On the nature of the anomalies in the supersymmetric kink

Kazuo Fujikawa

Department of Physics, University of Tokyo
Bunkyo-ku, Tokyo 113, Japan

Anton Rebhan

Institut für Theoretische Physik, Technische Universität Wien
Wiedner Hauptstr. 8–10, 1040 Vienna, Austria

Peter van Nieuwenhuizen

C.N. Yang Institute for Theoretical Physics
State University of New York, Stony Brook, NY 11794-3840, USA

Abstract

We discuss the possibility to absorb all anomalies in the supersymmetry algebra of the $N = (1, 1)$ Wess-Zumino model in $d = 1 + 1$ by a local counter term. This counter term corresponds to the change of the vacuum parameter $v_0^2$ in the model and the transition to an unconventional but admissible renormalization scheme. It does not modify the physical consequences such as BPS saturation, and thus the situation is rather different from gauge theory where local counter terms are required to absorb spurious gauge anomalies.

1 Introduction

In the past few years it has been established that the one-loop quantum corrections to the mass and central charge of the supersymmetric kink in $d = 1 + 1$ dimensions, which were studied long ago in [1, 2, 3], are non-vanishing [4, 5, 6, 7] and equal [8, 9, 10, 11, 12, 13]. The corrections to the mass involve various subtleties depending on the regularization methods, but the corresponding corrections to the central charge posed initially a particular problem. The central charge is the integral of a total derivative and it

\[^{1}\text{fujikawa@phys.s.u-tokyo.ac.jp}
^{2}\text{rebhana@hep.itp.tuwien.ac.at}
^{3}\text{vannieu@insti.physics.sunysb.edu} \]
seemed not to receive any corrections due to the presence of the kink soliton (in a minimal renormalization scheme with only mass renormalization such that tadpoles vanish) [4, 5]. On the other hand, although saturation of the BPS bound seemed likely [1], no formal proof was available at that time and it was conjectured that a new anomaly is responsible for the discrepancy of the quantum corrections [5]. Other authors [9, 10, 11, 12, 13] have since then established the existence of a central charge anomaly which is connected to the trace anomaly through supersymmetry and which is responsible for the saturation of the BPS bound at the quantum level.\footnote{1}

Recently, a superspace path integral analysis was made [14] according to which all anomalies reside in one super Jacobian [15, 16, 17, 18]. The simplicity of the results in the superspace formulation revealed that the anomaly in superspace (and thus also all the anomalies in $x$-space) can formally be removed by adding an order-$\hbar$ counter term to the action. In renormalizable gauge theories in 4-dimensions one must in such cases add such a counter term to the action, in order that higher-loop renormalizability be preserved. We argue, however, that in the present super-renormalizable model the situation is more subtle. In addition to anomalies, there is spontaneous $Z_2$ symmetry breaking and explicit conformal symmetry breaking present, and depending on whether or not one adds a counter term, the various quantum currents which one can define change. In fact, we derive below relations between conserved currents with the counter term present and non-conserved currents without the counter term. The various relations become quite clear if one uses path integrals, and explain why after adding the counter term one still has the same physical content while keeping the essence of the trace and central charge anomalies.

## 2 Previous results

The $N = (1,1)$ Wess-Zumino model in $d = 1 + 1$ dimensions is defined by

$$\int dx d^2\theta \mathcal{L}(x, \theta) = \int dx d^2\theta [\frac{1}{4}\overline{D} \phi(x, \theta) D \phi(x, \theta) + \frac{1}{3}g \phi^3(x, \theta) - g v_0^2 \phi(x, \theta)]$$

$$= \int dx \{\frac{1}{2}[FF - \partial^{\mu} \varphi \partial_{\mu} \varphi - \overline{\psi} \gamma^{\mu} \partial_{\mu} \psi] + gF \varphi^2 - g(\overline{\psi} \psi) \varphi - g v_0^2 F\}. \quad (2.1)$$

