Radion Induced Spontaneous Baryogenesis

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ABSTRACT: We describe a possible scenario for the baryogenesis arising when matter is added on the branes of a Randall-Sundrum model with a radion stabilizing potential. We show that the radion field can naturally induce spontaneous baryogenesis when the cosmological evolution for the matter on the branes is taken into account.

KEYWORDS: Baryogenesis, extra-dimensions.
1. Introduction

One of the most peculiar features of the Universe is the observed baryon asymmetry, that is the difference between the density of baryons and anti-baryons, conveniently characterized by the dimensionless number

\[ \frac{n_B}{s} \equiv \eta \simeq 10^{-10}, \quad (1.1) \]

where \( n_B \equiv n_b - n_\bar{b} \) is the difference between the baryon and anti-baryon density and \( s \) is the entropy density. The primordial nucleosynthesis, which is one of the most consistent and precise results in the standard model of cosmology, requires this value for \( \eta \) at the time when the light elements (\( ^3\text{He}, ^4\text{He}, ^7\text{Li} \)) were produced, and that value is believed not to have changed since.

The necessary conditions for generating the baryonic asymmetry were formulated by Sacharov in 1967 [1] (see also Ref. [2]) as:

1. Different interactions for particles and antiparticles, or, in other words, a violation of the C and CP symmetries;

2. Non-conservation of the baryonic charge;

3. Departure from thermal equilibrium.

The last condition results from an application of the CPT theorem. In fact, CPT invariance ensures that the energy spectra for baryons and anti-baryons are identical, leading consequently to an identical distribution in thermal equilibrium. This explains why the baryon number asymmetry was required to be generated out of thermal equilibrium.

The so called spontaneous baryogenesis mechanism [3] uses the natural CPT non-invariance of the Universe during its early history to bypass this third condition. We know that an expanding Universe at finite temperature violates both Lorentz invariance and time reversal, and this can lead to effective CPT violating interactions [4]. Thus the cosmological
expansion of the early Universe leads us naturally to examine the possibility of generating the baryon asymmetry in thermal equilibrium. The main ingredient for implementing this mechanism is a scalar field $\phi$ with a derivative coupling to the baryonic current. If the current is not conserved and the time derivative of the scalar field has a non-vanishing expectation value, an effective chemical potential with opposite signs for baryons and antibaryons is generated leading to an asymmetry even in thermal equilibrium.

The question we would like to address is whether this mechanism might arise naturally in brane-world models. Cosmological solutions in the model with two branes proposed by Randall and Sundrum (RS) in Ref. [5] have been examined in the case when matter is added on the two branes (see for example Ref. [6]). These models naturally contain a metric degree of freedom called the radion which determines the distance between the two branes and which appears as a scalar field on the branes. We will analyze the conditions under which the radion field might spontaneously induce the observed baryonic asymmetry $^1$.

In Section 2, we review in some details the spontaneous baryogenesis mechanism. In Section 3, the cosmological solutions in the RS framework are discussed and the radion driven spontaneous baryogenesis process is presented. We then conclude and comment on our results.

We shall use units with $c = \hbar = k = 1$, where $k$ is the Boltzmann constant, and denote by $M_{\text{Pl}}$ the four-dimensional Planck constant.

2. Spontaneous Baryogenesis

To illustrate the mechanism of spontaneous baryogenesis (see e.g. Refs. [3, 8, 9]) let us consider a theory in which a neutral scalar field $\phi$ is coupled to the baryonic current $J_B^\mu$ by the Lagrangian density

$$L_{\text{int}} = \frac{\lambda'}{M_c} J_B^\mu \partial_\mu \phi ,$$

(2.1)

where $\lambda'$ is a coupling constant and $M_c < M_{\text{Pl}}$ is a cut-off mass scale in the theory. Let us assume that $\phi$ is homogeneous, so that only the time derivative term contributes,

$$L_{\text{int}} = \frac{\lambda'}{M_c} \dot{\phi} n_B \equiv \mu(t) n_B ,$$

(2.2)

where $n_B = J_B^0$ is the baryon number density and $\mu(t)$ is to be regarded as an effective time-dependent chemical potential. This interpretation (see Ref. [10]) is valid if the current $J_B^\mu$ is not conserved (otherwise one could integrate the interaction term away) and if $\phi$ behaves as an external field which develops a slowly varying time derivative $\langle \dot{\phi} \rangle \neq 0$ as the Universe expands. Since the chemical potential $\mu$ enters with opposite signs for baryons and antibaryons, we have a net baryonic charge density in thermal equilibrium at the temperature $T$,

$$n_B(T; \xi) = \int \frac{d^3k}{(2\pi)^3} \left[ f(k, \mu) - f(k, -\mu) \right] ,$$

