On the Evidence for Clustering in the Arrival Directions of AGASA’s Ultrahigh Energy Cosmic Rays

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Abstract

Previous analyses of cosmic rays above $4 \cdot 10^{19}$ eV observed by the AGASA experiment have suggested that their arrival directions may be clustered. However, estimates of the chance probability of this clustering signal vary from $10^{-2}$ to $10^{-6}$ and beyond. It is essential that the strength of this evidence be well understood in order to compare it with anisotropy studies in other cosmic ray experiments. We apply two methods for extracting a meaningful significance from this data set: one can scan for the cuts which optimize the clustering signal, using simulations to determine the appropriate statistical penalty for the scan; alternatively, one can optimize the cuts with a first set of data, and then apply them to the remaining data directly without statistical penalty. While the former method is more useful in general, in the present case only the latter is an unbiased test of the clustering hypothesis. We find that the AGASA data is consistent at the 20% level with the null hypothesis of isotropically distributed arrival directions.

Key words: Ultrahigh Energy Cosmic Rays; Anisotropy of Cosmic Rays; Extensive Air Shower Arrays
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1 Introduction

The study of arrival directions of cosmic rays above $10^{19}$ eV (ultrahigh energy cosmic rays) is one of the most promising ways to gain insight into the origin of these particles. While a number of experiments have shown that the distribution of arrival directions is remarkably isotropic, evidence for small-angle clustering has been claimed, most notably by the AGASA [1] (Akeno Giant

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Air Shower Array) cosmic ray experiment [2–7]. This clustering signal, if confirmed, would give strong support to the idea that cosmic rays originate from compact sources [8].

The focus on small-angle anisotropies among the very highest energy events is well-motivated: if the cosmic ray particles are charged, then the highest energy ones are expected to suffer the smallest deflections while traversing Galactic and extragalactic magnetic fields. Their arrival directions are therefore the most likely ones to point back toward sources.

A search for clustering among the highest energy cosmic ray events must make choices for the minimum energy $E_c$ which defines the data set and the maximum angular separation $\theta_c$ which defines a pair. On the one hand, choosing a higher energy threshold $E_c$ should reduce deflections and allow clusters to show up within smaller angular separations $\theta_c$. This holds especially for a detector such as AGASA in which the angular resolution improves at higher energies. On the other hand, as a function of energy $E$ the cosmic ray flux drops at roughly the rate $E^{-3}$, so the statistical power of the available data quickly weakens with higher energy thresholds.

For a precise model of cosmic ray source distributions and Galactic and extragalactic magnetic fields, these competing forces would imply optimal choices for $E_c$ and $\theta_c$ to maximize the clustering signal. At present, however, not nearly enough is known about any of these to make a priori choices useful. Instead, what is done explicitly or implicitly is to scan over a range of values for $E_c$ and $\theta_c$, and identify the values which maximize the clustering signal. In this case, the final significance of the result must include a penalty factor for the a posteriori cuts arrived at by scanning.

It is the various ways of handling this penalty factor—or, in some cases, the failure to include it at all—which has led to a wide range of significances attached to the AGASA clustering signal.

Evaluating this significance rigorously is crucial for understanding the anisotropy results of new cosmic ray experiments. The world data set of detected cosmic ray particles above $10^{19}$ eV is currently dominated by the events observed by AGASA, which has been operated continuously since 1990. In the near future, a statistically independent data set from the currently operating HiRes (High Resolution Fly’s Eye) air fluorescence detector will become available. Already, first analyses by the HiRes collaboration indicate that a clustering signal is not present in the monocular [9,10] and stereo [11] HiRes data sets. The HiRes data set is especially interesting since the angular resolution of this detector running in stereo mode is below $1^\circ$, making it ideal for the study of small-angle clustering. In the more distant future, the Pierre Auger Array is expected to produce an even larger data set of ultrahigh energy cosmic rays.
To compare the anisotropy results of AGASA and these new experiments, the strength of the AGASA clustering signal must be well understood. We apply two methods for evaluating its significance. The first is a general method, applicable to upcoming searches by other experiments as well; the second is a specific test of the clustering hypothesis which is meaningful only in the context of the AGASA data set.

