United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency

THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

**LASER INDUCED AMPLIFICATION OF HYPERSOUND IN
NONDEGENERATE SEMICONDUCTOR SUPERLATTICES**

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MIRAMARE – TRIESTE

November 2002

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Abstract

Laser induced amplification of hypersound in nondegenerate semiconductor superlattice has been studied. It is shown that there exists a threshold laser field intensity E_0 for which the hypersound attenuation coefficient Γ_q may change sign and thus become amplified. It is also noted that the superlattice, while amplifying, also serves as a filter, i.e., allows waves of definite frequencies.

Introduction

The study of acoustic effects in semiconductors and particularly of acoustic wave amplification by drift carriers has attracted much attention [1 – 6]. It is known that, when an acoustic wave passes through a semiconductor, it may interact with various elementary excitations. In such an interaction the acoustic wave may lose or gain energy under certain circumstances. The latter is known as amplification and the former as attenuation of the acoustic wave. This idea of acoustic wave amplification was theoretically predicted in 1956 by Tolpygo and Uritskii [1] and by Weinreich [2] and experimentally observed for CdS by Hutson et al [3] and for n-Ge by Pomerantz [4]. These early investigations have stimulated a considerable research activity and the studies concentrated on both aspects of the acoustoelectric effect (ultrasonic amplification and non – Ohmic behaviour) in (II – VI) as well as (III – V) compounds such as GaAs, GaSb and InSb [7 – 11]. More recent work has been reported Gurevich et al [12] and for two – dimensional system by Govorov et al [13] and Komirenko et al [14, 15].

Amplification and generation of high frequency coherent phonons can be realized either under the electron transport with drift velocity exceeding the sound velocity [16] or due to the population inversion of electron states [14, 15]. In both cases amplification can be achieved using intensified laser or electric field; infact, with any field that can cause carrier drift, e.g. temperature gradient [17].

The advent of molecular beam epitaxy (MBE) and metal – organic chemical vapour deposition (MOCVD) technology have made it possible to fabricate multiquantum well structures (MQWS) and superlattice (SL) semiconductors. These materials owing to their interesting properties have been extensively studied, especially their transport and optical properties.

Concerning the acoustic properties, it is now receiving a lot of attention even though some works were done earlier [16 - 18]. The amplification of hypersound in SL in the presence of electric field has also been reported [16, 21]. It is worthy to note that the mechanism of amplification is due to Cherenkov emission of phonons by charge carriers whose drift velocity exceeds that of the phase velocity of sound. It has been established that three requirements are necessary for practical use of the Cherenkov effect: namely high electron mobilities, large electron densities and strong electron coupling between electrons and amplifying phonons. It is therefore not surprising that, SL is a good semiconductor material for acoustic amplification. In [19] Rodrigues et al studied “laser – dressed” sound absorption in SL. The results obtained agree with later results by Mensah et al [20]. However, in [19, 20] only the effect of the strong laser amplitude on the attenuation coefficient was considered, i.e, no photon absorption ($l = 0$, where l is the number of photons) by electrons in the SL were investigated.

In this paper, we report the possibility of hypersound amplification as a consequence of intraminiband photon absorption ($l \neq 0$) by electrons in SL. Intraminiband absorption of electromagnetic waves in SL has been studied extensively in [22-24]. It is known that intraband absorption of photons by charge carriers in

semiconductors is always followed by the emission or absorption of phonons so that the electrons can gain the necessary momentum for their transitions. In our work we want to emphasize that, the mechanism of amplification (generation) of phonons is due to the reverse Landau damping.

The motivation for such study is due to the numerous applications that can be derived from the study, e.g., phonon spectrometer, generation of high frequency electric oscillations, terahertz modulation of light, nondestructive testing of microstructures, and acoustic scanning.

The paper is organized in the following way: the second section is devoted to the theory, the third section to the Dispersion equation, the fourth section to the amplification of the hyper sound and finally, the last section to the discussion and conclusions.

