Higher-order non-symmetric counterterms in pure Yang-Mills theory

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Abstract

We analyze the restoration of the Slavnov-Taylor (ST) identities for pure massless Yang-Mills theory in the Landau gauge within the BPHZL renormalization scheme. The Zimmermann-Lowenstein IR regulator $M(s-1)$ is introduced via a suitable BRST doublet, thus preserving the nilpotency of the BRST differential. We explicitly obtain the most general form of the action-like part of the symmetric regularized action $\Gamma^s$, $s < 1$ obeying the ST identities and all other relevant symmetries of the model, to all orders in the loop expansion, and show that non-symmetric counterterms arise in $\Gamma^s$ starting from the second order in the loop expansion, unless a special choice of normalization conditions is done. We give a cohomological characterization of the fulfillment of BPHZL IR power-counting criterion, guaranteeing the existence of the physical limit $s \to 1$.

The technique analyzed in this paper is needed in the study of the restoration of the ST identities for those models, like the MSSM, where massless particles are present and no invariant regularization scheme is known to preserve all the relevant ST identities of the theory.

Keywords: Renormalization, BRST Symmetry.

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1 Introduction

In a preceding paper [1] the quantum restoration of the Slavnov-Taylor (ST) identities for anomaly-free gauge theories has been shown to be equivalent, in the absence of IR problems, to the recursive parameterization of the action-like part of the symmetric quantum effective action

\[ \Pi = \sum_{j=0}^{\infty} \Pi^{(j)} \]

in terms of suitable ST functionals, associated to the cohomology classes of the classical linearized ST operator \( S_0 \). In the above equation \( \Pi^{(j)} \) is the coefficient of order \( j \) in the \( \hbar \)-expansion of \( \Pi \). In particular, it has been shown [1] that also for those models where a regularization-invariant scheme exists, at orders higher than one non-symmetric counterterms enter in \( \Pi^{(j)}, j \geq 2 \), as a consequence of the bilinear form of the ST identities. The non-invariant counterterms can be put equal to zero only by a special choice of normalization conditions.

In this paper we would like to extend this investigation to the case of pure massless Yang-Mills theory in the Landau gauge. The extension to the massless case of the direct restoration of the ST identities [2] in the form of the ST parameterization of the quantum effective action [1] will provide a way to obtain the explicit form, to all orders in the loop expansion and for arbitrary normalization conditions, of the counterterms in the BPHZL scheme, required to fulfill the ST identities of the model.

The technique developed in this paper constitutes the building block to treat within the framework of the method discussed in [1, 2] those models, like the Minimal Supersymmetric Standard Model, where massless particles are present. It is worthwhile to point out that, due to the presence of the \( \gamma^5 \) matrix and of the completely antisymmetric tensor, no invariant regularization scheme is known, fulfilling all the relevant symmetries of the MSSM. Therefore in this case the explicit computation of the counterterms, required to fulfill the relevant ST identities, cannot be avoided.

The choice of the Landau gauge simplifies the computations, but similar results can be obtained for any Lorentz-covariant gauge. We choose to work within the BPHZL regularization scheme [5]-[7],[10], following the Lowenstein-Zimmermann prescription [9]-[11] to handle massless propagators. Therefore all massless fields are assigned a mass \( m^2 = M^2(s - 1)^2 \), where \( s \) is an auxiliary parameter ranging between 0 and 1. The relevant zero-momentum subtractions on 1-PI Green functions take place both at \( s = 0 \) and at \( s = 1 \), according to the prescriptions given in [9]-[11]. Both

\[ ^2 \text{For instance dimensional regularization [3] for non-chiral gauge models or the modified subtraction prescription given in [4] for some chiral non-supersymmetric models.} \]
subtractions are needed in order to guarantee the existence of the massless limit $s \to 1$.