First, we propose that a clustering signal among the highest energy events can be best evaluated by scanning simultaneously over energy thresholds and angular separations to find the values for $E_c$ and $\theta_c$ which optimize the signal. The chance probability of the signal is determined by counting the number of simulated data sets which yield a stronger signal under an identical scan. With this procedure, the statistical significance is determined without treating $a \ posteriori$ cuts as $a \ priori$ ones.

A bias remains in the case of the AGASA data set, however, due to the inclusion of the events that led to the clustering hypothesis in the first place. One can avoid this bias by removing the early data and only scanning over the events which have been detected since the original claim. Alternatively, one can test the AGASA clustering hypothesis by applying the original cuts to the newer events directly. Since the cuts are now $a \ priori$, this test requires no statistical penalty. It has the virtues of being simple and rigorously unbiased.

The paper is organized as follows. In Section 2, we summarize previous estimates of the significance of the AGASA clustering signal. In Section 3, we motivate and describe the autocorrelation scanning technique applied in our analysis. In Section 4, we perform this scan on the published AGASA data set and compare the result with previous estimates of the significance. In Section 5, we address the bias introduced by using the whole data set, and perform an analysis using only the unbiased event set. We present our conclusions in Section 6.

2 Small Scale Clustering in AGASA Cosmic Ray Data

The AGASA experiment reported possible clustering in the arrival directions of ultrahigh energy cosmic rays as early as 1996 [2], and has updated this data sample and analysis in several publications [3–6]. The first report of clustering in 1996 identified three pairs of events with angular separation less than 2.5° among the 36 events with energies above $4 \cdot 10^{19}$ eV. The corresponding chance probability was found to be 2.9%. It was noted that the angular separation of 2.5° is “nearly consistent with the measurement error ($\sqrt{2} \cdot 1.6$°)” [2]. The minimum energy of $4 \cdot 10^{19}$ eV was justified under the assumption that the Greisen-Zatsepin-Kuzmin (GZK) cutoff [12,13] should lead to an accumulation
of events around $4 \cdot 10^{19}$ eV, and therefore that events above this energy may point back to nearby sources. The values for $E_c$ and $\theta_c$ identified in this report set the stage for all analyses which followed.

In 1999, a new publication by AGASA [3] identified a stronger clustering signal using these cuts with an enlarged data set now containing 47 events. The following year, AGASA published an updated list with 57 events above $4 \cdot 10^{19}$ eV observed through May, 2000 [4]. There is also an additional event below $4 \cdot 10^{19}$ eV which was added to the list because it forms another doublet.¹ Not counting the extra event, there are four doublets and one triplet in this set.

This set was analyzed by Tinyakov and Tkachev [8], who calculated the chance probability as a function of the threshold energy $E_c$ of the data set, while keeping the angular bin size constant at $2.5^\circ$. The lowest probability was found to be less than $10^{-4}$ with $E_c = 4.8 \cdot 10^{19}$ eV. Since this probability was obtained by scanning over energies, it does not reflect the true significance of the clustering signal. To estimate the correct chance probability, the authors numerically calculated a correction factor by generating $10^3$ random sets of events which were then subjected to the same scanning in $E_c$. 27 (3) random samples had a probability of less than $10^{-2}$ ($10^{-3}$), and the authors concluded that the correction factor was of order 3. The final chance probability was given as $3 \cdot 10^{-4}$, considerably lower than the chance probability reported by the AGASA collaboration in the original publications [2,3].

A similar scan was then performed in the size of the angular bin, i.e. the maximum angular distance between events that defines a cluster. The probability shows a minimum at $2.5^\circ$, but since this was interpreted as the angular resolution of the experiment, no correction factor was applied to the final chance probability.

