Nuclear Physics from QCD

S.R. Beane

Institute for Nuclear Theory,
University of Washington, Seattle, Washington 98195-1550, USA.

Recent attempts to make direct contact between QCD and simple nuclear systems are reviewed.

1. Introduction

It is now accepted that QCD is the theory which underlies all of nuclear structure. A fundamental question we may then ask is: How do nuclear energy levels change as we vary the quark masses in the QCD Lagrangian? This is not solely an intellectual exercise; there are hints that the fundamental parameters of the standard model, such as $\alpha_{em}$ and $m_q$, may be time-dependent [1]. The successful predictions of Big-Bang nucleosynthesis (BBN) can be used to constrain the time dependence of these parameters and thereby search for physics beyond the standard model [2][3]. However in order to do so one must know how nuclear physics depends on the fundamental parameters. A more practical motivation for understanding the quark mass dependence of nuclear physics, and more generally of hadronic physics, is that in the near future it is through lattice QCD simulations that definitive predictions in nuclear physics will be made directly from QCD. And because lattice QCD is currently simulated with unphysically large quark masses, there is an inevitable extrapolation to physical quark masses. If rigorous predictions are to be had from the lattice, this extrapolation must be controlled in a precise way. Fortunately, the quark-mass dependence of few-nucleon systems can be studied by exploiting hierarchies of scales: i.e. by using effective field theory (EFT) [4][5][6]. Given the current state of technology, simulations of multi-nucleon systems are intractable, but realistic simulations of two-nucleon systems are feasible. Arguably, the most promising method is to calculate scattering phase shifts directly using Lüscher’s finite-volume algorithm, which, for instance, expresses the ground-state energy of a two-particle state as a perturbative expansion in the scattering length divided by the size of the box [7]. This method has been used successfully to study $\pi\pi$ scattering [8]. Attempts have been made to compute nucleon-nucleon (NN) scattering parameters in lattice QCD; Ref. [9] computes the $^1S_0$ and $^3S_1$ scattering lengths in quenched QCD (QQCD) using Lüscher’s method. Of course in the NN system, the scattering lengths are much larger than the characteristic physical length scale given by $\Lambda^{-1}_{QCD}$. Nevertheless one may expect that at some unphysical value of the pion mass used in a lattice simulation, the scattering lengths relax to natural values, thus allowing their determination from the lattice. This is yet another motivation for understanding the quark-mass dependence of few-nucleon systems.
Presently, unquenched lattice simulations with the physical values of the light-quark masses are prohibitively time-consuming, even on the fastest machines. While some quenched calculations can be performed with the physical quark masses there is no limit in which they reproduce QCD, and consequently they should be considered to be a warm-up exercise for the “real thing”. Relatively recently it was realized that partially-quenched (PQ) simulations, in which the sea quarks are more massive than the valence quarks, provide a rigorous way to determine QCD observables and are less time-consuming than their QCD counterparts. It is with PQ simulations that nuclear-physics observables will first be calculated rigorously from QCD. As PQQCD reproduces QCD only in the limit in which the sea-quark masses become equal to the valence-quark masses which, in turn, are set equal to the physical values of the light quark masses (we call this the QCD limit), there are some interesting features of the PQ theory that are distinct from nature away from the QCD limit. In QCD, the long-distance component of the NN potential is due to OPE, as discussed above. However, in PQQCD there is also a contribution from the exchange of the $\eta$-meson (in the theory with two flavors of light quarks). In QCD such an exchange is suppressed due to the large mass of the $\eta$ compared to the $\pi$. However, in PQQCD the $\eta$ propagator has a double-pole component that depends on the pion mass due to the hairpin interactions with a coefficient that depends upon the difference between the masses of the $\eta$ and $\pi$ quarks. Therefore, away from the QCD limit the long-distance component of the NN potential is dominated by one-eta exchange (OEE) and falls exponentially with a range determined by the pion mass, $\sim e^{-m_\pi r}$, as opposed to the familiar Yukawa type behavior [10]. All is not lost however: it is straightforward to develop the partially-quenched EFT which matches to a partially-quenched lattice simulation [11].