(2.3)

$^1$Other mechanisms for baryogenesis in the context of brane-world models have been recently analyzed, e.g. in Refs. [7].
where $\xi \equiv \mu / T$ is regarded as a parameter, and
\[
f(k, \mu) = \exp \left[ \left( \sqrt{k^2 + m^2} - \mu \right) / T \right] \pm 1
\] (2.4)
is the phase-space thermal distribution for particles with rest mass $m$ and momentum $k$. For $|\xi| \ll 1$ we may expand Eq. (2.4) in powers of $\xi$ to obtain
\[
n_B(T; \mu) = \frac{g T^3}{6} \xi + O(\xi^2),
\] (2.5)
where $g$ is the number of degrees of freedom of the field corresponding to $n_B$. Upon substituting in for the expression of $\mu$, one therefore finds
\[
n_B(T; \mu) \simeq \frac{\lambda' g}{6 M_c} T^2 \langle \dot{\phi} \rangle.
\] (2.6)

Whatever the mechanism of baryon number violation, we assume there is a temperature $T_F$ at which the baryon number violating processes become sufficiently rare so that $n_B$ freezes out (we will call $T_F$ the freezing temperature). Once this temperature is reached as the universe cools down, one is left with a baryon asymmetry whose value is given by Eq. (2.6) evaluated at $T = T_F$. The value of the parameter $\eta$ remains unchanged in the subsequent evolution.

3. Radion Induced Spontaneous Baryogenesis

We have discussed how the spontaneous baryogenesis mechanism may explain the observed baryon asymmetry. We would now like to assess how it might come out in the brane-world model. We consider the five-dimensional RS model of Ref. [5] perturbed by matter on both branes. The reader is referred to Ref. [6] for more details on the framework and notation used hereafter.

In this model the metric can be written in the form
\[
ds^2 = n^2(y, t) \, dt^2 - a^2(y, t) \left[ (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right] - b^2(y, t) \, dy^2
\equiv \tilde{g}_{AB}(x, y) \, dx^A \, dx^B,
\] (3.1)
where $t$ is the time, $x^i$ are the spatial coordinates along the branes and $y$ is the extra-dimensional coordinate. In this formalism, the Planck brane is located at $y = 0$ and the TeV brane at $y = 1/2$. The Einstein equations are given by $G_{AB} = \kappa^2 T_{AB}$, where $\kappa^2 = 1/(2M^3)$ and $M$ is the five-dimensional Planck scale. The energy-momentum tensor $T_{AB}$ picks up a contribution from the bulk cosmological constant $\Lambda$ of the form $T_{AB}^{\text{bulk}} = \Lambda \tilde{g}_{AB}$ and a contribution from the matter on the two branes,
\[
T_{AB}^{\text{branes}} = \frac{1}{b} \, \delta(y) \, \text{diag} [V_e + \rho_s, V_e - p_s, V_s - p_s, V_s - p_e, 0]
+ \frac{1}{b} \, \delta(y - 1/2) \, \text{diag} [V + \rho, V - p, V - p, V - p, 0],
\] (3.2)