We will perform the renormalization of the relevant ST identities for the intermediate regularized symmetric quantum effective action $\Gamma_s$, $0 \leq s < 1$, by combining a variant of the approach first pioneered in [8] with the use of a BRST-invariant IR regulator introduced via a BRST-doublet [12]. This allows to maintain nilpotency of the full BRST differential.

We explicitly derive the most general solution of the action-like part $t^4 \Gamma_s$ of $\Gamma_s$, by making use of the results given in [1]. We find that non-symmetric counterterms enter into $\Gamma_s^{(j)}$, $j \geq 2$, unless a special choice of normalization conditions has been done for $\Gamma_s^{(k)}$, $k < j$. The coefficients of the non-symmetric counterterms in $\Gamma_s^{(j)}$ can be parameterized in terms of the coefficients $\lambda^{(k)}_1, \rho^{(k)}_1$ of suitable invariant ST functionals appearing in $t^4 \Gamma_s^{(k)}$, $k < j$.

It turns out that the limit of $\Gamma_s$ for $s \to 1$ is well-defined and free of IR singularities, as a consequence of the fulfillment of the IR and UV power-counting criteria stated in [10, 11]. The fulfillment of the IR power-counting criterion, guaranteeing the absence of zero-mass singularities in the limit $s \to 1$, can be understood in terms of purely cohomological properties of the classical linearized ST operator $S_0$. This provides a somewhat novel cohomological interpretation of the IR power-counting criterion first introduced in [10, 11]. The quantum effective action $\Gamma$ for pure massless Yang-Mills model is obtained as

$$\Gamma = \lim_{s \to 1} \Gamma_s. \quad (1)$$

Physical unitarity stems from the ST identities obeyed by $\Gamma$.

We find that the non-symmetric counterterms entering into $\Gamma_s^{(j)}$ at order $j \geq 2$ do not vanish in the limit $s \to 1$. Hence they also appear in $\Gamma_s^{(j)}$, unless a special choice of normalization conditions has been done for $\Gamma_s^{(k)}$, $k < j$.

Finally we comment on the dependence of physical observables on the coefficients $\lambda^{(k)}_1, \rho^{(k)}_1$ parameterizing the non-invariant counterterms. This shows some of the advantages provided by the ST parameterization introduced in [1] in discussing the physical consequences of the non-invariant higher order counterterms.

The plan of the paper is the following. In Sect. 2 we discuss the ST identities for the model at hand and provide the most general solution to the symmetric regularized quantum effective action $\Gamma_s$, to all orders in the loop expansion. We show how non-invariant counterterms enter in $\Gamma_s^{(j)}$, $j \geq 2$ unless a special choice of normalization conditions is done for $\Gamma_s^{(k)}$, $k < j$. We also give a cohomological interpretation of the IR power-counting criterion stated in [10, 11]. In Sect. 3 we analyze the limit $s \to 1$ and discuss
the dependence of physical observables on the parameters controlling the non-invariant counterterms. Conclusions are presented in Sect. 4.

2 Higher-order non-symmetric counterterms

The ST identities for the classical action \( \Gamma^{(0)} \) (see Appendix A) of Yang-Mills theory in the Landau gauge with an IR regulator \( m \) introduced via a BRST doublet (\( \bar{\rho}, m \)) are

\[
\mathcal{S}(\Gamma^{(0)}) = \int d^4x \left( \Gamma^{(0)}(x, \Gamma^{(0)}) + m \frac{\partial \Gamma^{(0)}}{\partial \bar{\rho}} \right) = 0, \tag{2}
\]

where the bracket \( \langle \Gamma^{(0)}, \Gamma^{(0)} \rangle \) is defined according to

\[
(X, Y) = \int d^4x \left( \frac{\delta X}{\delta A^a_{\mu}} \frac{\delta Y}{\delta A^a_{\mu}} + \frac{\delta X}{\delta \omega^a} \frac{\delta Y}{\delta \omega^a} \frac{\delta X}{\delta \bar{\omega}^a} \frac{\delta Y}{\delta \bar{\omega}^a} \right). \tag{3}
\]