A second approach to the NN system on the lattice is to study the simplified problem of two interacting heavy-light particles [12]. It has been suggested that lattice QCD simulations of the potential between hadrons containing a heavy quark will provide insight into the nature of the intermediate-range force between two nucleons [13]. While the NN potential is not itself an observable, one may instead consider heavy systems. In the heavy-quark limit, the kinetic energy of the heavy hadrons is absent and the lowest-lying energy eigenvalues, which can be measured on the lattice, are given by the interaction potential. All discussions to date have addressed two heavy mesons. This case is somewhat complicated by the fact that there are degeneracies in the heavy-quark limit which require a coupled-channel analysis. By contrast, the $\Lambda Q\Lambda Q$ interaction (where the $\Lambda Q$ is the lowest-lying baryon containing one heavy quark, $Q$), does not suffer from this complication [14]. Moreover, since the $\Lambda Q$ is an isosinglet, there is no OPE, and the leading large-distance behavior is governed by two-pion exchange (TPE), which is physics analogous to the intermediate-range attraction in the NN potential.

In this proceedings I will review recent work in developing the EFTs relevant to understanding the quark-mass dependence of the NN system and the $\Lambda Q\Lambda Q$ potential, both in QCD and in PQQCD.
2. The Quark Mass Dependence of NN

During the last decade significant effort has been expended in constructing an EFT to describe nuclear physics. While it is straightforward to write down all possible terms in the effective Lagrangian for two or more nucleons, arriving at a consistent power-counting has proved to be a difficult task. Weinberg’s (W) original proposal [15] for an EFT describing multi-nucleon systems was to determine the NN potentials using the organizational principles of the well-established EFTs describing the meson-sector and single-nucleon sector (chiral perturbation theory, \(\chi\)PT), and then to insert these potentials into the Schrödinger equation to solve for NN wavefunctions. Observables are computed as matrix elements of operators between these wavefunctions. W power-counting has been extensively and successfully developed during the past decade to study processes in the few-nucleon systems [16]. This method is intrinsically numerical and is similar in spirit to traditional nuclear-physics potential theory. Unfortunately, there are formal inconsistencies in W power-counting [17], in particular, divergences that arise at leading order (LO) in the chiral expansion cannot be absorbed by the LO operators. Problems persist at all orders in the chiral expansion, and the correspondence between divergences and counterterms appears to be lost, leading to uncontrolled errors in the predictions for observables. This formal issue was partially resolved by Kaplan, Savage and Wise (KSW) who introduced a power-counting in which pions are treated perturbatively [18]. The NN phase-shifts and mixing angle in the \(^1S_0\) and \(^3S_1 - ^3D_1\) coupled-channels have been computed to next-to-next-to-leading order (N^2LO) in the KSW expansion by Fleming, Mehen and Stewart (FMS) [19] from which it can be concluded that the KSW expansion converges slowly in the \(^1S_0\) channel and does not converge in the \(^3S_1 - ^3D_1\) coupled-channels. Therefore, neither W or KSW power-counting provide a complete description of nuclear interactions.

Figure 1. The left (right) panel shows the scattering length in the \(^1S_0\)-channel (\(^3S_1\)-channel) as a function of the pion mass. The light gray region corresponds to \(\eta = 1/5\) and the black region corresponds to \(\eta = 1/15\). In the \(^3S_1\)-channel the parameter \(\tilde{d}_{16}\) is taken to be in the interval \(-2.61\ \text{GeV}^{-2} < \tilde{d}_{16} < -0.17\ \text{GeV}^{-2}\) and \(\tilde{d}_{18} = -0.51\ \text{GeV}^{-2}\).

The problems with W and KSW power-counting appear to have been resolved in Ref. [4], which from this point on I will refer to as BBSvK. It was realized in FMS that the contributions to the amplitude that lead to non-convergence in the \(^3S_1 - ^3D_1\) coupled-channels
persist in the chiral limit (it is the chiral limit of iterated one-pion-exchange (OPE) that is troublesome). Therefore, in BBSvK power-counting the scattering amplitude is an expansion about the chiral limit. This recovers KSW power-counting in the $^1S_0$ channel, where FMS found it to be slowly converging. However, in the $^3S_1-^3D_1$ coupled-channels, the chiral limit has contributions from both local four-nucleon operators and from the chiral limit of OPE. It is these two contributions that must be resummed using the Schrödinger equation to provide the LO scattering amplitude in the $^3S_1-^3D_1$ coupled-channels.

