Theoretical understanding of the nuclear incompressibility: where do we stand?

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The status of the theoretical research on the compressional modes of finite nuclei and the incompressibility $K_\infty$ of nuclear matter, is reviewed. It is argued that the recent experimental data on the Isoscalar Giant Monopole Resonance (ISGMR) allow extracting the value of $K_\infty$ with an uncertainty of about $\pm 12$ MeV. Non-relativistic (Skyrme, Gogny) and relativistic mean field models predict for $K_\infty$ values which are significantly different from one another, namely $\approx 220-235$ and $\approx 250-270$ MeV respectively. It is shown that the solution of this puzzle requires a better determination of the symmetry energy at, and around, saturation. The role played by the experimental data of the Isoscalar Giant Dipole Resonance (ISGDR) is also discussed.

\section{INTRODUCTION}

The quest for the value of the nuclear incompressibility $K_\infty$ is still continuing. Some significant progress in our understanding of how its value can be constrained have been achieved in recent times and this will constitute the subject of the present review. In this sense, this contribution is a continuation of those by J.P. Blaizot\cite{1} and by N. Van Giai \textit{et al.}\cite{2} in the previous conferences of the Giant Resonance series (Varenna 1998 and Osaka 2000).

It is well known that the energy per particle $E/A$ in nuclear matter, considered as a function of the density $\varrho$, exhibits a minimum at the saturation point $\varrho_0=0.17$ fm$^{-3}$. The nuclear matter incompressibility, defined as

\begin{equation}
K_\infty = 9\varrho_0^2 \frac{d^2 E}{d\varrho^2} \frac{E}{A}igr|_{\varrho=\varrho_0},
\end{equation}

provides a measure of the curvature of $E/A$ around $\varrho_0$. The interest of determining $K_\infty$ stems also from its impact on the physics of neutron stars.

Since we cannot directly create and probe nuclear matter in ordinary laboratories, the only way to extract a value for $K_\infty$ is by making contact with the phenomenology of the compressional modes in finite nuclei. The clearest example of compressional mode is the Isoscalar Giant Monopole Resonance (ISGMR), which is often called the nuclear
“breathing mode” and is excited by the operator

\[ \hat{M} = \sum_{i=1}^{A} r_i^2. \]  

(2)

The first evidences of this \( L = 0 \) mode date back to the 1970s. The presence, in the same energy region, of modes with different multipolarities (e.g., \( L = 2 \)) as well as of a non negligible background, makes the extraction of the monopole strength rather difficult. Over the last two decades, the experimental techniques have improved and our knowledge of the ISGMR properties has progressed. Many reactions have been employed to study this resonance, but inelastic \((\alpha, \alpha')\) scattering has been, from the beginning, one of the best tools. In the light nuclei this resonance is rather fragmented, while in the medium-heavy nuclei it corresponds to a single peak of energy \( E_{\text{ISGMR}} \sim 80 \cdot A^{-1/3} \) MeV. Nowadays, in the recent measurements of \(^{90}\text{Zr}, ^{116}\text{Sn}, ^{144}\text{Sn}\) and \(^{208}\text{Pb}\) performed at Texas A&M [3], the accuracy on the centroids of the ISGMR strength distribution has come down to about \( \pm 2\% \). The importance of this high accuracy for the extraction of \( K_\infty \) will be discussed below.

2. THE NUCLEAR INCOMPRESSIBILITY DEDUCED FROM THE ISGMR: BASIC FORMULAS AND PREVIOUS RESULTS

To perform the link between the properties of the ISGMR in finite nuclei and \( K_\infty \), the definition (1) must be complemented by an operative expression which contains quantities that can be measured in the laboratory. J.P. Blaizot [4] showed that a plausible definition of the finite nucleus incompressibility \( K_A \) is given by

\[ E_{\text{ISGMR}} = \sqrt{\frac{\hbar^2 K_A}{m \langle r^2 \rangle}}, \]  