The coupling constant $g$ has mass dimension one and $v_0$ is dimensionless. The superfield $\phi$ is defined by

$$\phi(x, \theta) = \varphi(x) + \overline{\theta} \psi(x) + \frac{1}{2} \overline{\theta} \theta F(x) \quad (2.2)$$

where $\theta^a$ is a Grassmann number, and $\theta^a$ and $\psi^a(x)$ are two-component Majorana spinors; $\varphi(x)$ is a real scalar field, and $F(x)$ is a real auxiliary field. We define $\theta = \theta^T C$ with $C$ the charge conjugation matrix, and the inner product for spinors is defined by

$$\overline{\theta} \theta \equiv \overline{\theta}^T C \theta = \theta^a C_{\alpha \beta} \theta^\beta \equiv \overline{\theta}_\alpha \theta^\beta \quad (2.3)$$

\footnote{1}{It should be noted, however, that [8] did not find an anomalous contribution to the central charge operator. According to [13], this is due to the fact that [8] employed a consistent regularization only when considering the kink mass, but not when relating the central charge operator to the Hamiltonian in a formal (unregularized) manner.}
with the Dirac matrix convention
\[
\gamma^0 = -\gamma_0 = -i, \quad \gamma^1 = \gamma_1 = \tau^3, \quad C = \tau^2, \quad \gamma_5 = \gamma^0 \gamma^1.
\] (2.4)
The \(\tau^a\) (a=1,3) are the usual Pauli matrices. The supersymmetry transformation is given by
\[
\begin{align*}
\delta \varphi &= \bar{\epsilon} \psi(x), \\
\delta \psi &= \partial_\mu \varphi(x) \gamma^\mu \epsilon + F(x) \epsilon = \bar{\varphi}(x) \epsilon + F(x) \epsilon, \\
\delta F &= \bar{\epsilon} \gamma^\mu \partial_\mu \psi = \bar{\epsilon} \partial_\mu \psi(x).
\end{align*}
\] (2.5)
The supercharge which generates (2.5)
\[
\bar{\epsilon} Q \equiv \bar{\epsilon}_\alpha \frac{\partial}{\partial \bar{\theta}_\alpha} - \bar{\epsilon} \gamma^\mu \theta \partial_\mu
\] (2.6)
and the covariant derivative
\[
\bar{\eta} D \equiv \bar{\eta}_\alpha \frac{\partial}{\partial \bar{\theta}_\alpha} + \bar{\eta} \gamma^\mu \theta \partial_\mu
\] (2.7)
commute with each other, \([\bar{\epsilon} Q, \bar{\eta} D] = 0\).

The present model accommodates the kink solution which has been studied by many authors in the past \([2, 3]\) with the conclusion reached by the majority of these authors that supersymmetry leads to vanishing corrections to mass and central charge. However, several years ago it was noted that the mass of the supersymmetric kink does receive quantum corrections, whereas the central charge (the integral of a total space derivative) did not seem to get corrected \([4, 5]\). This seemed to violate the BPS bound \([1]\). It was therefore conjectured that the kink system contains a new anomaly \([5]\), and the existence of a central-charge anomaly was first established in \([9]\). It belongs to an anomaly multiplet of which the superconformal anomaly and the trace anomaly are also partners. One can also define a multiplet of conformal anomalies, and then the central charge itself is the anomaly in the conservation of a corresponding conformal central charge current \([13]\).

Recently a path integral approach to the anomalies in superspace was undertaken \([14]\), and a Ward identity in superspace was derived \(^2\) in which the anomaly was due to the Jacobian of the path integral \([15, 16, 17, 18]\). Expanding this identity in terms of \(\theta\), the following set of \(x\)-space Ward identities was obtained
\[
\begin{align*}
(\gamma_\mu \tilde{J}^\mu)(x) &= (\gamma_\mu j^\mu)(x) - \frac{\hbar g}{\pi} \psi(x), \\
\tilde{T}^\mu_\mu(x) &= (T^\mu_\mu)(x) + \frac{\hbar g}{2\pi} F(x), \\
\tilde{\zeta}^\mu(x) &= (\zeta^\mu)(x) + \frac{\hbar g}{2\pi} \epsilon^{\mu\nu} \partial_\nu \varphi(x), \\
\partial_\mu j^\mu(x) &= \frac{\hbar g}{2\pi} \partial_\mu \psi(x).
\end{align*}
\] (2.8)

