Studying the role of the ’t Hooft interaction in QCD, by means of lattice simulations

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We report on a recent investigation on the dynamics of light quarks, at intermediate and low-energy. By measuring some appropriate correlation functions on the lattice, it is possible to probe the Dirac and flavor structure of the non-perturbative quark-quark interaction and look directly for signatures of the ’t Hooft Lagrangian. Results obtained with chiral fermions strongly support the instanton picture and the phenomenological parameters of the Instanton Liquid Model.

1. Introduction

Lattice QCD simulations have played a major role in improving our understanding of the quark dynamics. For example, by measuring the Wilson loop it was possible to extract the non-relativistic confining potential, between two static color sources. Unfortunately, the concept of a non-relativistic potential becomes useless, when the color sources involved are light or even massless dynamical quarks. Clearly, any information about the interaction between \( u \) and \( d \) quarks must be cast into a fully relativistic field-theoretic formalism. This makes the problem of understanding the light-quark dynamics much more complicated.

An important step forward in this direction was made by ’t Hooft [1], who derived a low-energy effective contact interaction between light quarks, by computing the Euclidean QCD partition function in the semi-classical limit (i.e. accounting for small perturbations around the instanton solution). In the \( N_f = 2 \) case, the ’t Hooft Lagrangian can be written as:

\[
L_{tH} = C_{\bar{n}, \bar{\rho}} \left( \frac{2 N_c - 1}{2 N_c} \right) \left[ (\psi^\dagger \tau_a^- \psi)^2 - (\psi^\dagger \gamma_5 \tau^- \psi)^2 \right] + \frac{1}{4 N_c} \left( \psi^\dagger \sigma_{\mu \nu} \tau_a^- \psi \right),
\]

where \( \tau^- = (\vec{\tau}, i) \) (\( \vec{\tau} \) are isospin Pauli matrices), and \( C_{\bar{n}, \bar{\rho}} \) is a coupling constant depending on the typical density of instantons in the vacuum \( \bar{n} \), and their typical size \( \bar{\rho} \).

The most important feature of the ’t Hooft interaction is that it is invariant under global \( SU_L(2) \times SU_R(2) \) rotations, but changes sign under \( U_A(1) \) transformations. Moreover, the vertex is active only between quarks of different flavor and different chirality. The finite size of the instanton field provides a natural cut-off scale for the interaction, \( \Lambda_{cut-off} \simeq 1/\bar{\rho} \).

Unfortunately, the instanton density and size distribution cannot be computed from first principles in QCD. However, the Dirac and flavor structure of the ’t Hooft Lagrangian is a model-independent prediction of the semi-classical approximation. The Instanton Liquid Model\(^2\) (ILM) [2] consists of assuming that the vacuum is saturated by instantons with typical size \( \bar{\rho} \simeq 1/3 \) fm and density \( \bar{n} \simeq 1 \) fm\(^{-4}\).

In the following, we shall report on a recent lattice-based investigation [5,6], which allows to check if effective quark-quark interaction in QCD has the specific Dirac and flavor structure predicted by the semi-classical expression \( L_{tH} \), and if the ILM phenomenological parameters are realistic.

\(^2\)In the last two decades it has been shown that the ILM can quantitatively explain important non-perturbative phenomena related to the structure of the QCD vacuum, as well as light hadrons masses and formfactors (see e.g. [2–4])
2. Dirac structure

In order to probe the Dirac structure of $L_{tH}$, we study the flavor Non-Singlet (NS) chirality-flip ratio, introduced in [5]:

$$R^{NS}(\tau) := \frac{A^{NS}_{\text{flip}}(\tau)}{A^{NS}_{\text{non-flip}}(\tau)} = \frac{\Pi_\pi(\tau) - \Pi_\delta(\tau)}{\Pi_\pi(\tau) + \Pi_\delta(\tau)},$$

(2)

where $\Pi_\pi(\tau)$ and $\Pi_\delta(\tau)$ are pseudoscalar (PS) and scalar (S) iso-triplet two-point correlators related to the currents $J_\pi(\tau) := \bar{u}(\tau) i \gamma_5 d(\tau)$ and $J_\delta(\tau) := \bar{u}(\tau) d(\tau)$. If the propagation is chosen along the time direction, $A^{NS}_{\text{flip(non-flip)}}(\tau)$ represents the probability amplitude for a $q\bar{q}$ pair with isospin 1 to be found after a “time” interval $\tau$ in a state in which the chirality of the quark and anti-quark is interchanged (not interchanged). Notice that the ratio $R^{NS}(\tau)$ must vanish as $\tau \to 0$ (no chirality flips), and must approach 1 as $\tau \to \infty$ (infinitely many chirality flips). It can be shown that the correlator (2) receives no leading perturbative contribution.

The two-point functions appearing in (2) have been recently calculated by one of the authors, using chiral (overlap) fermions [7]. Different results for $R^{NS}(\tau)$ are presented in Fig. 1. Notice that the lattice curve (square points) becomes considerably larger than one. This implies that, after few fractions of a fermi, the quarks are most likely to be found in a configuration in which their chiralities are flipped.