(3)

where \( m \) is the nucleon mass, and \( \langle r^2 \rangle \) the ground state mean square radius. Using the experimental ISGMR energies to deduce \( K_A \) for different values of \( A \), there have been attempts to make an extrapolation of \( K_A \) to \( A=\infty \). This was done by using a Weiszäcker-type formula for \( K_A \), namely

\[ K_A = K_\infty + K_{\text{surf}} A^{-1/3} + K_{\text{sym}} \frac{A^2}{Z^4/3} + K_{\text{Coul}} \frac{Z^4}{A^{4/3}}, \]  

(4)

where \( \alpha = \frac{N-Z}{A} \). M. Pearson [5] was the first to show that, in view of the correlations among the parameters of (4) and the scarcity of experimental data, trying to use these to perform a fit of Eq. (4) is statistically meaningless and would indeed leave \( K_\infty \) basically undetermined (fits which lead to 100 MeV or 400 MeV may be equally acceptable). Similar conclusions were reached by S. Shlomo and D. Youngblood [6]. Therefore, we will not discuss these so-called “macroscopic approaches” to \( K_\infty \). However, we remark that the theoretical values of the parameters entering Eq. (4) can be calculated, within the framework of different models. We will come back to this point in Sec. 4.

The procedure to obtain \( K_\infty \), which is nowadays believed to be physically sound, is the so-called “microscopic approach”. The basic idea consists in using energy functionals \( E[\varphi] \) which allow calculating nuclear matter and finite nuclei on the same footing.
In the non-relativistic case, the starting point is a two-body effective nucleon-nucleon interaction $V_{\text{eff}}$, whose parameters are adjusted to reproduce experimental data in a small set of nuclei. $E$ is written as the expectation value of $H_{\text{eff}} = T + V_{\text{eff}}$ on an independent particle wave function (i.e., a Slater determinant). In practice, the available functionals are based on the Skyrme and Gogny interactions.

In the relativistic models, nucleons are described as Dirac particles which interact by the exchange of effective $\sigma$, $\omega$ and $\rho$ mesons. In the limit of large meson masses, point coupling models are obtained. In some cases, the coupling constants are taken to be density-dependent. As in the cases of Skyrme and Gogny, the coupling constants are fitted. A specific model has been recently proposed [7], in which the main parameters are connected to more fundamental quantities, in particular to the so-called QCD sum rules and to the (iterated) pion exchange. In all models, the no-sea approximation is made and the energy functional is written in the Hartree (and not Hartree-Fock) form.

In both non-relativistic and relativistic cases, the second derivative of the energy functional can be calculated analytically for uniform nuclear matter and the value of $K_\infty$ associated to a given parametrization is therefore given. In the case of finite nuclei, one calculates the monopole excitation using self-consistent linear response theory. The system is perturbed with an (arbitrarily small) external field and the small oscillations around the ground state are governed by the residual force $\frac{\delta^2 E}{\delta \varrho^2}$. The theory is known as self-consistent Random Phase Approximation (RPA) and is well described in textbooks [8].

Then, the determination of $K_\infty$ proceeds as follows.

- Using a set of different parametrizations (within a given class of energy functionals) characterized by different values of $K_\infty$, self-consistent RPA calculations of the ISGMR are performed in a given nucleus. If the monopole strength has only one peak, $E_{\text{ISGMR}}$ is well defined and Eqs. (3-4) suggest that a relation of the type $E_{\text{ISGMR}} \sim \sqrt{K_\infty}$ can be expected. This can be verified empirically and indeed, relations of the type

$$E_{\text{ISGMR}} = a\sqrt{K_\infty} + b$$  \hspace{1cm} \text{(5)}$$

have been interpolated (see below).

- The experimental value of $E_{\text{ISGMR}}$ is inserted in Eq. (5) and the value of $K_\infty$ is deduced.

