Dynamical friction on Globular Clusters in core-triaxial galaxies: is it a cause of massive black hole accretion?

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ABSTRACT

The present work extends and deepens previous examinations of the evolution of globular cluster orbits in elliptical galaxies, by means of numerical simulations of a wide sample of orbits in 5 self-consistent triaxial galactic models characterized by a central core and different axial ratios. These models represent a valid and complete representation of regular orbits in elliptical galaxies. Dynamical friction is definitely shown to be an efficient cause of evolution for cluster system in elliptical galaxies of any mass and axial ratios. Moreover, our statistically significant sample of computed orbits confirm that the orbital decay times are, in many cases, much shorter than the age of the galaxies, so that many Globular Clusters have had enough time to reach the galactic centre where they may contributed significantly to the mass feeding of a compact nucleus therein.

Key words: galaxies: elliptical and lenticular, galaxies: star clusters, galaxies: nuclei, globular clusters: general; methods: numerical.

1 INTRODUCTION

The effects of dynamical friction (df) by field stars on the Globular Cluster (GC) orbits have been studied by many authors, since the pioneering work of Tremaine, Ostriker & Spitzer.
They deduced that df could have been responsible for the decay of a significant number of massive globulars into the inner region of M 31, with possible relevant consequences on the activity of a massive object therein. In more recent times, Ostriker, Binney & Saha (1989) and Pesce, Capuzzo–Dolcetta & Vietri (1992) (hereafter PCV) pointed out the role of triaxiality of the host galaxy in the enhancement of the effect of dynamical braking.

Actually, in triaxial galaxies, where the lack of symmetries in the potential implies the lack of conservation of any component of the angular momentum, one may expect that df is maximized in its importance. Indeed, in triaxial potentials the family of the so called box orbits is present; they have not a fixed pericentral distance and so objects moving on such orbits can penetrate deeply the inner regions of the galaxy (they densely fill their permitted region of the phase space). Box orbits are not rare, on the contrary they are substantial in the orbital structure of a triaxial galaxy. Indeed, it has been shown (Schwarzschild 1979; Gerhard & Binney 1985) that in triaxial potentials, at least those of moderate axial ratios, the family of box orbits constitutes the bone of the galactic orbital structure. If we, additionally, assume the framework of globular clusters as self-gravitating structures formed during the initial evolutionary phases of their mother galaxy, it turns out logical to expect an initial distribution of GC orbits biased towards low angular momenta, corresponding to small pericentric distances (in symmetric potentials) or to box orbits (in triaxial potentials).

All these points, when put together, suggest as extremely interesting a deep study of the evolution of GC orbits in axisymmetric potentials as well as in triaxial potentials. PCV were the first to analyze quantitatively the role of df in triaxial galaxies, hypothesized to be important by Ostriker et al. (1989) on the basis of their simplified scheme. PCV computed a set of orbits of GCs in the self-consistent triaxial model developed by Schwarzschild (1979) and de Zeeuw & Merritt (1983), well apt to represent a galaxy of moderate axis ratios (2:1.25:1). On the basis of the PCV results, Capuzzo-Dolcetta (1993) enlarged the data set of orbits in the same potential and studied the competitive effects of df and of the tidal disruption caused by a massive galactic nucleus in determining both the evolution of the radial distribution of GCs and the quantity of mass lost to the galactic center in form of orbitally decayed clusters. His results clearly indicate that the quantity of matter carried to the inner galactic regions in form of massive ($\geq 10^6 M_\odot$) globulars is relevant, with relevant consequences on the activity of a massive object in the galactic nucleus. The model is validated by that, taking into account both df and tidal effects acting on GCs, it is able
to explain the observed central flattening of the GC radial distribution in galaxies (see Capuzzo–Dolcetta & Tesseri 1997; Capuzzo–Dolcetta & Vignola 1997).

Given this scenario, the aim of this paper is a generalization to a wide set of triaxial models of different axial ratios of the results obtained in the case of just the Schwarzschild’s model.

The scheme of the paper is: in Sect. 2 we outline the frame of our work, in Sect. 3 we present the results of our numerical integrations of both planar and non-planar orbits of globulars in the various galactic models used; in Sect. 4 we apply our results to the study of the evolution of GC systems. Finally in Sect. 5 we present a general discussion and draw some conclusions.

2 THE MODEL

We study the orbital behaviour of globular clusters, seen as point-like mass objects influenced by the regular gravitational force of the background galaxy and by the df caused by the stars of the galactic field. Thus, the equation of motion of the GC of mass $M_{GC}$ is:

$$\ddot{r}_{GC} = -\nabla V + a_{df,G C}$$

where $r_{GC}$ is the position vector, $\nabla V$ is the gradient of the galactic gravitational potential and $a_{df,G C}$ accounts for the frictional deceleration and will be discussed in the next two subsections.

As usual, the second-order vector differential equation of motion is transformed into a system of first-order differential equations, to be numerically integrated. The numerical integration has been performed by mean of the Runge-Kutta-Merson method (Lance 1960); in particular, we use the subroutine $dDEQMR$ of the CERN Program Library. In absence of df, the energy is conserved better than at the 1 per cent level up to 1000 orbital crossing times. This level of accuracy in the numerical integration is likely conserved when df is included, as we checked by mean of an independent evaluation of the energy loss (i.e. by solving the differential equation $\dot{E} = a_{df,G C} \cdot \mathbf{v}$, where $E$ is the orbital energy per unit mass).