The linearized classical ST operator \( \mathcal{S}_0 \), given by

\[
\mathcal{S}_0 = \int d^4x \left( \frac{\delta \Gamma^{(0)}}{\delta A^a_{\mu}} \frac{\delta}{\delta A^a_{\mu}} + \frac{\delta \Gamma^{(0)}}{\delta A^a_{\mu}} \frac{\delta}{\delta A^a_{\mu}} + \frac{\delta \Gamma^{(0)}}{\delta \omega^a} \frac{\delta}{\delta \omega^a} + \frac{\delta \Gamma^{(0)}}{\delta \bar{\omega}^a} \frac{\delta}{\delta \bar{\omega}^a} \right) + m \frac{\partial}{\partial \bar{\rho}}, \tag{4}
\]

is nilpotent: \( \mathcal{S}_0^2 = 0 \). The BRST partner \( \bar{\rho} \) of the mass regulator \( m \) can be reabsorbed by the following antifield redefinition:

\[
A^a_{\mu} = A^a_{\mu} - \bar{\rho} m A^a_{\mu}, \quad \omega^{a'} = \omega^{a'} + \bar{\rho} m \omega^{a'}, \quad \bar{\omega}^{a'} = \bar{\omega}^{a'} - \bar{\rho} m \bar{\omega}^{a'}. \tag{5}
\]

This is a consequence of the fact that \( (\bar{\rho}, m) \) are cohomologically trivial, pairing into a BRST doublet. In the new variables in eq.(5) \( \Gamma^{(0)} \) becomes

\[
\Gamma^{(0)} = \int d^4x \left\{ -\frac{1}{4 g^2} G^a_{\mu \nu} G^a_{\mu \nu} - \bar{\omega}^a \partial_\mu (D_\mu \omega)^a + B^a \partial A^a + A^a_{\mu} (D^a)_{\mu}^a - \omega^{a'} + \frac{1}{2} f^{abc} \omega^{b} \omega^{c} + B^a + \frac{1}{2} m^2 (A^a_{\mu})^2 + m^2 \bar{\omega}^a \omega^a \right\}. \tag{6}
\]

The classical action \( \Gamma^{(0)} \) in eq.(6) obeys a set of additional symmetries:

\[
\frac{\partial \Gamma^{(0)}}{\partial \bar{\rho}} = 0, \tag{7}
\]

the \( B \)-equation

\[
\frac{\delta \Gamma^{(0)}}{\delta B^a} = \partial A^a + \bar{\omega}^{a'}, \tag{8}
\]
the ghost equation
\[ \frac{\delta \Gamma^{(0)}}{\delta \bar{\omega}^a} + \partial^\mu \frac{\delta \Gamma^{(0)}}{\delta A^{\mu a}} = m^2 \omega^a, \] (9)

and the anti-ghost equation
\[
\int d^4x \left( \frac{\delta \Gamma^{(0)}}{\delta \omega^a} - f^{abc} \omega^b \frac{\delta \Gamma^{(0)}}{\delta B^c} \right)
\[= \int d^4x \left( m^2 \bar{\omega}^a - f^{abc} A^a_{\mu} A^\mu_{c} + f^{abc} \omega^b \omega^c \right). \] (10)

Notice that the R.H.S. of eqs.(8)-(10) are linear in the quantum fields.

In order to carry out the renormalization of the model we choose the BPHZL regularization scheme [5]-[7],[10], following the Lowenstein-Zimmermann prescription [9]-[11] to handle massless propagators.

