The $D_s(2317)$ and $D_s(2463)$ Mesons asScalar and Axial-Vector
Chiralons in the
Covariant Level-Classification Scheme

Shin Ishida$^1$∗, Muneyuki Ishida$^2$, Toshihiko Komada$^3$, Tomohito Maeda$^1$,
Masuho Oda$^4$, Kenji Yamada$^3$ and Ichiro Yamauchi$^5$

$^1$ Research Institute of Quantum Science, College of Science and Technology
Nihon University, Tokyo 101-0062, Japan
$^2$ Department of Physics, Meisei University, Tokyo 191-8506, Japan
$^3$ Department of Engineering Science, Junior College Funabashi Campus,
Nihon University, Funabashi 274-8501, Japan
$^4$ Faculty of Engineering, Kokushikan University, Tokyo 154-8515, Japan
$^5$ Department of Mechanical Engineering, Tokyo Metropolitan College of Technology,
Tokyo 140-0011, Japan

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The new narrow mesons observed recently in the final states $D_s^+\pi^0$ and $D_s^{*+}\pi^0$ are pointed
out to be naturally assigned as the ground-state scalar and axial-vector chiralons in the $(cs)$
system, which would newly appear in the covariant hadron-classification scheme proposed a
few years ago.

Introduction

(Covariant Classification Scheme and Chiral states/Chiralons) A few years
ago we have proposed a covariant level-classification scheme for hadrons, unifying the
seemingly contradictory two, non-relativistic and extremely relativistic, viewpoints.
(Its essential points are reviewed by our review articles$^1$.) Here the framework is
manifestly Lorentz-covariant and the space for the static symmetry is extended from
that of non-relativistic (NR) scheme to

$$SU(6)_{SF} \bigotimes SU(2)_{\rho} \bigotimes O(3)_L,$$

where a new additional $SU(2)$-space for the $\rho$-spin ($\rho$- and $\sigma$- spin being Pauli-matrices
in the decomposition of Dirac $\gamma$ matrices: $\gamma \equiv \sigma \bigotimes \rho$) is introduced for covariant
description$^{**}$ of hadron spin-wave function (WF). The spin WF for the meson systems
are generally given by the Bargmann-Wigner (BW) spinors, and are represented
as the bi-Dirac spinors $W_{\alpha}^\beta = u_{\alpha} \bar{v}^\beta : \alpha = (\rho_3, \sigma_3), \beta = (\bar{\rho}_3, \bar{\sigma}_3)$, where $\alpha(\beta)$ denotes the
suffices of Dirac spinors of Heavy quarks(Light-anti-quarks) represented by the eigenvalues
of $\rho$-spin and $\sigma$-spin. In the HL meson system the states with $(\rho_3, \bar{\rho}_3) = (+, +)$
and $(+, -)$ are expected to be realized in nature, reflecting the physical situation that
the HL meson system has the non-relativistic $SU(6)_s$ spin symmetry (the relativistic
chiral symmetry) concerning the constituent Heavy quarks (Light quarks). Our relevant
$D_s(2317)/D_s(2463)$ mesons are naturally assigned as the scalar/axial-vector chiralons

$^{*)}$ Associate member.
$^{**)}$ It is to be noted that in our scheme the squared-mass spectra are globally $\tilde{U}(12)$-symmetric$^2$ and
the mass spectra themselves are able to be reconciled with the broken chiral symmetry.
in the \((c\bar{s})\) ground states with \((\rho_3, \bar{\rho}_3) = (+, -)\), and play the role of chiral partners of the already established Paulons \(D_s/D_s^*\), the states with \((\rho_3, \bar{\rho}_3) = (+, +)\).