2.Theory

In solving this problem, we shall use the Hamiltonian of the electron – phonon system in SL in second quantization notation ($\hbar=1$)

$$H = \sum_{p,s} \varepsilon^{(s)} \left(p - \frac{e}{c} A(t) \right) a_p^{(s+)} a_p^{(s)} + \sum_k \omega_k b_k^+ b_k + \frac{1}{\sqrt{N}} \sum_{p,k} \sum_{ss'} c_k m_{ss'}(k_z) a_p^{s+} a_{p-k+ng}^{s'} (b_k + b_{-k}^+) \quad (1)$$

where

$$\varepsilon^{(s)}(p) = \frac{p_{\perp}^2}{2m} + \Delta_s (1 - \cos p_z d) \quad (2)$$

$s = 1, 2, \dots$

Here d is the superlattice period, m the effective electron mass, Δ_s the half width of the s th allowed miniband, $p_{\perp}$ and p_z are the quasimomentum components across and along the SL axis, $A(t)$, which is directed along the SL axis, is the vector potential related to the electric field of the electromagnetic wave $E(t) = E_0 \sin \Omega t$ by the relation $E = -\frac{1}{c} \frac{\partial A}{\partial t}$;

$a_p^{(s)+}$ and $a_p^{(s)}$ are the creation and annihilation operators of an electron with quasimomentum p in the s -th miniband, b_k^+ and b_k are the phonon creation and annihilation operators, N is the number of SL periods; $g = (0, 0, 2\pi/d)$ is the reciprocal SL vector, $M_{ss'}(k_z)$ given by

$$M_{ss'}(k_z) = \int \varphi_s^*(z) \varphi_{s'}(z) e^{ik_z z} dz \quad (3)$$

where $\varphi_s(z)$ is the wave function of the s -th state in one of the one – dimensional potential wells from which the SL potential is formed; C_k is the matrix element of electron – phonon interaction (in homogeneous material) and is given as

$$C_k = \begin{cases} (2\rho\omega_k)^{-\frac{1}{2}} \frac{4\pi e\beta}{\epsilon_0} & \text{piezoelectric interaction} \\ (2\rho\omega_k)^{-\frac{1}{2}} \Lambda k & \text{deformation potential} \end{cases} \quad (4)$$

where ρ is the crystal density; ω_k is the phonon frequency with wave vector q ; Λ is the deformation potential constant; β is the piezoelectric modulus ϵ_0 is the lattice dielectric constant. The electromagnetic wave frequency is assumed to be large compared with the reciprocal of the electron mean free time τ^{-1} , and the wavelength is taken to be large compared with the SL period, electron mean free path and the de Broglie wavelength. This enables us to use the dipole approximation. Also, the plane electromagnetic wave of frequency Ω satisfies $\Omega/\omega_p > 1$, where ω_p is the plasma frequency. As for the phonons, we shall confine our considerations to those for which the wave vector q satisfies the conditions $ql \gg 1$, where l is the electron mean free path in the SL. Such phonons constitute well – defined elementary excitations of the system.

Consider $\Omega\tau \gg 1$ and $\Omega > \omega_p$ ensure that the electromagnetic wave should penetrate well into the sample and condition $ql \gg 1$ means that the hypersound wavelength is far smaller than the mean free path of the electron.

3. Dispersion equation

To calculate the attenuation (amplification) coefficient we write down the dispersion equation for the phonons. This is given by

$$i \frac{\partial}{\partial t} \langle b_q \rangle_t = \langle [b_q H] \rangle_t = \omega_q \langle b_q \rangle_t + \frac{1}{\sqrt{N}} c_{-q} \sum_p M_{ss'} (-q_z) \langle a_p^{(s)+} a_{p+q+ng}^{(s)} \rangle_t \quad (5)$$

Similarly for $\langle a_p^{(s)+} a_{p+q+ng}^{(s)} \rangle_t$ we have

$$\begin{aligned} \frac{\partial}{\partial t} \langle a_p^{(s)+} a_{p+q+ng}^{(s)} \rangle_t &= (\epsilon_{p+q}^{s'} - \epsilon_p^{(s)}) \langle a_p^{(s)+} a_{p+q+ng}^{s'} \rangle_t \\ &+ \frac{1}{\sqrt{N}} \sum_p \sum_{s'sn} C_k [M_{s's'}(q_z) \langle a_p^{(s)+} a_{p+q-k+(n+n')g} (b_k + b_{-k}^+) \rangle \\ &- M_{s's}(q_z) \langle a_{p+k-ng}^{s'} a_{p+q+ng}^{s'} (b_k + b_{-k}^+) \rangle_t] \quad (6) \end{aligned}$$