2.1 Dynamical Friction

The concept of dynamical friction has been introduced and developed by Chandrasekhar (1943). The classic Chandrasekhar’s formula for the df deceleration term has been extended by PCV to the triaxial case, in partial analogy with the Binney’s extension to the axysymmetric case (Binney 1977), obtaining:
\[ a_{gf,GC} = -\gamma_1 V_{1,GC} \hat{e}_1 - \gamma_2 V_{2,GC} \hat{e}_2 - \gamma_3 V_{3,GC} \hat{e}_3 \]  

(2)

where \( \hat{e}_i \) (\( i = 1, 2, 3 \)) are the eigenvectors of the velocity dispersion tensor and \( V_{i,GC} \) is the component of the GC velocity along the \( \hat{e}_i \) axis. The coefficients \( \gamma_i \) are (see PCV):

\[
\gamma_i = \frac{2\sqrt{2}\rho(r)}{\sigma_i^3} G^2 \ln \Lambda(m + M_{GC}) \int_{0}^{\infty} \frac{\sum_{k=1}^{3} \frac{v_{k,GC}^2/2\sigma_k^2}{\epsilon_i^2 + u}}{(\epsilon_i^2 + u)^{1/2}} du
\]

(3)

where: \( \rho(r) \) is the mass density of background stars of individual mass \( m \), \( \ln \Lambda \) is the usual Coulomb’s logarithm, \( M_{GC} \) is the globular cluster mass, \( G \) is the gravitational constant, \( \sigma_i \) is the eigenvalue, corresponding to \( \hat{e}_i \), of the velocity dispersion tensor, and \( \epsilon_i \) is the ratio between \( \sigma_i \) and the greatest eigenvalue set as \( \sigma_3 \).

2.2 The galactic models

There is a growing evidence that real elliptical galaxies are triaxial in shape and have a more or less bright central cusp in the light profile (smaller ellipticals showing, usually, steeper cusps). Indeed, contrarily to what believed for a long time, rarely the brightness profile of galaxies becomes perfectly flat in the inner region. Actually, the pioneering work of Schweizer (1979) found confirmation in the HST observations that show how the density distribution, when deprojected, shows usually a cuspy profile, at least within the resolution limit of the instrument (Lauer et al. 1995; Byun et al. 1996; Gebhardt et al. 1996; Faber et al. 1997) both in normal elliptical galaxies and in dwarf ellipticals (Stiavelli et al. 2001). Moreover, now we believe that most elliptical galaxies and bulges of spirals are at least moderately triaxial in shape (Franx, Illingworth & de Zeeuw 1991; Tremblay & Merritt 1995; Ryden 1996; Bak & Statler 2000).

For the purposes of this work, it is necessary to have both the matter density profile and the velocity distribution (i.e. the velocity dispersion tensor in Eq. 3); unfortunately, these latter data are not yet available for the models of self-consistent triaxial galaxies with density cusps developed so far (Merritt & Fridman 1996; Bockelmann et al. 2001). For this, we decided to rely on the large set of self-consistent triaxial models presently available (Statler 1987 and Statler private communication), that are based on the so-called ”perfect ellipsoid” models, i.e. those given by the mass density:

\[
\rho(m) = \frac{M}{\pi^2abc \left[1 + m^2\right]^2},
\]

(4)

where
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\[ m^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}. \]  

(5)

\((a > b > c)\) and \(M\) is the galactic mass. This family of density profiles is characterized by the central core, whose spatial size is \(\approx 0.64a\). Note that the density profile (4) has the same \(m^{-4}\) profile at large galactocentric distances of the cuspy Merritt & Fridman (1996) and Bockelmann et al. (2001) models (triaxial generalizations of the Hernquist 1990 and the Dehnen 1993 models), that do not account for the extended dark matter component that should be more gently decreasing. Given these limitations, the potential generated by the perfect ellipsoid has many advantages, as we now describe. Actually, the solution of the Poisson’s equation for the density law (4) is a potential in the so called Stäckel form (Weinacht 1924; de Zeeuw 1985).

\[
V(x, y, z) = -G \frac{M}{\pi} \times \int_0^\infty \frac{du}{(1 + \frac{x^2}{a^2 + u} + \frac{y^2}{b^2 + u} + \frac{z^2}{c^2 + u}) \sqrt{(a^2 + u)(b^2 + u)(c^2 + u)}}
\]

(6)

characterized by being fully integrable because the Hamilton-Jacobi equation separates with the choice of the cofocal ellipsoidal coordinates (denoted with \(\lambda, \mu, \nu\)). The model represented by the density law (4) is called "perfect ellipsoid" because it is the unique mass model having the potential in a Stäckel form in which the density is stratified on concentric ellipsoids (de Zeeuw & Lynden-Bell 1985). In addition to being fully integrable, the potential (6) is characterized by that all the orbits are regular, i.e. they respect three integrals of motion: the usual energy integral \((E)\) and two non classical integrals \((I_2, I_3)\) that converge, in the axisymmetric limit, to two of the three conserved components of the angular momentum. Consequently, there is a small number of well defined types of orbits while, in the cuspy models, chaotic (irregular) orbits are in a relevant fraction (Gerhard & Binney 1985; Merritt & Fridman 1996: Merritt & Valluri 1996). For these reasons, results obtained by mean of the perfect ellipsoid model (whose orbital properties have been fully studied by de Zeeuw 1985) constitute a deep analysis of the behaviour of regular orbits in triaxial potentials and give also results of astrophysical interest for what regards elliptical galaxies with a central density core.