In [5] in 2001, the AGASA group applied this scanning technique again to a data set which was now reported to include 59 events above $4 \cdot 10^{19}$ eV—essentially the same data set as the one published in 2000 [4], though it is unclear whether the one event below the energy cutoff was kept, or whether one or two new events were added. Five doublets and one triplet were reported in the sample. A scan over angular separations was again performed, showing the peak at $2.5^\circ$. Performing a scan over energies, the significance of the clustering above $4 \cdot 10^{19}$ eV was said to be $4.6 \sigma$, and above $4.5 \cdot 10^{19}$ eV it was said to be in excess of $5 \sigma$. No statistical penalties were applied for either the energy or angular separation scan.

¹ This is an unfortunate source of confusion. Like many authors, we do not include this extra event in our analysis because it is not clear how many additional events there are between it and $4 \cdot 10^{19}$ eV. However, it is sometimes included in other authors’ analyses to which we refer.
The most recently published study by AGASA [6] in 2003 recapitulates much of the above analysis. The same 59 events are analyzed, though the AGASA experiment has continued to observe ultrahigh energy cosmic rays and has reported 72 events above $4 \cdot 10^{19} \text{eV}$ seen through the end of July, 2002 [7]. Forgoing a scan over energies, the chance probability for all of the clusters (one triplet + five doublets = eight pairs) in the total set of 59 events is simply reported to be less than $10^{-4}$.

3 Scanning and Evaluation of Chance Probability

Given the competition between magnetic deflections and statistical power described earlier, scanning over small angles among the highest energy events to locate a signal is well motivated. This is only more true given the energy dependence of the AGASA angular resolution, as will be discussed shortly. However, the significance attached to such a result must correctly account for the scanning process itself.

In our analysis, we perform a scan simultaneously over energy thresholds and maximum separation angles to find the $E_c$ and $\theta_c$ which maximize the clustering signal, and then we perform identical scans over simulated data sets to evaluate the true significance.

In practice, rather than scanning directly over energy thresholds, we rank the events by energy and scan over events $N$. That is, for each value of $N$ and $\theta$, we restrict ourselves to the $N$ highest-energy events, and count the number of pairs $n_p$ separated by less than $\theta$.

Prior to scanning the data, we generate a large number ($10^7$) of simulated data sets with the same exposure as the detector, and use them to generate a table of values $P_{mc}$, where $P_{mc}(N, \theta, n)$ is the fraction of sets in which the first $N$ events contain exactly $n$ pairs separated by less than $\theta$.

For each $(N, \theta)$, the number of pairs $n_p$ is counted in the data, and the probability $P_{data}$ for observing $n_p$ or more pairs at $(N, \theta)$ is calculated as:

$$P_{data}(N, \theta) = \sum_{n=n_p}^{\infty} P_{mc}(N, \theta, n) = 1 - \sum_{n=0}^{n_p-1} P_{mc}(N, \theta, n).$$

For some combination $(N_c, \theta_c)$, $P_{data}$ has a minimum: $P_{min} = P_{data}(N_c, \theta_c)$. This identifies the location in the scan of the strongest potential clustering signal. To assess the true significance of this signal, we perform the same scan over $n_{mc}$ Monte Carlo data sets, identifying the minimum probability
\[ P_{\text{min}}^i = P_i(\theta_i^c, N_i^c) \] for each trial and counting the number of trials \( n_{mc}^* \) for which \( P_{\text{min}}^i \leq P_{\text{min}} \).

The chance probability of observing \( P_{\text{min}} \) in the scan is finally evaluated as:

\[ P_{\text{chance}} = \frac{n_{mc}^*}{n_{mc}}. \]

This scanning technique is essentially an auto-correlation analysis in which the angular size of the first bin and the energy threshold of the data set are varied to maximize the signal, and the final significance includes the correction factor for the scan over both variables.

4 Autocorrelation Scan of the AGASA Data Set

We perform this scan on the published AGASA data above \( 4 \cdot 10^{19} \text{ eV} \), which consists of 57 events [4]. To generate Monte Carlo events for determining the probabilities, we follow [8] in using a zenith angle \( (\theta_z) \) distribution \( dn \propto \cos \theta_z \sin \theta_z d\theta_z \), corresponding to geometric acceptance of isotropically distributed cosmic ray arrival directions. We use the same \( \theta_z < 45^\circ \) cut as employed by AGASA, and assign uniformly random arrival times, corresponding to the uniform exposure of AGASA in right ascension [4, 5]. We scan over angular separations from \( 0^\circ \) to \( 5^\circ \) in increments of \( 0.1^\circ \). The results of the scan are shown in Figure 1.

