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Nuclear Collective Vibrations in Extended Mean-Field Theory

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Abstract

The extended mean-field theory, which includes both the incoherent dissipation mechanism due to nucleon-nucleon collisions and the coherent dissipation mechanism due to coupling to low-lying surface vibrations, is briefly reviewed. Expressions of the strength functions for the collective excitations are presented in the small amplitude limit of this approach. This fully microscopic theory is applied by employing effective Skyrme forces to various giant resonance excitations at zero and finite temperature. The theory is able to describe the gross properties of giant resonance excitations, the fragmentation of the strength distributions as well as their fine structure. At finite temperature, the success and limitations of this extended mean-field description are discussed.

1 Introduction

In reaction to an external stress, systems with many degrees of freedom often self-organize in simple collective motion, while they could be naively expected to rather present disorder and chaos because of their intrinsic complexity. This paradox is present in different fields of physics. Indeed, collective oscillations have been observed in mesoscopic systems such as zero-sound phonons in helium-3 fluids or plasmons in metallic clusters. Also, the atomic nucleus is known to exhibit a large variety of collective vibrations usually called phonons. The latter case, is a perfect illustration of the coexistence of well-ordered and disordered motion in a mesoscopic system. Following the Bohr ideas, the compound nucleus resonances are the prototypes of quantum chaos. On the other hand, other experimental evidences, such as existence of magic numbers and presence of giant resonances even at high temperature, are pleading in favor of existence of ordered motions in such a system. The study of this

amazing self-organization and its transition from order to chaos is one of the important subject in modern nuclear physics.

From the very first experimental evidence of the giant dipole resonance due to Baldwin and Klaiber in 1947 [1], a large amount of data devoted to the study of collective vibrations in nuclei have been collected (for a recent review, see [2]). In the applications presented in this work, we are mainly concern with the "standard" electric vibrations, i.e. the isoscalar monopole (GMR), isovector dipole (GDR) and isoscalar quadrupole (GQR) motion. However, the formalism we are using, can be applied to any type of vibrations. A collective vibration is generally characterized by its energy, which varies smoothly along the nuclear chart. This is nicely illustrated by the GDR, which is well established as a general feature of all nuclei. Indeed, it has been observed in nuclei as light as ^3He and as heavy as ^{232}Th . The systematic study of the GDR has shown that the mass dependence of its energy (noted E_{GDR}) is intermediate between $A^{-1/6}$ and $A^{-1/3}$ and, as seen in figure (1), energy dependence can be reproduced by a two parameters expression $E_{GDR} = 31.2A^{-1/3} + 20.6A^{-1/6}$ MeV [3]. This smooth trend can be understood by considering nuclei as a liquid drop, thus underlying the macroscopic nature of nuclei, while quantum features are expected to give local fluctuations around the mean mass dependence. The collective energies are generally well understood within the standard Random-Phase-Approximation (RPA) by neglecting correlations beyond the RPA. In this approximation, the collective vibrations are generated as a coherent superposition of particle-hole ($p-h$) excitations.

From the experimental point of view, it is possible to access not only to the mean energy of the giant resonances (GR) but also the width of the resonance. Bottom of figure (1), represents the evolution of the GDR width as a function of the mass number. This figure illustrates a large variation of the width from one nucleus to another.

In dynamics of nuclear vibrations, one usually distinguishes damping due to the coupling to external and internal degrees of freedom. As giant resonances are excited at energies above particle emission threshold, we expect a direct damping due to coupling of the particle states with the continuum, which is usually called the escape width $\Gamma^\uparrow$. In addition, the collective response of nuclear systems may be highly fragmented due to spreading of collective modes on non-collective particle-hole (p-h) excitations. This is known as the Landau damping and it is mainly apparent in light nuclei. Also, we do expect a dissolution of the ordered motion due to coupling of collective vibrations to more complicated internal degrees of freedom, leading to the so-called spreading width $\Gamma^\downarrow$. While both evaporation of particles and Landau damping can be properly accounted for by the RPA approach, the spreading width requires a description of correlations beyond RPA, i.e. two-body and more generally many-body correlations.

Recent developments of high resolution experiments offer the opportunity to have a better understanding of collective vibrations in quantum fermionic system like nuclei. In particular, one is actually able to reach a good energy resolution (up to few keV) not only to have the energy and width of the collective vibrations but also to determine the fine structure of response, generally discussed in terms of the strength distributions. Besides study of fragmentation of the response, different experiments have revealed existence of small fluctuations on top of the global shape of the response [5, 6, 7, 8]. This situation is illustrated

Figure 1: Properties of the GDR as a function of the atomic mass. Top: Systematic dependence of the collective energy E_{GDR} , and (bottom) width Γ_{GDR} of the GDR as a function of the nuclear mass number A . (adapted from ref. [4]).

in figure (2) for the giant quadrupole response in ^{208}Pb . Structure of the response functions provide a unique tool to get information about the properties of complex internal degrees of freedom through their coupling to collective vibrations. Understanding of the fine structure of response functions is a major challenge [9, 10, 11] and it requires development of quantum transport theories [12, 13].

Figure 2: Giant quadrupole response deduced from inelastic proton scattering [14]. In this experiment the resolution is of the order of 50 keV. A focus on the collective energy region reveals fine structure at very small scales, which have been also observed by means of (e, e') experiments [15, 6].

Besides study of giant resonances in cold nuclei, there are extensive investigations to study giant resonances at finite temperatures. In 1955, D. Brink proposed, that giant resonances can be built on all nuclear states and that their properties should not depend strongly on details of the nuclear state. It is suggested that, these giant resonances have the same characteristics as the giant resonances built on the ground state but their energies are shifted according to the energies of the states on which they are built. This statement is known as the Brink-Axel hypothesis [16]. Since this pioneering work, many experiments have shown that the GDR persists as a collective motion under extreme conditions of excitation energies and angular momentum. We refer the reader to recent reviews on the subject (see for example [4, 17, 18, 19]). Damping properties of the giant resonances depend on temperature, and in general, width of the resonance, as seen for example for GDR, increases with temperature. However, a good understanding of the temperature dependence of GDR and its decay

properties is far from being complete [20]. Figure (3) illustrates the observed GDR width in ^{120}Sn as a function of the temperature. Comparison with models shows that while the global increase of the width between $T = 0$ to 2.5 MeV is reproduced, the exact temperature dependence is overestimated at low temperature.

Figure 3: GDR width in ^{120}Sn as a function of the temperature (taken from ref. [20]). Different points correspond to different experimental results. The shaded and solid lines are guide to eyes, the dashed line corresponds to recent calculations in which shape fluctuations are accounted for (ref. [21]).

According to the discussion above, descriptions of nuclear collective vibrations and their decay properties requires development of microscopic transport models, which account for quantum, dynamical and dissipative effects. Since a long time, giant resonances have been described using the small amplitude limit of the Time-Dependent Hartree-Fock (TDHF), which is equivalent to the RPA [22, 23]. Indeed, GR's correspond to response of the system to an external (collective) one-body field and mean-field approach are tailored to take care of such excitations. However, mean-field theories only account for the Landau damping and the escape width, which are not sufficient to describe experimental observations. In particular, it has been demonstrated for a long time [9] that two-body correlations, like coupling GR to low-lying vibrations, are one of the major effects and account for the main features of the fragmentation in cold nuclei. In this article, we present an extended mean-field theory, in which the evolution of single-particle density matrix is determined by a quantum transport equation that includes both the coherent (coupling to $p-h$ plus low-lying phonons) and the incoherent dissipation mechanisms (coupling to $2p-2h$). We review the linear response description that is obtained in the small amplitude limit of this extended mean-field theory. This theory provides a suitable framework to describe properties of giant resonances not only in cold but also in hot nuclei. We finally present a systematic comparison with experimental data for which a correct description is only achieved when both the coherent and the incoherent mechanisms are included. We also discuss specific aspects related to the response of nuclei, including the effect of correlation on the determination of incompressibility

modulus, fragmentation of the giant quadrupole resonances, appearance of fine structure and modification of the width due to the increase of temperature.

2 Extended Mean-Field Theory

2.1 General Many-Body dynamics

It is possible to derive an effective transport equation following the Martin-Schwinger hierarchy of Green functions [26, 27, 28], but we employ here density matrix formalism [24, 25, 13]. We consider a system of A interacting fermions described by an antisymmetrized many-body wave-function $|\Psi\rangle$. Its evolution is given by the time-dependent Schrödinger equation $i\hbar\partial_t|\Psi\rangle = H|\Psi\rangle$, where H is the many-body Hamiltonian. If we restrict to systems with two-body interaction only, the Hamiltonian reads,

$$H = \sum_{ij} t_{ij} a_i^\dagger a_j + \frac{1}{4} \sum_{ij,kl} \langle ij | v_{12} | kl \rangle_A a_i^\dagger a_j^\dagger a_l a_k \quad (1)$$

where $a_i^\dagger$ (a_i) are the creation (annihilation) operators of single-particle states $|i\rangle$, t_{ij} are the matrix elements of the kinetic operator $t = \frac{p^2}{2m}$, v_{12} is the two body interaction and $|kl\rangle_A = |kl\rangle - |lk\rangle$ represents the antisymmetrized two-particle states.