\(^2\)This Ward identity was derived by making a supersymmetry transformation with local (\(x\)-dependent and \(\theta\)-dependent) parameter. Because the action is only rigidly supersymmetric, the variation of the action produced the currents in (2.8), while the Jacobian yielded the anomalies (the last terms in (2.8)).
These are operator relations, to be used inside the correlation functions. They contain the supercurrent \( j^\mu \) (and \( \tilde{J}^\mu \)), energy-momentum tensor \( T_{\mu\nu} \) (and \( \tilde{T}_{\mu\nu} \)) and central charge current \( \zeta_\mu \) (and \( \tilde{\zeta}_\mu \)), respectively. The full expressions of these operators read [14]

\[
\begin{align*}
\hat{j}^\mu(x) &= -[\partial\varphi(x) + U(\varphi(x))] \gamma^\mu \psi(x), \\
\hat{J}^\mu(x) &= -[\partial\varphi(x) - F(x)] \gamma^\mu \psi(x) \\
&= j^\mu - \frac{\hbar g}{2\pi} \gamma^\mu \psi, \\
T_{\mu\nu}(x) &= \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} \eta_{\mu\nu} [(\partial^\rho \varphi)(\partial_\rho \varphi) - FU] \\
&\quad + \frac{1}{4} \bar{\psi}[\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu] \psi - \frac{1}{4} \eta_{\mu\nu} [\bar{\psi} \gamma^\rho \partial_\rho \psi + 2g \varphi \bar{\psi} \psi], \\
\tilde{T}_{\mu\nu}(x) &= \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} \eta_{\mu\nu} [(\partial^\rho \varphi)(\partial_\rho \varphi) + F^2] + \frac{1}{4} \bar{\psi}[\gamma_\mu \partial_\nu + \gamma_\nu \partial_\mu] \psi \\
&= T_{\mu\nu}(x) + \eta_{\mu\nu} \frac{\hbar g}{4\pi} F(x), \\
\zeta_\mu(x) &= \epsilon_{\mu\nu} \partial^\nu \varphi(x) U(\varphi), \\
\tilde{\zeta}_\mu(x) &= -\epsilon_{\mu\nu} \partial^\nu \varphi(x) F(x) \\
&= \zeta_\mu(x) + \frac{\hbar g}{2\pi} \epsilon_{\mu\nu} \partial^\nu \varphi(x)
\end{align*}
\]

(2.9)

with \( U(\varphi) = g(\varphi^2(x) - v_0^2) \) and \( \epsilon_{01} = -1 \). Another Ward identity in superspace was used to prove that \( F + U \) may be replaced by \( -\frac{\hbar g}{2\pi} \) inside composite operators.

It was shown in [14] that the currents \( \hat{J}^\mu, \tilde{T}_{\mu\nu} \) and \( \tilde{\zeta}_\mu \) are conserved at the quantum level and, being \( g \)-independent, are free from explicit symmetry breaking terms but contain anomalies. On the other hand, the currents \( j^\mu, T_{\mu\nu} \) and \( \zeta_\mu \) are not conserved (except for the topological current \( \zeta_\mu \)) and contain explicit symmetry breaking terms as is clear from the presence of \( g \)-dependent terms.

3 Local counter term and supersymmetry algebra

It was shown in [14] that the following supersymmetry algebra is satisfied by the conserved supersymmetry charges \( \tilde{Q}^\alpha = \int dx \tilde{J}^{\alpha,0}(x) \)

\[
\begin{align*}
\{ \tilde{Q}^\alpha, \tilde{Q}^\beta \} = -2(\gamma^\mu)^{\alpha\beta} \tilde{P}_\mu - 2\tilde{Z}(\gamma_5)^{\alpha\beta}
\end{align*}
\]

(3.1)

where we defined

\[
\begin{align*}
\tilde{P}_\mu &= \int dx \tilde{T}_{0\mu}(x), \quad \tilde{H} = \tilde{P}_0, \\
\tilde{Z} &= \int dx \tilde{\zeta}_0(x) = -\int dx \tilde{\zeta}_0(x).
\end{align*}
\]