One or few nuclei would be enough to apply this procedure, and $^{208}\text{Pb}$ is a typical system where the monopole strength has a well-defined peak and where recent experiments have reduced the sources of errors. From the theoretical point of view, the calculations must be free of the uncertainties associated to, e.g., the description of pairing or to anharmonic effects.

The procedure that we have described, was firstly applied by J.P. Blaizot and collaborators [9], by employing the Gogny interaction. The authors of [9] made use of the existing parametrizations and also built ad hoc new ones in order to cover more values of $K_\infty$. A value of about 230 MeV for the nuclear incompressibility can be extracted from the experimental $E_{\text{ISGMR}}$ of $^{208}\text{Pb}$. Exactly the same procedure was applied in the
case of the Skyrme forces (using only already existing parametrizations) [10,2]. From the experimental monopole energy of $^{208}$Pb a value of $K_\infty$ around 210 MeV is deduced, while using $^{90}$Zr the value is even lower (around 200 MeV). All the RPA calculations quoted are done using a discrete basis. However, the value of 210 MeV is consistent with what has been found by other authors within the framework of continuum-RPA [11].

Different calculations made by using the relativistic RPA gave instead larger values of $K_\infty$. Values of 250 MeV and 270 MeV were extracted from the experimental data in $^{208}$Pb and $^{144}$Sm respectively [12,13]. This model dependence in the extraction of $K_\infty$ has been, and partly it is still, the basic puzzle.

3. CONSISTENCY BETWEEN THE RESULTS FROM SKYRME AND GOGNY INTERACTIONS

Before proceeding, some quantitative statements concerning the numerical accuracy of the microscopic approach are in order. From Eq. (5), the relative errors on the ISGMR energy and on $K_\infty$ are related by

$$\frac{\delta K_\infty}{K_\infty} = 2 \frac{\delta E_{\text{ISGMR}}}{E_{\text{ISGMR}}}.$$ 

As a rule of thumb, let us keep in mind that sticking to the case of $^{208}$Pb, $\pm 150$ keV of uncertainty on $E_{\text{ISGMR}}$ result in about $\pm 5$ MeV uncertainty on $K_\infty$. The experimental measurement on $^{208}$Pb provides us with $14.17 \pm 0.28$ MeV [3] and therefore

$$\delta K_{\infty}^{\text{exp.}} \sim \pm 10 \text{ MeV}.$$ 

Theoretically, the best way to extract the monopole energy is by means of constrained Hartree-Fock (CHF) calculations [14]. These calculations provide the inverse energy-weighted sum rule $m_{-1}$ with a numerical error of the order of $\pm 3\%$ (see also Ref. [15]), and since the energy-weighted sum rule $m_1$ is known, the relation

$$E_{\text{ISGMR}} = \sqrt{\frac{m_1}{m_{-1}}}$$

implies

$$\delta K_{\infty}^{\text{th.}} \sim \pm 7 \text{ MeV}.$$ 

The two errors on $K_\infty$ are independent and should be added quadratically, so that

$$\delta K_\infty \sim \pm 12 \text{ MeV}.$$ (7)

We now discuss in more detail the calculations performed in Refs. [10,2]. One of their main limitations is the lack of complete self-consistency, as the residual p-h Coulomb and p-h spin-orbit interactions are dropped. We have analyzed the effect of this approximation in the nuclei $^{16}$O and $^{40}$Ca. The results are shown in Table 1. The Skyrme force used is SLy4 [16]. The results of (a) are obtained by dropping the Coulomb and spin-orbit terms both in HF and in RPA. In this sense, the calculation is fully self-consistent, and the fact that the RPA results agree with the CHF results within less than 1% suggests that the
Table 1
Values of the $m_{-1}$ sum rule in fm$^4$/MeV calculated using the interaction SLy4. In column (a), results obtained without the Coulomb and spin-orbit interaction are shown. In this case, the RPA is fully self-consistent. In column (b), the Coulomb and spin-orbit terms of the interaction are included in the CHF calculation and in the HF calculation on which RPA is based, but are excluded from the residual RPA interaction.