The velocity dispersion tensor needed for the computation of the df deceleration has been obtained by Statler (1987) who developed a numerical solution for the self-consistent problem.
Table 1. Galactic model characteristics. Models are numbered as in Statler (1987). \(a\), \(b\) and \(c\) are the axes and \(T\) is the triaxiality parameter (Statler 1991), \(E_{\text{min}}\) is the depth of the potential well and \(E_2\) and \(y_{\text{lim}}\) are the limiting values of the energy and of the \(y_0\), respectively, that generate a loop orbit. \(E_{\text{min}}\) and \(E_2\) are in units of \([GM^2a^{-1}]\).

<table>
<thead>
<tr>
<th>Model</th>
<th>(b/a)</th>
<th>(c/a)</th>
<th>(T)</th>
<th>(E_{\text{min}})</th>
<th>(E_2)</th>
<th>(y_{\text{lim}}/a)</th>
</tr>
</thead>
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<tr>
<td>04</td>
<td>0.625</td>
<td>0.500</td>
<td>0.81</td>
<td>-0.9074</td>
<td>-0.6660</td>
<td>0.78</td>
</tr>
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<td>06</td>
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<td>0.500</td>
<td>0.31</td>
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<td>-0.7330</td>
<td>0.48</td>
</tr>
<tr>
<td>16</td>
<td>0.250</td>
<td>0.125</td>
<td>0.95</td>
<td>-1.5296</td>
<td>-0.6606</td>
<td>0.96</td>
</tr>
<tr>
<td>19</td>
<td>0.625</td>
<td>0.125</td>
<td>0.62</td>
<td>-1.1347</td>
<td>-0.7766</td>
<td>0.78</td>
</tr>
<tr>
<td>21</td>
<td>0.875</td>
<td>0.125</td>
<td>0.24</td>
<td>-0.9855</td>
<td>-0.8747</td>
<td>0.48</td>
</tr>
</tbody>
</table>

of the "perfect ellipsoid". The numerical solution has been found for all the explored range of axis ratios, suggesting that exact self consistent solution exist for any axis ratios. Statler found the numerical solution for 21 different values of the axis ratios using an unbiased catalog of 1065 orbits and matching the density of the mass model at 240 grid points. The grid has been constructed dividing the space with 15 shells in \(\lambda\), and with four divisions for both \(\mu\) and \(\nu\). Statler (private communication) recomputed 5 out of the 21 different models of the 1987 paper with a finer grid (960 shells, constructed with 8 divisions both in \(\mu\) and in \(\nu\)) and using the improved Lucy’s method described in Statler (1991). The main characteristics of these models are reported in Table 1.

3 SIMULATION OF THE ORBITAL DECAY

The role of df in the core-triaxial galaxies modeled as described in Sect. 2 is studied here by numerical integration of a wide set GC orbits.

Hereafter, if not explicitly declared differently, we assume that all the quantities are expressed taking as length and mass units \(a\) and \(M\) respectively. Moreover setting \(G = 1\) implies

\[
\tau = G^{-1/2}a^{3/2}M^{-1/2} = 1.49 \cdot 10^6 \text{ yr} \left(\frac{M}{10^{11}M_\odot}\right)^{-1/2} \left(\frac{a}{1 \text{kpc}}\right)^{3/2} \tag{7}
\]

as unit of time.

To reduce the computational time, we exploit that the linear scaling of the df deceleration with \(M_{\text{GC}}\) allows us to fix a quite high mass for the GC. By the same token, the choice \(\ln \Lambda = 10\) is not limiting, because of the linear scaling with \(\ln \Lambda\) of the df braking term. In the
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following, if not explicity mentioned, we, indeed, refer always to a GC of $M_{GC} = 5 \times 10^{-4} M$; this corresponds to $M_{GC} = 5 \times 10^7 M_\odot$ for a typical elliptical galaxy with $M = 10^{11} M_\odot$.

3.1 The choice of the orbits

We have performed numerical integrations of many (about 850) bound orbits ($E_{min} \leq E_0 < 0$) of different types (radial, box and different kind of loop orbits of different elongation) at various energies, in the ranges $0.05 \leq E_0/E_{min} \leq 0.95$ for planar box orbits, $0.05 \leq E_0/E_{min} \leq E_2/E_{min}$ for planar loop orbits ($E_2$ is the limiting energy for planar loop orbits, see Fig. 1) and $0.05 \leq E_0/E_{min} \leq 0.6$ for all non-planar orbits $^1$. The choice of the lower limit ($0.05$) for $E_0/E_{min}$ is done to avoid an exceedingly large relative error in the energy as merely due to a value of energy too close to zero. On the other hand, the choices of $0.95$ as limiting energy value is to avoid a useless integration of orbits already confined in the very inner regions; the $0.6$ limit for non planar orbits is sufficient to have an adequate sampling of all the four orbital families.

Simulations were stopped at the ”orbital decay time”, $T_{df}$, when two conditions are fulfilled: i the df energy decrease in a time step is comparable to the numerical error in the energy and ii the GC is already confined in the inner galactic regions. In all the cases studied this corresponds to apocentric distances less than $0.01a$.

To select orbits, we exploit that all the planar orbits in a triaxial potential cross at least once the $y$ (or $x$) axis with the velocity vector perpendicular to that axis (Schwarzschild 1982). Thus, we always start the integration with the GC at $r_0=(0, y_0 > 0, 0)$, and with $v_0=(v_{x0} \geq 0, 0, 0)$ choosing $y_0$ and $v_{x0}$ such to obtain the required initial energy. With this choice of the initial conditions we can easily classify the morphology of the planar orbits (see Fig. 1): for any given energy $E_0$, box orbits have $y_0$ smaller than a fixed value $y_{lim}$ (a parameter that depends on the axis ratios of the models, see Table 1); just above $y_{lim}$ we find the most elongated loop orbits. A further increase of $y_0$ results in less elongated orbits up to the value corresponding to the (unique) closed orbit (dashed line of Fig. 1), that is quasi-circular at high energies. Above this value we re-obtain the same (previously found) elongated orbits (i.e. loop orbits cross the $y$ axis with the velocity vector perpendicular to the axis itself twice). Fig. 1 allows to generate all the possible planar orbits.