(3.2)
It was also noted in [14] that the same supersymmetry algebra is satisfied by the naive (Noether) supersymmetry charges $Q^\alpha = \int dx j^{0,\alpha}(x)$

$$i\{Q^\alpha, Q^\beta\} = -2(\gamma^\mu)^{\alpha\beta} P_\mu - 2Z(\gamma_5)^{\alpha\beta}$$  \hspace{1cm} (3.3)

where we defined

$$P_\mu = \int dx T_{0\mu}(x), \quad H = P_0, \quad Z = \int dx \zeta_0(x) = -\int dx \zeta_0(x).$$  \hspace{1cm} (3.4)

The second algebra (3.3) was dismissed in [14] because $j^{\mu,\alpha}(x)$ and $T_{\mu\nu}(x)$ are not conserved. The BPS bound is then saturated in the algebra (3.1) by the trace and central charge anomalies\(^3\).

What we would like to clarify in this note is the nature of the anomalous contributions appearing in (2.8). The superspace analysis of [14] as summarized in the preceding chapter allows one to recognize that the precise form of anomalies is modified if one adds the counter term

$$\mathcal{L}_{\text{counter}} = -\frac{\hbar g}{2\pi} \phi(x, \theta)$$  \hspace{1cm} (3.5)

to the original Lagrangian (2.1) where $c$ stands for a numerical constant. In particular, for $c = 1$ the counter term

$$\mathcal{L}_c = -\frac{\hbar g}{2\pi} \phi(x, \theta)$$  \hspace{1cm} (3.6)

removes all anomalies in (2.8) and (2.9) proportional to $\hbar$.

The theory with extra local counter term is defined by

$$\mathcal{L}_{\text{total}} = \mathcal{L} + \mathcal{L}_c = \frac{1}{4} \bar{D}_\alpha \phi D^\alpha \phi + \frac{1}{3} g \phi^3 - gv_0^2 \phi - \frac{\hbar g}{2\pi} \phi(x, \theta).$$  \hspace{1cm} (3.7)

The usual counter term on the other hand is contained within $v_0^2 = v^2 + \delta v^2$, and we adopt the following minimal renormalization condition [4, 7]

$$U = g(\varphi^2 - v_0^2) = g(\varphi^2 - v^2 - \delta v^2) = g([\varphi^2]_{\text{ren}} - v^2)$$  \hspace{1cm} (3.8)

at the one-loop level in the trivial vacuum; the counter term $\delta v^2$ is defined to remove the one-loop tadpole of $\varphi$ prior to the addition of $\mathcal{L}_c$. Here $[\varphi^2]_{\text{ren}}$ stands for the renormalized operator which gives rise to a finite quantity when inserted into Green’s functions.

The Lagrangian (3.7) can be obtained from the one without extra local counter term by the replacement $v_0^2 \rightarrow v_0^2 + \frac{\hbar}{2\pi}$. It is shown that both of the above supersymmetry algebras (3.1) and (3.3) hold in the theory with the extra local counter term if one makes the above replacement of the vacuum parameter. This is intuitively understood by noting

\(^3\)The trace of stress tensor is given by $\bar{T}_{\mu\nu} = -\bar{T}_{00} + \bar{T}_{11} = -\bar{T}_{00}$ since $\langle \bar{T}_{11} \rangle = 0$, and yields the total mass as a sum of regular and anomalous contributions. For the supersymmetric kink, the regular quantum corrections cancel if one uses minimal renormalization with only a mass counter term such that tadpoles vanish.
that the canonical equal-time commutators are independent of the vacuum parameter. In the theory with extra local counter term, the conserved quantities are given by (using the notation in (2.9))

\[ \tilde{J}^{\mu,\alpha}(x)_{v_0^2 \rightarrow v_0^2 + \frac{\hbar}{2\pi}}, \quad \tilde{T}_{\mu\nu}(x)_{v_0^2 \rightarrow v_0^2 + \frac{\hbar}{2\pi}}, \quad \tilde{\zeta}_\mu(x)_{v_0^2 \rightarrow v_0^2 + \frac{\hbar}{2\pi}} \]  