We identify
\[ m = M(s-1), \] (11)

where \(0 \leq s \leq 1\) and \(M\) is a constant with the dimension of a mass. The subtraction operator \(t_\gamma\) for a given divergent 1-PI graph or subgraph \(\gamma\) involves both a subtraction around \(p = 0, s = 0\) and around \(p = 0, s = 1\) [8, 9, 10, 11]:
\[(1 - t_\gamma) = (1 - t^{\rho(\gamma)-1}_{p,s=1})(1 - t^{\delta(\gamma)}_{p,s}),\] (12)

where \(\rho(\gamma)\) is the IR subtraction degree and \(\delta(\gamma)\) the UV subtraction degree for \(\gamma\) [8, 9, 10, 11]. Both subtractions around \(s = 0\) and \(s = 1\) are needed in order to guarantee the absence of IR singularities of the 1-PI Green functions in the physical limit \(s \to 1\) \((m \to 0)\).

The assignments of UV dimension for the fields and external sources, required to compute \(\rho(\gamma)\) and \(\delta(\gamma)\) for a given graph \(\gamma\) involving the fields and the antifields of the model, are as follows: \(A^a_{\mu}, \omega^a, \bar{\omega}^a\) have UV dimension 1, \(B_a, A^a_{\mu}, \omega^{a*}\) and \(\bar{\omega}^{a*}\) have UV dimension 2. The IR dimension coincides with the UV dimension.

We denote by \(\Gamma_s\) the symmetric quantum effective action fulfilling the ST identities
\[ \mathcal{S}(\Gamma_s) = \int d^4x \left( \frac{\delta \Gamma_s}{\delta A^a_{\mu}} \frac{\delta \Gamma_s}{\delta A^{\mu a}} + \frac{\delta \Gamma_s}{\delta \omega^{a*}} \frac{\delta \Gamma_s}{\delta \omega^a} + \frac{\delta \Gamma_s}{\delta \bar{\omega}^{a*}} \frac{\delta \Gamma_s}{\delta \bar{\omega}^a} \right) + m \frac{\partial \Gamma_s}{\partial \bar{\rho}} = 0. \] (13)

During the renormalization procedure we will always keep \(s < 1\). Only in the very end we will take the physical limit \(s \to 1\). We remark that for \(s \neq 1\) the ST identities in eq.(13) give rise to a violation of physical unitarity, due to the soft breaking term
\[ M(s-1) \frac{\partial \Gamma_s}{\partial \bar{\rho}}. \] (14)
This can be explicitly verified by using methods close to the one discussed in [14]. We recover physical unitarity in the limit $s \to 1$.

It can be proven by using the standard methods discussed e.g. in [13] that the functional identities in eqs. (7)-(10) can be restored at the quantum level. So we assume that they are also fulfilled by the symmetric quantum effective action $\Gamma_s$:

$$
\frac{\partial \Gamma_s^{(j)}}{\partial \rho} = 0 , \quad \frac{\delta \Gamma_s^{(j)}}{\delta B^a} = 0 , \quad \frac{\delta \Gamma_s^{(j)}}{\delta \bar{\omega}^a} + \partial^\mu \frac{\delta \Gamma_s^{(j)}}{\delta A^{a\mu}} = 0 ,
$$

$$
\int d^4x \left( \frac{\delta \Gamma_s^{(j)}}{\delta \bar{\omega}^a} - f^{abc} \bar{\omega}^b \frac{\delta \Gamma_s^{(j)}}{\delta B^c} \right) = 0 , \quad j \geq 1 . \quad (15)
$$

From the third equation in the first line of eq.(15) we conclude that $\Gamma_s^{(j)}$, $j \geq 1$ depends on $\bar{\omega}^a$ only through the combination

$$
\hat{A}_{\mu}^{a\prime} = A_{\mu}^{a\prime} + \partial_\mu \bar{\omega}^a . \quad (16)
$$

2.1 One-loop order

At one-loop order the ST identities read

$$
S_0(\Pi_s^{(1)}) = 0 . \quad (17)
$$

The action-like part of the most general solution $\Pi_s^{(1)}$ to eq. (17), compatible with the additional symmetries in eq. (15), is constrained to have the form [1]

$$
t^4 \Pi_s^{(1)} = \lambda_1^{(1)} \int d^4x G^a_\mu G^a_\mu + \rho_1^{(1)} S_0(\int d^4x A_{\mu}^{a} \hat{A}_{\mu}^{a}) , \quad (18)
$$

where $t^4$ is the projection operator on the sector of dimension $\leq 4$ in the fields, the antifields and their derivatives. $t^4 \Pi_s^{(1)}$ exists since $s < 1$.

