter is governed by the relative values of the fission barrier $B_f$ and the neutron separation energy $S_n$. The fission probability $P_f$ is then approximately given by [17]

$$
P_f = \frac{1}{1 + 4(\frac{m_n}{h^2})R^2T \exp \{ (B_f - S_n)/T \}}
$$

where $m_n$ is the nucleon mass and $R = 1.2 \cdot A^{1/3}$ is the nuclear radius. This formula assumes that the decaying nucleus is excited at energies $E$ which are significantly larger than $B_f$ and $S_n$. This is true in $(\nu_e, e^-)$ reactions with supernova neutrinos on heavy neutron-rich nuclei where $E \sim 25$ MeV. Such excitation energies correspond to nuclear temperatures of $T \approx 1$ MeV in nuclei with $A \sim 230 – 270$. For simplicity we have assumed that $T = 1$ MeV in the following. Eq. (1) yields $P_f \sim 1/6$ if $B_f = S_n$, and $P_f = 0.5$ if $B_f - S_n = -1.6$ MeV. The difference $U = B_f - S_n$ is strongly dependent on the excitation energy. In fact, Eq. (1) is derived from statistical considerations involving the level density at vanishing nuclear deformation (for the neutron emission probability) and at the saddle points of the double-humped fission barriers. The latter corresponds to a sizable nuclear deformation, where the level density increases faster than at vanishing deformation. This reduces $U$ with increasing excitation energy, enhancing the fission probability relative to neutron emission [9,18]. This is consistent with the fact that the fission barrier in heavy nuclei is strongly influenced by shell effects [19,17], and these are washed out with increasing excitation energy.

Using Eq. (1) we have inverted our calculated fission probabilities to obtain $U(T) = B_f(T) - S_n(T)$, assuming $T = 1$ MeV. The desired quantity $\Delta U(T) = U(T = 0) - U(T)$ is plotted in Fig. 3, where $U(0)$ has been derived from the compiled fission barriers [7] and neutron separation energies [8]. We note that the energy reduc-

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**FIG. 1.** Neutrino-induced charged-current cross sections on selected Po (upper panel), Rn (second panel), Ra (third panel), Th (fourth panel) and U (lower panel) nuclides. The total cross sections are shown by circles and the partial fission cross section by squares. A Fermi-Dirac spectrum with temperature $T_\nu = 4$ MeV and zero chemical potential has been assumed for the $\nu_e$ neutrinos.

**FIG. 2.** Total $(\nu_e, e^-)$ (circles) and partial $(\nu_e, e^- n)$ (triangles) and neutrino-fission cross sections (squares) for selected Th (left panels) and U (right panels) nuclides. The calculations have been performed for Fermi-Dirac neutrino spectra with temperature $T_\nu = 4$ MeV and $8$ MeV and zero chemical potential.
tion is significant, amounting to about 4 MeV on average. This result is in good agreement with earlier estimates for heavy nuclei [9]. Although Fig. 3 shows some scatter among the studied nuclei caused by structure effects, we will assume that $U(T)$ for supernova ($\nu_e, e^-$) reactions on neutron-rich nuclei is lowered by 4 MeV compared to its ground state value. This allows for some interesting conclusions, which are rather independent of the chosen fission barriers and neutron separation energies. For charged-current reactions with supernova $\nu_e$ neutrinos one then has $P_f = 0.5$ for $B_f - S_n \sim 2.4$ MeV and $P_f = 0.2$ for $B_f - S_n \sim 3.7$ MeV, where $B_f$ and $S_n$ are the tabulated values appropriate for low excitation energies. Note that the predicted fission barrier heights vary quite significantly where modern evaluations (i.e. [20]) give higher barriers for neutron-rich nuclei than did earlier work (i.e. [7]). The recent fission barriers of [20] predict a fission probability $P_f > 0.2$ for most nuclei with $Z > 92$ and $A > 230$, with the exception of nuclei with lower $Z$-values around the potentially magic neutron number $N = 184$. The fission barriers of M"oller and Howard [7] allow for significant spontaneous fission probabilities for nuclei with $Z > 87$ and $A > 230$, including those around $N = 184$. However, nuclei with $A < 230$ have a smaller fission probability in neutrino-induced reactions than tentatively assumed by Qian [3].

