We study brane recombination for supersymmetric configurations of intersecting branes in terms of the world-volume field theory. This field theory contains an impurity, corresponding to the degrees of freedom localized at the intersection. The Higgs branch, on which the impurity fields condense, consists of vacua for which the intersection is deformed into a smooth calibrated manifold. We show this explicitly using a superspace formalism for which the calibration equations arise naturally from F- and D-flatness. HU-EP-03/44

1 D-branes, Supersymmetric effective theories, brane dynamics in gauge theories, tachyon condensation
   document

Introduction

It is well known that a supersymmetry preserving set of intersecting branes may merge to form a single brane on a smooth calibrated surface (see Smith for a review). This is expected to occur via the condensation of world-volume degrees of freedom localized at the intersection. However, the manner in which the Higgs branch of the world-volume gauge theory describes smooth calibrated surfaces has not been studied explicitly in much detail. Filling this gap is the aim of this note. We shall do so using a superspace for which calibrated geometries on the Higgs branch arise naturally as solutions of F- and D-flatness conditions. The gauge theories which we write down are suitable for describing both the Coulomb branch on which the branes are separated and the Higgs branch on which the branes have merged on a calibrated surface.

These gauge theories contain impurities, given by degrees of freedom constrained to the lower-dimensional subspace where the branes intersect. It is convenient to describe such a field theory by a superspace which spans the space-time directions of the impurity. This superspace may also be used to described the fields in the ambient space which are not localized at the impurity. The formalism in which degrees of freedom in higher dimensions are described in a lower-dimensional superspace was developed originally in MSS,AHGW and has been applied to study a number of theories, including the defect conformal theories which arise for systems of intersecting branes EGK,CEGK1,CEGK2,CEGK3.

In the context of brane intersections preserving eight supercharges, the holomorphic curves which arise on the Higgs branch are solutions of the F- and D-flatness conditions for a superspace which spans the mutual coordinates of the intersection. For triple (and quadruple) intersections preserving four supercharges, we will find F- and D-flatness corresponding to special Lagrangian conditions. These conditions will take the form of the equations of motion of an abelian Chern-Simons theory and a gauge fixing respectively.

For a particular triple intersection of D6-branes preserving four supercharges, the Higgs branch solutions of the F- and D-flatness conditions correspond to the union of a special Lagrangian plane and a holomorphic curve times a line. This configuration lifts to a $G_2$ manifold in M-theory which shares many features with $G_2$-manifolds discussed in AW,GT.

There are a number of reasons to be interested in the process of brane recombination. In the context of intersecting brane-worlds BGKL1,BGKL2,AFIRU, the Higgs mechanism is believed to be realized by brane-recombination Ibanez. In this setting intersecting branes merge via open string tachyon condensation. The supersymmetric triple D6-brane intersection which we will study provides a controlled toy model with which to explicitly demonstrate the brane-world Higgs mechanism. A precise treatment of recombination by tachyon condensation has not been given and would seem to require string field theory, except in certain small angle approximation Hashimoto. Approximate treatments have been given using effective tachyon field theories Hashimoto,Huang,HT. For the supersymmetric brane recombination which we will describe, the field theory treatment suffices. Recombination in a supersymmetric case was also considered in HT.
Another reason to be interested in brane recombination is that it is closely related (via U-dualities) to processes such as the brane collision in ekpyrotic Khoury:2001wf or cyclic universes ST. In this context it is useful to have a description which allows to study the dynamics of transitions between the Coulomb branch, with separated branes, and the Higgs branch on which the branes have recombined.

Branes touching and the intersection being resolved resolved

The organization of this paper is as follows. In section 2, we discuss intersecting pairs of branes which preserve eight supercharges and wrap holomorphic curves on the Higgs branch. This section reviews and expands upon results in CEGK1. In section 3, we consider intersections preserving four supercharges, for which the $F$ and $D$-flatness conditions are the the special Lagrangian conditions. We study the particular example of three (and four) intersecting D6-branes. Finally, in section 4, we numerically compute correlations of fields on the different asymptotic regions of branes which have recombined into a holomorphic curve $xy=c$. We find that the correlation vanishes in the singular limit $c \to 0$ in which the branes can separate.

Holomorphic curves from intersecting branes preserving eight supercharges

The low energy dynamics of intersecting branes is described by a field theory with impurities. Some of the earliest studies of such impurity field theories may be found in S,GS,KS. In many instances, this theory is a non-trivial defect conformal field theory KR,DFO,AdWFK,EGK,CEGK1,CEGK2,CEGK3), or dCFT. In this section we review the superspace description of defect field theories and expand upon some results CEGK1 concerning the Higgs branch of the dCFT describing intersecting D3-branes. Although the world-volume of this dCFT is the singular space $xy=0$, we will explicitly see that the classical Higgs branch has a geometric interpretation as the smooth resolution $xy=c$.