In recent papers by Savage and the author [5] and also by Epelbaum, Glöckle and Meißner [6] EFT was used to determine the $m_q$-dependence of scattering in the two-nucleon sector. Remarkably, in the $^1S_0$-channel KSW power-counting can be used to derive an analytic expression for the scattering length,

$$a(^1S_0) = \gamma + \frac{g^2 A N}{8 \pi f^2} \left[ m^2 \log \left( \frac{\mu}{m_\pi} \right) + (\gamma - m_\pi)^2 - (\gamma - \mu)^2 \right] - \frac{M N m^2}{4 \pi} (\gamma - \mu)^2 D_2 , \quad (1)$$

where $\gamma$ is a LO constant and $D_2(\mu)$ is a combination of coefficients of operators with a single insertion of $m_q$. Unfortunately, $D_2$ is presently unknown (in the $^3S_1-^3D_1$ channel $D_2$ contributes to, for instance, $\pi$-deuteron scattering, however only at one order beyond the current state-of-the-art [20][21]). The best that one can do at present is to use naive dimensional analysis (NDA) to estimate a range of reasonable values for $D_2$, defined by a parameter $\eta \ll 1$ [5]. The results of NDA are shown in Fig. 1. We use scatter plots as the point density represents the probability associated with a particular set of low-energy constants. NDA suggests that the di-neutron remains unbound in the chiral limit, while a relatively small increase in $m_q$ could lead to a bound di-neutron.

In the $^3S_1-^3D_1$ coupled channels the situation is somewhat more complicated. At NLO in BBSvK counting there is not only OPE, but also the chiral limit of TPE. As a
consequence, there are additional counterterms in the single nucleon sector that contribute in this channel but do not contribute in the $^1S_0$ channel, in particular $\mathcal{F}_{18}$ and $\mathcal{F}_{16}$ associated with the pion-nucleon interaction, and $\mathcal{F}_4$ associated with $f_\pi$. This is in addition to the $D_2$ contribution in the $^3S_1$ channel. The allowed regions for $\mathcal{F}_{18}$ and $\mathcal{F}_{16}$ are given in Ref. [22], and $\mathcal{F}_4$ is known. Fig. 1 shows the presently allowed values of the scattering length in the $^3S_1$ channel where we again have used NDA to estimate the possible values for $D_2$. It is clear that for the range of parameters considered the deuteron could be bound or unbound in the chiral limit, and at present one cannot make a more definitive statement. This last statement is in disagreement with the conclusion of Ref. [6]; a discussion of this disagreement is given in Ref. [5]. The quark-mass dependence of the deuteron binding energy is shown in Fig. 2. Notice that the $^3S_1$ scattering length relaxes to natural values of $\sim 1$ fm as the pion mass is increased beyond $\sim 200$ MeV (see Fig. 1). One anticipates similar behavior in the partially-quenched theory. Given current uncertainties in strong interaction parameters, particularly $D_2$, it is at present unclear whether the same is true in the $^1S_0$ channel (see Fig. 1).

As an interesting application of these results, Ref. [2] derive constraints on the time variation of the Higgs vacuum expectation value $\langle \phi \rangle$ through the effects on BBN, including the effect of the change in the deuteron binding energy, which alters both the $^4$He and deuterium abundances significantly. See Fig. 3.

![Figure 3](image-url)

Figure 3. The solid curves represent the constraints from the $^4$He and deuterium abundances, assuming no change in the deuteron binding energy (left panel) and a change in the deuteron binding energy as shown in Fig. 2. The region allowed by BBN is shaded.

It is quite simple to construct the partially-quenched effective field theory from the known QCD results. And too, it is gratifying to see that one can obtain analytic results for many observables in the $^1S_0$ channel and in the higher partial waves. For instance, the $^1P_1$ scattering volume is given by

$$a(^1P_1) = \frac{g_A^2 M_N}{4 \pi f^2 m_\pi^2} + \frac{g_0^2 M_N}{12 \pi f^2 m_\pi^2} \frac{m_{SS}^2 - m_\pi^2}{m_\pi^2},$$

(2)

where $g_0$ is an axial coupling and $m_{SS}$ is a Goldstone boson mass containing two sea quarks. Notice that the QCD limit agrees with the well-known results [23]. One should
be concerned about the range of sea and valence quark masses for which this theory converges. In QCD it is found that the NN EFT converges for $m_\pi$ and momenta less than of order $\Lambda_{NN} \sim 300$ MeV, and one suspects that the same radius of convergence will exist in the partially-quenched theory. If this is indeed the case, lattice calculations will be required with meson masses of less than $\sim 300$ MeV in order to match to the EFT and use it to make predictions about nature. This is somewhat more restrictive than in the meson and single nucleon sectors and therefore one would like to see convergent results in those sectors before being confident in results obtained in the multi-nucleon sectors.