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<tr>
<td>$^{16}$O</td>
<td>CHF</td>
<td>14.45</td>
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<td></td>
<td>RPA</td>
<td>14.49</td>
</tr>
<tr>
<td>$^{40}$Ca</td>
<td>CHF</td>
<td>75.31</td>
</tr>
<tr>
<td></td>
<td>RPA</td>
<td>75.91</td>
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approximations done in the RPA (neglect of the continuum and truncation of the discrete basis) do not affect seriously the values of $m_{-1}$. On the other hand, the results of (b) are obtained by including the Coulomb and spin-orbit terms in the mean field and not in the residual RPA interaction. The difference between the CHF and RPA results is larger than the CHF intrinsic uncertainty and can therefore be a meaningful indication of the error induced by the lack of full RPA self-consistency. Consequently, we have compared CHF and RPA in the nuclei already considered for the extraction of $K_\infty$, that is, $^{208}$Pb and $^{90}$Zr. Note that in the case of $^{208}$Pb and $^{90}$Zr the CHF result for $m_{-1}$ is larger than the RPA result, contrarily to the case of $^{16}$O and $^{40}$Ca. The results are displayed in Figs. 1 and 2.

The values of 235 MeV from $^{208}$Pb and 220 MeV from $^{90}$Zr for $K_\infty$ which are extracted from the Skyrme-CHF calculations, are consistent with each other and with the values deduced using the Gogny interaction. This is the first important conclusion of the present paper.

We conclude this Section with a brief justification of the mean field approximation in the present calculations. Firstly, it has been shown in Ref. [17] that including the coupling of the RPA states with more complicated configurations of 2 particle-2 hole (2p-2h) type, shifts the ISGMR in $^{208}$Pb to lower energy by only $\approx 500$ keV. This number is smaller than the uncertainties discussed above. Secondly, according to the arguments of Sec. 2, in order to use the result for $E_{ISGMR}$ obtained beyond mean field, a calculation for nuclear matter performed on the same footing is needed. The development of a suitable energy functional is still a challenge for nuclear structure theory.

4. HOW TO RECONCILE THE RELATIVISTIC MEAN FIELD WITH SKYRME AND GOGNY ?

As far as the relativistic models are concerned, there has been a recent suggestion [18] that the different outcome for $K_\infty$ (compared with Skyrme and Gogny) is originated by the different density dependence of the symmetry energy $S(\rho)$ predicted by different models. It is true that the measurements of the ISGMR are done in a system ($^{208}$Pb) with a finite value of $(N-Z)/A$. To illustrate the possible influence of the density dependence of
Figure 1. Monopole energies in $^{208}\text{Pb}$ (defined as in Eq. (6)) obtained from RPA (crosses) and CHF (stars) calculations which employ different Skyrme forces, plotted as a function of the associated $K_\infty$. The lines are fits of the type (5). The dashed lines indicate the extracted values of $K_\infty$. The value of $K_\infty$ resulting from CHF is in agreement with that extracted from the Gogny calculations of [9].

$S(\rho)$ on the extraction of $K_\infty$, J. Piekarewicz [18] has built parametrizations of effective Lagrangians whose symmetry energy has different density dependences (this is easy to achieve since the $\rho$ coupling constant is an adjustable parameter) and he finds that the extracted $K_\infty$ indeed differ and can even become close to Skyrme force values. To achieve this, it is necessary to soften the function $S(\rho)$ and in Ref. [18] this was done by lowering the symmetry energy at the saturation point, $a_\tau$. However, it has been pointed out in Ref. [19] that, in the type of model used in Ref. [18] parametrizations with $a_\tau$ lower than 36 MeV cannot describe satisfactorily $N \neq Z$ nuclei. A complementary attempt has been made in [15], by constructing Skyrme forces with associated values of $a_\tau$ up to 38 MeV. In this case, finite nuclei have been carefully considered in the fits. By calculating the ISGMR centroid energy, a very weak dependence on $a_\tau$ has been found.