Let us comment on the R.H.S. of eq.(18). At one-loop level only $S_0$-invariant terms appear in $t^4 \Pi_s^{(1)}$. Moreover, we notice that they are all IR-safe (all monomials entering into the R.H.S. of eq.(18) have IR degree equal to 4). This follows since

$$
S_0(\Phi^{(1)}) = \frac{\delta \Gamma_0^{(0)}(\Phi^{(1)})}{\delta \Phi} \quad (19)
$$

for $\Phi^{(1)} = \hat{A}_{\mu}^{a\prime}, \bar{\omega}^{a\prime}, \bar{\omega}^{a\prime}$. $\lambda_1^{(1)}$, $\rho_1^{(1)}$ are free parameters entering into the solution, unconstrained by the ST identities and the additional symmetries in eq.(15). They can be fixed by providing a choice of normalization conditions.
As an example, one might choose

$$\xi_{G_{\mu
u}^a G_a}^{(1)} = 0, \quad \xi_{\hat{A}_{\mu'}^a \partial \mu \omega^a}^{(1)} = 0.$$  \hspace{1cm} (20)$$
yielding

$$\lambda_1^{(1)} = 0, \quad \rho_1^{(1)} = 0.$$  \hspace{1cm} (21)$$

In the following we will not restrict ourselves to a special choice of normalization conditions, so we will keep \(\lambda_1^{(1)}, \rho_1^{(1)}\) free.

### 2.2 Higher orders

At orders higher than one the ST identities read

$$S_0(\Pi^{(n)}) = - \sum_{j=1}^{(n-1)} (\Pi^{(n-j)}_s, \Pi^{(j)}_s). \hspace{1cm} (22)$$

The brackets are given in eq.(3). Eq.(22) is an inhomogeneous linear equation whose unknown is the action-like part of \(\Pi^{(n)}_s\). The solution is [1]

$$t^4 \Pi^{(n)}_s = \lambda_1^{(n)} \int d^4x G_{\mu
u}^a G_a^{\mu\nu} + \rho_1^{(n)} \int d^4x \hat{A}_{\mu'}^a \hat{A}_{\mu}^a$$

\[
\begin{align*}
&\quad + \int d^4x A_{\mu}^a \frac{\delta}{\delta A_{\mu}^a} \left[ \sum_{j=1}^{(n-1)} \rho_1^{(n-j)} \lambda_1^{(j)} \int d^4y G_{\rho\sigma}^b G_{\rho\sigma}^b \right] \\
&\quad + \sum_{j=1}^{(n-1)} \rho_1^{(n-j)} \int d^4x A_{\mu}^a \frac{\delta}{\delta A_{\mu}^a} S_0(\int d^4y \hat{A}_{\nu}^{b\nu} A_{\nu}^{vb}) \\
&\quad - \sum_{j=1}^{(n-1)} \rho_1^{(n-j)} \int d^4x \frac{1}{g^2} (\Box A_{\rho}^d - \partial_{\rho} (\partial A)^d) A_{\rho}^d \\
&\quad - \int d^4x \frac{1}{g^2} \int d^4y A_{\sigma}^q (\partial_{\rho} A_{\sigma}^m - \partial_{\sigma} A_{\rho}^m) A_{\rho}^d \\
&\quad + \int d^4x \frac{1}{4g^2} f^{\rho q k} f^{k r d} A_{\sigma}^q A_{\sigma}^r A_{\rho}^d A_{\rho}^d). \hspace{1cm} \text{ (23)}
\end{align*}
\]