Also to obtain non planar orbits we fix the initial energy ($E_0$) and the initial position on the

$^1$ $E_0/E_{min} = 0$ corresponds to the asymptotically free orbit, while $E_0/E_{min} = 1$ to a GC at quiet in the origin.

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$y_0$

![Diagram of regions delimited by box and loop orbits](image)

**Figure 1.** Curves delimiting the regions of the $(E_0, y_0)$ plane where box and loop orbits are permitted. The dashed curve is $y_{0cl}(E)$, that corresponds to the initial values of $y_0$ giving rise to the particular case of closed, periodic loops.

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$y$ axis ($x_0 = 0, y_0 > 0, z_0 = 0$): then we choose the initial velocity orthogonal to the $y$ axis, pointing in the positive $x$ and $z$ directions ($v_{x0} = v_0 \cos \phi > 0, v_{y0} = 0, v_{z0} = v_0 \sin \phi > 0$) letting the inclination angle $\phi$ over the $x-y$ plane assume the five values $15^\circ, 30^\circ, 45^\circ, 70^\circ$ and $90^\circ$ (the latter value corresponds to an orbit in the $y-z$ plane).

This way, a sample of orbits sufficiently wide to draw conclusions about differences in the evolution of orbits limited to a plane and moving in 3D is obtained.

We checked that the particular choice of the initial condition influences slightly (less than 5% for the highest energies and even less for lower energies) the orbital decay time, in the sense that two GCs starting on two different points of a given orbit have very similar decay times. This ensures that the results on the role of df presented here are quite general.

### 3.2 Quasi-radial orbits

To understand better the characteristics of the evolution of orbits in presence of df it is useful to have a glance at radial orbits. Obviously, in a triaxial galaxy, radial orbits are a-priori allowed along the axes, only. We checked that in our models radial orbits are stable
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Figure 2. The ratios of the df decay times of radial orbits along the $y$ (squares) and $z$ (triangles) axis to that of orbits in the $x$ direction, as functions of the initial orbital energy. Results are relative to model 04.

for any initial energy just along the major axis ($x$); those along $y$ and $z$ are stable at low energies, only, while for higher energies they quickly develop an $x$-oscillation. The decay times of orbits along the 3 axes are similar at low energies, when the orbits are quasi-radial along all the 3 axes, while they increasingly differ at higher energies (see Fig. 2). Actually, the efficiency of df as dynamical braking mechanism is greater for greater values of the phase-space density (PCV) which roughly follows the behaviour of the space density, $\rho(r)$, that (along an axis of symmetry) is flatter (greater) in the direction of greater length scale parameter ($a > b > c$) at a given distance from the origin: high enough energies are required for a GC to sample a spatial region where the density difference is appreciable along the various axes.

3.3 Planar orbits

To resume the general characteristics of the shape-evolution of orbits subjected to df, we show in Fig. 3 some time-snapshots of three representative orbits: a closed loop, an almost
Figure 3. Every column represents three time snapshots corresponding to an initial, intermediate and final phases of the evolution (time flows downward) of three orbits subjected to dynamical friction in the case of model 04; the left column refers to a typical loop; the central and right columns refer to a very few elongated box and a typical (thin) box, respectively.

"squared" box and an elongated (typical) box. The evolution of the loop orbit (initially quasi-circular) shows an increasing collimation in the $y-$direction (left column of Fig. 3). This is due to indirect ($i$) and direct ($ii$) effects of df: ($i$) the initial reduction of the orbital energy due to df braking transforms the circular shape into that of a typical low-energy loop, which is characterized by being elongated in the $y-$direction (de Zeeuw, 1985); ($ii$) a further reduction of energy leads, eventually, the orbit to become a very thin (quasi-radial) box. This happens at an energy greater than $E_2$ (the minimum energy allowed to loop orbits), thus we consider the latter a direct effect of df because it corresponds to an actual change of the type
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Figure 4. Panel (a): the energy decay times of a GC with $M_{GC} = 5 \times 10^{-5} M$ (upper curves) and of a GC with $M_{GC} = 5 \times 10^{-4} M$ (lower curves) as functions of the initial energy. Panel (b): the energy decay times of a GC of given arbitrary mass vs the mean radius of the corresponding unperturbed (i.e. without df) orbit. $T_{df}$ is in arbitrary units. Both panels refer to model 04 and different curves refer to the different kinds of planar orbits (solid: closed loops; dashed: least elongated boxes; dotted: $x-$ radial orbits). Horizontal lines mark the age of the universe (assumed 12 Gyr) corresponding to three choices of $a$ and $M$: dot-dashed is for a typical dwarf elliptical ($a = 0.1$ kpc, $M = 10^7 M_\odot$), dashed for a normal elliptical ($a = 1$ kpc, $M = 10^{11} M_\odot$) and dotted for a compact bulge ($a = 0.2$ kpc, $M = 3 \times 10^9 M_\odot$).

of orbit structure. A different effect of df in the two coordinate directions was also expected simply by that $a > b$, that corresponds to a matter density distributed flatterly along the $x$ direction, meaning a stronger deceleration along $x$.

The time evolution of both the box orbits in Fig. 3 is mainly that of shrinking in the two directions, even with a slight flattening along the $x-$direction.