The Schrödinger equation is equivalent to the Liouville-von Neumann equation for the density matrix $D = |\Psi\rangle\langle\Psi|$,

$$i\hbar \frac{dD}{dt} = [H, D] \quad (2)$$

which contains all the information on the system. A natural way, to access one, two... k -body effects consists in focusing to the k -body density matrix $\rho_{1,\dots,k}$, for which, matrix elements in a given basis are defined as

$$\langle 1, \dots, k | \rho_{1,\dots,k} | 1', \dots, k' \rangle = \langle \Psi | a_1^\dagger \dots a_k^\dagger a_{k'} \dots a_{1'} | \Psi \rangle.$$

With this definition, the different density matrices are connected through the relations,

$$\rho_{1,\dots,k} = \frac{1}{A-k} \text{Tr}_{k+1} \rho_{1,\dots,k+1} = \frac{A!}{(A-k)!} \text{Tr}_{k+1,\dots,A} D$$

where Tr_k denote the partial trace on the k^{th} particles while $\text{Tr}_{k,\dots,A}$ is the partial trace on the $k^{th}, \dots, A^{th}$ particles. By applying the successive trace to the Liouville von-Neumann

eq. (2), we obtain a hierarchy of equations of motion for the density matrices,

$$\left\{ \begin{array}{l} i\hbar\partial\rho_1/\partial t = [t_1, \rho_1] + \text{Tr}_2([v_{12}, \rho_{12}]) \\ i\hbar\partial\rho_{12}/\partial t = [t_1 + t_2, \rho_{12}] \\ \quad \quad \quad + [V_{12}, \rho_{12}] + \text{Tr}_3([v_{13} + v_{23}, \rho_{123}]) \\ \quad \quad \quad \vdots \\ i\hbar\partial\rho_{1\dots k}/\partial t = [\sum_{i=1}^k (t_i + \sum_{j<i}^k v_{ij}), \rho_{1\dots k}] \\ \quad \quad \quad + \sum_{i=1}^k \text{Tr}_{k+1}([v_{ik+1}, \rho_{1\dots k+1}]) \end{array} \right. \quad (3)$$

which is strictly equivalent to the Liouville-von Neumann equation. This hierarchy is known as the Bogolyubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy[29, 30, 31]. Note that in this expression, we use the convention of ref. [13] where the indices denote the particles to which the operators acts, for instance, $\langle ij | t_1 + t_2 | kl \rangle = \langle i | t | k \rangle + \langle j | t | l \rangle$.

In nuclear systems, we are interested in problems where one-body effects plays a predominant role and where correlations act as a perturbation. In general, a measure of k-body correlation is given by the cumulant expansion of the density matrix. This leads for instance the correlated part of the two-body density matrix ρ_{12} , noted C_{12} , is expressed as,

$$C_{12} = \rho_{12} - \rho_1\rho_2(1 - P_{12}) \quad (4)$$

and for the three-body correlations C_{123} to

$$\begin{aligned} C_{123} = & \rho_{123} - \rho_1 C_{23} (1 - P_{12} - P_{13}) - \rho_2 C_{13} (1 - P_{21} - P_{23}) \\ & - \rho_3 C_{12} (1 - P_{31} - P_{32}) - \rho_1 \rho_2 \rho_3 (1 - P_{13}) (1 - P_{12} - P_{23}). \end{aligned} \quad (5)$$

In the following, we use the notation $\widetilde{\rho_1\rho_2} = \rho_1\rho_2(1 - P_{12})$ for the antisymmetrized product of two single particle density matrices.

Eq. (3) can then be replaced by a set of equations coupling the single-particle density to two-body correlations, two-body correlations to three-body correlations, etc... The one-body density matrix evolves according to the first equation of this hierarchy,

$$i\hbar\frac{\partial}{\partial t}\rho_1 - [h_1, \rho_1] = \text{Tr}_2[v_{12}, C_{12}] \quad (6)$$

where $h_1 = p_1^2/2m + U(\rho_1)$ is the mean-field Hamiltonian with an effective mean-field potential $U(\rho_1) = \text{Tr}(v_{12}\widetilde{\rho_1\rho_2})$.

2.2 First Order Truncation: Mean-Field Approximation

At low energies, the Pauli blocking is very effective, and as a result the effect of short-range correlations on scattering of nucleon is weak [32]. Therefore, as a first approximation, we can neglect two and higher order correlations and assume that nucleons move under an effective

mean-field potential without seen each other [12, 33], i.e. $C_{12}(t) = 0$. In this mean-field approximation, the one-body density matrix is thus determined by,

$$i\hbar \frac{\partial}{\partial t} \rho_1 - [h_1, \rho_1] = 0, \quad (7)$$

which is equivalent to Time-Dependent Hartree-Fock (TDHF) theory in which the time-dependent many-body wave-function is assumed to be a Slater determinant at all times [23]. Mean-field approximation and more generally single-particle picture is widely used in nuclear physics. It has been applied with great success for description of nuclear structure as well as nuclear dynamics [23]. In particular, the linearized version of eq. (7) leads to the standard RPA. The latter theory is widely used for the study of collective vibrations in nuclei. However, the mean-field approximation breaks down when dissipation and correlations plays an important role in nuclear dynamics, such as damping of collective vibrations and heavy-ion collisions at intermediate energies. In order to provide a realistic description of such dynamics, one-body transport description must be improved beyond mean-field approximation.

2.3 Second Order Truncation: Beyond Mean-Field

A systematic extension beyond the mean-field approximation can be achieved by incorporating two-body correlations into the description. This extension has been carried out in the Time Dependent Density Matrix theory (TDDM). The two-body correlation C_{12} is evolving according to the second equation of the BBGKY hierarchy, which is coupled to the three-body correlation. In order to have a closed set of equations for ρ_1 and C_{12} in the TDDM theory, the BBGKY hierarchy is truncated at the second order by neglecting coupling to three-body correlation C_{123} . As a result, in the TDDM approximation, the one-body density matrix evolves according to eq. (6) while the two-body correlation is determined by [34],

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} C_{12} - [h_1 + h_2, C_{12}] = & (1 - \rho_1 - \rho_2) v_{12} \widetilde{\rho_1 \rho_2} - \widetilde{\rho_1 \rho_2} v_{12} (1 - \rho_1 - \rho_2) + \\ & (1 - \rho_1 - \rho_2) v_{12} C_{12} - C_{12} v_{12} (1 - \rho_1 - \rho_2) + \\ & \text{Tr}_3 [v_{13}, (1 - P_{13}) \rho_1 C_{23} (1 - P_{12})] + \\ & \text{Tr}_3 [v_{23}, (1 - P_{23}) \rho_1 C_{23} (1 - P_{12})]. \end{aligned} \quad (8)$$

However, applications of this theory require a large numerical effort. Therefore, only a few applications have been carried out so far for collective vibrations [35, 36], and very recently for nuclear collisions [37]. We note also that the small amplitude limit of the TDDM gives rise to the second Random-Phase Approximation (Second RPA)[10].

2.4 Extended Time-Dependent Hartree-Fock

In order to have a relatively manageable extended mean-field theory, further approximations should be incorporated into the description. At intermediate heavy-ion collisions energy,

dissipation by incoherent nucleon-nucleon collisions play an important role. For these situations, it is possible to simplify the TDDM approach further by retaining the lowest order terms in the residual interactions, i.e. the Born terms in eq. (8) [26, 27, 28, 38, 39, 40]. Then, the equation for the two-body correlation takes a relatively simple form,

$$i\hbar \frac{\partial}{\partial t} C_{12} - [h_1 + h_2, C_{12}] = F_{12} \quad (9)$$

where F_{12} denotes the first two terms in the right side of eq. (8),

$$F_{12} = (1 - \rho_1)(1 - \rho_2) v_{12} \widetilde{\rho_1 \rho_2} - \widetilde{\rho_1 \rho_2} v_{12} (1 - \rho_1)(1 - \rho_2) \quad (10)$$

which acts as a source for generating correlations. Solving this equation formally, we can express development of correlations over a time interval from an initial time t_0 to time t as,

$$C_{12}(t) = -\frac{i}{\hbar} \int_{t_0}^t U_{12}(t, s) F_{12}(s) U_{12}^\dagger(t, s) ds + \delta C_{12}(t) \quad (11)$$

where $U_{12}(t, s)$ represents the independent particle propagation of two particles, $U_{12} = U_1 \otimes U_2$ with $U(t, s) = \exp\left(-\frac{i}{\hbar} \int_s^t h(\rho(t')) dt'\right)$. In expression (11), the first term represents correlations developed by the residual interactions during the time interval. The second term describes propagation of the initial correlations $C_{12}(t_0)$ from t_0 to t ,

$$\delta C_{12}(t) = U_{12}(t, t_0) C_{12}(t_0) U_{12}^\dagger(t, t_0). \quad (12)$$