*Description of HL Mesons* The WF of HL mesons are described as

\[
\Phi_A^B(x, y) \sim \psi_{Q, A}(x) \tilde{\phi}^B(y)
\]

\(A = (\alpha, a)\), \(B = (\beta, b)\); \(\alpha, \beta = (1 \sim 4); a = (c \text{ or } b), b = (u, d, s)\),

and are assumed to satisfy the master Klein-Gordon equation of Yukawa-type\(^3\). The squared-mass operator is assumed to contain no light-quark Dirac matrices \(\gamma(q)\) in the ideal limit, leading to the chiral symmetric global structure of squared-mass spectra. The WF is separated into the two parts, the one of plane-wave center of mass motion and the other of internal WF: The internal WF with definite total spin \(J\) is expanded in terms of respective eigen functions \(W(P)\) on spinor-space and \(O(P_N, r)\) on internal space-time, where \(W^\beta_\alpha(P_N)\) and \(O(P_N, r)\) are covariant tensors respectively, in the \(\hat{U}(4)_{D.S.}\) (pseudo-unitary Dirac spinor) space and the \(O(3, 1)_L\) (Lorentz-space).

*Spin WF/BW-spinors* As the complete set of spinor-space eigen-functions we choose the BW spinors, which are defined as solutions of the (local) Klein-Gordon equations. For the HL-mesons we have the two physical solutions:

\[
U_\alpha^\beta(P) \equiv u_\alpha^{(Q)}(P)\tilde{v}_\beta^{(q)}(P); \quad C_\alpha^\beta(P) \equiv u_\alpha^{(Q)}(P)\tilde{v}_\beta^{(q)}(-P),
\]

As is evidently seen from Eq. (3), through the chiral transformation on light anti-quarks \(\tilde{v}(P)\gamma_5 = \tilde{v}(-P)\), the former is changed into the latter as \(U(P)\gamma_5 = C(P)\). They are decomposed into the pseudo-scalars/vectors, and scalars/axial-vectors, respectively, as

\[
U_\alpha^\beta(v) = 1/2\sqrt{2} \left(1 - iv \cdot \gamma\right) [i\gamma_5 P_\alpha(P) + i\tilde{\gamma}_\mu V_\mu(P)],
\]

\[
C_\alpha^\beta(v) = 1/2\sqrt{2} \left(1 - iv \cdot \gamma\right) [S(P) + i\gamma_5 \tilde{\gamma}_\mu A_\mu(P)], \quad (P_\mu \tilde{\gamma}_\mu = 0, \ v_\mu \equiv P_\mu/M).
\]

*Internal space-time WF/Yukawa oscillators* As the complete set of space-time eigen-functions we choose the covariant, 4-dimensional Yukawa oscillator functions. By imposing the freezing relative-time condition they become effectively the conventional, 3-dimensional oscillators:

\[
\langle P_\mu r_\mu \rangle = \langle P_\mu p_\mu \rangle = 0 \Rightarrow O(3, 1)_L \approx O(3)_L.
\]

*Mass spectra for low-lying \(D\) and \(D_s\)-mesons* Since of the static symmetry (1) the global mass spectra are given by

\[
M^2_N = M^2_0 + N\Omega, \quad N \equiv 2n + L,
\]

leading to phenomenologically well-known Regge trajectories. The masses of ground state mesons, \(P_s, V_\mu, S\) and \(A_\mu\) are degenerate in the ideal limit, and they are split with each others between chiral partners (spin partners) by the bilinear scalar-quark condensates (the perturbative QCD spin-spin interaction) as

\[
M_0(0^-/1^-) \lesssim M_0(0^+/1^+) < M_1(L = 1).
\]

As for the splittings between chiral partners we obtain, applying the linear SU(3) \(\sigma\)-model, the universal relation\(^4\) as

\[
\Delta M_X = M_0(0^+) - M_0(0^-) = M_0(1^+) - M_0(1^-)
\]
within the same quark-configuration mesons; and the relation of those between the different light-quark configurations as \( \Delta M_X(c\bar{c})/\Delta M_X(c\bar{s}) = a/b \), where the \((a, b)\) is defined by \( \langle \sigma^2 \lambda^I/\sqrt{2} \rangle \equiv \text{diag}(a, a, b) \), and \( a \equiv f_\pi/\sqrt{2} \), \((a + b)/2 \equiv f_K/\sqrt{2} \) \( (f_\pi f_K) \) being the decay constant of \( \pi^0 \) \( (K) \) . Through the experimental values of \( \Delta M_X(c\bar{s}) = 350\text{MeV}/c^2 \) and \( a/b = 1.144 \), we predict \( \Delta M_X(c\bar{c}) = 240\text{MeV}/c^2 \). In Fig. 1 (a) and (b) we show, respectively, the low-lying \( D \) meson and \( D_s \) meson mass spectra, presently known and/or predicted through the above relations.