The first quantitative results of our simulations are summarized in Figs. 4 and 5 that show the dependence of the decay times of the various families of orbits upon their initial energy and also (just for the model 04) upon the mean radius of the corresponding unperturbed orbit (i.e. without df). For any initial energy, closed loop orbits have longest decay times, while the $x-$radial orbits have them shortest. Because our orbital integrations show that box orbits with larger $y_0$ have longer decay times, it is possible to conclude that all box orbits are confined between the dotted and the dashed lines in the figures. In analogy, more elongated orbits are more powerfully decelerated than circular orbits, because the presence of the density term in the expression of df deceleration makes its effect more evident in
the inner regions of the galaxy where elongated orbits plunge. Consequently, all loop orbits should be confined between the dashed and the solid lines; it is worth noting that there are, instead, few very eccentric loop orbits that do not follow this rule and that decay in the innermost galactic region in a time shorter than that of few less elongated box orbits. Anyway, we found that just loops with $y_{lim} \leq y_0 \leq 1.002 \, y_{lim}$ are fasterly decaying than box
orbits with $0.998 \leq y_0 \leq y_{\text{lim}}$. This very narrow range of initial conditions means that the number of loop orbits evolving faster than box orbits is almost negligible.

The behaviour of the decay times as functions of the mean galactocentric radius of the corresponding unperturbed orbits (Fig. 4b) shows that the scaling of the decay time with the mean galactocentric radius is different for the different families of orbits, being always an increasing function of the mean galactocentric radius.

One of the scopes of this paper is the exam of the dependence of the df decay time on the axial ratios of elliptical galaxies. In the galactic models used here, for a given total galactic mass there is an obvious inverse linear dependence of the df efficiency on the axes lengths through the central density ($\rho_0 = M/(\pi^2abc)$) that is partly counterbalanced by that increasing $a$, $b$ or $c$ the size of the core increases and df is larger in larger cores (this dependence on the mass density is just slightly modified by variations of the velocity dispersion tensor with $a$, $b$ and $c$).

We found a quasi-linear dependence of $\log T_{df}$ on $b^n/c$, as clearly shown in Fig. 6, for the whole range of orbital energies investigated, with $n = 0.36$ for closed loop orbits, $n = 0.25$ for the lesser elongated boxes and $n = -0.32$ for $x$–radial orbits.

### 3.4 Non planar orbits

Of course, a reliable application of the results to the study of the orbital evolution of GCs in presence of df needs the computation of a large set of orbits sampling as completely as possible the allowed phase-space. In Subsect. 3.3 we have deeply examined the orbital evolution in one of the coordinate planes; obviously, this is not an exhaustive representation of all the possible orbits of clusters in their host galaxy but it suffices to outline some general conclusions on the role of df in the global evolution of Globular Cluster Systems (GCSs) in galaxies of various size and mass, size and axis ratios. We postpone to a following paper a thorough investigation of the df effects on the evolution of a GCS in an elliptical galaxy by mean of the analysis of a complete sample of orbits, limiting here ourselves to results concerning some selected 3D orbits.

Fig. 7 shows the energy-dependence of the df decay time for 3D orbits. The decay time is increasing with the initial inclination angle over the $x–y$ plane making the $y–z$ planar orbits a lower limit to the df efficiency; this is due to that the $z$–axis has the shortest space scale parameter ($c$). Correspondingly, the density along this axis decreases more steeply than
along $y$ and $x$, respectively, and, for what explained in the sub Sect. 3.2, $d_f$ is thus maximized on the $x - y$ plane. A remarkable feature of Fig. 7 is that the ratio between the $T_{df}$ time of orbits moving on the $y - z$ plane ($\phi = 90^\circ$) and that of orbits of the same energy (and type) limited to the $x - y$ plane ($\phi = 0^\circ$) is much larger for models with larger $b/c$ ratios (indeed, for fixed $a$ we already noticed that a larger axis length implies a shorter $T_{df}$).

Because of this, the modelization (that we present in the next section) of a GCS as composed just by objects moving on the $x - y$ plane leads to a slight overestimate of the role of $d_f$.
on the GCS evolution. A quantitative evaluation of this overestimate is straightforwardly obtained by Fig. 7 itself because, by the same arguments above, $y - z$ is the plane where $df$ is minimized.

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**Figure 7.** Left column: energy decay times vs the initial inclination angle $\phi$ (in degrees) for three galactic models (models 04, 16 and 21, bottom to up). Black squares represent orbits starting with $y_0 = y_{lim}$ and open circles those with $y_0 = y_{0lc}$; the initial orbital energy $E_0$ is set to 0.2 $E_{min}$. Right column: the dependence of the energy decay time upon the initial energy for orbits at different angles $\phi$ over the $x - y$ plane. Orbits have $y_0$ corresponding to that of the closed loop. For the sake of presentation, we report just the $\phi = 0^\circ$ (empty circles), $45^\circ$ (crosses) and $90^\circ$ (filled squares) cases. Orbits decaying in a time longer than $6 \times 10^4 \tau$ are not reported.
Figure 8. The fraction (to the total) of orbits not yet frictionally decayed at time $t$, for a single-mass GCS population ($M_{GC} = 5 \times 10^{-5} M$). Solid lines refer to the whole GCS; dashed lines to the sub-sample of loop orbits; dotted lines to that of box orbits. Panel (a): the GCS energy distribution is Gaussian with $<E>/E_{\text{min}} = 0.1$; Panel (b): as in panel (a), but $<E>/E_{\text{min}} = 0.2$; Panel (c): as in panel (a), but $<E>/E_{\text{min}} = 0.8$; Panel (d): the energy distribution is uniform over the whole allowed range of E. All the plots refer to the model 04.