Solving Eq (6) with the initial condition $\langle a_p^{(s)+} a_{p+q+ng}^{s'} \rangle_{t=0} = 0$ and substituting into Eq (5) we obtain

$$\begin{aligned} \frac{\partial}{\partial t} \langle b_q \rangle_t &= \omega_q \langle b_q \rangle_t - \sum_p \sum_{ss'n} C_{-q} M_{ss'} (-q) \\ &\times \int_{-\infty}^t e^{-\int_{t'}^t (\epsilon_{p+q}^{s'} - \epsilon_p^{(s)}) dt''} dt' \sum_{s'n'} C_k [M_{s's'}(k_z) \langle a_p^{(s)+} a_{p+q-k+(n'+n)g}^{(s')} \\ &\times (b_k + b_{-k}^+) \rangle_{t'} - M_{s's}(k_z) \langle a_{p+k-ng}^{(s')} a_{p+q+ng}^{(s')} (b_k + b_{-k}^+) \rangle_{t'}] \quad (7) \end{aligned}$$

Taking the electron – phonon interaction as weak, we decouple the right – hand side of Eq (7)

$$\langle a_p^{(s)+} a_{p'}^{(s')} b_k \rangle = \delta_{pp'} \delta_{ss'} \langle b_k \rangle_k n_p^s$$

where $n_p^s = \langle a_p^{(s)+} a_p^{(s)} \rangle_t$ is the electron distribution function. We obtain

$$\begin{aligned} \frac{\partial}{\partial t} \langle b_q \rangle_t + i\omega_q \langle b_q \rangle_t &= \sum_p \sum_{ss'n} C_{-q} C_{q+ng} M_{s's}(-q_z) M_{s's}(q_z + ng_z) [n_p^{s'} - n_{p+q}^{s'}] \\ &\times \int_{-\infty}^t dt' \left(\langle b_{q+ng} \rangle_{t'} + \langle b_{-q-ng}^+ \rangle_{t'} \right) \exp[i(\epsilon_p^s - \epsilon_{p+q}^{s'})(t-t')] \\ &- \frac{eE_0 d \Delta}{\Omega} \{ \sin(p_z + q)d - \sin p_z d \} (\sin \Omega t - \sin \Omega t') \end{aligned} \quad (8).$$

Eliminating $\langle b_{-q-ng}^+ \rangle_t$ with the help of the conjugate equation and taking the Fourier

component $B_q(\omega) = \int_{-\infty}^{\infty} \langle b_q \rangle_t e^{i\omega t} dt$

We obtain

$$(\omega - \omega_q) B_q(\omega) = \sum_{k=-\infty}^{\infty} \sum_{ss'n} C_{-q} C_q M_{ss'}(-q_z) M_{s's}(q_z - ng_z) \frac{2\omega_{q+ng}}{\omega + \omega_{q+ng} - \Omega k} B_q(\omega - k) M_k(q\omega) \quad (9)$$

where

$$m_k(q, \omega) = \sum_{l=-\infty}^{\infty} J_l(\xi) J_{l+k}(\xi) \Pi(q, \omega + l\Omega) \quad (10)$$

$$\Pi_{ss'}(q, \omega + \Omega l) = \sum_p \frac{n_p^s - n_{p+q}^{s'}}{\epsilon_{p+q}^{(s')} - \epsilon_p^{(s)} - \omega_q} \quad (11)$$

and $\xi = \frac{eE_0 d \Delta}{\Omega} [\sin(p_z + q)d - \sin p_z d]$ (12)