Consider two stacks of D3 branes, one of which spans the directions $x^{0,1,2,3}$, while the other spans $x^{1,4,5,6}$. The two stacks are at the origin in the transverse $x^{6,7,8,9}$ directions. In addition to the $N=4$ gauge theory that lives on each stack of parallel D3-branes, there are additional massless fields that arise from open strings stretching between the orthogonal branes. These fields are localized at the 1+1 dimensional intersection. There is an unbroken $(4,4)$ supersymmetry which includes translations in the 0,1 directions. The $(4,4)$ algebra is a common subalgebra of the two four-dimensional $N=4$ algebras associated with each parallel stack of D3-branes.

Although the theory contains both two- and four-dimensional degrees of freedom, one can write the action using a two-dimensional superspace. The degrees of freedom which propagate in four dimensions are described by superfields with continuous indices which parameterize the world-volume directions transverse to the intersection. The directions parallel to the intersection are included in the superspace. In CEGK1 the action for this theory was constructed using two-dimensional $(2,2)$ superspace. The formalism which we will use below is trivially generalized to intersecting D-branes of other dimensions, such as D5-branes intersecting over four dimensional $N=4$ superspace.

The superfields on the first D3-brane are functions of $(x^0,x^1,\theta,\bar{\theta},w,\bar{w})$. The superspace is spanned by $(x^0,x^1,\theta,\bar{\theta},w,\bar{w})$, while $w = x^2 + ix^3$ should be thought of as a continuous index. The necessary $(2,2)$ superfields are a vector superfield $V$ and three chiral superfields $\Phi$, $Q_1$, and $Q_2$. While this resembles the four-dimensional $N=1$ superfield content of the $N=4$ theory, the component fields are distributed very differently. The gauge connections $A_{0,1}$ are contained in $V$, while $A_2 + iA_3$ is the lowest component of a chiral superfield $\Phi$. This chiral superfield transforms inhomogeneously under U(N) gauge transformations with non-trivial dependence on the index $w$. Gauge transformations are parameterized by families of chiral superfields $\Lambda$ labelled by $w, \bar{w}$. Two of the six adjoint scalars are contained in $V$ or, equivalently, the lowest component of a twisted chiral superfield, which (in the abelian case) is $\Sigma = D_+D_-V$. The four remaining adjoint scalars comprise the lowest components of the chiral superfields $Q_1$ and $Q_2$. In $(2,2)$ superspace, the four-dimensional $N=4$ action is equation split $S_{D3} = 1g^2 \int d^2x d^2\theta\, d^4\psi\, e^\psi \bar{\psi} \bar{\Sigma} + (\partial_w + g\Phi) e^\psi \bar{\psi} \bar{\Sigma} + g\Phi e^{-\bar{\psi} \bar{\psi}}$

The remaining degrees of freedom are strictly two-dimensional and arise from strings stretched between the orthogonal stacks of D3-branes. These are described by two chiral superfields $B$ and $\bar{B}$ in the $(N,N')$ and $(\bar{N},N')$ representations of $U(N) \times U(N')$. Together these form a $(4,4)$ hypermultiplet. The part of the action containing these fields is equation defect split $S_{D3-D3'} = \int d^2x d^4\theta \left( e^{-\bar{\psi} \bar{\psi}} \bar{B} e^{\bar{\psi} \bar{\psi}} B + e^{-\bar{\psi} \bar{\psi}} \bar{B} e^{\bar{\psi} \bar{\psi}} \bar{B} \right)$

The lowest components of $Q^1$ and $\Sigma$, as well as their primed counterparts, correspond to adjoint scalars describing fluctuations in the directions $x^{6,7,8,9}$ transverse to both stacks of D3-branes. This can be seen by noting that expectations values for these fields give mass to the defect fields $B$ and $\bar{B}$. The lowest component
of $Q_2$ describes fluctuations of the first stack of D3-branes in the $x^{4,5}$ directions which are tangential to the other stack of D3-branes (the D3$'\,$branes). The lowest component of $Q'_2$ describes fluctuations of the D3$'$-branes in the directions $x^{2,3}$ tangential to the D3-branes.

Supersymmetric solutions: Branches of the moduli space

To determine the supersymmetric vacua, we look for the gauge equivalence classes of solutions to the D- and F-flatness equations. Earlier investigations of moduli spaces of theories with impurities appeared in KS,BGK. The vanishing of the F-terms in this theory requires: align $F_{Q_1} = \partial \bar{w} q_2 + [\phi, q_2] + \delta^{(2)}(w) \bar{b} b = 0 \, f q_1$.

First consider the D-term equation (Dterm). Assuming that the $q$-fields are regular the coefficient of the $\delta$-function has to vanish. Thus we find equation $\bar{b} b^\dagger = b^\dagger b$, backandforth