**Decay properties of \( D_s \)-mesons**

The observed properties of \( D_s \)-mesons to be examined are as follows\(^5\)\(^7\):

\[
\begin{align*}
D_s(0^+; 2.32) & \rightarrow D(0^-; 1.97) + \pi^0 \quad \text{observed}, \\
D_s(1^+; 2.46) & \rightarrow D(1^-; 2.11) + \pi^0 \quad \text{observed}, \\
R(0^+) & \equiv \frac{\text{Br}(D_{s,0} \rightarrow D_s^0\gamma)}{\text{Br}(D_{s,0} \rightarrow D_s^\pi\gamma)} < 0.078 \quad \text{(CLEO)}. \tag{10}
\end{align*}
\]

\[
\begin{align*}
R(1^+) & \equiv \frac{\text{Br}(D_{s,1} \rightarrow D_s\gamma)}{\text{Br}(D_{s,1} \rightarrow D_s^\pi\gamma)} = 0.47 \pm 0.10 \quad \text{(Belle)}. \tag{11}
\end{align*}
\]

\[
\Gamma_{\gamma}[D_s(0^+; 2.32)], \quad \Gamma_{\gamma}[D_s(1^+; 2.46)] < 7\text{MeV}. \tag{12}
\]

**(Pionic transitions)** The observed processes (9) are iso-spin violating and considered to occur by the mixing of intermediate \( \eta \) meson with \( \pi \)-meson. From this picture and Eq. (8) we get the relation\(^4\)\(^9\),

\[
\Gamma(D_s^0(0^+) \rightarrow D_s(0^-)\pi^0) = \Gamma(D_s^0(1^+) \rightarrow D_s(1^-)\pi^0), \tag{13}
\]

which is consistent with the property (12). We can estimate phenomenologically the value of mixing parameter \( \sin\theta \), by using the experimental branching ratio\(^8\) of \( D_u(c\bar{u}) \) meson to the iso-spin violating decay channel as

\[
(sin\theta)^2_{\text{exp}} \approx \frac{\text{Br}(D_{s}^+ \rightarrow D_{s}^{*+}\pi^0)(M_{D_s^*}^2/q^3)}{\text{Br}(D_{s}^{*+} \rightarrow D_{s}^{\pi}\gamma)} / \frac{\text{Br}(D^{*+} \rightarrow D^{+}\pi^0)(M_{D^{*+}}^2/q^3)}{\text{Br}(D^{+} \rightarrow D^{+}\gamma)}
\]

\[
= (0.9 \pm 0.4) \cdot 10^{-3}. \tag{14}
\]

This seems to be of reasonable order of magnitude as due to the virtual EM-interaction. In order to estimate the absolute magnitude of the width (13) in relation with those of
the other HL-mesons, we shall set up the chiral symmetric interaction Lagrangian, applying the linear $\sigma$ model in the framework of covariant oscillator quark model (COQM)$^{10}$, as

$$S_L^I = \int d^4x_1d^4x_2\mathcal{L}(x_1,x_2) \equiv \int d^4X\mathcal{L}_I(X), \quad \mathcal{L} = \mathcal{L}^{ND} + \mathcal{L}^{AX}$$

(15)

$\mathcal{L}^{ND} = g_{ND}\langle \Phi(x_1,x_2)M(x_2)\Phi(x_1,x_2)\rangle$.