\(^3\)The notation is as follows. We expand \(t^4 \Pi^{(j)}_s\) into a sum of linearly independent, Lorentz-invariant action-like functionals \(M_j(x)\) in the fields, the antifields and their derivatives, providing a basis for the space to which \(t^4 \Pi^{(j)}_s\) belongs, and write accordingly \(t^4 \Pi^{(j)}_s = \sum_j \int d^4x \xi_{M_j}^{(j)}(x)\). \(\xi_{M_j}^{(j)}\) is the coefficient of \(M_j(x)\) in this expansion. As an example, \(\xi_{G_{\mu\nu}^a G_a}^{(1)}\) is the coefficient of \(G_{\mu\nu}^a G_a^{\mu\nu}\), \(\xi_{\hat{A}_{\mu'}^a \partial \mu \omega^a}^{(1)}\) the coefficient of \(\hat{A}_{\mu'}^a \partial \mu \omega^a\) in the expansion of \(t^4 \Pi^{(1)}_s\).
In contrast with one-loop level, non-symmetric counterterms enter in $t^4 \Pi^{(n)}_s$. They appear due to the inhomogeneous term in the R.H.S. of eq.(22), which depends on $\Pi^{(k)}_s$, $k < j$.

Again one can verify that all monomials in the R.H.S. of eq.(23) are IR safe by eq.(19).

The non-symmetric counter-terms, depending on the lower order contributions, disappear if one chooses to impose the following normalization condition for $t^4 \Pi^{(j)}_s$:

$$\rho^{(j)}_1 = 0, \quad j = 1, 2, \ldots, n - 1$$

(24)

equivalent to

$$\xi^{(j)}_{\hat{A}^a \mu, \partial^\mu \omega_a} = 0, \quad j = 1, 2, \ldots, n - 1.$$  

(25)

We might supplement it by the choice

$$\lambda^{(j)}_1 = 0, \quad j = 1, 2, \ldots, n - 1,$$  

(26)

equivalent to

$$\xi^{(j)}_{G^a_{\mu \nu}, G^{\mu \nu} a} = 0, \quad j = 1, 2, \ldots, n - 1.$$  

(27)

Eqs. (25) and (27) extend the one-loop normalization conditions in eq.(20).

We wish to comment on the IR-safeness of the monomials entering into $t^4 \Pi^{(n)}_s$. In the present approach this property stems from the fact that the classical linearized ST operator $S_0$ in eq.(4) has in the primed variables in eq.(5) definite degree +1 with respect to the counting operator of the fields, the antifields and their derivatives. The existence of the antifield redefinition in eq.(5) is in turn a consequence of the fact that $(\bar{\rho}, m = M(s - 1))$ form a BRST doublet. This provides a sufficient condition to guarantee the fulfillment of the IR power-counting criteria given in [10, 11] on a pure cohomological basis.

3 The limit $s \to 1$

In the previous section we have derived the most general form of the action-like part of $\Gamma_s$, $s < 1$ to all order in the loop expansion. We have checked that all action-like terms in $\Gamma_s$, $s < 1$ are IR-safe. UV convergence criteria are also satisfied. This is a sufficient condition [8, 9, 10, 11] to guarantee the existence of the limit $s \to 1$:

$$\Pi = \lim_{s \to 1} \Pi_s.$$  

(28)
In the limit $s \to 1$ ($m \to 0$) the primed antifields in eq.(5) reduce to their unprimed counterparts. The ST identities obeyed by $\Gamma$ read

$$\mathcal{S}(\Gamma) = \int d^4x \left( \frac{\delta \Gamma}{\delta A^a_{\mu}} \frac{\delta \Gamma}{\delta A^a_{\mu}} + \frac{\delta \Gamma}{\delta \omega^a} \frac{\delta \Gamma}{\delta \omega^a} + \frac{\delta \Gamma}{\delta \bar{\omega}^a} \frac{\delta \Gamma}{\delta \bar{\omega}^a} \right) = 0. \quad (29)$$