There is one more conceptual remark. Starting from (4), one would expect a dependence of $K_A$, and consequently of the monopole energy, on a parameter like $K_{sym}$ more than on $a_\tau$. That is, on the second derivative of the symmetry energy more than on its value at saturation. In fact, it has been found - by studying a semi-infinite slab of nuclear matter
- that both in relativistic and in non-relativistic models the quantity $K_{surf}$ is essentially given by $c \cdot K_{\infty}$ with $c \approx -1$ [4,20]. Therefore, we can write

$$K_A^{(\text{non rel.})} \sim K_{\infty}(1 - A^{-1/3}) + K_{\text{sym}}^{(\text{non rel.})} \alpha^2 + K_{\text{Coul}} A^2 \frac{Z^{2/3}}{A^{1/3}},$$

$$K_A^{(\text{rel.})} \sim K_{\infty}(1 - A^{-1/3}) + K_{\text{sym}}^{(\text{rel.})} \alpha^2 + K_{\text{Coul}} A^2 \frac{Z^{2/3}}{A^{1/3}}. \quad (8)$$

We argue that the difference between relativistic and non-relativistic models is mainly due to the different values of $K_{\text{sym}}$. In fact, this parameter is typically -150 MeV for the Skyrme forces employed in [10,2], and -700 MeV for the most used relativistic functionals [20]. This large difference of 550 MeV, multiplied by $\alpha^2=0.04$ (the $^{208}\text{Pb}$ value) gives about 20 MeV difference in $K_A$, which translates into a difference of approximately 1 MeV in $E_{\text{ISGMR}}$. This is indeed the difference which emerges from numerical calculations of the ISGMR using a Skyrme (or Gogny) functional and a relativistic one, with similar associated values of $K_{\infty}$. This is shown in Fig. 3.
5. THE ISOSCALAR GIANT DIPOLE RESONANCE

The ISGDR is a non-isotropic compressional mode, which is excited by the operator

\[
\hat{D} = \sum_{i=1}^{A} r_i^3 Y_{1M}(\hat{r}_i),
\]

and which provides in principle a further way to extract the value of \( K_\infty \). There are many reasons why this task is more challenging in the case of the dipole than in the case of the monopole.

From the theoretical point of view, one difficulty arises from the fact that the spurious center-of-mass translation (associated with the operator \( \sum_{i=1}^{A} r_i Y_{1M}(\hat{r}_i) \)) carries the same quantum numbers as the ISGDR. In principle, a sharp spurious state at zero energy should result from an ideal self-consistent RPA calculation. However, in practice this is not the case due to approximations and numerical inaccuracy. Consequently, the resulting states are not orthogonal to the spurious state and one has to correct for this. The spurious
transition density is expected to be of the type $\sim \frac{d\rho}{dr}$ where $\rho_0$ is the ground state density. It is possible to project out the spurious component from each excited state (cf., e.g., Ref. [21]). An equivalent procedure [22] consists in using instead the modified operator

$$\hat{D}_{\text{eff}} = \sum_{i=1}^{A} (r_i^3 - \eta r_i) Y_{1M}(\hat{r}_i),$$

where $\eta = \frac{5}{3} < r^2 >$. A recent analysis of the accuracy of these techniques, as well as a discussion about some different prescriptions to subtract the spurious state, can be found in Ref. [23]. There have been other discussions in the recent literature [24], also in connection with semiclassical models [25].

On the experimental side, different ($\alpha, \alpha'$) measurements have been performed over the years but the problem of disentangling the ISGDR strength from the other multipoles (and from the IVGDR) is far from being trivial. The ISGDR lies at higher energies than the ISGMR (approximately $110A^{-1/3}$ MeV). An accurate determination of its high-energy tail is therefore more difficult. On the low-energy side, a sizeable amount of fragmented strength is found. These issues are discussed in these Conference proceedings [26,27].