$\mathcal{L}^{AX} = g_{AX}\langle \Phi(x_1,x_2)(\partial_{2,\mu}^+ + i\sigma_{\mu\nu}\partial_{2,\nu})[\partial_{2,\mu}M(x_2)]\Phi(x_1,x_2)\rangle$.

$$M \equiv s - i\gamma_5\phi \quad (s \equiv s^a\lambda^a/\sqrt{2}, \quad \phi \equiv \phi^a\lambda^a/\sqrt{2}),$$

$$\Phi \propto (1 - iv\cdot\gamma)(i\gamma_5D + i\tilde{\gamma}_\nu D^\nu + D^0 + i\gamma_5\tilde{\gamma}_\nu D^{\nu,0}),$$

$$\Phi \equiv \gamma_4\Phi^1\gamma_4, \quad D = (D^0, D^+, D^+_\gamma) \text{ etc.}$$

(16)

where only the Yukawa interaction of the scalar ($s$) and pseudo-scalar ($\phi$) nonets with the light quarks in the HL-meson is taken into account. The interaction (15) consists of the two terms:

Firstly the $g_{ND}$ term concerns with the mass-splitting between chiral partners, and gives dominant (compared to the $g_{AX}$ term) contribution to the (quark-) spin non-flip processes. Accordingly, by fixing the coupling parameter $g_{ND}$ from the experimental value of $\Delta M^X(e\bar{s}) = 350\text{MeV}$, we can predict the absolute values of the relevant pionic decay widths as

$$\Gamma(D_{0,0}^X \to D_n\pi) = \Gamma(D_{n,1}^X \to D_n^0\pi) = 133\text{MeV} \text{,}$$

(17)

$$\Gamma(D_{s,0}^X \to D_s\pi^0) = \Gamma(D_{s,1}^X \to D_s^0\pi^0) = 122 \pm 54\text{keV} \text{,}$$

(18)

where, in deriving Eq. (18), the estimated value Eq. (14) is used. The value of width (18) is consistent with the experiment (12).

Secondly the $g_{AX}$ term in the interaction (15) corresponds to the extended PCAC term, and concerns dominantly (compared to the $g_{ND}$ term) to the spin-flip processes. Accordingly we can determine the coupling parameter $g_{AX}$ by fitting the experimental decay width $\Gamma(D^{*+} \to D^0\pi^+) = (96 \pm 23) \times 0.68\text{keV}$, and predict the $\pi$- (and/or $\sigma$-) mesonic decay widths of the other HL-mesons.

(Radiative decay) In order to treat systematically all the radiative transitions between the HL-mesons we shall set up the basic EM-interaction Lagrangian in the framework of COQM, as

$$S_L^{EM} = \int d^4x_1d^4x_2 \sum_{i=1,2} j_{i,\mu}(x_1,x_2)A_\mu(x_i) = \int d^4X \sum_i J_{i,\mu}(X)A_\mu(X),$$

(19)

$$j_{i,\mu}(x_1,x_2) = -ie_i ((m_1 + m_2)/m_i)(\bar{\Phi}_U(\partial_{i,\mu} + ig_M\sigma_{\mu\nu}^{(i)}\partial_{i,\nu})\Phi_U),$$

$$\Phi_U \equiv \Phi_U(-i \bar{\nu} \cdot \gamma), \quad \Phi_U \equiv \Phi_U(-i \bar{\nu} \cdot \gamma),$$

where $\Phi_U$ is the unitary correspondent of $\Phi$, so defined as $\langle \Phi_U\Phi_U \rangle \to \langle \Phi^\dagger\Phi \rangle$ at the rest frame. Here it is to be noted that our effective current $J_{i,\mu}(X)$ is obtained through the “minimal substitution” of $(\partial_{i,\mu} \to \partial_{i,\mu} - ie_iA_\mu(x_i))$, and accordingly it is conserved in the ideal limit.

Our effective current has also another remarkable feature due to the covariant nature of our scheme. The spin-current interaction (the second term in Eq. (19)) leads
to the Hamiltonian

\[ \mathcal{H}_{\text{spin}}^{(i)} = J_{\mu}^{(i)\text{spin}} A_\mu = \mu^{(i)}(\sigma^{(i)}) \cdot B + d^{(i)}(\rho^{(i)}_1 \sigma^{(i)}) \cdot E, \quad \mu^{(i)} = \epsilon^{(i)} = e_i/2m_i. \]  

(20)

This shows that our Hamiltonian contains the interaction through the “intrinsic electric dipole” \( d_\rho \sigma \) as well as the one through the magnetic dipole \( \mu \sigma \). The “intrinsic dipole” gives contributions only for the transitions between chiralons and Paulons, while does none for the other transitions.

