Cosmological gravitino problem confronts electroweak physics

Gi-Chol Cho$^a$ and Yosuke Uehara$^{b,c}$

$^a$ Department of Physics, Ochanomizu University, Tokyo 112-8610, Japan
$^b$ Theory Group, KEK, Ibaraki 305-0801 Japan
$^c$ Department of Physics, University of Tokyo, Tokyo 113-0033, Japan

Abstract

A generic feature of gauge-mediated supersymmetry breaking models is that the gravitino is the lightest supersymmetric particle (LSP). In order not to overclose the universe, the gravitino LSP should be light enough ($\lesssim 1$keV), or appropriately heavy to be consistent with the reheating temperature of the inflation. We study constraints on the mass of the gravitino from experiments below the electroweak scale – muon $g-2$, electroweak precision measurements, and the direct search experiments of supersymmetric particles at LEP2. We find that the heavy gravitino of $O$(GeV) is strongly disfavored from the lower mass bound on the next-to-LSP, unless $\tan \beta$ is large, say $\tan \beta \sim 50$. The sufficiently light gravitino, on the other hand, has rather sizable allowed parameter space for $\tan \beta \gtrsim 10$. 
Although the standard model (SM) of particle physics has been shown a good agreement with the results of high energy collider experiments, we expect that new physics beyond the SM lies in TeV scale, which stabilizes the weak scale by protecting the Higgs boson mass from the radiative correction. The Minimal Supersymmetric extension of the SM (MSSM) is the most promising candidate of new physics beyond the SM. In the MSSM, the quadratic divergence in the radiative correction of the Higgs boson mass is canceled between contributions from the particles in the SM and those from their supersymmetric partners. However, no supersymmetric particle has not been found yet, so SUSY must be broken softly. Therefore it is important to understand the mechanism of SUSY breaking and find constraints on the soft SUSY breaking terms from phenomenological point of view. One of the serious constraints on the SUSY breaking parameters comes from the processes mediated by the flavor changing neutral current (FCNC) such as $K^0-\bar{K}^0$ mixing, which require high degeneracy of the sfermion masses in the flavor space.

There are a few classes of SUSY breaking scenarios. Among them, gauge mediated SUSY breaking (GMSB) models [1] have been motivated to satisfy the phenomenological constraints on the soft SUSY breaking parameters from the FCNC processes. In general, GMSB consists of (i) a secluded sector where the supersymmetry is dynamically broken, (ii) the visible sector in which all the MSSM fields live, and (iii) the messenger fields that transmit the effect of SUSY breaking from the secluded sector to the visible sector via the ordinary gauge interactions. As a result, since the gauge interaction is flavor blind, there is no dangerous flavor violating source in the SUSY breaking parameters, and the phenomenological constraints from the FCNC on the SUSY breaking parameters are satisfied.

The most striking feature of GMSB is that the gravitino is the Lightest Supersymmetric Particle (LSP)\(^1\). In general, the energy density of the stable gravitinos may exceed the critical density of the universe, which is so called the cosmological gravitino problem [2]. Since the gravitinos are produced more at higher temperature, the gravitino problem leads to the upper bound on the temperature, \(T_{\text{max}}\). The upper bound on \(T_{\text{max}}\) for different mass scale of the gravitino is given by [3, 4]

\[
T_{\text{max}} \lesssim \begin{cases} 
100\text{GeV} - 1\text{TeV} & \text{for } 1\text{keV} \lesssim m_{3/2} \lesssim 100\text{keV} \\
10^9\text{GeV} \times \left( \frac{m_{3/2}}{1\text{GeV}} \right) \left( \frac{m_{\tilde{B}}}{100\text{GeV}} \right)^{-2} & \text{for } m_{3/2} \gtrsim 100\text{keV}
\end{cases}
\]

where \(m_{\tilde{B}}\) being the bino mass. In the inflationary universe, \(T_{\text{max}}\) corresponds

\(^1\)We assume the \(R\)-parity conservation.
to the reheating temperature $T_R$. Then, the temperature $T_{\text{max}}$ for lighter gravitino mass region is incompatible with the inflation scenario where the reheating temperature is typically $T_R \gtrsim 10^8 \text{GeV}$. The heavier gravitino LSP is, therefore, favored rather than the lighter one unless a certain substantial entropy production mechanism below $T_R$ is introduced [3]. It should be noted that the overclosure problem via the gravitino LSP disappears if the gravitino mass is small enough, say, $m_{3/2} \leq 1 \text{keV}$ [2].