Different theoretical calculations [21,28], have clarified that the low-energy part of the ISGDR strength is formed by non-collective states. The separation of a "high-energy" region and a "low-energy" region in the ISGDR strength distribution, emerges systematically from the calculations of Ref. [21] as illustrated here in Fig. 4. One of the main indications about the different character of the two parts of the strength, comes from the fact that the centroids of the high-energy regions, calculated with different Skyrme forces in a given nucleus, scale with the corresponding $K_\infty$ (which testifies to their compressional nature), whereas the centroids of the low-energy regions do not. The same pattern is found in the relativistic calculations.

More detailed considerations concerning the wave functions of the low-lying states have been done in Ref. [30] where the authors suggest that the low-lying strength may correspond to a "toroidal" resonance, which can be visualized - in a simple way - by thinking of a particle current bent into a torus. It is excited by the vector operator

$$\hat{T} = \sum_{i=1}^{A} \vec{\nabla} \times (\hat{r}_i \times \vec{\nabla}) r_i^2 Y_{1M}(\hat{r}_i)$$ (10)

which couples to the transition current. To verify the toroidal nature of the states, some other probe than the $\alpha$-particles should be tried, and ($e, e'$) experiments would be probably useful in this respect.

In Table 2 we report the results of the various self-consistent RPA calculations of the ISGDR in $^{208}$Pb. The first remark is that the low-energy part is of course significantly dependent on the model and on the specific functional used, as expected due to its lack of collectivity. As far as the high-lying centroid is concerned, the Skyrme results of Refs. [31,21] (continuum and discrete RPA respectively) are in good agreement with each other. In Ref. [34] it has been shown that coupling the RPA states with 2p-2h type configurations, shifts the ISGDR centroid down to 22.9 MeV, in very good agreement with experiment (this coupling also produces a conspicuous spreading width of about 6 MeV).
Figure 4. ISGDR strength functions in different nuclei, calculated in RPA using the Skyrme interaction SGII [29] and corrected for center-of-mass effects. In the case of $^{208}$Pb the dashed line corresponds to a calculation where the spurious center-of-mass state is not subtracted. Here as well as in Fig. 5 the discrete RPA states have been smeared out by means of a Lorentzian of 1 MeV width. Taken from Ref. [21].

In Table 2, also the results of the most recent measurement performed at RCNP in Osaka (making use of $\alpha$-particles at incident energy of 400 MeV) as well as the new findings of the Texas A& M group, are shown. The experimental energies seem to converge towards each other (compared to the previous experiments of the same groups, where the discrepancies were larger). Nevertheless, a point should be made concerning the experimental analysis. The directly measured quantity is the double-differential cross section, $\frac{d^2\sigma}{d\Omega dE}$. The multipole decomposition of this cross section is done by relying on Distorted Wave Born Approximation (DWBA) calculations where the radial form factors for the various multipole excitations are the same at all energies. This is somehow in contrast with the outcome of theory, as we have discussed above.

We conclude the discussion on $^{208}$Pb with a statement about the deduction of $K_{∞}$. If this quantity is derived from a plot similar to those of Figs. 1 and 2, i.e., by using the “microscopic approach” described in Sec. 2 in connection with the ISGMR but using now the ISGDR data, one finds a value around 205 MeV which is still (marginally) compatible with the value 220-235 MeV discussed above (cf. Fig. 2 of [21]). This means that there is no basic contradiction between the present ISGDR and ISGMR data, as far as the
Table 2
Self-consistent (relativistic and non-relativistic) RPA calculations performed for the ISGDR in $^{208}$Pb compared with the most recent experimental data. The two columns report the centroid energies (in MeV) of the low-energy and high-energy regions discussed in the text.