3. The $\Lambda_Q\Lambda_Q$ Potential

The lowest-lying baryons containing a single heavy quark can be classified by the spin of their light degrees of freedom (dof), $s_l$, in the heavy-quark limit, $m_Q \to \infty$. Working with two light flavors, $u$ and $d$ quarks, the light dof of the isosinglet baryon, $\Lambda_Q$, have $s_l = 0$, while the light dof of the isotriplet baryons, $\Sigma_Q^{\pm1,0}$ and $\Sigma_Q^{\pm1,0*}$, (the superscript denotes the third component of isospin) have $s_l = 1$. In the heavy-quark limit the spin-$\frac{1}{2}$ $\Sigma_Q^{\pm1,0}$ baryons are degenerate with the spin-$\frac{3}{2}$ $\Sigma_Q^{\pm1,0*}$ baryons, but are split in mass from the $\Lambda_Q$ by an amount that does not vanish in the chiral limit. As the light dof in the $\Lambda_Q$ have $s_l = 0$ in the heavy-quark limit, the light-quark axial current matrix element vanishes, and thus there is no $\Lambda_Q\Lambda_Q\pi$ interaction at leading order in the heavy quark expansion. This means that there is no OPE (or OEE) contribution to the $\Lambda_Q\Lambda_Q$ potential in QCD and PQQCD, and therefore there is no long-distance component in the $\Lambda_Q\Lambda_Q$ potential. It is the TPE box and crossed-box diagrams that provide the longest-distance interaction between two $\Lambda_Q$'s. In addition, there are local four-$\Lambda_Q$ operators at the same order in the chiral expansion but such local interactions give coordinate-space delta-functions. Analogous diagrams in the two-nucleon sector provide part of the intermediate-range component of the NN potential. However, it is important to realize that there are additional interactions that contribute to the intermediate-range component of the NN potential in the chiral expansion, for instance contributions from the Weinberg-Tomazawa term and also from higher-dimension $NN\pi\pi$ vertices. Therefore, while the $\Lambda_Q\Lambda_Q$ potential provides a window into the nature of the intermediate-range NN interaction, it certainly does not provide a complete description.

If the $\Lambda_Q$ and $\Sigma_Q^{(*)}$ were degenerate, we would be required to solve the coupled-channel system with $\Lambda_Q\Lambda_Q$ and $\Sigma_Q^{(*)}\Sigma_Q^{(*)}$ coupled to $I = 0$. In the charmed sector the $\Sigma_c - \Lambda_c$ mass splitting is $\Delta = 167.1$ MeV and the $\Sigma_c^* - \Lambda_c$ mass splitting is $\Delta = 232.7$ MeV, and we use the spin-weighted average of these splittings to estimate $\Delta \sim 211$ MeV in the heavy-quark limit. There is no symmetry reason for this mass-splitting to vanish in the chiral limit, and hence there is no infrared divergence that requires a coupled-channel analysis. In the power-counting we treat $\overline{\Delta} \sim m_\pi$ and take the $M_{\Lambda_Q}, M_{\Sigma_Q} \to \infty$ limit in evaluating the diagrams in Fig. 4. With this power-counting, one can directly use the Feynman rules of Heavy-Baryon Chiral Perturbation Theory (HB$\chi$PT) to describe the low-momentum dynamics of the nucleon and $\Delta$-resonance, without the need to resum the baryon kinetic energy term as is the case for the box and crossed-box diagrams in the

\footnote{For three light flavors the baryons fall into a $6 \oplus 3$ of SU(3).}
nucleon sector. Evaluating the diagrams in Fig. 4 and then Fourier transforming them to position space is straightforward [14]. For asymptotically large distances, the potential is well-approximated by

\[ V^{\text{QCD}}(r) \rightarrow -\frac{3}{16} \frac{g_3^4 m_\pi^{9/2}}{\pi^{5/2} f_\pi^2} e^{-2m_\pi r} + \ldots , \]  

(3)

which exhibits the expected fall off with a length scale set by twice the mass of the pion and where the dots represent subleading contributions in the large-distance expansion.