In addition to the bound on $T_{\text{max}}$, the heavier gravitino LSP ($m_{3/2} \gtrsim 100 \text{MeV}$) has another constraint associated with the Next-to-LSP (NLSP). The lifetime of NLSP may be comparable with the Big-Bang Nucleosynthesis (BBN) era, so that the decay of NLSP might affect the abundance of the light elements. The constraints on $m_{3/2}$ and $T_{\text{max}}$ are examined in ref. [4] taking into account of the abundance via the NLSP decay and found the allowed region as $m_{3/2} = 5 - 100 \text{GeV}$ and $T_R = 10^9 - 10^{10} \text{GeV}$ when the stau is NLSP. If the neutralino is NLSP it gives rise to more severe constraint on the reheating temperature because of its small annihilation cross section and relatively larger abundance as compared to the stau NLSP.

In this letter, we study constraints on the parameter space of GMSB, taking into account of the results of muon $g - 2$ experiments at BNL [5] and the electroweak precision measurements at LEP and SLC [6]. We would like to pay a special attention to that if there are further constraints on the gravitino mass scale from these experimental data, in addition to the cosmological constraints. In the following, we do not assume any entropy production mechanisms below $T_R$, so that the cosmologically favored gravitino mass scale is limited to $m_{3/2} \leq 1 \text{keV}$ or $m_{3/2} \gtrsim O(\text{GeV})$. The former is the upper bound of the gravitino mass without the gravitino problem [2] while the latter respects the study of ref. [4]. We will show that the allowed region of the gravitino mass is sensitive to the muon $g - 2$ and the NLSP search experiments, and the heavy gravitino might be allowed only in a small corner of the parameter space.

Let us first briefly review the parameter set of GMSB model to fix our notation. The fundamental parameters in GMSB model can be summarized as follows [7]:

\begin{equation}
M_m, \Lambda, k, N_m, \tan \beta, \text{sgn}(\mu).
\end{equation}

The first four parameters in (2) are related to the SUSY breaking sector and the messenger sector. $M_m$ is the mass scale of messenger fields and $\Lambda$ denotes the scale of soft SUSY breaking parameters in the MSSM. The positivity of the messenger
squared mass requires $\Lambda < M_m$ \cite{7}. The dimensionless parameter $k(\leq 1)$ is the ratio of the fundamental scale of SUSY breaking and the SUSY breaking scale felt by the messenger fields. For the messenger sector, we assume the simplest structure and that the gauge coupling unification is preserved. Thus the integer $N_m$ represents the number of messenger fields which transform as $\mathbf{5} + \mathbf{\bar{5}}$ (or $\mathbf{10} + \mathbf{\bar{10}}$) in SU(5). $\tan \beta$ is defined by the ratio of two vacuum expectation values $v_u$ and $v_d$, which corresponds to the Higgs fields with the hypercharge $Y = 1/2$ and $-1/2$, respectively. The last parameter in (2) is the sign of the higgsino mass $\mu$. Then, the soft SUSY breaking parameters in the MSSM at a certain high energy scale are expressed in terms of (2), and those at the weak scale can be obtained by solving the renormalization group equations (RGE). The RGE for SUSY breaking parameters in GMSB can be found, for example, in ref. [7]. The gravitino mass $m_{3/2}$ is given by using $M_m$, $\Lambda$ and $k$ as follows:

$$m_{3/2} = \frac{\Lambda M_m}{k \sqrt{3} M_{Pl}},$$

(3)

where $M_{Pl}$ is the reduced Planck mass.