The time interval cannot be taken arbitrary large, but should be taken sufficiently small to justify neglect of explicit coupling to the three-body correlations in eq. (8) during the time interval. However, the dominant effect of the correlations is still accounted for by the initial correlation term $C_{12}(t_0)$, which, in principle, contains all order correlations that are accumulated up to time t_0 . In the extended TDHF theory, one considers an ensemble of identical systems that are prepared with slightly different initial conditions at the remote past, and a statistical averaging is carried out over such an ensemble. It is assumed that the exact two-body correlations accumulated until t_0 exhibits random fluctuations. As a result, the average value of the initial correlations vanishes. This assumption is known as the "molecular chaos assumption" in classical transport theory and it corresponds to factorization of two-particle phase-space density before each binary collision [41, 42]. As a result, in the extended TDHF theory, the evolution of the average single-particle density matrix is determined by a transport equation,

$$i\hbar \frac{\partial}{\partial t} \rho_1(t) - [h(\rho_1), \rho_1(t)] = K_I(\rho_1) \quad (13)$$

where the term on the right hand side represents a binary collision term given by,

$$K_I(\rho_1) = -\frac{i}{\hbar} \int_{t_0}^t ds \text{Tr}_2 [v_{12}, U_{12}(t, s) F_{12}(s) U_{12}^\dagger(t, s)]. \quad (14)$$

Eq. (13), is known as the extended TDHF with a non-markovian collision term. $K_I(\rho_1)$ corresponds to the mixing of single particle configurations induced by incoherent nucleon-nucleon collisions and will be called incoherent collision term in the following. Note that, expression (13) is sometimes obtained in different way by assuming an initial uncorrelated state at time t_0 . In this case $\delta C_{12}(t) = 0$ while the correlation entering in (6) is reduced to the correlation built with F_{12} during the time $(t - t_0)$. Although both derivation gives the same evolution for the first instant of the dynamics, the extended mean-field dynamics described by eq. (13) for long time evolution is justified by invoking ensemble averages and "molecular chaos assumption".

2.4.1 Time-scales and approximate expression for the collision term

Expression (13) as often served as a starting point for extension of quantum mean-field dynamics. It has however been rarely directly applied because of the numerical effort required. In order to illustrate the difficulties, let us introduce the single-particle basis $|\alpha(t)\rangle$ that diagonalizes the one body density $\rho_1(t)$ at a given time:

$$\rho(t) = \sum |\alpha(t)\rangle n_\alpha(t) \langle \alpha(t)|. \quad (15)$$

this basis which depends on time will be called "natural" basis (this basis is sometimes called "canonical" basis). As we do expect from nucleon-nucleon collisions, the incoherent collision term induces a mixing of single-particle degrees of freedom during time evolution [43, 44]. Indeed, if we consider a time $t' \geq t$, we can define the propagated basis $|\bar{\alpha}(t')\rangle = U(t', t) |\alpha(t)\rangle$. It should be noticed that $|\bar{\alpha}(t')\rangle \neq |\alpha(t')\rangle$ since the mean-field propagation of $|\alpha\rangle$ is not the extended TDHF propagation of the natural basis. Using these notations, matrix elements of the incoherent collision terms expresses as

$$\langle \alpha | K_I | \alpha' \rangle (t) = -\frac{1}{2\hbar^2} \left\{ \int_{t_0}^t ds \sum \left\langle \bar{\alpha}\delta | v_{12} | \bar{\lambda}\beta \right\rangle_A \Big|_t \left\langle \bar{\lambda}\beta | v_{12} | \bar{\alpha}'\delta \right\rangle_A \Big|_s \right. \\ \left. (n_\alpha n_\delta (1 - n_\lambda) (1 - n_\beta) - n_\lambda n_\beta (1 - n_\alpha) (1 - n_\delta)) \Big|_s + h.c. \right\} \quad (16)$$

where the sum runs over all indices but α and α' and where we have introduced the notation $\langle . \rangle_{t'}$ to express the fact that the matrix elements are taken at time t' . The two major difficulties in the application of ETDHF are the integral in time and the summation over many indices in eq. (16).

The collision term essentially involves two different characteristic times. The first one is the correlation time τ_{cor} , If we note $V_{12}(t) = \langle \bar{\alpha}\beta | v_{12} | \bar{\lambda}\delta \rangle_A \Big|_t$, the correlation time is defined as

$$\langle V_{12}(t) V_{12}(s) \rangle \propto e^{-|t-s|/\tau_{cor}} \quad (17)$$

where we use the notation $\langle . \rangle$ for the average over all single-particle states. This time, characteristic of the residual interaction, is directly related to the mean energy Δ exchanged during nucleon-nucleon collisions through the relation $\tau_{cor} = \hbar/\Delta$ [45, 46]. The second characteristic time, called relaxation time τ_{rel} , corresponds to the time-scale associated to

the reorganization of the occupation numbers. This time is discussed more extensively in appendix A.

These two time-scales correspond in the semi-classical limit respectively to the average duration time of binary collisions and to the mean-free-time between collisions. Here, we consider the weak coupling regime specified by $\tau_{cor} \ll \tau_{rel}$, which is valid for a sufficiently dilute system when the binary collisions are well separated in time [45, 46]. In this case, the decay time of the collision kernel is determined by the correlation time, and the memory effects associated with the variation of the occupation numbers over this time may be neglected. Using $U(t, s)\rho_1(s)U^\dagger(t, s) \approx \rho_1(t)$, we can make the following substitution in the collision term,

$$U_{12}(t, s)F_{12}(s)U_{12}^\dagger(t, s) = (1 - \rho_1)(1 - \rho_2)v_{12}(t, s)\widetilde{\rho_1\rho_2} - \widetilde{\rho_1\rho_2}v_{12}(t, s)(1 - \rho_1)(1 - \rho_2) \quad (18)$$

where $v_{12}(t, s) = U_{12}(t, s)v_{12}U_{12}^\dagger(t, s)$, and all density matrices are evaluated at time t . In the following, we assume that we are in the weak coupling regime.

2.5 Stochastic Transport Theory

The incoherent damping mechanism in the extended TDHF approach is very important at relatively high-energy nuclear collisions to convert the collective energy of relative motion into incoherent excitations and thermalize the system. However, at low energies including giant resonance excitations, the incoherent mechanism is not effective due to long nucleon mean-free-path. Therefore, for a proper description of damping mechanism at low energies, the coherence between the particle-hole pairs should be taken into account [9, 47]. For this reason, it is highly desirable to improve the TDHF theory by incorporating a coherent collision term into the equation of motion. One possibility is provided by the TDDM approach: the resultant coupled equations for the one-body density matrix and the two-body correlations take into account for the coherence effects in particle-particle, hole-hole and particle-hole channels. Another possibility is provided by the stochastic transport approach in which the effects of correlations are incorporated into the equation of motion by a stochastic mechanism according to the generalized Langevin description of Mori [48, 49, 50]. In the stochastic transport description, the BBGKY hierarchy is truncated at the lowest order as in the extended TDHF, but the initial correlation term δC_{12} given by eq. (12) is retained, and it is treated as a random quantity specified by a Gaussian distribution [13, 51, 52, 53] determined by its first and second moments. If the indice $\delta C_{12}^{(n)}(t_0)$ denotes a specific value of the random operator δC_{12} , the average value of the initial correlations is assumed to vanish

$$\overline{\delta C_{12}^{(n)}(t_0)} = \frac{1}{N} \sum_n \delta C_{12}^{(n)}(t_0) = 0 \quad (19)$$

where N denotes the number of stochastic values, while $\overline{\delta C_{12}^{(n)}(t_0)\delta C_{12}^{(n')}(t_0)}$ is given in ref. [54]. As a result, similar to the Langevin description of a Brownian motion, the ensemble of one-body density matrices $\rho^{(n)}$ evolves according to a stochastic transport equation,

$$i\hbar \frac{\partial}{\partial t} \rho^{(n)} - [h(\rho^{(n)}), \rho^{(n)}] = K_I(\rho^{(n)}) + \delta K^{(n)}(t) \quad (20)$$

Here, the first term on the left-hand-side is the incoherent binary collision term, which has the same form as in eq. (14) but express in terms of fluctuating quantities. The second term arises from the initial correlations,

$$\delta K^{(n)}(t) = \text{Tr}_2[v_{12}, \delta C_{12}^{(n)}(t)] \quad (21)$$

and it describes the stochastic part of the collisions. According to the stochastic properties of the initial correlations, $\delta K^{(n)}$ also has a Gaussian distribution with zero mean and a second moment determined by its auto-correlation function [54].