Subsequently, we shall confine our selves to the case when the electron gas is non-degenerate and only the lowest miniband is filled, i.e., $n_p^s = n_p$ and also consider the situation when the characteristic width of the potential well is small compared with the SL period in this case $M(q_z) = M(q_z - ng) = 1$. We shall also limit ourselves to $k = 0$ since $k \neq 0$ in the summation give terms of higher perturbation. Then for $n = 0$ we get from Eq (9)

$$\omega^2 - \omega_q^2 - 2\omega_q C_q^2 M_0(q, \omega) = 0 \quad (13)$$

This can further be simplified to

$$\omega - \omega_q - C_q^2 \sum_{l=-\infty}^{\infty} J_l^2(\xi) \Pi_q(\omega + \Omega l) = 0 \quad (14)$$

Then the phonon attenuation in the presence of the electromagnetic wave will be given by

$$\Gamma_q = -\text{Im} \omega = \sum_{l=-\infty}^{\infty} J_l^2(\xi) \Gamma_q^0(\omega_q + \Omega l) \quad (15)$$

where

$$\Gamma_q^0(\omega) = \pi C_q^2 \sum_p (n_p - n_{p+q}) \delta(\epsilon_{p+q} - \epsilon_p - \omega_q)$$

i.e., the imaginary part of the polarization vector.

In Eq (15) $J_l(x)$ is the Bessel function of order l and argument x . The expression for $\Gamma_q^0(\omega)$ formally coincides with a similar expression for the sound attenuation coefficient in the absence of the laser field the only difference is the shift in the argument of Γ_q by $l\Omega$ which permits the possibility of absorption or emission of l field quanta by conduction electrons upon interaction with phonons. The case $l = 0$ is the one discussed in Refs. [19, 20].

It follows from Eq (15) that if $\Gamma_q > 0$ we have hypersound attenuation, whereas if $\Gamma_q < 0$ we have hypersound amplification (negative attenuation) as a result of the absorption ($l > 0$) and emission ($l < 0$) of $|l|$ photons from the intense laser field.

4. Hypersound Amplification

In this section, we shall calculate Γ_q in the intense field regime of laser field and show that under certain conditions the hypersound attenuation may change sign (amplification). As stated in [25], the regime of intense laser field, i.e., $\xi \gg \Omega$, only the electron - phonon collisions with the absorption or emission of $l \gg 1$ photons are significant. Accordingly, in the case of $\xi \gg \Omega$ the argument of the Bessel function $J_l(\xi)$ is large. For large values of the argument, the Bessel function $J_l(\xi)$ is small except when the order is equal to the argument.

Therefore from Eq. (15) i.e.,

$$\Gamma_q = \pi c_q^2 \sum_p \sum_{l=-\infty}^{\infty} J_l^2(\xi) (n_p + n_{p+q}) \delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q - l\Omega) \quad (16)$$

where
$$\xi = \frac{eE_0 d^2 \Delta}{\Omega} q.$$

We take the sum over l using the approximation [26, 27]

$$\sum_{l=-\infty}^{\infty} J_l^2(\xi) \delta(E - l\Omega) \cong \frac{1}{2} [\delta(E - \xi) + \delta(E + \xi)] \quad (17)$$

where
$$E = \varepsilon_{p+q} - \varepsilon_p - \omega_q$$

The attenuation coefficient Γ then becomes

$$\frac{\pi c_q^2}{2} \sum_p (n_p - n_{p+q}) \{ \delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q - \xi) + \delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q + \xi) \} \quad (18)$$

The first δ -function in Eq (18) corresponds to the emission and the second to the absorption of photons.

The number of photons absorbed or emitted is the same order of magnitude as the ratio of the classical oscillatory energy of the electron to the phonon energy, namely [26] $\hbar = 1$,

$$l = \frac{2e^2 E_0^2 / m \Omega^2}{\Omega} \quad (19)$$

where e is the electron charge, m is the effective mass of the electrons. Therefore, multi-photon absorption or emission processes ($l \gg 1$) are valid for laser fields such that

$$E_0 \gg \left(\frac{m\Omega^3}{2e^2} \right)^{\frac{1}{2}}$$

We now assume that the electron temperature is low. Then $k_B T \ll \xi$ and the emission term is negligible compared to the absorption term.