$^2$Of course, the plus sign is for fermions and the minus sign for bosons.
where \( V_\ast \) is the (positive) tension of the Planck brane and \( V \) is the (negative) tension on the TeV brane. We have correspondingly denoted by \( \rho_\ast \) and \( p_\ast \) the density and pressure of the matter localized on the positive tension (Planck) brane (assuming an equation of state of the form \( p_\ast = w_\ast \rho_\ast \)) and by \( \rho \) and \( p \) the density and pressure of the matter on the negative tension (TeV) brane. We have correspondingly denoted by \( \rho_\ast \) and \( p_\ast \) the density and pressure of the matter localized on the positive tension (Planck) brane (assuming an equation of state of the form \( p_\ast = w_\ast \rho_\ast \)) and by \( \rho \) and \( p \) the density and pressure of the matter on the negative tension (TeV) brane. Once a stabilizing potential for the radion is included, the stress-energy tensor picks up an additional term and the solution of the Einstein equations may be written as a perturbation of the usual RS solution,

\[
n(y) = a(y) = e^{-m_0 b_0 |y|},
\]

(3.3)

with \( V_\ast = 6 m_0 / \kappa^2 = -V \) and \( \Lambda = -6 m_0^2 / \kappa \). We also recall that the constants \( b_0 \) and \( m_0 \) determine the effective four-dimensional Planck scale as \((8 \pi G_N)^{-1} = M_{Pl}^2 = (1 - \Omega_0^2) / \kappa^2 m_0 \), where \( \Omega_0 \equiv e^{-m_0 b_0} / 2 \).

In order to obtain an effective action for the four-dimensional theory, one therefore perturbs the metric about the RS solution in the form

\[
a(t, y) = a(t) e^{-m_0 b(t) |y|} [1 + \delta a(y, t)]
\]

\[
n(t, y) = e^{-m_0 b(t) |y|} [1 + \delta n(y, t)]
\]

(3.4)

\[
b(t, y) = b(t) [1 + \delta b(y, t)]
\]

and drops the metric perturbations which contribute only to second order in \( \delta a, \delta n \) and \( \delta b \). It is then useful to introduce the notation \( \Omega(y, b(t)) = e^{-m_0 b(t) |y|} \) and \( \Omega_0 \equiv \Omega(1/2, b(t)) \) (\( \Omega_0 \) evaluated at \( b = b_0 \) is then given by \( \Omega_0 \)). By integrating over the fifth dimension, one obtains an effective action for the radion field. Further, by examining the equations of motion for \( b(t) \), one can note that, due to the dependence of \( \Omega_0 \) on \( b \), the presence of matter on the two branes generates an effective potential for \( b(t) \) whose explicit expression is given by

\[
V_{\text{eff}} (b) = V_r (b) + \frac{f^4 (b)}{4} \left[ \rho_\ast - 3 p_\ast + (\rho - 3 p) \Omega_0^4 \right],
\]

(3.5)

with

\[
f(b) = \left( \frac{1 - \Omega_0^2}{1 - \Omega_b^2} \right)^{1/2}.
\]

(3.6)

The function \( V_r = V_r (b(t)) \) is the potential which would stabilize the radion at the value \( b = b_0 \) in the absence of matter. It can therefore be expanded near its minimum as

\[
V_r (b) = \frac{1}{4} m_r^2 \left( \frac{m_0 b_0}{1 - \Omega_0^2} \right)^2 \Omega_0^2 M_{Pl}^2 \left( \frac{b - b_0}{b_0} \right)^2,
\]

(3.7)

\footnote{This expression follows from Eq. (4.12) of Ref. \cite{[Ref]} by defining \( \sqrt{3/2} \phi / \Lambda_W = m_0 b \) (where \( \Lambda_W = \Omega_0 M_{Pl} \simeq 1 \text{ TeV} \).}

\[
\]

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where
\[
m_r^2 = \frac{2}{3 m_0^2} \frac{\left( f^4 V_r \right)^{\prime\prime} (b_0)}{M_{Pl}^2 \Omega_0^2} \left( 1 - \Omega_0^2 \right)^2 ,
\]
and \(m_r\) is the effective radion mass. Thus the radion, in the presence of matter on the two branes, is stabilized to a value \(b_0 + \Delta b\) determined by
\[
\frac{\Delta b}{b_0} = \frac{1}{3 m_0 b_0} \frac{1 - \Omega_0^2}{\Omega_0^2} \frac{\rho - 3p + \rho_0^2 \left( \rho_* - 3p_* \right)}{\rho_*} ,
\]
where \(\Delta b\) is the distance between the minima of the effective potential \(V_{\text{eff}}\) with and without matter.