Nuclei on the r-process path have $S_n \sim 1.5 - 2.5$ MeV. Such nuclei fission after excitation by neutrinos with probability $P_f > 0.2$ if $B_f \sim 5.2 - 6.2$ MeV. This condition is satisfied for some nuclei on the r-process path with $Z \geq 96$ for the fission barriers of [20] and for $Z \geq 88$ for those of [7], which are, however, likely too small for neutron-rich nuclei [6]. This would imply that neutrino-induced reactions can initiate a fission cycle, if the r-process production of superheavy elements occurs in a noticeable neutrino fluence. Such a scenario might be conceivable, if neutrino oscillations occur. In contrast, the fission barriers of [20] are too high to allow for fission cycling during the r-process by $\beta$-delayed or neutron-induced fission.

We note that the typical fission cross section for Th and U isotopes ($\sim 400 \cdot 10^{-42}$ cm$^2$) corresponds to a half-life of $\sim 0.08$ s, assuming a neutrino reaction at a radius of 100 km above the neutron star and a typical supernova $\nu_e$ luminosity of $10^{52}$ erg s$^{-1}$. Such a half-life is shorter than the expected half-lives for the r-process waiting point nuclei with $N = 126, A > 195$ [15] (and also with $N = 184, A > 280$ [16]). These typical half-lives are also comparable and may be shorter than the typical $\sim 0.1$ s expansion timescale in “slow” neutrino-driven wind models. Thus, if the r-process occurs in a strong neutrino fluence neutrino-induced fission on the progenitor nuclei during the decay to stability might affect the relative Th/U r-process abundance. This abundance ratio is a necessary theoretical ingredient if one wants to deduce an age limit for the universe from the recently observed Th/U abundance ratios in old galactical halo stars [10].

The leverage that neutrino capture-induced fission has in an r-process set in a neutrino-driven wind is dependent on the $\nu_e$ energy spectrum and on the neutrino fluxes at the position where the neutrons are captured. Models with an extremely fast outflow rate (\cite{11,12}) generally have neutron capture occurring far from the neutron star where neutrino fluxes are low and, hence, neutrino capture-induced fission effects could be scant, though post-processing fission could still be significant.

Models with a slow outflow rate suffer from a deficit of neutrons [2] associated with the “alpha effect.” However, these models can yield a viable r-process close to the neutron star if neutrino flavor mixing effects are invoked (\cite{13,14}). A hierarchical neutrino energy spectrum, one where the mu and tau flavor neutrinos are more energetic than the electron neutrinos remains a possibility for at least some epochs following the bounce of the supernova core. In this case, matter-enhanced neutrino flavor transformation can play an important role in determining the efficacy of neutrino capture-induced fission, by making the average energies of the electron neutrinos larger and, hence, boosting fission probabilities in the region where neutrons are being captured in the r-process. (This is demonstrated in Fig. 2 for a T=8 MeV neutrino spectrum.) Crudely, the relationship between radial distance $r_6$ from the neutron star’s center in units of 10 km and the temperature $T_9$ in billions of Kelvins is $r_6 \approx 22.5/(S_{100}T_9)$, where $S_{100}$ is the entropy per baryon in units of hundreds of Boltzmann’s constant. Typically, neutron capture in the “slow outflow” schemes takes place in the region where $1 < T_9 < 3$. The loca-
Here neutrino capture-induced fission alone is problematic. If, in steady state flow, every seed nucleus is brought by neutrino capture to a nuclear mass where the fission cross section is greater than some threshold value, $\sigma_f^{th}$, then fission of this nucleus will result. Over a time $\Delta t \sim 2\tau_{dyn}$ there will be only some $\sim 72 (\lambda_f/300 \text{s}^{-1}) (\Delta t/0.03 \text{s}) (N/8)$ neutrons liberated per threshold nuclear mass, where $N$ is the assumed number of neutrons liberated per fission. Sustaining steady state fission cycle flow would require the liberation of some 70 to 100 neutrons per fission fragment (mass $\sim 130$) and this is clearly untenable. Nevertheless, a more modest number of neutrons liberated per fission coupled with the large rate of mass 130 fission fragment production could represent a significant alteration in the r-process flow. At the very least it shows that the mass 130 and 195 peaks should have comparable abundances.