(3.9)

but these operators are expressed in terms of the operators in (2.9) for the theory without the extra counter term as follows

\[ \tilde{J}^{\mu,\alpha}(x)_{v_0^2 \rightarrow v_0^2 + \frac{\hbar}{2\pi}} = j^{\mu,\alpha}(x)_{v_0^2 \rightarrow v_0^2 + \frac{\hbar}{2\pi}} - \frac{h g}{2\pi} \gamma^{\mu}\psi = \tilde{j}^{\mu,\alpha}, \]

\[ \tilde{T}_{\mu\nu}(x)_{v_0^2 \rightarrow v_0^2 + \frac{\hbar}{2\pi}} = T_{\mu\nu}(x)_{v_0^2 \rightarrow v_0^2 + \frac{\hbar}{2\pi}} + \eta_{\mu\nu} \frac{h g}{4\pi} F(x) = \tilde{T}_{\mu\nu}(x)_{v_0^2}, \]

\[ \tilde{\zeta}_\mu(x)_{v_0^2 \rightarrow v_0^2 + \frac{\hbar}{2\pi}} = \zeta_\mu(x)_{v_0^2 \rightarrow v_0^2 + \frac{\hbar}{2\pi}} + \frac{h g}{2\pi} \epsilon_{\mu\nu} \phi^{\nu} \varphi(x) = \tilde{\zeta}_\mu(x)_{v_0^2}. \]  

(3.10)

We thus have

\[ \tilde{T}_{\mu\nu}(x)_{v_0^2} = T_{\mu\nu}(x)_{v_0^2}. \]  

(3.11)

We note that the conservation conditions of \( \tilde{T}_{\mu\nu}(x)_{v_0^2} \) and \( T_{\mu\nu}(x)_{v_0^2} \) are expressed in the path integral formulation as

\[ \langle \partial^\mu \tilde{T}_{\mu\nu}(x)_{v_0^2} \rangle = \int D\phi \partial^\mu \tilde{T}_{\mu\nu}(x)_{v_0^2} \exp \{ i \int dx L \} = 0 \]  

(3.12)

and we obtain, replacing \( v_0^2 \) by \( v_0^2 + \frac{\hbar}{2\pi} \) (which replaces \( L \) by \( L + L_c \)), and using (3.11)

\[ \langle \partial^\mu T_{\mu\nu}(x)_{v_0^2} \rangle = \int D\phi \partial^\mu T_{\mu\nu}(x)_{v_0^2} \exp \{ i \int dx [L + L_c] \} = 0. \]  

(3.13)

The operator \( T_{\mu\nu}(x)_{v_0^2} \), which is not conserved for the theory defined by \( L \), becomes conserved for the theory specified by \( L + L_c \); different equations of motion give rise to different conservation properties.

As for the trace anomaly, we note that

\[ T_{\mu}^{\mu}(x)_{v_0^2} = [g F(\varphi^2 - v_0^2) - g \varphi \bar{\varphi} \psi] \]

\[ = [g F(\varphi^2 - v_0^2) - g \varphi \bar{\varphi} \psi]_{v_0^2 \rightarrow v_0^2 + \frac{\hbar}{2\pi}} + \frac{h g}{2\pi} F(x) \]

\[ = T_{\mu}^{\mu}(x)_{v_0^2 \rightarrow v_0^2 + \frac{\hbar}{2\pi}} + \frac{h g}{2\pi} F(x) \]  

(3.14)

and thus the trace of the conserved tensor \( T_{\mu\nu}(x)_{v_0^2} \) in the theory with the counter term in fact contains the trace anomaly in addition to \( T_{\mu}^{\mu}(x)_{v_0^2 \rightarrow v_0^2 + \frac{\hbar}{2\pi}} \) which is the explicit breaking term for the theory with the counter term. In connection with this observation, we emphasize that if one makes the replacement \( v_0^2 \rightarrow v_0^2 + \frac{\hbar}{2\pi} \) for the combination of the explicit symmetry breaking term and the trace anomaly for the theory \textit{without} the counter term, the cancellation of the trace anomaly by a variation of the explicit symmetry
breaking term takes place regardless of regularization schemes. Our analysis is thus not specific to the path integral formulation.