This is justified provided $\Delta \gg k_B T$ as being considered here. Eq (18) becomes

$$\Gamma_q = \frac{\pi c_q^2}{2} \sum_p (n_p - n_{p+q}) \delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q + \xi) \quad (20)$$

In general, both mechanism of electron – phonon interaction, i.e, the deformation potential and piezoelectric mechanisms, can lead to the phonon amplification effects. However, for high frequency phonons the main mechanism of electron – phonon coupling is the deformation potential. We therefore substitute the value from C_q in Eq (4) into Eq (20) and change the summation to integral because of the continuity of the carrier states as a function of the momentum p . As a result we obtain

$$\Gamma_q = \frac{|\Lambda|^2 q}{4\pi^2 \rho v_s} \int (n_p - n_{p+q}) \delta(\varepsilon_{p+q} - \varepsilon_p - \omega_q + \xi) d^3 p \quad (21)$$

For a nondegenerate semiconductor SL n_p becomes the Boltzmann's distribution function which is given after normalization as

$$n_p = \frac{\pi d n}{mkT e^{\frac{\Delta}{kT}} I_0\left(\frac{\Delta}{kT}\right)} \exp - \frac{\varepsilon_p}{kT} \quad (22)$$

where k is the Boltzmann's constant and $I_0(x)$ is the modified Bessel function.

Substituting Eq (2) and Eq (22) into Eq (21), using cylindrical coordinates and after cumbersome calculations we obtain

$$\Gamma_q = \frac{|\Lambda|^2 q n \theta (1-b'^2)}{\rho v_s^2 \Delta \sin\left(\frac{1}{2} qd\right) \sqrt{1-b'^2}} \sinh\left\{ \frac{\omega_q}{kT} \left(1 - \frac{v_d}{v_s} \right) \right\} \cosh\left\{ \frac{\Delta}{kT} \cos qd \sqrt{1-b'^2} \right\} \quad (23)$$

where $v_d = \frac{eE_0 \Delta d^2}{\Omega}$ and $b' = \frac{\omega_q \left(1 - \frac{v_d}{v_s} \right)}{2\Delta \sin \frac{qd}{2}}$, note that $\hbar = 1$.

Eq (23) can finally be written as

$$\Gamma = \frac{\Gamma_0}{\sqrt{1-b'^2}} \sinh\left\{ \frac{\omega_q}{kT} \left(1 - \frac{v_d}{v_s} \right) \right\} \cosh\left\{ \frac{\Delta}{kT} \cos qd \sqrt{1-b'^2} \right\} \quad (24)$$

$$\text{where } \Gamma_0 = \frac{|\Lambda|^2 q n \theta (1-b^2)}{\rho v_s^2 \Delta \sin\left(\frac{1}{2} q d\right)}. \quad (25)$$

5. Discussion and Conclusions

As can be seen from Eq (23), Γ_q becomes negative whenever $v_d > v_s$, this means amplification of the acoustic phonons. This occurs because the laser field gives to the electrons a drift velocity $v_d = \frac{e E_0 \Delta d^2}{\hbar^2 \Omega}$. This laser induced drift is analogous to the reversed Landau damping mechanism of plasma in which plasma electrons drift the plasma waves under the action of static field. Whenever the drift velocity v_d of the electrons as imposed by the external field is greater than the phase velocity v_ϕ of the plasma wave, energy is given to the plasma wave by the electrons and the wave amplifies. The reverse situation, when $v_d < v_\phi$ and energy flows from the plasma waves to the electrons is the Landau damping. It is worth noticing that whenever

$$\omega_q \gg \frac{2\Delta \sin \frac{qd}{2}}{\left(1 - \frac{v_d}{v_s}\right)} \text{ there appears a "transparency window", i.e., } \Gamma_q = 0. \text{ This is the}$$

consequence of the Laws of Conservation. Hence the phonon amplification is obtained for a particular band of phonon wavevectors. This behaviour of the SL enables it to be used as a phonon filter. Again Γ_q can be quite large when ω_q is reasonably close

$$\text{to } \frac{2\Delta \sin \frac{qd}{2}}{1 - \frac{v_d}{v_s}}. \text{ Near resonance, } \omega_q \left(1 - \frac{v_d}{v_s}\right) = 2\Delta \sin \frac{qd}{2} \text{ the absorption coefficient}$$

becomes so large that the initial criteria are broken down. Therefore this situation is not considered here.