4 AN APPLICATION TO THE EVOLUTION OF GLOBULAR CLUSTER SYSTEMS

It is well known that GCSs in galaxies evolve in time (Fall & Rees 1977; Capuzzo–Dolcetta & Tesser 1997; Murali & Weinberg 1997a,b; Baumgardt 1998; Capuzzo–Dolcetta & Tesser...
Figure 9. The left column (panels (A), (B) and (C)) shows the time evolution of the mass in form of globular clusters dragged into the innermost galactic zone ($r \leq 0.01a$) for the model 21 (the model where df is more efficient) and for three different power law mass spectra of the GCS (see text). Solid, dotted and dashed lines refer to GCS gaussian energy distributions with $<E>/E_{\text{min}} = 0.8$, 0.2 and 0.1, respectively. Vertical lines mark 1 Gyr with symbols as in Figs. 4-5. Panel (A): In the mass function: $s = 0$, $M_{\text{min}} = 5 \times 10^{-7} M$, $M_{\text{max}} = 5 \times 10^{-3} M$, $N_{\text{tot}} = 150$. Panel (B): $s = 0$, $M_{\text{min}} = 5 \times 10^{-7} M$, $M_{\text{max}} = 5 \times 10^{-4} M$, $N_{\text{tot}} = 1500$; Panel (C): $s = 2$, $M_{\text{min}} = 5 \times 10^{-7} M$, $M_{\text{max}} = 5 \times 10^{-4} M$, $N_{\text{tot}} = 1500$; Right column (panels (a), (b) and (c)): time evolution of the rate of GC mass falling onto the galactic centre; all parameters and symbols as in the left column. Note that a rate of $1 M_\odot/\text{yr}$ corresponds to $1.5 \times 10^{-5} M_\odot/\text{yr}^{1/2}$.

1999; Vesperini 2001) because of the individual internal evolution of globulars and because they, as satellites of a galaxy, have an orbital evolution caused by both a direct and indirect interaction with the external field, via dynamical friction and tidal effects. Of course, in a disk galaxy the role of the flattened gas-star distribution may be relevant and may overcome
the role of the bulge; analogously, the tidal effect of a massive central object, if present, may overwhelm the tidal disruptive effect of the regular large scale star distribution.

Here we limit ourselves to isolate and evaluate the role of the dynamical friction caused by the regular triaxial distribution of stars in a galaxy modeled as described in Sect. 2 on the overall GCS evolution. Note that neglecting evolutionary effects other than df does not introduce an excessively large approximation relatively to the evolution of high-mass GCS, for which the df effect is in any case overwhelming (see for instance Capriotti & Hawey 1996). This study is a first generalization of previous results (Capuzzo–Dolcetta 1993) that showed how crucial is dynamical braking in the determination of the observed characteristics of GCS in galaxies.

The results we present in this section are for clusters on planar orbits: this allows an easier presentation and discussion of some general aspects of the GCS evolution, without claiming to be a complete and exhaustive approach (that will be presented in a forthcoming paper).

We assume an initial distribution of the orbital energies of the globular clusters and a GCS mass spectrum, with the aim to follow the evolution of the GCS over a time of the order of the Hubble time.

More specifically, we assume an initial population of the different types of orbits (loops and boxes of different elongation) given by:

\[ dN_{0,i} = f_{0,i}(E)g_0(M)dEdM \]  

where \( i \) identifies the orbital type and the normalization is such to have a total number of GC equal to \( N_{tot} \). For the sake of simplicity, we assume that the number of clusters initially moving on box and loop orbits is the same, equal to \( N_{tot}/2 \). Regarding the shapes of the initial energy distributions, \( f_{0,i}(E) \), we make two different choices: \( i \) uniform (flat) over the whole permitted interval of bound energies and \( ii \) gaussian of chosen mean and variance.

For the initial mass spectrum, \( g_0(M) \), of the GCS we make different choices: \( i \) a single mass population (all the GCs have the same mass \( M_{GC} = 5 \times 10^{-5}M \)); \( ii \) two uniform mass distributions between a fixed low mass cutoff, \( M_{min} = 5 \times 10^{-7}M \), and two different high mass cutoffs, \( M_{max} = 5 \times 10^{-4}M \) and \( M_{max} = 5 \times 10^{-3}M \); \( iii \) a power law \( \propto M^{-2} \) cut at \( M_{min} = 5 \times 10^{-7}M \) and at \( M_{max} = 5 \times 10^{-4}M \).

The fraction of "surviving" clusters (in the case of single-mass population) at various ages is shown in Fig. 8, where the fraction of GCs moving on box and loop orbits is given, too. In this case of single mass GCS, the df "disruptive" mechanism acts quite abruptly.
The steep descent of the number of surviving clusters reflects the steepness shown by orbital decay time vs energy relations of Figs. 4-5. Moreover, we note an inversion of the relative abundance of GCs on box and loop orbits at sufficiently old ages: for an initial distribution of GC orbital energies peaked at high energies, more clusters moving on loop orbits are expected at larger ages (see panels (a), (b) of Fig. 8), while at "low" orbital energies the opposite is seen in panels (c) and (d) of Fig. 8. Of course, this feature could constitute a way to deduce information about the initial distribution function of the GCS, whenever reliable velocity and position data of extragalactic globular clusters will be available.