Next we summarize the set of experimental data which we adopt in our analysis. The anomalous magnetic moment $(g - 2)$ of the muon has been measured precisely at BNL. Using the convention $a_\mu = (g - 2)/2$, the current data is given as \cite{5} 

$$a_\mu(\text{expt}) = 11659203(8) \times 10^{-10},$$

(4)

while the SM prediction is

$$a_\mu(\text{th}) = 11659177(7) \times 10^{-10}.$$ 

(5)

Theoretical prediction on $a_\mu$ has a large uncertainty due to the hadronic contributions. Although there are still a number of estimation on the hadronic contributions using various methods, and those results are not converged, we use eq. (5) as the SM prediction in our study. Then the difference between the experimental data and the SM prediction is given as

$$\Delta a_\mu = 26(10) \times 10^{-10},$$

(6)

which shows 2.6-$\sigma$ discrepancy, and we adopt this as a constraint on the SUSY contribution.

The supersymmetric contributions to the muon $g - 2$ come from the 1-loop diagrams mediated by (i) chargino-sneutrino exchange and (ii) neutralino-smuon...
exchange. The size of effects from these diagrams is proportional to $\tan \beta$, while
the sign is consistent with (6) if the sign of $\mu$-parameter is positive [8].

The electroweak precision measurements, i.e., $Z$-pole observables from LEP1
and SLC, and the $W$-boson mass from LEP2 and Tevatron, may also constrain
the parameter space of GMSB. The electroweak data which we use in our study consists
of 17 $Z$-pole observables and the $W$-boson mass. The $Z$-pole observables include
8 line-shape parameters $\Gamma_Z, \sigma_k^b, R_\ell, A_{FB}^{0,\ell}(\ell = e, \mu, \tau)$, two asymmetries from the $\tau$-
polarization data ($A_\tau, A_e$), the decay rates and the asymmetries of $b$- and $c$-quarks
($R_b, R_c, A_{FB}^{0,b}, A_{FB}^{0,c}$) and the asymmetries measured at SLC ($A_{L,R}^0, A_b, A_c$). The
experimental data of these observables which we use in our analysis are summarized
in ref. [6]. Taking into account the $m_t$ data from the Tevatron[9], $\alpha_s(m_Z)$ [10] and
$\alpha(m_Z^2)$[11], we find that the SM best fit gives $\chi^2/\text{(d.o.f.)} = 21.4/(21 - 4)$ (21% CL).
The supersymmetric particles affect the electroweak observables radiatively
through the universal gauge-boson propagator corrections (oblique corrections)
and process specific vertex/box corrections. It has been shown that, the contributions
from squark and sleptons to the electroweak observables always make the fit
to the experimental data worse than the SM if they are as light as $O(100\text{GeV})$ [12].

In GMSB, the NLSP is either the lightest neutralino $\tilde{\chi}_1^0$ or the lighter stau $\tilde{\tau}_1$.
As already mentioned, the BBN constraint favors the stau NLSP rather than the
neutralino when the gravitino is rather heavy, $m_{3/2} \gtrsim 100\text{MeV}$ [4]. The lower mass
bounds on the NLSP at direct search experiments are given as [13]

$$m_{\text{NLSP}} > \begin{cases} 
55\text{GeV} & \text{for } \tilde{\chi}_1^0 \text{ NLSP} \\
77\text{GeV} & \text{for } \tilde{\tau}_1 \text{ NLSP}
\end{cases} \quad (7)$$