Figure 4. The box and crossed-box diagrams that give the longest-distance component of the $\Lambda_Q \Lambda_Q$ potential. Heavy-quark symmetry forbids $\Lambda_Q \Lambda_Q$ intermediate states in the box and crossed-box diagrams.

The extension of the heavy-baryon sector from QCD to PQCD is straightforward and the leading modifications to the $\Lambda_Q \Lambda_Q$ potential are easily computed [14]. Fig. 5 exhibits the potential as a function of $m_{SV}$, the mass of the Goldstone boson consisting of a valence quark and a sea quark, for two values of $r$. One may wonder whether “hairpin” contributions may enter in such a way as to dominate the potential. The leading hairpin

![Figure 5](image-url)

Figure 5. The left panel shows $V^{\text{PQ}}(r)$ evaluated at $r = 1$ fm as a function of the meson mass $m_{SV}$, while the right panels shows $V^{\text{PQ}}(r)$ evaluated at $r = 2$ fm. The vertical axis is in units of MeV. When $m_{SV} = m_\pi$ the value of $V^{\text{PQ}}(r)$ is equal to $V^{\text{QCD}}(r)$. 
contribution enters through the diagram in Fig. 6. At asymptotically large distances this contribution to the potential becomes

$$V_{HP}(r) \rightarrow -c_1^2 \frac{(m_{SS}^2 - m_{q}^2)^2}{64 \pi^{5/2} \Lambda_{\chi}^6} \frac{e^{-2m_{\pi}r}}{\sqrt{r}} + \ldots$$  \hspace{1cm} (4)$$

where the dots represent subleading contributions in the large-distance expansion. While at asymptotically large distances this contribution is larger than that from the box and crossed-box diagrams, asymptopia finally sets in at distances at which all contributions are numerically insignificant.

4. Conclusions

Understanding how nuclei and nuclear interactions depend upon the light-quark masses is a fundamental aspect of strong-interaction physics. Recent work has been able to explore the \(m_q\)-dependence of two-nucleon systems using a recently-developed effective field theory and naive dimensional analysis. In the \(1S_0\)-channel we expect that di-nucleon systems, such as the di-neutron, are unbound for all values of \(m_q\) less than their physical values. However, for \(m_q\) larger than their physical values both bound and unbound systems are presently consistent with data and NDA. In the \(3S_1 - 3D_1\) coupled-channels, where the deuteron resides for the physical values of the quark masses, the deuteron may or may not be bound in the chiral limit. A more definitive statement can only be made with a more precise determination of the \(\pi N\) coupling \(d_{16}\) and a determination of the coefficients of the leading \(m_q\)-dependent four-nucleon operators, \(D_2\). As discussed in Ref. [5], it is likely that a determination of \(D_2\) will require a future lattice QCD calculation.

Recent work has computed the potential between two \(\Lambda_Q\)'s at leading order in effective field theory in both QCD and PQQCD. The size of the leading contribution from hairpin interactions in PQQCD has been estimated. Evidently the partially-quenched \(\Lambda_Q\Lambda_Q\) potential does not suffer from some of the (partial-) quenching problems that plague the NN potential due to the absence of single pseudo-Goldstone exchange. The computed potentials will allow for the chiral extrapolation of lattice calculations performed with unphysically large sea quark masses. As these potentials fall off with a mass scale set by \(\sim 2m_{\pi}\), they are quite small for baryon separations greater than \(r \sim 1.5\) fm. Therefore, the theoretical advantages of studying this system to learn about the NN potential may be undermined by the difficulties in extracting a signal from lattice simulations. However,
the simplifications introduced by only having two light quarks, and a single infinitely-
massive quark to fix the inter-baryon separation makes this system a prime candidate for
studying inter-baryon interactions. It is very exciting indeed to be so close to making
fundamental statements about nuclear physics.

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