Fig. 9 gives information about the mass dragged in the innermost galactic region in form of frictionally decayed GCs ($M_d$). The left column shows the time evolution of $M_d$, while the right column gives the time-rate of this phenomenon ($\dot{M}_d$). The earlier steepening of the $M_d(t)$ function in panel (A) with respect to panel (B) (which obviously corresponds to an earlier peak of the mass accretion rate) is simply due to the larger value of the average GC mass, implied by the greater high mass cutoff ($M_{\text{max}}$). We note that $M_{\text{max}}$ a factor ten larger corresponds to about the same factor of reduction of the time of the peak of the mass accretion rate; actually, the inverse proportionality between $\dot{M}_d$ and $M_{\text{max}}$ simply reflects the behaviour of the df decay time vs $M_{\text{GC}}$. The smaller slope of $M_d(t)$, and thus the broader mass accretion rate in the two panels (C) and (c), respectively, is explained by the $-2$ slope of the power law GCS mass function that implies a larger relative number of light clusters, that decay later.

Let’s note that Fig. 9 gives important hints on the relative role played by the various free parameters into the accretion of the galactic central region due to decayed GCs. It results that: i) the time of the accretion burst is mainly determined by the value of the mass of the heaviest clusters and by the mean value of their orbital energy; ii) the height of the peak of the mass accretion rate increases with the abundance of the GC sample and decreases with its orbital average energy (see the various curves in each panel); iii) the length in time of the (effective) accretion process mainly depends on the spread in the mass distribution of the GCS.

To resume, the efficiency of the accretion mechanism is maximum for populous GCSs of low orbital energies and large individual mass. As a consequence, with the choice of few parameters it is possible to change the behaviour of the decay rate in the galactic center (i.e. the time of the maximum and the duration of the whole process).
5 CONCLUSIONS

The evolution of the GCS of a galaxy is determined by various concurrent causes. In elliptical galaxies, due to the absence of the flattened disk component it is easier to isolate the relevant evolutionary effects. A part from internal mechanisms (mass loss due to stellar evolution, dynamical evaporation, etc.), the evolutionary effects are mainly consisting in the collective role played by the galactic stellar distribution through tidal distortion and dynamical friction, as well as by the tidal interaction between clusters and massive objects (mainly super-massive black holes sited in the galactic centers). In this paper we have enlarged and deepened previous theoretical and numerical investigations of the role of dynamical friction extending them to a significant range of axial ratios of triaxial self-consistent models of core elliptical galaxies. As galactic models, we have used 5 self consistent triaxial configurations, computed by Statler (1987 and private communication) with the method of maximum entropy applied to Stäckel potentials. The galactic models are characterized by the perfect ellipsoid density profiles, having a central core and decaying as $r^{-4}$ at large radii. While these profiles do not fit with the cuspy density behaviour observed at high resolution in many elliptical galaxies, so that they are not very realistic in the central region, they have the advantage of being appreciably simple and apt to a deep study of the evolution of regular orbits in triaxial potentials. We computed a remarkably large set of orbits on the $x - y$ coordinate plane of these galaxies; fully 3D orbits have been examined, too, but not yet deeply enough to draw fully general conclusions.

The main results of this paper may be resumed as follows: 1) df is confirmed to be important in inducing orbital evolution of sufficiently massive globular clusters in all of the models studied; 2) triaxiality plays a role, in that box orbits (absent in the axysimmetric case) suffer more of the frictional deceleration than the usual loop orbits (the ratio of the loop- to box-decay time ranges from 1 to 10 in the interval of orbital energies studied, rapidly increasing with the energy value); in any case the initial orbital energy is the most relevant parameter, more than the large scale galactic phase-space structure; 3) the df braking efficiency scales with the square root of the average mass density of the galaxy (so to expect a larger variation in time of the GCS distribution in more compact galaxies); 4) with regard to the axis ratio, we saw that for a fixed average mass density, orbits of the same type suffer more of df in more roundish galaxies.

We have studied the df effect (alone) on the global evolution of a GCS composed by a
Dynamical friction on Globular Clusters in core-triaxial galaxies.

set of globulars of different masses and different initial conditions on position and velocity. A relevant result is that, under general assumptions, the phase-space GCS distribution function is heavily modified by df. Indeed, in GCSs having an initial distribution peaked at low values of orbital energy, all the loop orbits are transformed into boxes before they decay completely leaving, eventually, the galaxy spoiled of globulars. This effect corresponds to a time evolution of the relative abundance of loop and box orbits; of course, this is far from being observable with the present day instrumentation.

Another important consequence of dynamical friction on a GCS is the feedback it has on the galaxy due to the accumulation of mass in form of orbitally decayed clusters in the central regions, as it was first suggested by Capuzzo–Dolcetta (1993) on the basis of calculations based on a single galactic model. The present paper indicates that the results of Capuzzo–Dolcetta (1993) can be generalized to a large interval of types of the host galaxies (of different masses, surface brightness, axial ratios, etc.): df has actually been able to carry into the inner few parsecs of the galaxy an amount of massive GCs such to guarantee the formation of a "reservoir" of mass around a compact nucleus in a time scale compatible with the red-shift distribution of AGNs. The modes and details of the actual feeding of the central black hole by means of this accreted mass have not been studied here, as well as the modes of conversion into electromagnetic energy of the gravitational energy. Here we have just identified and examined the role of some of the free parameters of the model in determining the time of the mass accretion burst, the peak of the mass accretion rate and the length in time of the mass accretion process. Qualitatively: the larger the typical value of the GC mass and/or the "colder" the GCS, the sooner the occurrence of a substantial mass accretion into the inner galaxy region; the more numerous the GCS and/or the colder the GCS, the higher the value of the GCS mass loss time rate; the larger the spread of the mass distribution of the GCS, the longer the (possible) induced duration of the nuclear activity. Quantitatively speaking, the GCSs characterized by the highest value of the average cluster mass ($< M_{GC} > = 2.5 \cdot 10^{-3} M$) studied here all decay in a time much shorter than the Hubble time, making the central mass accretion rate rise up to hundreds of solar masses per year (having assumed an initial number of GCs $N_{tot} = 150$). On the other hand, GCSs characterized by a flat mass function with $< M_{GC} > = 3.5 \cdot 10^{-6} M$ are not able to carry mass to the centre in a quantity and with a time rate (it is always less than $1 M_{\odot}/yr$) large enough to ignite a central gravitational engine and to sustain an AGN activity (even assuming an initial number of $N_{tot} = 1500$ GCs). The relevant final result is that, in the
range of the parameters of the orbital energy and mass distribution of the GCSs explored here, a huge region of reasonable values does exist that corresponds to high mass accretion rates ($\gtrsim 10 \, M_\odot/yr$) and to a huge amount of mass ($\sim 10^8 \, M_\odot$) accumulated in few $10^8 \, yr$ into the nuclear zone to account for the fueling and growth of a central massive black hole. All this convinces us of the importance to examine in a deeper detail the actual fate of clusters interacting among themselves and with the external field in the inner galactic regions where they are eventually confined by dynamical friction.