Let us now consider the result for $R^{NS}(\tau)$ obtained in the Random Instanton Liquid Model (circles), which accounts for the ‘t Hooft interaction to all orders, but neglects quark loops (quenched approximation). The agreement with the lattice results is impressive. The presence of a maximum in $R^{NS}(\tau)$ and the subsequent fall-off toward 1 have a very simple explanation in the ILM: if quarks propagate in the vacuum for a time comparable with the typical distance between two neighbor instantons (i.e. two consecutive ‘t Hooft interactions), they have a large probability of crossing the field of the closest pseudo-particle. If so happens, they must necessarily flip their chirality, due to the S and PS structure of the ‘t Hooft vertex. So, after some time, the quarks are most likely to be found in the configuration in which their chirality is flipped. On the other hand, if one waits for a time much longer than 1 fm, then the quarks will “bump” into many other such pseudo-particles, experiencing several more chirality flips. Eventually, either chirality configurations will become equally probable and $R^{NS}(\tau)$ will approach 1.

From the agreement between ILM and lattice data one may argue that the phenomenological values $\bar{\rho} \simeq 1/3$ fm, $\bar{n} \simeq 1$ fm$^{-4}$ are indeed realistic.

3. Flavor structure

In this section, we shall introduce another correlator, which can be used to infer simultaneously information about the chiral and flavor structure of the effective vertex. We define:

$$R^{tH}(\tau) := \frac{(\Pi_\pi(\tau) - \Pi_\delta(\tau)) + (\Pi_\rho(\tau) - \Pi_{\rho'}(\tau))}{(\Pi_\pi(\tau) - \Pi_\delta(\tau)) - (\Pi_\rho(\tau) - \Pi_{\rho'}(\tau))}.$$  

(3)

On the one hand, the numerator receives no perturbative contribution, but couples maximally to the ‘t Hooft interaction. On the other hand, the denominator does not receive perturbative contribution, and does not receive contribution from the single-instanton ‘t Hooft interaction, as well. Hence, if instantons were the only sources of non-perturbative interactions in the light-quark sector of QCD, then $R^{tH}(\tau)$ should display a dramatic
enhancement (divergence), in the limit \( \tau \to 0 \) (i.e. when the single-instanton contribution becomes dominant over many-instanton effects). On the contrary, if the driving non-perturbative interaction does not have the specific chiral/flavor structure of \( L_{tH} \), there should be no enhancement in the short distance limit, because the denominator in \( R^{tH}(\tau) \) does not vanish. As a result, the magnitude of \( R^{tH}(\tau) \) near the origin represents a model-independent estimate of the strength of the \( 't \) Hooft instanton-induced interaction, relative to other sources of non-perturbative dynamics. We suggest that this quantity should be investigated on the lattice.

4. Instanton, unitarity and quenching effects

We recall that lattice results reported in section 2 have been obtained in the quenched approximation. It is important to ask what differences should be expected in full QCD. It is immediate to show that \( R^{NS}(\tau) > 1 \) if and only if \( \Pi_3(\tau) < 0 \). The negativity of such a two-point function is a reflection of the fact that the unitarity of the theory is lost.

In terms of chirality flipping amplitudes, we see that the \( \Pi_3(\tau) \geq 0 \) constraint implies that quarks must never be more likely to be found in the flipped chirality configuration than in the original configuration. Hence, we can conclude that the fermionic determinant suppresses some chirality flipping events, which are otherwise allowed in the quenched approximation. Such a dramatic qualitative difference between quenched and full QCD calculations of (2) can be exploited in order to test any phenomenological description of the non-perturbative dynamics. In fact, a realistic model must reproduce a dramatic enhancement of the chirality flipping amplitude, when quark loops are suppressed.

Indeed, this phenomenon is naturally explained in the instanton picture [6]. Quark loops generate attraction between instantons and anti-instantons leading to a screening of the topological charge. As a result, quarks crossing the field of an instanton are very likely to find, in the immediate vicinity, an anti-instanton which restores their initial chirality configuration. With the inclusion of the fermionic determinant, the unitarity condition \( R^{NS}(\tau) < 1 \) is actually restored. We stress that, although such a restoration must necessarily take place in QCD, it represents a remarkable success of the ILM, which is not a unitary field theory.

5. Conclusions

We have presented a study of the light quark dynamics at low energy, based on the results of lattice QCD simulations with chiral fermions. Our analysis provides an indication that the low-energy effective quark-quark interaction has the Dirac structure predicted by the semi-classical the \( 't \) Hooft interaction, and that the phenomenological parameters of the ILM are realistic. We identified a correlator, \( R_{tH}(\tau) \), which allows to probe also the chiral/flavor structure of the instanton-induced vertex. We suggest that this quantity should be measured on the lattice.

We studied the contribution of quark loops to chirality flipping dynamics, by comparing the results of quenched and full simulations. We observed dramatic quenching effects that can be naturally explained by instantons and suggests an interesting link between unitarity and topology.

REFERENCES