The soft-breaking term in eq.(14) has disappeared. This ensures the physical unitarity of the model [15]-[18]. The non-symmetric counterterms entering into $\Pi_s^{(j)}$ at order $j \geq 2$ do not vanish in the limit $s \to 1$. Hence they also appear in $\Pi^{(j)}$, unless the special choice of normalization conditions in eq.(25) has been done for $\Pi_s^{(k)}$, $k < j$.

We wish to comment on the dependence of physical observables on the parameters $\rho^{(j)}$. $\rho^{(j)}$ enters in $\Pi_s^{(j)}$ as the coefficient of the $S_0$-exact functional $\mathcal{S}_0(\int d^4x A^a_{\mu} A^a_{\mu})$. Hence physical observables should not depend on $\rho^{(j)}$. The study of the dependence of the Green functions of local BRST-invariant operators on these parameters can be carried out according to the standard procedure [13, 19], relying on the extension of the BRST differential $s$ in such a way to incorporate $\rho^{(j)}$ into a BRST doublet together with its partner $\theta^{(j)}$:

$$s \rho^{(j)} = \theta^{(j)}, \quad s \theta^{(j)} = 0. \quad (30)$$

The corresponding ST identities yield for $\Pi$

$$\mathcal{S}'(\Pi) = \mathcal{S}(\Pi) + \sum_j \theta^{(j)} \frac{\partial \Pi}{\partial \rho^{(j)}} = 0, \quad (31)$$

where $\mathcal{S}(\Pi)$ is given in eq.(29). Upon taking the Legendre transform $W$ of $\Pi$ with respect to the quantized fields of the model the ST identities read for the connected generating functional $W$:

$$\mathcal{S}'(W) = -\int d^4x \left( J^a_{\mu} \frac{\delta W}{\delta A^a_{\mu}} + J^a_{\omega} \frac{\delta W}{\delta \omega^a} + J^a_{\bar{\omega}} \frac{\delta W}{\delta \bar{\omega}^a} \right) + \sum_j \theta^{(j)} \frac{\partial W}{\partial \rho^{(j)}} = 0. \quad (32)$$

In the above equation $J^a_{\mu}$ is the external source coupled to $A_{\mu}$, $J^a_{\omega}$ the source coupled to $\omega^a$ and $J^a_{\bar{\omega}}$ the source coupled to $\bar{\omega}^a$. Now we differentiate eq.(32) with respect to $\theta^{(j)}$ and with respect to the sources $\beta_1(x_1), \ldots, \beta_n(x_n)$, coupled to local BRST-invariant operators $\mathcal{O}_1(x_1), \ldots \mathcal{O}_n(x_n)$ and go on-shell ($J = \beta = \theta = 0$). We obtain

$$\delta^{(n+1)}W = \left. \frac{\delta^{(n+1)}W}{\delta \rho^{(j)} \delta \beta_n(x_n) \ldots \delta \beta_1(x_1)} \right|_{o.s.} = 0. \quad (33)$$

Therefore the Green functions of local BRST-invariant operators are $\rho^{(j)}$-independent, as a consequence of the ST identities. The non-symmetric
counterterms in eq.(23) are required in order to ensure the fulfillment of the ST identities at orders $n \geq 2$. They must be included in order to guarantee the validity of eq.(33).

4 Conclusions

In this paper we have analyzed the restoration of the ST identities for pure massless YM theory in the Landau gauge within the BPHZL renormalization scheme and the Zimmermann-Lowenstein prescription for handling massless propagators. We have explicitly obtained the most general form of the action-like part of the symmetric regularized action $\mathbb{I}_s$, $s < 1$, to all orders in the loop expansion, and we have shown that non-symmetric counterterms arise in $\mathbb{I}_s^{(j)}$, $j \geq 2$, unless a special choice of normalization conditions for $\mathbb{I}_s^{(k)}$, $k < j$ is done.