Similarly, for the $\gamma$-trace of the central charge current (which is equivalent to the current itself) we have the anomaly relation

$$\zeta_{\mu}(x)_{\nu_0^2} = \zeta_{\mu}(x)_{\nu_0^2} - v_0^2 + \frac{\hbar g}{2\pi} \epsilon_{\mu \nu} \partial^\nu \varphi(x). \quad (3.15)$$

In the theory with extra local counter term, the relevant conserved quantities are thus given by the operators appearing on the right-hand sides of (3.10). The supersymmetry algebra satisfied by the conserved quantities is then given by the naive algebra (3.3). The BPS bound in the theory with the extra local counter term is thus formally satisfied without anomaly contributions.

However, we note some differences between the local counter term in the present context and counter terms in ordinary gauge theories. For example, the main physics issue in the present context is the saturation of the BPS bound, which is saturated in both cases with or without extra local counter term, though the actual manner of saturation differs in these two cases with or without the extra local counter term. Also, the inevitable trace anomaly for the conserved energy-momentum tensor, which has been established by a variety of ways [5]~[13], is present in the theory with or without the extra local counter term.

In the present problem, the kink mass is another important physical quantity, and we want to comment on how it remains invariant under the addition of extra local counter terms. The fact that the counter term (3.6) removes all the anomalies suggests that all the anomalies are generated if one adds an extra local term

$$\mathcal{L}_{\text{anomaly}} = \frac{\hbar g}{2\pi} \phi(x, \theta) \quad (3.16)$$

to the Lagrangian, by pretending as if the path integral measure generates no non-trivial Jacobians. The term $\mathcal{L}_{\text{anomaly}}$ plays a role of a "Wess-Zumino term". This picture is convenient when one analyzes the effects of the anomalies in the framework of the effective Lagrangian and the effective potential.

In the above picture with $\mathcal{L}_{\text{anomaly}}$, the original theory is effectively described by

$$\mathcal{L}_{\text{eff}} = \mathcal{L} + \mathcal{L}_{\text{anomaly}} = \frac{1}{4} \bar{D}_\alpha \phi D^\alpha \phi + \frac{1}{3} g \phi^3 - g v_0^2 \phi + \frac{\hbar g}{2\pi} \phi(x, \theta) \quad (3.17)$$

combined with the naive path integral measure. The net effect of the anomalies is thus the replacement of the vacuum value $v_0^2 \rightarrow v_0^2 - \frac{\hbar}{2\pi}$, or after the renormalization

$$v^2 \rightarrow v^2 - \frac{\hbar}{2\pi}. \quad (3.18)$$

The justification of this picture is given when one starts with the naive operators

$$j^\mu(x) = -[\partial_\mu \varphi(x) + U(\varphi(x))] \gamma^\mu \psi(x),$$
\[ T_{\mu\nu}(x) = \partial_{\mu}\varphi \partial_{\nu}\varphi - \frac{1}{2} \eta_{\mu\nu}[(\partial^{\rho}\varphi)(\partial_{\rho}\varphi) - FU] \]
\[ + \frac{1}{4} \bar{\psi} [\gamma_{\mu}\partial_{\nu} + \gamma_{\nu}\partial_{\mu}] \psi - \frac{1}{4} \eta_{\mu\nu} [\bar{\psi} \gamma^{\rho} \partial_{\rho} \psi + 2g \varphi \bar{\psi} \psi], \]
\[ \zeta_{\mu}(x) = \epsilon_{\mu\nu} \partial^{\nu}\varphi(x) U(\varphi), \]
(3.19)

with \( U(\varphi) = g(\varphi^2(x) - v_0^2) \) and \( \epsilon_{01} = -1 \). When one makes the above replacement \( v_0^2 \to v_0^2 - \frac{h}{2\pi} \) in these naive operators, all the operators with tilde in (2.9), which give the correct anomalies, are generated.