In Fig. 1 we show that the allowed region on the $(\Lambda, M_m)$ plane from the
direct search experiments of NLSP (7). In practice, we fix the parameters $k$ and
$N_m$ in eq. (2) by $N_m = k = 1$ for simplicity. We also choose $\mu > 0$ to be
consistent with the muon $g - 2$ constraint (6). The $\tan\beta$ dependence is examined
for $\tan\beta = 3, 10, 30$ and 50. In the figure, the solid line denotes $M_m = \Lambda$, and we
consider the region that satisfies $M_m > \Lambda$ [7]. The dark region (labelled “EWSB”)
is excluded since the electroweak symmetry is not broken radiatively. The excluded
regions from the $\tilde{\chi}_1^0$ or $\tilde{\tau}_1$ NLSP search experiments (7) are shown by shaded areas
with different pattern. The blank space corresponds to the region where the direct
search bound on the $\tilde{\chi}_1^0$ NLSP (7) is satisfied. In the analysis the lower mass
bounds on the lighter chargino, $m_{\tilde{\chi}_1^\pm} > 104\text{GeV}$ [14], and the lightest Higgs boson,
Figure 1: Allowed region on the \((\Lambda, M_{m})\) plane from the radiative electroweak symmetry breaking condition and the NLSP direct search for \(\tan \beta = 3\) (a), \(10\) (b), \(30\) (c) and \(50\) (d). The solid line in each graph shows \(M_{m} = \Lambda\). The dark shaded region (labeled “EWSB”) denotes that the electroweak symmetry is not broken radiatively. The excluded region from the direct search limit on the NLSP (\(\tilde{\chi}_{1}^{0}\) or \(\tilde{\tau}_{1}\)) are shown explicitly. In the blank region above the \(M_{m} = \Lambda\) line, the NLSP is the neutralino. One can find the allowed region from the direct search limit on the stau NLSP \([13]\) in (c) and (d).

\(m_{h} > 91\,\text{GeV}\) \([15]\) from the LEP2 experiments are included, and they do not reduce the allowed region of \(\tilde{\chi}_{1}^{0}\) or \(\tilde{\tau}_{1}\) NLSP in Fig. 1. It is remarkable that the allowed region of the stau NLSP appears only when \(\tan \beta\) is rather large (Figs. 1(c) and (d)), so that the heavier gravitino \(m_{3/2} = 5 - 100\,\text{GeV}\) associated with the cosmological gravitino problem \([4]\) is strongly constrained from the stau NLSP search experiments.

Let us examine constraints on GMSB models from the muon \(g - 2\) and the electroweak precision data for \(\tan \beta = 3\) and \(10\) in Fig. 2. In addition to the NLSP constraint, we superpose the gravitino mass range \((1\,\text{eV} < m_{3/2} < 1\,\text{keV})\).
Figure 2: Constraints on the $(\Lambda, M_m)$ plane from the electroweak precision measurements and the muon $g - 2$ experiments for $\tan \beta = 3$ (a) and 10 (b). The gravitino mass range is shown for $1\text{eV} \leq m_{3/2} \leq 1\text{keV}$ and $5\text{GeV} \leq m_{3/2} \leq 100\text{GeV}$, respectively. The enclosed regions by the dotted line denotes $\Delta \chi^2 < 4$ while those by the long-dashed line denotes $\Delta \chi^2 < 1$ for the electroweak precision data. The 2-$\sigma$ allowed region of the muon $g - 2$ experiments is shown explicitly in (b). In (a), the allowed region of the muon $g - 2$ is hidden by the $\tilde{\chi}_1^0$ NLSP excluded region.

and $5\text{GeV} < m_{3/2} < 100\text{GeV}$), the 2-$\sigma$ allowed region of the muon $g - 2$ data, and the contours for $\Delta \chi^2 = 1$ and 4 for the electroweak data onto the $(\Lambda, M_m)$ plane. It is easy to see that there is no allowed region of the muon $g - 2$ data in Fig. 2(a) ($\tan \beta = 3$). As is already mentioned, the SUSY contribution to the muon $g - 2$ is proportional to $\tan \beta$. When $\tan \beta$ is small, therefore, relatively light SUSY particles are required for sizable contributions to the muon $g - 2$, and such parameter region in Fig. 2(a) is inconsistent with the direct search limit on the $\tilde{\chi}_1^0$ NLSP mass. When $\tan \beta$ is larger, Fig. 2(b), we find the allowed region for the light gravitino with $m_{3/2} < 1\text{keV}$, where constraints from the muon $g - 2$, the electroweak precision measurements, and the direct search on the $\tilde{\chi}_1^0$ NLSP are satisfied simultaneously.