ACKNOWLEDGMENTS

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REFERENCES

APPENDIX A: FITTING FORMULAS

Even in the limits of planar orbits, for practical use is convenient to have fitting formulas for the estimate of the df decay time.

"Synthetic" (even if not very accurate) expressions to obtain an estimate of the df decay time of different families of orbits ($x$-radial, less elongated box and closed loop orbits respectively) involving the initial orbital energy ($E_0$) and axial ratios ($b$ and $c$) are:

\[ T_{\text{rad}}^{\text{df}} (yr) = 1.57 \times 10^4 \cdot \frac{(E_0/E_{\text{min}})^{-0.475}}{e^{0.080 \frac{b^{0.080}}{c^{0.080}}}} \times \]
\[ \left( \frac{M}{10^{11} M_\odot} \right)^{-\frac{1}{2}} \left( \frac{a}{10^3 \text{ pc}} \right)^{\frac{3}{2}} \left( \frac{M}{M_{\text{GC}}} \right) \ln \Lambda \] \hfill (A1)

\[ T_{\text{box}}^{\text{df}} (yr) = 1.40 \times 10^4 \cdot \frac{(E_0/E_{\text{min}})^{-0.705}}{e^{0.104 \frac{b^{0.25}}{c^{0.25}}}} \times \]
\[ \left( \frac{M}{10^{11} M_\odot} \right)^{-\frac{1}{2}} \left( \frac{a}{10^3 \text{ pc}} \right)^{\frac{3}{2}} \left( \frac{M}{M_{\text{GC}}} \right) \ln \Lambda \] \hfill (A2)

\[ T_{\text{loop}}^{\text{df}} (yr) = 1.35 \times 10^3 \cdot \frac{(E_0/E_{\text{min}})^{-2.134}}{e^{0.130 \frac{b^{0.38}}{c^{0.38}}}} \times \]
\[ \left( \frac{M}{10^{11} M_\odot} \right)^{-\frac{1}{2}} \left( \frac{a}{10^3 \text{ pc}} \right)^{\frac{3}{2}} \left( \frac{M}{M_{\text{GC}}} \right) \ln \Lambda \] \hfill (A3)

More accurate fits may be obtained exploiting the existence of the integrals of motion.
Table A1. Values of the coefficients of the approximation formulae (A4,A5) for the different galactic models.

<table>
<thead>
<tr>
<th>Mod</th>
<th>A</th>
<th>α</th>
<th>B</th>
<th>β</th>
</tr>
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<tr>
<td></td>
<td>([G^{-2}a^4M^{-3}])</td>
<td>([(GM)^{-1/2}a^{3/2}])</td>
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<td>04</td>
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<td>-1.60</td>
<td>40.46</td>
<td>-0.656</td>
</tr>
</tbody>
</table>

for the Stäckel potential \((E_0, I_2 \text{ and } I_3, \text{ de Zeeuw 1985})\). Because in the case of planar orbits \(I_3 \equiv 0\), we have a fitting formula depending just on \(E_0\) and \(I_2\):

\[
T_{df}(E_0, I_2) = m(E_0) \cdot I_2 + q(E_0) \tag{A4}
\]

where \(m\), and \(q\) depends on \(E_0\) as power laws:

\[
m(E_0) = A \cdot (E_0/E_{\text{min}})^\alpha \quad q(E_0) = B \cdot (E_0/E_{\text{min}})^\beta \tag{A5}
\]

with \(A, B, \alpha\) and \(\beta\) constants, reported in Table A1. These fits give the df decay time, in units of \(\tau\) (Eq. 7), for a GC with mass \(M_{GC} = 5 \times 10^{-4}M\) and with the choice of \(\ln \Lambda = 10\). Of course the fits are valid just in the energy range explored in our simulations: they fails just for the lowest permitted values of \(I_2\) at the lowest energies (i.e. for very elongated box and \(x\)-radial orbits for energies \(E_0/E_{\text{min}} = 0.05\)). Under the choice of initials conditions done in this paper \((x_0 = z_0 = 0, y_0 \geq 0, v_{x0} \geq 0, v_{y0} = v_{z0} = 0)\) the expression for \(I_2\) (equation 49 in de Zeeuw 1985) reduces to:

\[
I_2 = \frac{1}{2}(y_0^2 - a^2 + b^2)v_{x0}^2 \tag{A6}
\]

For their use in Eq. A4 \(a, b, y_0\) and \(v_{x0}\) must enter in the units used throughout this paper.

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