Since Eq (19) defines a critical laser field for multiphonon absorption processes it follows that one has amplification of hypersound provided $E_0 \geq E_c$ where

$$E_c = \left(\frac{m \hbar \Omega^3}{2e}\right)^{\frac{1}{2}}. \text{ In fact, } E_0 = \frac{v_s \hbar^2 \Omega}{e \Delta d^2} \text{ has the sense of a threshold field. To get a}$$

numerical estimate of E_0 for a typical GaAs/AlGaAs superlattice, the following parameters are chosen: $v_s = 4.7 \times 10^3$; $\Delta = 0.01 \text{ eV}$; $d = 100 \text{ \AA}$; $\Omega = 10^{13} \text{ s}^{-1}$. For these values we obtained $E_0 = 1.7 \times 10^2 \text{ Vm}^{-1}$ which is obtainable practically.

For further analysis of our results, we plotted $\frac{\Gamma_q}{\Gamma_0}$ against $\frac{v_d}{v_s}$ for various fixed values of ω_q as well as three dimensional plots of $\frac{\Gamma_q}{\Gamma_0}$ against $\frac{v_d}{v_s}$ and ω_q . It is observed that when the frequency is about 1THz and below, $\frac{\Gamma_q}{\Gamma_0}$ is linear dependent on both $\frac{v_d}{v_s}$ and ω_q (see figs. 1 and 2). In this range amplification occurs for all q as noted in [25].

However, as the frequency increases above 1THz the linear dependence collapses and the amplification coefficient slowly changes with $\frac{v_d}{v_s}$ and frequency and then suddenly shoots down (become strongly enhanced) (see figs.2 and 4). When the drift velocity is reversed the same behaviour is observed for the attenuation coefficient (see fig. (5)). More importantly, (amplification) absorption does not occur for all frequencies. The SL behaves like a filter.

It is worthy to note that Γ_q depends on the SL parameters Δ and d in a more complicated form. Hence to obtain a better amplification over the bulk semiconductor will require optimization of Δ and d . Finally, our result is similar to that of Komirenko et al [15], and when the laser is switched off, i.e, $v_d = 0$ to that of [16].

Finally, we want to indicate that our calculations do not take into accounts phonon losses which can arise due to phonon – phonon scattering, phonon absorption due to nonelectronic mechanism, phonon decay due to anharmonicity of the lattice, etc. Nevertheless, we believe that the phonon losses may not considerably affect the results.

In conclusion, we have calculated the amplification of hypersound due to laser field and have noted that the SL can, while amplifying, filter as well phonons of definite frequencies.

Acknowledgements

The authors would like to thank the Abdus Salam ICTP and UNESCO for hospitality. This work was done within the framework of the Associateship Scheme of the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy. Financial support from the Swedish International Development Cooperation Agency is acknowledged.

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Plot of $\frac{\Gamma_2}{\Gamma_0}$ against $\frac{v_d}{v_s}$ for (1) $\omega_q = 5 \times 10^{11}\text{Hz}$, (2) $\omega_q = 10^{12}\text{Hz}$, (3) $\omega_q = 2 \times 10^{12}\text{Hz}$.

fig.1

Plot of $\frac{\Gamma_2}{\Gamma_0}$ against $\frac{v_d}{v_s}$ for (1) $\omega_q = 4 \times 10^{12}\text{Hz}$, (2) $\omega_q = 6 \times 10^{12}\text{Hz}$, (3) $\omega_q = 8 \times 10^{12}\text{Hz}$.

fig.2

Plot of $\frac{\Gamma_2}{\Gamma_0}$ against $\frac{v_a}{v_s}$ and ω_q .

fig.3

Plot of $\frac{\Gamma_2}{\Gamma_0}$ against $\frac{v_d}{v_s}$ and ω_q .

fig.4

Plot of $\frac{\Gamma_2}{\Gamma_0}$ against $\frac{\nu_d}{\nu_s}$ and ω_q .

fig.5