In this review article, we concentrate on the ensemble averaged evolution associated to eq. (20). We consider the transport equation for evolution of the ensemble averaged density matrix $\rho(t) = \overline{\rho^{(n)}(t)}$. The average of eq. (20) can be calculated by expressing the mean-field and the density matrix as

$$\begin{cases} h(\rho^{(n)}) = h(\rho) + \delta h^{(n)}(t) \\ \rho^{(n)}(t) = \rho(t) + \delta \rho^{(n)}(t) \end{cases} \quad (22)$$

where $\delta h^{(n)}(t) = (\partial h / \partial \rho) \cdot \delta \rho^{(n)}(t)$ and $\delta \rho^{(n)}(t)$ represent the fluctuating parts of the mean-field and the density matrix, respectively. Noting that, the ensemble average of the noise $\overline{\delta K^{(n)}(t)}$ vanishes, the evolution of the average density matrix is governed by the transport equation [54, 55, 56, 57],

$$i\hbar \frac{\partial}{\partial t} \rho(t) - [h(\rho), \rho(t)] = K_I(\rho) + K_C(\rho). \quad (23)$$

The first term $K_I(\rho)$ on the right hand side is the usual binary collision term of the extended TDHF given by eq. (14). The second term $K_C(\rho)$ on the right hand side represents a coherent dissipation mechanism due to coupling of single-particle excitations with collective density fluctuations,

$$K_C(\rho) = \overline{[\delta h^{(n)}(t), \delta \rho^{(n)}(t)]} \quad (24)$$

and it is referred to as the coherent collision term. For small fluctuations around the average evolution, it is possible to derive an explicit expression of the collision term by analyzing the small density fluctuations in terms of time dependent RPA approach. Then, the coherent term takes the form known as a particle-phonon collision term [54, 57],

$$K_C(\rho) = -\frac{i}{\hbar} \int_{t_0}^t ds \sum_{\lambda} [h_{\lambda}^{\dagger}(t), U(t, s) F_{\lambda}(s) U^{\dagger}(t, s)] - h.c. \quad (25)$$

where the sum runs over the time-dependent collective RPA modes labelled by λ and

$$F_{\lambda}(s) = (N_{\lambda} + 1)(1 - \rho(s))h_{\lambda}(s)\rho(s) - N_{\lambda}\rho(s)h_{\lambda}(s)(1 - \rho(s)). \quad (26)$$

In this expressions, $h_{\lambda}^{\dagger}(t) = (\partial h / \partial \rho) \cdot \rho_{\lambda}^{\dagger}(t)$ represents the fluctuating part of the mean-field in the corresponding RPA mode and N_{λ} indicates the occupation number of RPA phonons. The collision term $K_C(\rho)$ provides an approximate treatment of the coherence between particle-hole pairs in terms of time dependent RPA phonons, and requires much less numerical effort than the TDDM approach.

It is seen that the first and second terms in the coherent collision term correspond to absorption and excitation of RPA phonons with rates, which are proportional to $N_\lambda + 1$ and N_λ , respectively. The amplitudes of the mean-field fluctuations in the coherent collision term are self-consistently determined by the time-dependent RPA equations,

$$i\hbar \frac{\partial}{\partial t} \rho_\lambda^\dagger(t) - [h(\rho), \rho_\lambda^\dagger(t)] - [h_\lambda^\dagger(t), \rho(t)] = 0. \quad (27)$$

In general, it is difficult to determine the time evolution of the occupation factors $N_\lambda(t)$ of the RPA functions, since they depend on rather complex damping mechanism of the mean-field fluctuations (see discussion of appendix B). In this report, since we are concern with small fluctuations around thermal equilibrium, we assume that the phonon occupation numbers are determined by their thermal equilibrium values, $N_\lambda = 1/[\exp(\omega_\lambda/T) + 1]$. Both the incoherent and the coherent collision terms in eq. (23) are valid in the weak-coupling limit. The collision terms involve memory effects due to the time integration over the past history from an initial time t_0 to the present time t . The memory effects in the collision terms are essential to give rise the correct energy conservation and for correct counting of the phase space of mixing between the collective state and the doorway states. Note also that, as for the incoherent collision term, we can make the following substitution in the coherent collision term,

$$U(t, s) F_\lambda(s) U^\dagger(t, s) = (N_\lambda + 1)(1 - \rho(t)) h_\lambda(t, s) \rho(t) - N_\lambda \rho(t) h_\lambda(t, s) (1 - \rho(t)) \quad (28)$$

where $h_\lambda(t, s) = U(t, s) h_\lambda(s) U^\dagger(t, s)$.

3 Linear Response Based on Extended Mean-Field Theory

In this section, we consider the small amplitude limit the transport eq. (23) and give a brief description of the linear response formalism including both the incoherent and the coherent damping mechanisms. A details description of the formalism can be found in recent publications [57, 54, 58, 59] (see also [60, 61]).

3.1 Linearization of mean-field equations

The linearization of mean-field equation is found in many textbook [22, 23]. In this section, we recall briefly equations and notations used. The linear response of the system to an external perturbation can be described by the small amplitude limit of the transport eq. (7). The small deviations of the density matrix $\delta\rho(t) = \rho(t) - \rho_0$ around a finite temperature equilibrium state ρ_0 defined by $[h_0, \rho_0] = 0$ are determined by the linearized form of the TDHF equation,

$$i\hbar \frac{\partial}{\partial t} \delta\rho - [\delta h, \rho_0] - [h_0, \delta\rho] = 0. \quad (29)$$

We analyze the linear response of the system to the external perturbation by expanding the small deviation $\delta\rho(t)$ in terms of finite temperature RPA operators $O_\lambda^\dagger$ and O_λ associated with creation and annihilation of RPA modes,

$$\delta\rho(t) = \sum_{\lambda>0} z_\lambda(t)\rho_\lambda^\dagger + z_\lambda^*(t)\rho_\lambda, \quad (30)$$

where $z_\lambda(t)$ and $z_\lambda^*(t)$ denote the amplitudes associated with the RPA modes (noted by λ), $\rho_\lambda^\dagger = [O_\lambda^\dagger, \rho_0]$. If we introduce the notation $\delta Q_0(t) = \sum z_\lambda(t)O_\lambda^\dagger - z_\lambda^*(t)O_\lambda$, we obtain the relation

$$\delta\rho(t) = [\delta Q_0(t), \rho_0(t)]. \quad (31)$$

The RPA modes $\rho_\lambda^\dagger$ are determined by the finite temperature RPA equations,

$$\hbar\omega_\lambda\rho_\lambda^\dagger - [h_0, \rho_\lambda^\dagger] - [h_\lambda^\dagger, \rho_0] = 0 \quad (32)$$

where $h_\lambda^\dagger(t) = (\partial h / \partial \rho) \cdot \rho_\lambda^\dagger(t)$ represents the fluctuating part of the mean-field in the corresponding RPA mode. In order to study collective motions, we consider an external harmonic perturbation $A(t) = A[\exp(-i\omega t) + \exp(+i\omega t)]$ applied to the system. The linear transport equation (29) is modified by the replacement $\delta h \rightarrow \delta h + A$. The amplitudes $z_\lambda(t)$ evolve according to

$$i\hbar \frac{d}{dt} z_\lambda - \hbar\omega_\lambda z_\lambda = A_\lambda \exp(-i\omega t) \quad (33)$$

where the driving term is determined by the external perturbation $A_\lambda = \text{Tr}[O_\lambda, A]\rho_0$. We thus see that for vibrations around equilibrium, RPA functions becomes harmonic $\rho_\lambda^\dagger(t) = \rho_\lambda^\dagger \exp(-i\omega_\lambda t)$. The positive frequency part of the induced density vibrations is given in terms of solution of eq. (32) by

$$\delta\rho(\omega) = \sum_{\lambda>0} z_\lambda^+ \rho_\lambda^\dagger + z_\lambda^- \rho_\lambda \quad (34)$$

where we use the notation $z_\lambda^+ = z_\lambda(\omega)$ and $z_\lambda^- = z_\lambda^*(-\omega)$. Then, the response of the system to the external perturbation A is determined according to, $\delta\rho(\omega) = R(\omega, T) \cdot A$, where $R(\omega, T)$ denotes the RPA response function at finite temperature.

3.2 Linearized Transport Equation with collision terms

In this section, we consider the complete transport eq. (23) including the coherent and the incoherent collision terms. The linearization around a finite temperature equilibrium state gives rise to,

$$i\hbar \frac{\partial}{\partial t} \delta\rho - [\delta h, \rho_0] - [h_0, \delta\rho] = \delta K_I(\rho) + \delta K_C(\rho) - h.c. \quad (35)$$

where $\delta K_I(\rho)$ and $\delta K_C(\rho)$ denote the linearized incoherent and coherent collision terms respectively. In the following sections, we present explicit expressions for the collision terms.

