Vacuum Condensates and Dynamical Mass Generation in Euclidean Yang-Mills Theories

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Abstract

Vacuum condensates of dimension two and their relevance for the dynamical mass generation for gluons in Yang-Mills theories are discussed.

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1 Introduction

Vacuum condensates of dimension two in Yang-Mills theories have witnessed increasing interest in recent years, both from the theoretical point of view as well as from lattice simulations, which have provided rather strong evidence for an effective dynamically generated gluon mass. Here we shall report on our current work on these condensates. Our aim is that of defining a renormalizable effective potential and evaluating it in analytic form. The condensates arise thus as nontrivial solutions of the gap equation corresponding to the minimization of the effective potential. The content of this work is as follows. In Sect. 2 we review the gluon condensate $\langle A^2 \rangle$ in the Landau gauge. In Sect. 3 the mixed gluon-ghost condensate $\langle 1/2 A^a_\mu A^a_\mu + \xi c^a c^a \rangle$ is analysed in the Maximal Abelian and Curci-Ferrari gauges. Sect. 4 contains a few remarks on the gauge (in)dependence of the obtained effective gluon mass and on possible relationships with lattice simulations. We conclude with a short discussion on the operator $A^2$ in linear covariant $\alpha$-gauges.

2 The condensate $\langle A^2 \rangle$ in the Landau gauge

The gluon condensate $\langle A^2 \rangle = \langle A^a_\mu A^a_\mu \rangle$ in the Landau gauge has been introduced [1, 2] in order to account for the discrepancy between the expected perturbative behavior and the lattice results concerning the two and three point functions in pure Yang-Mills theories. The lattice estimate for the condensate is [1] $\langle A^2 \rangle \approx (1.64 \text{ GeV})^2$. A simple argument shows that $\int d^4 x A^2$ is invariant under infinitesimal gauge transformations, $\delta A^a_\mu = -(D_\mu \omega)^a$, in the Landau gauge, $\partial A = 0$, namely $\delta \int d^4 x A^2 = 0$. In the BRST framework, it turns out that $\int d^4 x A^2$ is BRST invariant on shell

$$s \int d^4 x A^2 = 0 + \text{Eqs. motion} \quad (1)$$

This property ensures that the local operator $A^2$ is multiplicatively renormalizable to all orders of perturbation theory [3]. Its anomalous dimension $\gamma_{A^2}$ can be expressed [3] as a combination of the gauge beta function $\beta$ and of the anomalous dimension $\gamma_A$ of the gauge field $A^a_\mu$, i.e.,

$$\gamma_{A^2} = -\left( \frac{\beta(a)}{a} + \gamma_A(a) \right), \quad a = \frac{g^2}{16\pi^2} \quad (2)$$

Concerning the analytic evaluation of $\langle A^2 \rangle$, the two-loop effective potential for $\langle A^2 \rangle$ has been obtained [4] in pure Yang-Mills theory by combining the Local Composite Operators technique with the Renormalization Group Equations. This has led to a gap equation whose weak coupling solution gives a nonvanishing condensate, resulting in an effective gluon mass $m_{\text{gluon}} \approx 500 \text{MeV}$. Recently, the inclusion of massless quarks has been worked out [5]. It is worth mentioning that an effective gluon mass has been reported in lattice simulations in the Landau gauge [6], yielding $m_{\text{gluon}} \approx 600 \text{MeV}$. 
3 Other Gauges

3.1 The Maximal Abelian Gauge

The so called Maximal Abelian gauge (MAG) plays an important role for the dual superconductivity picture for confinement. In the MAG, the gauge field is decomposed according to the generators of the Cartan subgroup of the gauge group. For SU(2), we have $A_\mu^a T^a = A_\mu T^3 + A_\mu^a T^a$ with $\alpha = 1, 2$. The gauge fixing term in the MAG is given by

$$
\int d^4x \left[ \frac{1}{2\xi} F^\alpha F_\alpha - \tau^{\alpha\beta} M^{\alpha\beta} c^\beta - g^2 \xi \left( \tau^{\alpha\beta} c^\beta \right)^2 \right]
$$

(3)

where $\xi$ is the gauge parameter and

$$
M^{\alpha\beta} = (D^\alpha A_\mu^\beta + g^{\alpha\beta} A_\mu A^\beta)
$$

(4)

The MAG allows for a residual local $U(1)$ invariance, which has to be fixed later on. Lattice simulations have shown that the off-diagonal components $A_\mu^\beta$ acquire a mass [7, 8], reporting $m_{\text{off-diag}} \approx 1.2 GeV$. The operator $A^2$ can be generalized to the MAG [9]. Indeed, the gluon-ghost dimension two operator $O_{\text{MAG}} = \left( \frac{1}{2} A^2 + \xi c^a c^a \right)$ turns out to be BRST invariant on-shell, namely

$$
\int d^4x \, O_{\text{MAG}} = 0 + \text{Eqs. motion}
$$

(6)

The condensate $\langle O_{\text{MAG}} \rangle$ could thus provide effective masses for the off-diagonal components. However, at present, very little is known about the condensate $\langle O_{\text{MAG}} \rangle$. Concerning the UV properties of $O_{\text{MAG}}$, it has been proven [10, 11] to be multiplicatively renormalizable. Its anomalous dimension can be expressed as [10, 11]

$$
\gamma_{O_{\text{MAG}}} = -2 \left( \beta(a) \frac{a}{2} + \gamma_{c\text{diag}} \right)
$$

(7)

where $\gamma_{c\text{diag}}$ is the anomalous dimension of the diagonal ghost field.

