Heuristic approach to the Schwarzschild geometry
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abstract In this article I will present a simple Newtonian heuristic for “deriving” a weak-field approximation for the spacetime geometry of a point particle. The heuristic is based on Newtonian gravity, the notion of local inertial frames [the Einstein equivalence principle], plus the use of Galilean coordinate transformations to connect the freely falling local inertial frames back to the “fixed stars”. Because of the heuristic and quasi-Newtonian manner in which the spacetime geometry is obtained, we are only justified in expecting it to be a weak-field approximation to the true spacetime geometry. However, in the case of a spherically symmetric point mass the result is an exact solution of the vacuum Einstein field equations — it is the Schwarzschild geometry in Painlevé–Gullstrand coordinates.

This result is much stronger than the well-known result of Michell and Laplace whereby a Newtonian argument correctly estimates the value of the Schwarzschild radius — using the heuristic of this article one obtains the entire Schwarzschild geometry. Unfortunately the heuristic construction does not seem to generalize; it does not give the correct result for the Reissner–Nordström geometry (though it gets rather close), and does not seem capable of generating the Kerr geometry. Thus it is at this stage still somewhat unclear as to whether there is anything deeper to the heuristic than a remarkable but fortuitous coincidence.

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