3.2.1 Incoherent contribution

According to eq. (14), small deviations of the incoherent term are given by,

$$\delta K_I(\rho) = -\frac{i}{\hbar} \int_{t_0}^t ds \text{Tr}_2[v_{12}, \delta \mathcal{C}_{12}(\rho)] \quad (36)$$

where $\delta \mathcal{C}_{12}(\rho)$ corresponds to the linearization of the correlation with respect to a small density fluctuation $\delta \mathcal{C}_{12} = \delta \{U_{12} F_{12} U_{12}^\dagger\}$. The quantity $\delta \mathcal{C}_{12}(\rho)$ involves two different contributions: one part is arising from small deviations of the mean-field propagator $U_{12}(t, s)$ around its equilibrium value $U_{12}^0 = U_1^0 \otimes U_2^0$ with $U^0 = \exp(-\frac{i}{\hbar}(t-s)h_0)$, and another part coming from the small amplitude density vibrations in $F_{12}(s)$. Treating the vibrating field $\delta h(t)$ as a perturbation, we obtain

$$\begin{aligned} \delta \mathcal{C}_{12}(t) = & U_{12}^0(t, s) \delta F_{12}(s) U_{12}^{0\dagger}(t, s) \\ & - \frac{i}{\hbar} \int_s^t dt' [U_{12}^0(t, t') (\delta h_1(t') + \delta h_2(t')) U_{12}^{0\dagger}(t, t'), U_{12}^0(t, s) F_{12}^0(s) U_{12}^{0\dagger}(t, s)] \end{aligned} \quad (37)$$

where $\delta F_{12} = F_{12}(\rho) - F_{12}(\rho_0)$

The evolution of the density vibrations over short time intervals, by neglecting the right hand side of eq. (35), can be expressed as

$$\delta \rho(t) = U_0(t, s) \delta \rho(s) U_0^\dagger(t, s) - \frac{i}{\hbar} \int_s^t dt' [U_0(t, t') \delta h(t') U_0^\dagger(t, t'), \rho_0] \quad (38)$$

This results can be expressed in the form of $\delta \rho(t) = [\delta Q_0(t), \rho_0]$, where

$$\delta Q_0(t) = U_0(t, s) \delta Q_0(s) U_0^\dagger(t, s) - \frac{i}{\hbar} \int_s^t dt' U_0(t, t') \delta h(t') U_0^\dagger(t, t') \quad (39)$$

combining this expression with expression (37) leads to

$$\delta \mathcal{C}_{12}(t) = U_{12}^0 \delta F_{12} U_{12}^{0\dagger} + [\delta Q_0(t), U_{12}^0 F_{12} U_{12}^{0\dagger}] - U_{12}^0 [\delta Q_0(s), F_{12}] U_{12}^{0\dagger} \quad (40)$$

where δQ_0 is related to the density vibrations without the damping in eq. (35) according to $\delta \rho(t) = [\delta Q_0(t), \rho_0]$. We can approximate the collision term further by assuming that the collective density vibrations can be expressed in the same form even if the damping is included $\delta \rho(t) = [\delta Q(t), \rho_0(t)]$. In the collision term, these two expressions are essentially equivalent in the weak coupling limit, $Q(t) \approx Q_0(t)$. Then, the contribution arising from δF_{12} in the first term in eq. (40) can be combined with the contribution coming from the mean-field propagator in the third term to give

$$\delta \mathcal{C}_{12}(t) = (1 - \rho_1^0)(1 - \rho_2^0) U_{12}^0(t, s) [v_{12}, \delta Q(s)] U_{12}^{0\dagger}(t, s) \rho_1^0 \rho_2^0. \quad (41)$$

This quantity also has a hermitian conjugate part, which is already incorporated in eq. (35). There is another contribution arising from the second term in the expression (40). Since, the collective density fluctuations are not correlated in time with the 2p-2h excitations, this contribution is expected to be small and neglected here.

3.2.2 Coherent contribution

Now we turn our attention to the small amplitude limit of the coherent collision term. From eq. (25), we have

$$\delta K_C(\rho) = -\frac{i}{\hbar} \int_{t_0}^t ds \sum_{\lambda} [h_{\lambda}^{\dagger}(t), \delta\{UF_{\lambda}U^{\dagger}\}]. \quad (42)$$

Similar to the treatment of the incoherent collision term, there are two contributions in $\delta\{UF_{\lambda}U^{\dagger}\}$, coming from the mean-field vibrations through $\delta\{U(t, s)h_{\lambda}U^{\dagger}(t, s)\}$ and from the density vibrations $\delta\rho(t)$, which should be combined in a consistent manner. Using again eq. (39), the contributions through the mean-field propagator can be expressed as,

$$\delta\{U(t, s)h_{\lambda}U^{\dagger}(t, s)\} = [\delta Q_0(t), U_0 h_{\lambda} U_0^{\dagger}] - U_0 [\delta Q_0(s), h_{\lambda}] U_0^{\dagger}. \quad (43)$$

As before, assuming collective density vibrations can be expressed in the same form when damping is included, the first term in the expression (43) cancels out with the contributions coming from the density vibrations. As a result, the quantity $\delta\{UF_{\lambda}U^{\dagger}\}$ in the coherent collision term becomes,

$$\begin{aligned} \delta\{UF_{\lambda}U^{\dagger}\} = & (N_{\lambda} + 1)(1 - \rho_0)U_0(t - s)[h_{\lambda}(s), \delta Q(s)]U_0^{\dagger}(t - s)\rho_0 - \\ & N_{\lambda}\rho_0 U_0(t - s)[h_{\lambda}(s), \delta Q(s)]U_0^{\dagger}(t - s)(1 - \rho_0). \end{aligned} \quad (44)$$

Substituting this result into eq. (42) we obtain an expression for $\delta K_C(\rho)$ in terms of $\delta Q(s)$.

3.2.3 Response to an external field

For vibrations around equilibrium induced by an external field $A(t)$, we can express $\delta Q(t)$ in terms of the RPA states, $\delta Q(t) = \sum z_{\lambda}(t)O_{\lambda}^{\dagger} - z_{\lambda}^*(t)O_{\lambda}$, where $z_{\lambda}(t)$ denotes the amplitudes associated with the RPA modes, $z_{\lambda}(t) = \text{Tr}O_{\lambda}\delta\rho(t)$. Using the orthonormality relation $\text{Tr}[O_{\lambda}, O_{\mu}^{\dagger}]\rho_0 = \delta_{\lambda\mu}$, we can project the linearized transport eq. (35). We find that the amplitudes of the RPA modes execute forced harmonic motion, (see discussion in appendix B) [62, 63, 64]

$$i\hbar \frac{d}{dt} z_{\lambda} - \hbar\omega_{\lambda} z_{\lambda} - A_{\lambda} \exp(-i\omega t) = \sum_{\mu} \int_{t_0}^t ds \{K_{\lambda\mu}(t - s)z_{\mu}(s) + \widetilde{K}_{\lambda\mu}(t - s)z_{\mu}^*(s)\} \quad (45)$$

where, as in eq. (33), the driving term is determined by the external perturbation $A_{\lambda} = \text{Tr}[O_{\lambda}, A]\rho_0$. Here, we include only the positive frequency part of the driving force, which is sufficient for calculating response function. The right hand side describes the self-energy of the collective mode due to coherent and incoherent coupling mechanisms. In general, the self-energy is not diagonal but contains contributions due to off diagonal parts arising from coupling through the intermediate doorway states between positive frequency RPA modes, $K_{\lambda\mu}(t) = K_{\lambda\mu}^I(t) + K_{\lambda\mu}^C(t)$, and between positive and negative frequency RPA modes, $\widetilde{K}_{\lambda\mu}(t) = \widetilde{K}_{\lambda\mu}^I(t) + \widetilde{K}_{\lambda\mu}^C(t)$, each having coherent and incoherent contributions. Following sections 3.2.1 and 3.2.2, these self-energies are given by

$$\begin{aligned} K_{\lambda\mu}^I(t - s) &= \text{Tr}\{[v_{12}, O_{\lambda}]\delta\mathcal{C}_{12}(\rho)\} \\ &= \text{Tr}\{[v_{12}, O_{\lambda}](1 - \rho_1^0)(1 - \rho_2^0)U_{12}^0(t, s)[v_{12}, O_{\mu}^{\dagger}(s)]U_{12}^{0\dagger}(t, s)\rho_1^0\rho_2^0\} \end{aligned} \quad (46)$$

and

$$K_{\lambda\mu}^C(t-s) = \sum_{\nu} \text{Tr} O_{\lambda} \left[h_{\nu}^{\dagger}(t), (N_{\nu}+1)(1-\rho_0)U_0(t-s)[h_{\nu}(s), O_{\mu}^{\dagger}]U_0^{\dagger}(t-s)\rho_0 - N_{\nu}\rho_0 U_0(t-s)[h_{\nu}(s), O_{\mu}^{\dagger}]U_0^{\dagger}(t-s)(1-\rho_0) \right]. \quad (47)$$

3.2.4 Linear Response to Harmonic Perturbation around thermal equilibrium

In order to study the response of nuclei at finite temperature, we concentrate on harmonic perturbation $A(t)$ acting on a system at temperature T . In that case, the equilibrium density ρ_0 and mean-field Hamiltonian h_0 can be expressed in terms of the Hartree-Fock states at finite temperature as

$$\begin{cases} \rho_0 = \sum |i\rangle n_i \langle i| \\ h_0 = \sum |i\rangle \varepsilon_i \langle i| \end{cases} \quad (48)$$

where occupation numbers are given by Fermi-Dirac distribution $n_i = 1/[\exp(\varepsilon_i - \varepsilon_F)/T + 1]$ with ε_F as the Fermi energy. In addition, occupation probabilities for RPA states are given by the equilibrium Bose-Einstein distribution $N_{\lambda} = 1/[\exp(\hbar\omega_{\lambda}/T) + 1]$.