The strongest clustering signal is contained within the \( N_c = 36 \) highest-energy events, where there are \( n_p = 6 \) pairs separated by less than \( \theta_c = 2.5^\circ \). (The energy threshold corresponding to this subset is \( 4.89 \cdot 10^{19} \text{ eV} \).) At this spot \( P_{\text{data}} = P_{\text{min}} = 8.4 \cdot 10^{-5} \), that is, 839 out of \( 10^7 \) MC data sets had the same or greater number of pairs at the same values for \( N \) and \( \theta \). This value for \( P_{\text{min}} \) (not \( P_{\text{chance}} \)) is essentially the same as the \( 10^{-4} \) probability found by Tinyakov and Tkachev [8] for the same energy threshold and angular separation.

To evaluate the significance of this result, we perform the same scan over simulated AGASA data sets and count how many simulated sets have \( P_{\text{min}}^{MC} \leq P_{\text{data}} \). We find that 3475 out of \( 10^6 \) simulated sets meet this condition, implying a chance probability of 0.35%. Figure 2 illustrates how \( P_{\text{chance}} \) varies as a function of \( P_{\text{min}} \) for the simulated AGASA sets.

In performing the data scan and the simulated scans, it is necessary to choose three parameters which can affect the final result: \( N_{\text{max}} \), the total number of events included in the scan; \( \theta_{\text{max}} \), the angular extent of the scan; and \( \Delta \theta \), the size of the angular binning. In Table 1 we show a range of values for these
parameters and the effect on the final value of $P_{\text{chance}}$. We motivate our choices for each of the parameters as follows:

In choosing the extent of the scan in $N_{\text{max}}$ and $\theta_{\text{max}}$, it is clear that the significance would be biased if one scanned out precisely to the maximum clustering signal and no further. An investigator who reports a clustering signal in a scan can reasonably be expected to have scanned out at least twice as far in search of an even stronger signal; hence a reasonable estimate of $P_{\text{chance}}$ should extend $N_{\text{max}}$ to $\sim 2 \cdot N_c$, and further, if $N_c$ is very small. The same can be said for $\theta_{\text{max}}$ with respect to $\theta_c$. Finally, it can be seen in Table 1 that reducing the angular bin size $\Delta \theta$ has a negligible effect at small scales. Our choices for each of the parameters are indicated in bold.

We note that over each of the ranges shown in Table 1—a factor of three in event number, a factor of 16 in angular area, and a factor of 25 in angular
Fig. 2. $P_{\text{chance}}$ as a function of $P_{\text{min}}$ for the AGASA data, with $N_{\text{max}} = 57$ and three different values of the scan parameter $\theta_{\text{max}}$.

binning—the chance probability remains within $2 \cdot 10^{-3}$ to $6 \cdot 10^{-3}$. Therefore our result does not depend sensitively on these scanning parameters.

The value of 0.35% we calculate for $P_{\text{chance}}$ is 10 times larger than that calculated by Tinyakov and Tkachev in [8]. Although they use an angular scan to demonstrate that the separation angle $2.5^\circ$ maximizes the signal, they nevertheless treat the choice as an a priori one and make no correction for it.