From the effective currents \( J_\mu \) in Eq. (19) we can calculate the relevant decay widths. In Table I we have given the predicted widths for all the radiative spin-flip transitions between ground state \( D_s \) mesons, and, for reference, the width for the transition of \( D_n \) meson, \( D_2^+ (1^-) \rightarrow D_2^+ (0^-) \gamma \), in comparison with experiments. There, we have also shown the predicted values by the other chiral model.

<table>
<thead>
<tr>
<th>Processes</th>
<th>( \gamma )-decays</th>
<th>( \Gamma ) (keV)</th>
<th>( \Gamma ) (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_s (1^-) \rightarrow D_s (0^-) \gamma )</td>
<td>( P \rightarrow P )</td>
<td>0.33</td>
<td>0.43</td>
</tr>
<tr>
<td>( D_s (1^+) \rightarrow D_s (0^+) \gamma )</td>
<td>( \chi \rightarrow \chi )</td>
<td>0.26</td>
<td>0.43</td>
</tr>
<tr>
<td>( D_s (0^+) \rightarrow D_s (1^-) \gamma )</td>
<td>( \chi \rightarrow P )</td>
<td>22</td>
<td>1.74</td>
</tr>
<tr>
<td>( D_s (1^+) \rightarrow D_s (0^-) \gamma )</td>
<td>( \chi \rightarrow P )</td>
<td>82</td>
<td>5.08</td>
</tr>
</tbody>
</table>

\[ D_n^+ (1^-) \rightarrow D_n^+ (0^-) \gamma \] \( P \rightarrow P \), 1.93 (theor) \( \leftrightarrow \) 1.54 \( \pm \) 0.53 (exp)

Table I. \( \gamma \)-decay widths for spin-flip transitions between the ground state \( D_s \) mesons. The values of parameters are taken as \( g_{c,n} = 1 \), \( m_c = M_\psi/2 \), \( m_n(s) = M_\rho(M_\phi)/2 \).

From the results in Table I we see that our model gives the much larger widths for the transitions, (c) and (d), from chiralons to Paulons, compared to the other chiral model, reflecting the above mentioned feature (20) of our currents, while does the width of almost the same amount for transitions, (a) (and (b)), from Paulons(chiralons) to Paulons(chiralons). This difference is considered to come from the different identification of the relevant mesons in the two cases: The narrow \( D_s \) mesons are assigned as the conventional \( P \)-wave excited states in the other model, while they are the \( S \)-wave chiral states other than the \( P \)-wave Pauli-states in our scheme.

\( \text{(Branching ratios between radiative to pionic decay widths)} \) From the predicted values of pionic (Eq. (10)) and radiative (Table I) decay widths we obtain the ratios between them as follows:

\[ R(0^+) = 0.18 \pm 0.10, \quad R(1^+) = 0.67 \pm 0.40, \]  

(21)

which seems to be consistent with the experiments Eqs. (10) and (11).

Concluding Remarks

\( \circ \) The \( D_s (2317) \) and \( D_s (2463) \) mesons are shown consistently assigned as the chiralons with \( J^P = 0^+ \) and \( 1^+ \) in the \((c \bar{s})\) ground states.

\( \circ \) The decay width of \((D_n^+(1^+) \rightarrow D_n^+(1^-)\pi)\) is predicted as \( \Gamma \simeq 130 \text{MeV} \), and the radiative decay widths of chiral states into Pauli-states are predicted to be remarkably larger than those estimated in other works. These are to be checked experimentally.

\( \circ \) Further experimental search for chiralons are desirable.

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References

1) M. Ishida, in the proceedings of Hadron '03: S. Ishida, KEK Proceedings '03; NUP-B-'03.
3) H. Yukawa, Phys. Rev. 91 (1953), 415, 416.
7) P. Krokovny, in the proceedings of Hadron '03.