Using the same notations as in section (3.1), steady state solution of the amplitudes are easily obtained by Fourier transform of eq. (45) and determined by a set of coupled equations,

$$(\hbar\omega - \hbar\omega_{\lambda})z_{\lambda}^{+} - \sum_{\mu} \{K_{\lambda\mu}(\omega)z_{\lambda}^{+} - \widetilde{K}_{\lambda\mu}(\omega)z_{\lambda}^{-}\} = +A_{\lambda} \quad (49)$$

and

$$(\hbar\omega + \hbar\omega_{\lambda})z_{\lambda}^{-} + \sum_{\mu} \{K_{\lambda\mu}^{*}(-\omega)z_{\lambda}^{-} - \widetilde{K}_{\lambda\mu}^{*}(-\omega)z_{\lambda}^{+}\} = -A_{\lambda}^{*} \quad (50)$$

This set of coupled equations for the amplitudes can be recast into a matrix form [59],

$$(\hbar\omega I - \hbar\bar{\omega} - \Sigma(\omega)) \begin{pmatrix} z^{+} \\ z^{-} \end{pmatrix} = \begin{pmatrix} +A \\ -A^{*} \end{pmatrix} \quad (51)$$

where z^{+} and z^{-} are the amplitude vectors with components z_{λ}^{+} and z_{λ}^{-} , and A is the forcing vector with components A_{λ} . The quantities I , $\bar{\omega}$ and $\Sigma(\omega)$ denote, respectively, the unit matrix, the frequency matrix $\hbar\bar{\omega}$

$$\hbar\bar{\omega} = \begin{pmatrix} \hbar\Omega & 0 \\ 0 & -\hbar\Omega \end{pmatrix} \quad (52)$$

with $\hbar\Omega_{\lambda\mu} = \delta_{\lambda\mu}\hbar\omega_{\lambda}$ and the self-energy matrix, which can be expressed as,

$$\Sigma_{\lambda\mu}(\omega) = \begin{pmatrix} K_{\lambda\mu}(\omega) & \widetilde{K}_{\lambda\mu}(\omega) \\ -\widetilde{K}_{\lambda\mu}^{*}(-\omega) & -K_{\lambda\mu}^{*}(-\omega) \end{pmatrix}. \quad (53)$$

In the Hartree-Fock representation, the elements of the incoherent self-energy are given by,

$$K_{\lambda\mu}^I(\omega) = -\frac{1}{4} \sum_{ijkl} \frac{\langle k\ell | [O_{\lambda}, v] | ij \rangle \langle ij | [O_{\mu}^{\dagger}, v] | k\ell \rangle}{\hbar\omega + i\eta - \Delta\varepsilon_{ijkl}} \mathcal{N}_{ijkl} \quad (54)$$

and

$$\widetilde{K}_{\lambda\mu}^I(\omega) = \frac{1}{4} \sum_{ijkl} \frac{\langle k\ell | [O_\lambda, v] | ij \rangle \langle ij | [O_\mu, v] | k\ell \rangle}{\hbar\omega + i\eta - \Delta\varepsilon_{ijkl}} \mathcal{N}_{ijkl} \quad (55)$$

with $\mathcal{N}_{ijkl} = (1 - n_i)(1 - n_j)n_k n_\ell - n_i n_j(1 - n_k)(1 - n_\ell)$ and $\Delta\varepsilon_{ijkl} = \varepsilon_i + \varepsilon_j - \varepsilon_k - \varepsilon_\ell$, and η is a small energy averaging interval. The incoherent self-energy describes the collisional damping of the collective vibrations due to coupling with $2p-2h$ states. At low temperature, in particular for light and medium weight nuclei, the incoherent damping mechanism has a sizeable influence on the strength functions, and it becomes more important at higher temperature [62, 63, 64, 65]. In a similar manner, the elements of the coherent self-energy are given by

$$\begin{aligned} K_{\lambda\mu}^C(\omega) = & - \sum_{vij} \frac{\langle i | [Q_\lambda, h_\nu^\dagger] | j \rangle \langle j | [Q_\mu^\dagger, h_\nu] | i \rangle}{\hbar\omega + i\eta - \hbar\omega_\nu - \varepsilon_j + \varepsilon_i} \mathcal{M}_{\nu,ij} \\ & + \sum_{vij} \frac{\langle i | [Q_\lambda, h_\nu] | j \rangle \langle j | [Q_\mu^\dagger, h_\nu^\dagger] | i \rangle}{\hbar\omega + i\eta + \hbar\omega_\lambda - \varepsilon_j + \varepsilon_i} \mathcal{M}_{\nu,ji} \end{aligned} \quad (56)$$

and

$$\begin{aligned} \widetilde{K}_{\lambda\mu}^C(\omega) = & \sum_{vij} \frac{\langle i | [Q_\lambda, h_\nu^\dagger] | j \rangle \langle j | [Q_\mu, h_\nu] | i \rangle}{\hbar\omega + i\eta - \hbar\omega_\nu - \varepsilon_j + \varepsilon_i} \mathcal{M}_{\nu,ij} \\ & - \sum_{vij} \frac{\langle i | [Q_\lambda, h_\nu] | j \rangle \langle j | [Q_\mu, h_\nu^\dagger] | i \rangle}{\hbar\omega + i\eta + \hbar\omega_\lambda - \varepsilon_j + \varepsilon_i} \mathcal{M}_{\nu,ji} \end{aligned} \quad (57)$$

where $\mathcal{M}_{\nu,ij} = (N_\nu + 1)(1 - n_j)n_i - N_\nu n_j(1 - n_i)$. The coherent mechanism describes damping of collective vibrations due to coupling with low-lying phonons and $p-h$ states (noted $p-h \otimes \text{phonon}$).

Employing the solution of the matrix equation (51), the strength function can be expressed in terms of a compact notations as,

$$S(\omega, T) = -\frac{1}{\pi} \text{Im} \left(\begin{pmatrix} A^* & A \end{pmatrix} (\hbar\omega I - \hbar\bar{\omega} - \Sigma(\omega))^{-1} \begin{pmatrix} A \\ -A^* \end{pmatrix} \right). \quad (58)$$

where the self-energy contains both incoherent and coherent components, $\Sigma(\omega) = \Sigma_I(\omega) + \Sigma_C(\omega)$.

In general, the self-energy of a collective mode is non-diagonal and it couples different RPA modes. This coupling and its possible influence on the response are discussed in appendix D. In this appendix, it is shown that this coupling does not play an important role in nuclear collective response. For this reason, in most of the examples we present in this report we neglect this coupling and retain only the diagonal elements $\Sigma_{\lambda\lambda}(\omega)$.

The extended mean-field theory, which includes two-body effects, offers the possibility to understand different effects appearing in the damping of collective vibrations. In the following sections, we first discuss applications of usual mean-field and focus on its limitations. The modification of the response induced by the introduction of nucleon-nucleon collisions and by phonon coupling are then discussed step by step. Finally, some applications from medium to heavy nuclei are presented.

4 Applications of RPA : success and limitations

4.1 Numerical details

In this chapter, we illustrate applications of the zero and finite temperature RPA. Starting from mean-field, the linearized equations have been derived in the previous chapter (eq. (32)). In the Hartree-Fock basis, the finite temperature RPA states are obtained by solving the equation

$$(\hbar\omega_\lambda - \epsilon_i + \epsilon_j) \langle i|O_\lambda^\dagger|j \rangle + \sum_{l \neq k} \langle ik|v_{12}|jl \rangle_A (n_l - n_k) \langle l|O_\lambda^\dagger|k \rangle = 0. \quad (59)$$

In this expression, the indices $i, j, \dots$ represent all single particle quantum numbers including spin-isospin, and n_k denotes finite temperature Fermi-Dirac occupation numbers of the Hartree-Fock states. At zero temperature these occupation numbers are zero or one, so that the RPA operators $O_\lambda^\dagger$ have only particle-hole ($p-h$) and hole-particle ($h-p$) matrix elements. At finite temperatures the RPA functions involve more configurations including particle-particle ($p-p$) and hole-hole ($h-h$) states.

Eq. (59) is solved according to the following procedure: the Hartree-Fock states are first obtained by diagonalizing the static mean-field in r -space. In order to reduce the numerical effort, only the components of the states in a large but finite harmonic oscillator representation are retained [69]. The basis includes respectively 12 major shells for ^{40}Ca and 15 major shells for other nuclei. The RPA solutions are then obtained in this bases by solving eq. (59). In order to avoid the structure in the strength due to the finite number of states of the harmonic oscillator basis, we introduced a small imaginary part η to the frequency. The strength functions obtained in this manner are in agreement with the continuum RPA calculations at zero and finite temperature of ref. [70]. In applications presented in this article, two different Skyrme forces have been used: the SGII force [69] that was optimized to reproduced some properties of giant resonances in spherical nuclei and the effective Skyrme force SLy4[71] that gives a better parameterization away from the the stability line. Parameters of these forces are reported in Appendix C.