3.2 The Curci-Ferrari gauge

The Curci-Ferrari gauge shares similarities with the MAG, providing useful information on the gluon-ghost condensate. The gauge fixing term is

$$
\int d^4x \left[ \frac{(\partial A_a)^2}{2\xi} + c^a \partial_\mu D_\mu c^a + \frac{\xi g}{2} f^{abc} \partial A_\mu^{a-b} c^c - \frac{\xi g^2}{16} f^{abc} c^b f^{mnc} c^m c^n \right]
$$

(8)

where $a, b, c = 1, ..., (N^2 - 1)$ for $SU(N)$. For the gluon-ghost operator[9] in the CF gauge we have $O_{\text{CF}} = \left( \frac{1}{2} A^2 + \xi c^a c^a \right)$. The operator $O_{\text{CF}}$ is renormalizable to all orders, its anomalous dimension being given by [11]

$$
\gamma_{O_{\text{CF}}} = -2 \left( \gamma_c + \gamma_{ge^2} \right)
$$

(9)
where \( \gamma_{g^2} \) is the anomalous dimension of the composite operator corresponding to the BRST variation of the ghost \( c \), i.e., \( sc = \frac{g}{2} f^{abc} \delta_{bc}c \). Recently, the effective potential for \( O_{CF} \) has been obtained [12], leading to a nonvanishing condensate \( \langle O_{CF} \rangle \) and to a dynamical gluon mass. Something similar is expected to happen in the MAG.

4 Open problems and future perspectives

Many aspects concerning the gauge condensates of dimensions two are still open, deserving a better understanding. Two important questions which arise almost immediately are:

- 1) are these condensates related to the effective gluon masses observed in lattice simulations?
- 2) to what extent are these condensates and the related gluon masses gauge (in)dependent?

Concerning the first question, it could be useful to observe that, to the first approximation, the condensates \( \langle A^2 \rangle, \langle A^2/2 + \xi cc \rangle \) modify the euclidean gluon propagator according to

\[
\frac{1}{k^2} \rightarrow \frac{1}{k^2 + m_{\text{eff}}^2}
\]

This suggests that the lattice data could be fitted with a formula which contains a Yukawa type term. This seems to be the case for the off-diagonal masses in the MAG [7, 8]. Also, it is worth remarking that the fitting formula employed in the lattice simulations for the gluon propagator in the Landau gauge [6]

\[
\sum_{a, \mu} \langle A^a_\mu(k) A^a_\mu(-k) \rangle = \frac{N}{k^2 + m_A^2} \left[ \frac{1}{k^4 + m_A^2} + \frac{s}{(\log\left(m_L^2 + k^2\right))^{13/22}} \right]
\]

contains in fact a Yukawa term. Expression (11) fits well the lattice data with \( m_A = 0.64 \text{GeV}, \ m_2 = 1.31 \text{GeV}, \ m_L = 1.23 \text{GeV}, \ s = 0.32, \ N = 8.1 \)

Concerning the second question, we look at other gauges allowing for a local renormalizable operator which could condense, providing thus an effective gluon mass. That is why we are studying in detail the local operator \( A^2 \) in linear covariant \( \alpha \)-gauges, whose gauge fixing term is

\[
\int d^4x \left( \frac{1}{2\alpha} (\partial A^a)^2 - \partial_\mu \bar{c}^a D_\mu c^a \right)
\]

Notice that in this gauge the operator \( A^2 \) is not BRST invariant on-shell. Nevertheless, we have strong indications of its multiplicative renormalizability to all orders of perturbation theory [13]. There is a simple understanding of this property. In linear \( \alpha \)-gauges, due to the shift symmetry of the antighost, i.e. \( \bar{c} \rightarrow \bar{c} + \text{const.} \), the operator \( A^2 \) cannot mix with the other dimension two ghost composite operator \( \bar{c}c \), which cannot show up due to the above symmetry. The renormalizability of \( A^2 \) is the first step towards the construction of a renormalizable effective potential in order to study the possible condensation of \( A^2 \) and the related dynamical mass generation [13]. Although this doesn’t yet answer the question of the gauge (in)dependence, it might increase the number of gauges displaying such an interesting phenomenon. Finally, it is worth underlining that an effective gluon mass has been reported in the Coulomb gauge [14] as well as in the lattice Laplacian gauge [15].
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