The elementary fermion or boson mass parameter in the asymptotic region \( m = 2gv \), which is not the pole mass itself but nevertheless defines a physical parameter, is thus given by the above replacement as

\[ m = 2g \sqrt{v^2 - \frac{h}{2\pi}}. \]
(3.20)

On the other hand, it was shown in [14] that the total central charge, which is equal to the total energy of the kink vacuum in the theory without the extra local counter term, is given by

\[ M = \frac{4}{3} g (v^2 - \frac{h}{2\pi})^{3/2} \]
(3.21)

where \( -\frac{h}{2\pi} \) arises from the effect of the central charge anomaly. This kink mass is written in terms of the modified fermion mass parameter as

\[ M = \frac{1}{6g^2 m^3} \]
(3.22)

which has the same form as the naive kink mass expressed in terms of the naive fermion or boson mass parameter, as in the case in the theory with the extra local counter term which removes all the anomalies. We thus have the same physical relation between physical masses with or without the extra local counter term.

It should be noted, however, that the transition from the original Lagrangian to the one given by (3.7) corresponds to a modification of the renormalization scheme in the present model. With (3.7) the condition which determines \( \delta v^2 \) is in effect one where the tadpole diagrams are renormalized precisely such as to cancel additional terms in (3.22).\(^4\)

Thus physical quantities as well as the property of BPS saturation are left intact simply because a change of renormalization prescriptions does not change physics; yet, on a formal level the anomalous contributions to both sides of the BPS bound are modified.

### 4 Discussion and conclusion

We have shown that the extra local counter term (3.6) formally removes all the anomalies from the supersymmetry algebra in the model (2.1). We have also shown that the physical

\(^4\)If one would like to have a renormalization scheme where all tadpoles vanish even after the transition to (3.7), this can be achieved by allowing for a finite renormalization of \( g \) such that (3.22) is maintained (see Ref. [7] for a detailed discussion of possible renormalization schemes in this model).
consequences are not modified by the counter term. It is possible to show this equivalence for a more general choice of the counter term. At the same time we explained that the trace of the conserved energy-momentum tensor consists of the explicit symmetry breaking terms plus a unique trace anomaly regardless of counter terms, and similarly the $\gamma$-trace of the central charge current.

In the context of the present problem we thus conclude that the issue whether the anomalies are spurious or not is immaterial. Rather, we have seen that different descriptions based on different bare Lagrangians give rise to the same physical consequences. In particular, the original description with equal anomalies in the trace and central charge provides a consistent and well-defined description of the quantized supersymmetric kink model. This description is quite informative with respect to physics, such as the inevitable appearance of a trace anomaly for the conserved energy-momentum tensor. The quantum modification of the central charge, as manifested as the central charge anomaly, may take place for a more general class of Abelian topological quantities. On the other hand, the description with the extra counter term which removes all the anomalies is less informative though the physical content of the model is succinctly expressed.\(^5\) Another way to understand why the extra local counter term, which formally removes explicit anomalies from the supersymmetry algebra, does not change physics, is to recognize, as we have discussed, that it corresponds to a change of the renormalization scheme, and this does not change physics. Only the formal analysis of how the BPS bound is saturated is changed.

This is in sharp contrast to the case in gauge theory where the description with counter terms which eliminate all the (spurious) gauge anomalies is the unique description of the physical theory.

Acknowledgments

One of us (KF) thanks all the members of the C.N.Yang Institute for Theoretical Physics at Stony Brook for their hospitality.

References


\(^5\)The continuous number of ways to describe the same physics, which is parametrized by the counter term (3.5), is to some extent analogous to the $R_\xi$-gauge formulation of the Higgs mechanism. The unitary gauge $\xi = 0$ succinctly expresses the physical content, but other gauges are more informative about the essence of the Higgs mechanism. The would-be Nambu-Goldstone bosons appear in general gauge conditions though they disappear in the unitary gauge. The question if the Nambu-Goldstone bosons are “spurious” or not in the Higgs mechanism may be compared to the question if the anomalies are “spurious” or not in the analysis of BPS bound in the present kink model. One may say that the anomalies are essential to understand the quantized supersymmetric kink properly, even if they do not explicitly appear in the description with $c = 1$, just like the Nambu-Goldstone bosons in the Higgs mechanism.