4.1.1 Excitation operators and sum-rules

We consider the response of the system to standard multipole excitation operators A_{LM} (for a recent review see [72]). For isoscalar (IS) and isovector (IV) excitations (for $L \geq 1$), we have respectively,

$$A_{LM}^{IS} = \frac{Z}{A} \sum_{i=1}^A r_i^L Y_{LM}$$

$$A_{LM}^{IV} = \frac{N}{A} \sum_{i=1}^Z r_i^L Y_{LM} - \frac{Z}{A} \sum_{i=1}^N r_i^L Y_{LM}$$

where Y_{LM} are the spherical harmonics. For the isoscalar giant monopole resonance, the excitation operator is

$$A_{00}^{IS} = \frac{Z}{A} \sum_{i=1}^A r_i^2 Y_{00}.$$

In order to discuss, the main features of the response function, it is useful to introduce the associated moments,

$$m_k(T) = \int_0^\infty \omega^k d\omega S(\omega, T) \quad (60)$$

The first moment, which can be written as $m_1(T) = \sum_{\lambda>0} \omega_\lambda |<[O_\lambda, A]>_0|^2$, is of particular interest. It can be expressed in terms of the excitation operator as,

$$m_1(T) = \frac{1}{2} <[A^\dagger, [H, A]]>_0 \quad (61)$$

which is known as the energy weighted sum rules (EWSR). It has been demonstrated that it is also valid at finite temperature [73]. For the isoscalar giant monopole (GMR), isovector dipole (GDR) and isoscalar quadrupole (GQR) resonances, the EWSR can be directly related to the properties of the Hartree-Fock ground state according to,

$$\begin{cases} m_1^{GMR} = \frac{2\hbar^2}{m} \frac{Z^2}{A} \langle r^2 \rangle_{HF} \\ m_1^{GDR} = \frac{9}{4\pi} \frac{\hbar^2}{2m} \frac{NZ}{A} (1 - \kappa) \\ m_1^{GQR} = \frac{50}{4\pi} \frac{\hbar^2}{2m} \frac{Z^2}{A} \langle r^2 \rangle_{HF} \end{cases}$$

where $\langle r^2 \rangle_{HF}$ denotes the root-mean square radius (rms) of the Hartree-Fock state. Note that, the EWSR calculated in mean-field theory for the GDR differs by a factor $(1 - \kappa)$ from the usual Thomas-Reich Kuhn (TKR) sum rule [70, 74], where κ is given by

$$\kappa = \frac{2m}{\hbar^2} \left[t_1 \left(1 + \frac{1}{2} x_1 \right) + t_2 \left(1 + \frac{1}{2} x_2 \right) \right] \frac{1}{A} \int \rho_n(r) \rho_p(r) d^3r, \quad (62)$$

and where ρ_n and ρ_p are the neutron and proton one-body densities. We report in table (1) the EWSR associated to the GMR, GDR and GQR resonances.

Besides the EWSR, moments of the response are also useful to discuss its global shape. In the following, we use these moments to estimate the mean collective energy according to,

$$\langle E \rangle = (m_1/m_0). \quad (63)$$

4.1.2 RPA at zero temperature

An example of the response obtained with RPA for the different multipolarities is given in figure (4) for ^{40}Ca nucleus. We have also performed the RPA calculations for GMR, GDR and GQR in ^{40}Ca , ^{90}Zr , ^{120}Sn and ^{208}Pb . In figure (5), the calculated energy of the main peaks of these RPA modes are displayed by circles as a function of the nuclear mass number, and they compared with the experimental results (squares). This figure illustrates that mean-field theory gives a reasonable estimate of the mean-energy of collective vibrations. The situation is generally less satisfactory as far as the width of giant resonances are concerned [10]. In general, mean-field theory underestimates the fragmentation of the strength. As

Nucleus	$\langle r^2 \rangle_{HF}^{1/2}$ (fm)	κ	GMR (MeV. fm ⁴)	GDR (MeV. fm ²)	GQR (MeV. fm ⁴)
⁴⁰ Ca	3.40	0.160	9290	144	9226
⁹⁰ Zr	4.26	0.177	26025	325	22556
¹²⁰ Sn	4.70	0.176	37026	427	26477
²⁰⁸ Pb	5.55	0.180	84572	748	76034

Table 1: Root mean square radius and κ values obtained in the Hartree-Fock calculation with the Sly4 force for the different nuclei. The associated sum-rules for the GMR, GDR and GQR are also reported.

Figure 4: Strength distribution obtained with the RPA method at zero temperature for ^{40}Ca as a function of the energy. From left to right the isoscalar monopole, isovector dipole and isoscalar quadrupole response are presented.

we see in following sections, the experimental GQR response in ^{40}Ca splits into two main components with energies around 14 and 18 MeV. However, the RPA response displayed in figure (4) has a single peak centered at $E = 17$ MeV. The RPA description contains dissipative effects that are in one-body nature, the escape width due to particle emission and the Landau damping that appears when a large number of $p - h$ states are placed in the vicinity of collective energy [47]. The calculated GMR in ^{40}Ca illustrates this situation.

4.1.3 Finite temperature response

Besides applications at zero temperature, the RPA approach can also be used to study collective response at finite temperature. An example of the finite temperature RPA calculation is given in figure (6), where the GDR response of ^{120}Sn and ^{208}Pb is presented at two different temperatures $T = 0$ MeV and $T = 4$ MeV. It is seen that the RPA response depends very weakly on temperature. This behavior has already been noted in early applications of RPA at finite temperature [70], and is also true for other multipolarities [62]. This is however in contradiction with experimental observation (figure (3)). For these nuclei, experimental strength functions become broader as thermal excitation increases. For instance, it is observed that the width increases by a factor of two in the GDR in ^{120}Sn when temperature increases from $T = 0$ to 2.5 MeV [20]. Again, such a behavior is absent when correlation beyond RPA are neglected.

5 Application of Extended Mean-field Theory

As seen in chapter 2, it is possible within the extended mean-field theory to incorporate two-body correlation effects. This leads to additional terms in the equation of motion, which describe the coherent and the incoherent dissipation mechanisms (eq. (54) to (58)). In applications to collective vibrations, it is sufficient to consider the linearized limit of the extended mean-field theory. In this case, the incoherent mechanism induces coupling between the collective states and the $2p - 2h$ configurations, while the coherent mechanism gives a coupling with the $1p - 1h \otimes \text{phonon}$ states. This coupling is expected to give rise to decay of collective states towards the decay channels. Accordingly, we do expect a modification of the response with an increase of the spreading. Let us illustrate this effects by considering a state that is coupled to a uniformly spaced ensemble of states. In this case, the decay width is given by a simple golden rule expression, $\Gamma = 2\pi v^2/\Delta E$, where ΔE is the mean level spacing and v is the average matrix element of a typical coupling interaction. Although, the situation is more complicated in damping of nuclear collective vibrations, both eq. (54) to (57) can be regarded as a generalized version of the coupling to more complex states taking into account the blocking coming from the Pauli principle. More generally, the fragmentation of the collective response gives information about the complex structure of internal degrees of freedom. The understanding of such fragmentation is thus of particular interest, since it may directly give information on correlations in nuclei [11].

Figure 5: The main peak energy of the isoscalar GMR (top), isovector GDR (middle) and isoscalar GQR (bottom) response obtained in the RPA are presented by circles as a function of the nuclear mass number. For comparison, the corresponding experimental energies are shown by squares. For the RPA response, when the strength function is highly fragmented, the mean-energy around the resonance is taken for the peak value. Note also that, for the GQR in ^{40}Ca , two experimental points are presented due to the presence of two well separated peaks in the observation. The data points are taken from: for the GMR ref. ^{40}Ca [75], ^{90}Zr [76], ^{120}Sn [76] and ^{208}Pb [76, 5]. For the GDR : ^{40}Ca [77], ^{90}Zr [3], ^{120}Sn [3] and ^{208}Pb [3]. For the GQR : ^{40}Ca [78, 79], ^{90}Zr [80], ^{120}Sn [80] and ^{208}Pb [80].

Figure 6: Strength distribution obtained with the RPA method at $T = 0$ MeV (thin line) and $T = 4$ MeV (thick line) for the GDR of ^{120}Sn (left) and ^{208}Pb as a function of the energy (adapted from [64]).

5.1 Response with incoherent damping

In this section, we present results of the extended mean-field including incoherent mechanism. The collisional self-energy is calculated by constructing the $2p - 2h$ phase-space from the set of the Hartree-Fock single-particle states. The use of a Skyrme force to estimate the incoherent contribution is not appropriate. Indeed, the momentum dependence of the force, which represents the finite range effect of the interaction gives an overestimation of coupling matrix element entering in eq. (54) and (55). In order to avoid this problem, we employ a modified version of the Skyrme force to estimate of the incoherent self-energy. More discussion on the finite range effects and on the modified force are given in Appendix C.