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<th>High-energy centroid</th>
<th>Low-energy centroid</th>
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<tr>
<td>Hamamoto et al. [31]</td>
<td>23.4</td>
<td>$\sim$ 14</td>
</tr>
<tr>
<td>Colò et al. [21]</td>
<td>23.9</td>
<td>10.9</td>
</tr>
<tr>
<td>Vretenar et al. [28]</td>
<td>26</td>
<td>10.4</td>
</tr>
<tr>
<td>Piekarewicz [32]</td>
<td>24.4</td>
<td>$\sim$ 8</td>
</tr>
<tr>
<td>Shlomo and Sanzhur [24]</td>
<td>$\sim$ 25</td>
<td>$\sim$ 15</td>
</tr>
<tr>
<td>Uchida et al. [33,26]</td>
<td>23 ± 0.3</td>
<td>12.7 ± 0.2</td>
</tr>
<tr>
<td>Lui et al. [27]</td>
<td>21.7</td>
<td>12.6</td>
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deduction of the nuclear incompressibility is concerned. However, it should be clear from all our discussion, that we still need to await for improvements in the ISGDR studies in order to reach the same confidence that we have in $K_\infty$ extracted from the ISGMR.

We end this Section by showing the result of a Skyrme-RPA calculation for a lighter system, namely $^{58}$Ni (see Fig. 5). It can be noticed, by comparison with Fig. 4, that although the centroid of the ISGDR is still in the region around $110 \cdot A^{-1/3}$ MeV, the strength is more fragmented than in the heavier systems. Also, the distinction between a low- and a high-energy regions is less evident. A comparison with experiment would be useful. In any case, as it was said for the monopole case, this picture confirms that medium-mass systems are less suitable for studying the nuclear incompressibility.

6. CONCLUSIONS

In recent years, there have been significant progresses both in the experimental techniques aimed to extract with good precision the moments of the monopole strength function, and in the theoretical models - especially in those based on relativistic functionals which can be nowadays discussed on the same footing as the well tested Skyrme and Gogny functionals. Still, it is shown in this paper that it is in general possible to determine a delicate quantity like the nuclear incompressibility $K_\infty$ only within $\pm 12$ MeV.

Many discussions have been devoted to the fact that the values of $K_\infty$ depend on the model through which they have been extracted, either Skyrme, Gogny or relativistic functionals. In this contribution, we show that the discrepancy between Skyrme and Gogny does not exist and, using the ISGMR data in $^{208}$Pb the value of $K_\infty$ lies between 220 MeV and 235 MeV. On the other hand, the relativistic calculations point to larger values, of the order of 250-270 MeV. This puzzle is still unsolved, but we argue in this work that the reason has to be found probably in the different features that the asymmetry energy curve has in the relativistic and non-relativistic models, respectively. The relativistic models
are characterized by significantly larger values of the symmetry energy at saturation, and of its first and second derivatives. The quest about the proper values of the asymmetry energy has of course very general implications on the nuclear phenomenology, and new works on the subject are in progress.

As far as the isoscalar dipole is concerned, there has been also definite experimental improvements in the techniques aimed to reduce or eliminate the background, and in the data analysis. While in the past the results from different groups were in disagreement among themselves, and with theory, this does not seem to be the case for the new experiments. The degree of accuracy that can be expected in extracting $K_\infty$ is smaller in the dipole than in the monopole case. However, there does not seem to exist a basic incompatibility between the values of the incompressibility deduced using for instance the Skyrme forces, in the monopole and in the dipole case. Among the perspectives, it can be said that new $(\alpha, \alpha')$ cross section calculations based on a fully microscopic input may be of great help to reduce some of the uncertainties which still exist. On the other hand, it is unlikely that the ISGDR data can help in solving the discrepancy between the non-relativistic and relativistic models as far as the value of $K_\infty$ is concerned.

Figure 5. ISGDR Strength distribution in $^{58}$Ni calculated using Skyrme-RPA with the SGII force.
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26. See the contribution by U. Garg in this volume.
27. See the contribution by Y.-W. Lui in this volume.