From the above expression, we see that, since the trace of the stress-energy tensor vanishes for radiation, even if the Universe on the TeV brane is in a radiation dominated era \((\rho \simeq 3p)\), the radion evolution is determined by the behavior of matter on the TeV brane and by the behavior of matter on the Planck brane. This is the essential ingredient which allows for the possibility to induce spontaneous baryogenesis. We note that the factor \(\Omega_0^2\) in front of \(\rho_*\) would make this term negligible for comparable energy densities on the TeV and Planck branes, but the fact that the natural energy scale on the Planck brane is of the order of \(M_{Pl}\) may nonetheless allow for a relevant contribution to the baryogenesis process from the Planck brane.

Let us now assume that the high energy Lagrangian contains an interaction term of the form given in Eq. (2.2) (see Eq. (4.30) in Ref. [6]),
\[
L_{\text{int}} = \lambda' m_0 \dot{b} n_B ,
\]
where \(\dot{b}\) now plays the role of \(\langle \dot{\phi} \rangle\). By using Eq. (3.9) to estimate \(\dot{b} \simeq \dot{\Delta} b\), we obtain
\[
\eta = \frac{m_0 \dot{b}}{T} = \frac{1}{m_r^2 \Omega_0^2 M_{Pl}^2} \frac{1}{T} \frac{1}{dt} \left[ \left( \rho - 3p \right) + \Omega_0^2 \left( \rho_* - 3p_* \right) \right] .
\]
Due to the Universe expansion, the time derivative on the right hand side of the above equation might get a non vanishing expectation value. We also note that the radion field is likely very massive \(^4\), implying that one may assume that the radion follows instantaneously any changes of the matter density.

We can now describe the generated baryon asymmetry by evaluating Eq. (2.3) at the freezing temperature \(T_F\) in three different scenarios:

1. If the effect of matter on the Planck brane is negligible in Eq. (3.11), the condition to generate the observed baryon asymmetry can be estimated as
\[
\frac{1}{T} \left. \frac{dT}{dt} \right|_{T_F} (\rho - 3p) \gtrsim 10^{-10} \text{TeV}^4 .
\]

\(^4\)For example, assuming a Goldberger-Wise mechanism for the stabilization of the radion \([4]\), one has \(m_r \simeq 1\) TeV.
Let \( \rho_m \) be the energy density of any non-traceless component of the energy momentum tensor. By using the continuity equation and the Friedmann equation for a radiation dominated Universe up to an energy density of the order of \( \text{TeV}^4 \), which is the limit of validity for the RS model, one obtains the requirement

\[
\rho_m \gtrsim 10^{-6} \text{TeV}^4
\]

for the mechanism to produce sufficient baryonic asymmetry.

2. Since the matter on the Planck brane remains hidden, one can allow for the term proportional to \( \dot{\rho}_s \) in Eq. (3.11) to be significantly large (we recall that matter energy on the Planck brane is allowed up to the Planck scale). The radion velocity would hence be larger than in the previous case, and the freezing temperature correspondingly lower. The bound in this case is

\[
\frac{1}{T} \frac{d}{dt} (\rho_s - 3 p_s) \bigg|_{T_F} \gtrsim 10^{-42} M_{Pl}^4.
\]

3. Another possibility is to consider the stage when the radion is still stabilizing towards the value \( b_0 \). In this case, the effect of matter on the branes is negligible and a typical radion velocity would be larger than the previous cases, \( m_0 \dot{b} \simeq H(T) M_{Pl}/\Lambda_W \). One then finds

\[
T \gtrsim 10^2 \text{eV},
\]

which allows the widest range of temperature among the three possibilities outlined.

4. Conclusions

We have shown that the perturbations induced by the addition of matter on the two branes of a cosmological RS model naturally lead to a non-vanishing expectation value for the velocity of the radion field. This in turn may induce the onset of the spontaneous baryogenesis mechanism and produce the observed baryonic asymmetry. We have also briefly described a variety of possible scenarios with different initial conditions and matter distributions on the two branes.

References