In Fig. 3, we show constraints on the model parameter space for $\tan \beta = 30$ (a) and 50 (b). When $\tan \beta = 30$, we find that, in sizable region, the lighter gravitino is consistent with whole experimental constraints. The heavier gravitino, however, is again disfavored because of the lower mass bound on the stau NLSP from collider experiments. Fig. 3(b) shows that the fit to the electroweak precision data at the lighter gravitino region may be worse ($\Delta \chi^2 > 4$) than the case for smaller
Figure 3: Constraints on the \((\Lambda, M_m)\) plane from the electroweak precision measurements and the muon \(g-2\) experiments for \(\tan \beta = 30\) (a) and 50 (b). The gravitino mass range is shown for \(1\text{eV} \leq m_{3/2} \leq 1\text{keV}\) and \(5\text{GeV} \leq m_{3/2} \leq 100\text{GeV}\), respectively. The enclosed regions by the dotted line denotes \(\Delta \chi^2 < 4\) while those by the long-dashed line denotes \(\Delta \chi^2 < 1\). The \(2\sigma\) allowed region of the muon \(g-2\) experiments is shown explicitly. The enclosed region by the thick solid-line in (b) denotes that the \(\tilde{\tau}_1\) mass satisfies the direct search limit of the NLSP, \(i.e., m_{\tilde{\tau}_1} > 77\text{GeV}\).

tan \(\beta(\leq 30)\). For the heavier gravitino, on the other hand, there is very small region in which the bounds from the stau NLSP and the muon \(g-2\) are compatible. From these analysis, we find that the lower mass bound on the stau NLSP is most stringent constraint for the heavier gravitino, and which could be possible only when \(\tan \beta\) is large, say \(\tan \beta \sim 50\).

We have so far performed our analysis by fixing the parameters \(k\) and \(N_m\) in (2) to be one. It may be helpful to mention about the \((k, N_m)\) dependence of our analysis. First, the \(k\)-parameter is related to the gravitino mass through (3). When \(k\) is smaller than 1, the gravitino mass increases for fixed \(\Lambda\) and \(M_m\). This means that the gravitino mass range on the \((\Lambda, M_m)\) plane in our study is lowered for \(k < 1\), in parallel with the range for \(k = 1\). It is easy to see that the constraints on both the heavier and lighter gravitinos are not altered so much for \(k < 1\). The dependence on \(N_m\) of the result is rather complicated because it reflects the detail of the SUSY breaking sector. In general, the soft SUSY breaking parameters tend to be large as \(N_m\) increases, so that the constraints on \((\Lambda, M_m)\), \(i.e.,\) the NLSP mass, may be weaker when \(N_m > 1\).
To summarize, we have studied constraints on the model parameter space of the GMSB taking into account of the muon $g-2$ experiment, the electroweak precision measurements and the direct search experiments on the NLSP. The main interests of our study is to learn that if these experimental results affect the gravitino mass scale which are allowed from the cosmological gravitino problem without any entropy production mechanism below the reheating temperature of the inflation. In our study, we focused on two different gravitino mass scales, $m_{3/2} < 1$keV and $5$GeV $< m_{3/2} < 100$GeV. The former is free from the cosmological gravitino problem while the latter has been obtained taking into account of the BBN constraint on the NLSP decay [4]. We find that both possibilities are disfavored from the muon $g-2$ data and/or the NLSP direct search experiments when $\tan \beta = 3$. For $\tan \beta > 10$, the model parameter space of the light gravitino mass can be compatible with the low-energy experiments in sizable parameter region. On the other hand, the heavier gravitino is strongly disfavored from the lower mass bound on the stau NLSP and the muon $g-2$ experiments, and could be allowed only when $\tan \beta$ is large enough, say $\tan \beta \sim 50$. The possibility of heavier gravitino of $O$(GeV), therefore, is pushed to a small corner of the parameter space.

The authors thank K. Hamaguchi, M. Fujii for stimulating discussions. We are also grateful to C. Kao for allowing us to use his computer program to solve the RGE of GMSB. Y.U. also thank Japan Society for the Promotion of Science for financial support. The work of G.C.C. is supported in part by the Grant-in-Aid for Science Research, Ministry of Education, Science and Culture, Japan (No.15740146).

References