In [5], the AGASA collaboration analyzes the same data set and finds that at $4 \cdot 10^{19}$ eV the significance of the clustering signal is $4.6\sigma$, and that at a slightly higher energy threshold it is “$5\sigma$ or more”. These results imply chance probabilities of $4.2 \cdot 10^{-6}$ and $5.7 \cdot 10^{-7}$, respectively—three to four orders of magnitude lower than the probability we have presented here. This overestimation of the significance of the clustering signal arises in part from the application of Gaussian statistics to a non-Gaussian distribution: these significances are obtained by measuring the excess clustering signal in units of standard deviations, $(N_{\text{obs}} - N_{\text{exp}})/\Delta N_{\text{exp}}$, when in fact this distribution is not Gaussian for the small numbers of clusters observed. Having cited Tinyakov and Tkachev [8] and made use of their technique, the authors ignore their
Table 1

<table>
<thead>
<tr>
<th>$N_{\text{max}}$</th>
<th>$\theta_{\text{max}}$</th>
<th>$\Delta \theta$</th>
<th>$P_{\text{chance}}$</th>
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<td>3.48 \cdot 10^{-3}</td>
</tr>
<tr>
<td>36</td>
<td>5°</td>
<td>0.1°</td>
<td>2.40 \cdot 10^{-3}</td>
</tr>
<tr>
<td>100</td>
<td>5°</td>
<td>0.1°</td>
<td>5.63 \cdot 10^{-3}</td>
</tr>
<tr>
<td>57</td>
<td>2.5°</td>
<td>0.1°</td>
<td>2.00 \cdot 10^{-3}</td>
</tr>
<tr>
<td>57</td>
<td>10°</td>
<td>0.1°</td>
<td>5.77 \cdot 10^{-3}</td>
</tr>
<tr>
<td>57</td>
<td>5°</td>
<td>0.5°</td>
<td>2.31 \cdot 10^{-3}</td>
</tr>
<tr>
<td>57</td>
<td>5°</td>
<td>0.02°</td>
<td>4.05 \cdot 10^{-3}</td>
</tr>
</tbody>
</table>

The effect on $P_{\text{chance}}$ due to variations in the scan parameters. In each case, $P_{\text{chance}}$ was determined using $10^6$ Monte Carlo data sets. The values in bold indicate those used in this paper.

warning on exactly this point. Furthermore, they ignore the statistical penalty involved in scanning over energy thresholds, and they do not consider a penalty for the choice of angular separations.

5 Unbiased Test of AGASA Clustering Hypothesis

It is worth considering in more detail the claim that the AGASA data should, \textit{a priori}, cluster at 2.5°. The claim is based on the angular resolution of AGASA at $4 \cdot 10^{19}$ eV, estimated by comparing the true and reconstructed arrival directions of Monte Carlo events generated with this energy. It is found that 68% of these events have reconstructed angular errors $\theta_{\text{err}} < 1.8°$ [3,5]. This value is then multiplied by $\sqrt{2}$ to “make excess regions clearer” [3].

However, when scanning over energy thresholds for a clustering signal, it must be kept in mind that the angular resolution of the AGASA detector continues to shrink with increasing energy. At $10^{20}$ eV, AGASA reports $\theta_{\text{err}} < 1.2°$ [3]. Eight of the 57 events in the data set are in fact above this energy. Given the improved AGASA angular resolution at higher energies, and given the original motivation for scanning over higher energies to decrease deflections by magnetic fields, there is no \textit{a priori} justification for the choice of 2.5° to maximize the clustering signal in this data set.

Rather than constituting \textit{a priori} choices, the values of $4 \cdot 10^{19}$ eV and 2.5° are justified \textit{a posteriori} in conjunction with the observation in 1996 that they lead to a clustering signal [2]. They can only be treated as \textit{a priori} for a
data set independent of the one which was used to derive them. We can do this by dividing the AGASA data into an “original data set” comprising the events observed through October 1995 which formed the basis of the original clustering claim, and a “new data set” comprising the events which have been observed since then. Using the list of events published in 2000 [4], there are 30 events in the original set and 27 in the new one.  

Because the new data set is independent, we can test the original clustering hypothesis directly without the need for any statistical penalties. We simply count the number of pairs of events using $E_c = 4 \cdot 10^{19}$ eV and $\theta_c = 2.5^\circ$, and we find one pair. The chance probability for one or more pairs among 27 events is 28%.