We have verified that both UV and IR power-counting criteria are fulfilled for $t^4 \mathbb{I}_s$, thus guaranteeing the existence of the physical limit $\mathbb{I} = \lim_{s \to 1} \mathbb{I}_s$ (absence of zero-mass singularities).

We have provided a cohomological interpretation of the IR power-counting criterion, by noticing that it follows from the fact that the IR regulator $m = M(s - 1)$ enters into the classical action, together with its BRST partner $\bar{\rho}$, only via a cohomologically trivial term.

We have shown that the non-symmetric counterterms appearing in $\mathbb{I}_s^{(j)}$ at orders $j \geq 2$ do not vanish in the limit $s \to 1$. By making use of the ST parameterization of $t^4 \mathbb{I}_s$ we have been able to analyze the dependence of the Green functions of physical observables on the coefficients $\rho_1^{(k)}$ entering into the parameterization of the non-symmetric counterterms.

The proper inclusion of these non-symmetric counterterms is strictly necessary in order to guarantee the fulfillment of the ST identities at orders higher than one in the loop expansion, for general lower-orders normalization conditions.

The technique developed in this paper constitutes the building block to treat within the framework of the method discussed in [1, 2] those models, like the Minimal Supersymmetric Standard Model, where massless particles are present and the explicit computation of the counterterms, required to fulfill the relevant ST identities, cannot be avoided, due to the lack of an invariant regularization scheme.

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A The classical action

The classical action of pure massless Yang-Mills theory in the Landau gauge with an IR regulator \( m \) introduced via the BRST doublet \((\bar{\rho}, m)\) is [1]

\[
\Gamma^{(0)} = \int d^4x \left\{ -\frac{1}{4g^2} G_{\mu\nu}^a G^{\mu\nu a} - \bar{\omega}^a \partial_\mu (D^\mu \omega)^a + B^a \partial A^a + A^a_{\mu} (D^\mu \omega)^a \\
- \omega^a \frac{1}{2} f^{abc} \omega^b \omega^c + \bar{\omega}^a B^a \right\} + \int d^4x s \left( \frac{1}{2} \bar{\rho} m (A^a_{\mu})^2 + \bar{\rho} m \bar{\omega}^a \omega^a \right) \\
= \int d^4x \left\{ -\frac{1}{4g^2} G_{\mu\nu}^a G^{\mu\nu a} - \bar{\omega}^a \partial_\mu (D^\mu \omega)^a + B^a \partial A^a + A^a_{\mu} (D^\mu \omega)^a \\
- \omega^a \frac{1}{2} f^{abc} \omega^b \omega^c + \bar{\omega}^a B^a + \frac{1}{2} m^2 (A^a_{\mu})^2 + m^2 \bar{\omega}^a \omega^a \\
- \bar{\rho} m A^a_{\mu} \partial_\mu \omega^a - \bar{\rho} m B^a \omega^a - \frac{1}{2} \bar{\rho} m \bar{\omega}^a f^{abc} \omega^b \omega^c \right\} .
\]

\( G_{\mu\nu}^a \) is the field strength

\[
G_{\mu\nu}^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu .
\]

\( \omega^a \) is the ghost field, \( \bar{\omega}^a \) the antighost field, \( B^a \) the associated Nakanishi-Lautrup multiplier field.

BRST transformations

\[
s A^a_\mu = (D_\mu \omega)^a = \partial_\mu \omega^a + f^{abc} A^b_\mu \omega^c, \quad s \omega^a = -\frac{1}{2} f^{abc} \omega^b \omega^c, \\
s \bar{\omega}^a = B^a, \quad s B^a = 0, \\
s \bar{\rho} = m, \quad s m = 0 .
\]

References


