Deconfinement transition in 2+1-dimensional SU(4) lattice gauge theory

Philippe de Forcrand*, ETHZ Institute for Theoretical Physics, ETH Zürich, CH-8093 Zürich, Switzerland

CERN Theory Division, CH-1211 Geneva 23, Switzerland

and Oliver Jahn

A missing piece is added to the Svetitsky-Yaffe conjecture. The spin model in the same universality class as the (2 + 1)d SU(4) theory, the 2d Ashkin-Teller model, has a line of continuously varying critical exponents. The exponents measured in the gauge theory correspond best to the Potts point on the Ashkin-Teller line.

1. FRAMEWORK

The SU(4) Yang-Mills theory in (2 + 1) dimensions has a special status: its critical exponents at the finite temperature deconfinement transition are not specified by the Svetitsky-Yaffe (S-Y) conjecture [1]. By measuring them, we can learn about the couplings of the equivalent spin model, i.e. of the effective Polyakov loop model.

The S-Y conjecture says that, if the deconfinement transition of the gauge theory is second-order, then it is in the same universality class as the spin model having the symmetry-breaking pattern of the Polyakov loop.

Assume that the (2 + 1)d SU(4) deconfinement transition is second-order – we will investigate this issue numerically next. Then the equivalent 2d spin model has Z(4) spins, with orientation \{0, ±\pi/2, \pi\} corresponding to the 4 Polyakov loop sectors and the 4 perturbative vacua of the gauge theory. A nearest-neighbour Hamiltonian, which gives the same critical properties as the SU(4) theory according to S-Y, will have 3 possible energy levels for each link: $E_0$ for parallel spins $\uparrow\uparrow$, $E_1$ for perpendicular spins $\uparrow\rightarrow$, and $E_2$ for antiparallel spins $\uparrow\downarrow$. Remarkably, the critical exponents of this spin system are known to vary continuously with the ratio $\rho \equiv \frac{E_2 - E_1}{E_1 - E_0}$ and the corresponding couplings of the Hamiltonian. Therefore, the S-Y conjecture does not tell us what the SU(4) critical exponents are. Measuring them fixes the couplings of the effective Hamiltonian.

A Z(4) spin model with 3 energy levels per link is called a symmetric Ashkin-Teller model [2]. It is the symmetric case $J = J'$ of the Ashkin-Teller model, which describes 2 coupled Ising systems $\{\sigma_i, \tau_i\}$ on a square lattice, with Hamiltonian:

2. NUMERICAL STUDY

We simulate the (2 + 1)d SU(4) gauge theory using the Wilson plaquette action on a cubic grid of size $L^3 \times N_t$, with $N_t = 2, 3, 4$ and $L$ up to 40. To accelerate Monte Carlo evolution, we use as elementary update a mixture of pseudo-heatbath (in all 6 SU(2) subgroups) and overrelaxation in the full SU(4) group [4,5]. The latter requires a similar amount of work to 6 SU(2) overrelaxation steps, but gives a larger step size. As a result, the number of sweeps needed to decorrelate the Polyakov loop is reduced by a factor $\sim 3$. Up to $10^6$ sweeps are performed on each volume. We analyze results for various couplings $\beta$ near criticality together with multihistogram reweighting. This model gives a clear first-order transition on $N_t = 1$ lattices. On $N_t = 2$, an old Monte Carlo study found a second-order deconfinement transition [6]. Unfortunately, the larger sizes which we simulated revealed that the transition is in fact first-order. Distortions of the plaquette distribution towards a double-peak structure appear in Fig. 1 for our largest sizes. Also, the
Binder cumulant $1 - \frac{\langle O^4 \rangle}{6 \langle O^2 \rangle^2}$ for the plaquette extrapolates as $L \to \infty$ to a value below 2/3. A likely explanation of these first-order transitions is the vicinity of a sharp, bulk crossover at $\beta \sim 13.5$, whereas the $N_t = 1$ and 2 transitions occur at $\beta_\epsilon \approx 8.67$ and 14.87 respectively.

This forced us to consider $N_t = 3$ lattices, with correspondingly higher $\beta_\epsilon$. No double-peak structure is visible on the largest size considered ($L = 32$). However, the $L \to \infty$ extrapolation of the Binder cumulant still misses 2/3 by a small but somewhat significant amount. It may well be that the transition is still weakly first-order. Our current $N_t = 4$ results are consistent with a second-order transition, but do not reach as large volumes yet. Therefore, we present the critical exponents analyzed from the $N_t = 3$ data.

The effect of logarithmic corrections to scaling can be seen in Fig. 2. They limit the usefulness of accurate, small-size data.

The bulk crossover at $\beta \sim 13.5$ has a more pernicious effect. It dominates the behaviour of the specific heat over the singular piece, leading to an exponent $\alpha \approx 0$ (Fig. 3 left). We subtracted the bulk specific heat, measured on a $4^4$ lattice, to isolate the singular contribution. The exponent then becomes consistent with the Potts case (Fig. 3 right).

All other exponents also favor the Potts case. Fig. 4 shows the scaling of $\frac{\Delta U}{T^2}$, where $U$ is the Polyakov loop Binder cumulant. $\nu$ remains consistent with the Potts value 2/3, even allowing for a large (> 2σ) variation of $\beta_\epsilon$ by ±0.01 and logarithmic finite-size corrections.

Agreement among all observables is shown in Fig. 5. The pseudo-critical $\beta$'s obtained on various volumes all extrapolate to a common thermodynamic value $\beta_\epsilon = 20.4356(41)$, with corrections of the form $aL^{-\frac{7}{4}}(1+b/L)$, and $\nu = 2/3$, the Potts value.

3. CONCLUSION

The unexpected $N_t = 2$ first-order transition, together with expected but unwelcome logarithmic finite-size corrections, have turned this simple problem into a numerically challenging one.

Our results need to be made more precise, by simulating larger volumes. They also must be confirmed on a finer lattice, $N_t \geq 4$, where the transition presumably is second-order. In practice, we can never exclude a weak first-order transition. What we want is to reach physical volumes large enough to reliably determine effective criti-

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**Figure 1.** Distribution of the plaquette for various lattice sizes $L^2 \times 2$. Deviations towards a double peak characteristic of a first-order transition start to appear at the largest sizes.

**Figure 2.** Check of magnetic susceptibility exponent $\frac{\gamma}{\nu} = \frac{7}{4}$, without (left) and with (right) logarithmic corrections to scaling.

**Figure 3.** $\frac{\alpha}{\nu}$ determined from the scaling of the specific heat, without (left) and with (right) subtraction of smooth background caused by nearby crossover.
Figure 4. Correlation length exponent $\nu$ determined from the Polyakov loop Binder cumulant. The Potts value is strongly favored, even allowing for an error of $\pm 0.01$ in the determination of $\beta_c$.

Figure 5. Consistency of all determinations of $\beta_c$. $\nu$ has the Potts value $2/3$; corrections to scaling are fitted.

Potts model for $(3 + 1)d SU(N)$ would lead to $k$-string tensions independent of $k$ at $T_c$.

REFERENCES