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\[ T_{9,MSW} \approx 1.3(20 \text{MeV}/E_{\nu})^{1/3}(0.42/(Y_e + Y_{\nu}))^{1/3} \times S_{100}^{1/3}(3 \times 10^{-3}\text{eV}^2/\delta m^2 \cos 2\theta)^{1/3} \]

where $\delta m^2$ is the relevant difference of the squares of the vacuum neutrino mass eigenvalues (scaled here by the atmospheric neutrino value) and $\theta$ is the effective two-neutrino vacuum mixing angle, which for the $\nu_e \leftrightarrow \nu_{\mu,\tau}$ transformation channel is a strictly three-neutrino mass/mixing scheme would be roughly $\theta_{13} < 0.15$. The experimental upper limit on this mixing angle precludes an adiabatic transformation at resonance in a straight MSW scheme, but we note that large effective matter-mixing could occur (depending on entropy and overall neutrino luminosity) on account of the flavor basis off-diagonal neutrino-neutrino forward scattering contributions to the weak potential which determines neutrino effective masses and matter mixing angles. In this equation $Y_e$ is the electron fraction and $Y_{\nu}$ is the effective neutrino number fraction which enters into the neutrino forward scattering potential. Note that $Y_{\nu}$ can be negative when the neutrino background is important. In fact, the neutrino background potential can lead to near maximal flavor mixing in medium. Either way, the above expression and the implied location of significant flavor transformation is conservative: we may actually have a more energetic electron neutrino spectrum on account of "chaotic" maximal mixing well below this position. Crudely, the neutrino flux is $(5 \times 10^{42} \text{cm}^{-2} \text{s}^{-1})^{1/2} (10 \text{MeV}/\langle E_{\nu_e} \rangle) L_{\nu_e}^{51}$. Here $L_{\nu_e}^{51}$ is the effective electron neutrino luminosity in units of $10^{51} \text{ergs s}^{-1}$. If the entropy per baryon is $S_{100} = 2$, then the radius where a neutrino of energy $E_{\nu} = 20 \text{MeV}$ transforms is $r_6 \approx 7$ (corresponding to $T_9 \approx 1.6$) and we would expect the typical lifetime against fission per big nucleus (inverse fission rate) to be $\lambda_{f}^{-1} \approx 0.05 s/L_{\nu_e}^{51}$ where $\alpha = e, \mu, \tau$ is the flavor of the progenitor of the electron neutrino when it leaves the neutrino sphere; whereas, if $Y_e + Y_{\nu} = 0.1$, a possibility if neutrino mixing has been augmented by the neutrino background potential(s), then $r_6 \approx 4.3$ and $T_9 \approx 2.6$ (for $S_{100} = 2$) so that $\lambda_{f}^{-1} \approx 0.02 s/L_{\nu_e}^{51}$. In either case, these lifetimes are shorter than typical waiting point r-process beta decay lifetimes and are shorter than at least a plausible range of expansion time scales, $\tau_{dyn} \sim 0.15 s$. This implies that the neutron capture flow could proceed out to some threshold nucleide mass in the 195 peak or just beyond, whereupon fission sets in, producing two fission fragments in the 130 peak, as outlined by Qian.

To establish a steady state fission cycling scenario with neutrino capture-induced fission alone is problematic. If, in steady state flow, every seed nucleus is brought by neutron capture to a nuclear mass where the fission cross section is greater than some threshold value, $\sigma_f^{th}$, then fission of this nucleus will result. Over a time $\Delta t > 2\tau_{dyn}$ there

\[ \delta m^2 \text{MHz} \approx 2.5 \times 10^{-3} \text{eV}^2 \]

with $\cos 2\theta = 1$.