We can investigate whether there is a better choice of $E_c$ and $\theta_c$ for the new data set by performing an autocorrelation scan. Figure 3 shows the results of scanning over both the old and new data sets separately. The strongest clustering signal in the original set has a chance probability $P_{ch} = 4.4\%$ and occurs for $\theta_c = 2.4^\circ$ and $E_c = 4.35 \cdot 10^{19}$ eV (with 3 pairs among the 26 highest energy events, and a minimum probability $P_{min} = 0.33\%$ in this bin). This confirms that the cuts originally selected in 1996 were nearly optimal for that data set. However, when the new events are scanned, there is no hint of clustering at the 2.5° scale or any other angular separation. The “strongest” clustering signal occurs at $\theta_c = 4.7^\circ$ with $P_{ch} = 27\%$.

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2 In [2] (1996), the original data set is said to contain 36 events above $4 \cdot 10^{19}$ eV. However, the lists [3,4] published in 1999 and 2000 contain only 30 events during this same time period, due to a reevaluation of the energies (according to Uchihori et al. [15]). The three original clusters are present in all sets.
There is an even larger set of new events which we can use to test the original clustering hypothesis. While AGASA has not published an event list for the data taken since May 2000, it has made public a summary of events seen through July 2002 [7]. The sample now includes a total of 72 events above $E_c = 4 \cdot 10^{19}$ eV, and contains one new doublet since May 2000. Therefore, the independent data sample since October 1995 now contains 42 events, with two doublets separated by less than 2.5°. While there is not enough information in this summary to perform an autocorrelation scan, there is enough to test the clustering hypothesis directly by counting doublets with $E_c = 4 \cdot 10^{19}$ eV and $\theta_c = 2.5^\circ$ as before. The chance probability for finding two or more pairs among 42 events is 19%. This test, using an independent data set 40% larger than the original, is the soundest statistical test of the clustering hypothesis.

The fact that an analysis using only the later data yields a chance probability on the order of 20%, while the autocorrelation scan over the whole data set yields a chance probability on the order of 0.5% merits some attention. Part of the discrepancy arises from cross-correlation between the two sets—in particular one event in the new data set correlates with two events in the old data set, forming the so-called “triplet.” A second round of Monte Carlo tests can be invoked to measure the effect of cross-correlations between the two sets, but more important is the self-correlation in the first set. The clustering signal in the first data set contributes to the final clustering signal of the whole data set, which biases the final significance. Any number of anomalies in the arrival directions might have been observed in the original data set and motivated other analyses. Thus an additional statistical penalty is necessary to correct for the possibility of having observed any of these other anomalies. Given the impossibility of calculating this a posteriori, the only completely unbiased test is one using independent data. For AGASA, this requires setting aside the events which played a part in formulating the initial hypothesis.

6 Conclusions

Taken at face value, an autocorrelation scan of the published AGASA data set finds a chance probability around 0.35% for the clustering signal claimed previously. At this level, the observed clustering could be a hint of real small-scale anisotropy, or it could well be a chance fluctuation in an isotropic distribution. To investigate the possibility that it is a fluctuation, and that the significance of the scan is artificially high because it includes the data which led to the clustering hypothesis in the first place, we test the original claim with inde-

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3 There are a total of five doublets and one triplet in this sample. Five doublets and one triplet were also claimed previously in [4], but one of these doublets involves an event below the energy cutoff. This doublet is not included in the 2002 summary.
pendent data consisting only of the AGASA events observed after the claim. The cuts which were identified originally can now be applied \textit{a priori} in an unbiased test. We find the chance probability for the clustering signal in the new set to be 19\%. Hence, using an independent sample 1.4 times the size of the original, no evidence is found to support the original claim of clustering in arrival directions.

We conclude that the evidence for clustering in the AGASA data set is weaker than has sometimes been claimed, and in fact is consistent with the null hypothesis of isotropically distributed arrival directions. Nevertheless, small-scale anisotropy and clustering searches remain of great value in the study of ultra-high energy cosmic rays. Identification of compact sources would be a tremendous breakthrough for the field, and it is quite possible that with the increased statistics and improved angular resolution of experiments like HiRes and the Pierre Auger Array, compelling evidence may yet be discovered. And, it is worth remembering that even isotropy can tell us something important about the highest energy cosmic rays.

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References