5.1.1 Influence of the incoherent mechanism on collective properties in the pole approximation

The effect of the collisional damping due to coupling of the collective excitation with the incoherent $2p - 2h$ states is conveniently illustrated using an approximate calculation, which is referred as the pole approximation. If we neglect the coupling between collective modes through the incoherent term, i.e. $K_{\lambda\mu}^I = 0$ if $\lambda \neq \mu$, eq. (54) reduces to

$$K_{\lambda\mu}^I(\omega) = -\frac{1}{4} \sum_{ijkl} \frac{|\langle ij | [O_{\mu}^{\dagger}, v_{12}] | k\ell \rangle_A|^2}{\hbar\omega + i\eta - \Delta\varepsilon_{ijkl}} \mathcal{N}_{ijkl} \quad (64)$$

In this case, the expression of the strength function becomes,

$$S(\omega, T) = \frac{1}{\pi} \sum_{\lambda>0} |A_{\lambda}|^2 \{D_{\lambda}(\omega) - D_{\lambda}(-\omega)\}$$

Figure 7: The quadrupole strength in ^{40}Ca obtained in RPA (thin line) is compared with the strength function of the extended RPA obtained in the pole approximation, when only the imaginary part of the self energy is included (dashed line), and when both the real and imaginary parts of the self-energy are included (thick lines). The arrows indicate the induced shift in the mean-energy Δ_λ and the damping width Γ_λ in the latter case.

where

$$D_\lambda(\omega) = \frac{\Gamma_\lambda(\omega)/2}{(\hbar\omega - \hbar\omega_\lambda - \Delta_\lambda(\omega))^2 + (\Gamma_\lambda(\omega)/2)^2}. \quad (65)$$

Here, the quantities $\Delta_\lambda(\omega)$ and $\Gamma_\lambda(\omega)$ represent the real and imaginary parts of the self-energy, $K_{\lambda\lambda}^I(\omega) = \Delta_\lambda(\omega) - \frac{i}{2}\Gamma_\lambda(\omega)$, which describe the shift and the broadening of the strength distribution.

In the pole approximation, the complicated frequency dependence of the self-energy in eq. (64) is neglected by making the substitution $\hbar\omega \rightarrow \hbar\omega_\lambda + \Delta_\lambda - i\Gamma_\lambda/2$. Accordingly, the real and imaginary part of the self-energies, Δ_λ and Γ_λ , of the collective modes are self-consistently determined by the coupled equations [63]:

$$\Delta_\lambda = \frac{1}{4} \sum | \langle ij | [O_\lambda^\dagger, v_{12}] | kl \rangle_A |^2 \frac{\hbar\omega_\lambda + \Delta_\lambda - \Delta\epsilon_{ijkl}}{(\hbar\omega_\lambda + \Delta_\lambda - \Delta\epsilon_{ijkl})^2 + (\Gamma_\lambda/2)^2} \cdot \mathcal{N}_{ijkl} \quad (66)$$

and

$$\Gamma_\lambda = \frac{1}{2} \sum | \langle ij | [O_\lambda^\dagger, v_{12}] | kl \rangle_A |^2 \frac{\Gamma_\lambda/2}{(\hbar\omega_\lambda + \Delta_\lambda - \Delta\epsilon_{ijkl})^2 + (\Gamma_\lambda/2)^2} \cdot \mathcal{N}_{ijkl} \quad (67)$$

which can be solved iteratively [62]. Then, the strength function is calculated with eq. (65) by employing approximate expression for $D_\lambda(\omega)$ function,

$$D_\lambda(\omega) \simeq \frac{\Gamma_\lambda/2}{(\hbar\omega - \hbar\omega_\lambda - \Delta_\lambda)^2 + (\Gamma_\lambda/2)^2}. \quad (68)$$

In this manner, energy dependence of the self-energy is neglected, and the strength distribution around each collective mode acquires a Lorentzian shape. In figure (7), we show an example of the pole approximation for the incoherent damping in the quadrupole response of ^{40}Ca , while in figure (8), we compare the pole approximation with the strength function obtained by including the full energy dependence of the incoherent self-energy. In figure (8), we observe that the pole approximation gives a smoothed strength distribution. Figure (8) also indicates that the pole approximation provides a rather good approximation of the strength when structure in the strength is disregarded. In addition, within this approximation, it appears clearly that Δ_λ is the shift of the mean collective energy, while Γ_λ is the collisional damping width due to the coupling with $2p - 2h$ states. In this manner, the pole approximation gives an intuitive picture of the coupling effect and help to explain the gross properties of incoherent damping mechanism.

In figure (7), we see that due to effect of the incoherent self-energy, the strength distribution becomes broader and shifts towards lower energies. In addition, it is possible to show that the EWSR is satisfied as it does in the RPA. When the full energy dependence of the self-energy is included into the description, the strength function exhibits fine structure as a direct consequence of the fluctuations in the density of $2p - 2h$ states and the fluctuations magnitude of coupling matrix elements (figure (8)).

Figure 8: Quadrupole response in ^{40}Ca when the frequency dependence of the self-energy is taken into account (thick solid line). In thin solid line, we present the strength obtained when the real part is neglected. In both cases, the corresponding strength with pole approximation are presented by dashed lines. Note that, when both real and imaginary part of the self-energy with or without the pole approximation are included, we observed similar mean-energy between the two calculations while the widths are slightly different.

5.1.2 2p-2h decay channels and coherent self-energy

As indicated by eq. (58), the incoherent mechanism modifies the collective response through collisional self-energy. An explicit expression of the collisional self-energy is given in eq. (54). Two examples of the real part (top panel) and imaginary part (bottom panel) of the self-energy are illustrated in figure (9) for the dipole and quadrupole vibrations in ^{40}Ca . From the bottom panel of the figure, we notice that the imaginary part of the self-energy remains essentially zero until a threshold energy of 11 MeV for the GQR and 20 MeV for the GDR. Then, it increases strongly with energy due to the fact that number of $2p - 2h$ decay channel grows rapidly with increasing energy. Both real and imaginary part of the self-energy present fluctuations as a function of energy that follows from fluctuations of the number of decay channels and also fluctuations of the coupling matrix elements. As a result, the strength distributions exhibit fine structure due to mixing of the collective excitation with the 2p-2h states [81, 82, 11].

Figure 9: Real (top) and imaginary (bottom) part of the incoherent self-energy obtained at zero temperature for the GDR (right) and GQR (left) of ^{40}Ca as a function of energy. In both cases, the horizontal lines indicate the mean energy of the collective RPA response.

In figure (9), we also indicate the mean collective energies obtained with usual RPA. We note that the collective energy is in the vicinity of the threshold energy for the GQR,

while it is below the threshold energy for the GDR leading to a zero imaginary part of the self-energy. This behavior reflects a particular quantum feature due to shell effects in the extended mean-field calculation of double magic light nuclei. Indeed, in a simplified picture, the energy of the $2p - 2h$ that could be coupled to odd parity collective modes should correspond at least to three times the major shell spacing, while two times the major shell spacing is possible for even parity modes. This property is illustrated in figure (10), where the density of the $2p - 2h$ states involved in decay of the dipole (right) and quadrupole (left) states are displayed as a function of energy. The threshold energy in the self-energy could thus be directly assigned to a threshold in the $2p - 2h$ energies.

Figure 10: Density of $2p - 2h$ states involved in decay of the quadrupole (left) or dipole (right) collective states in ^{40}Ca as a function of energy.

Although the $2p - 2h$ states has a dense spectrum at rather large energy, due to structure effects, only few states can couple to collective modes in the collective energy region. Since the imaginary part of the self-energy is generally associated to the damping width of the collective mode, one can thus expect only weak effect of the incoherent damping mechanism at low temperatures. On the other hand, the real part of self-energy $\Delta_\lambda(\omega)$ has rather large

value in the region of mean resonance energy, which indicates that the shift of collective energy due to collisional self-energy is expected to be significant.

5.1.3 Illustration of the response with incoherent damping

In figure (11), we illustrate effects of the incoherent collision term in the ^{40}Ca at zero temperature. In this figure, the extended RPA results with incoherent damping are compared to the RPA case in the monopole, dipole and quadrupole excitations. The change in the response directly reflects the properties of the incoherent self-energy. In all cases, the incoherent damping is largely inhibited due to the small number of $2p - 2h$ channels able to couple in the collective energy region (figure (10)). In fact, in the GDR, the width does not change as compared to the RPA result, while in the other modes some broadening of the strength is visible. As discussed in the pole approximation, in addition to a collisional broadening, we observe a shift of the mean-energy in all three modes shown in this figure. The shift in energy is sometimes accompanied by a further fragmentation of the strength, which uncovers the underlying structure of the incoherent coupling. This is clearly seen in the case of the GQR of ^{40}Ca . In that case, the usual RPA gives a single highly collective state. When only the imaginary part of the self-energy is included into the description, the strength is centered at the same collective energy with a width of the order of 2 MeV. When in addition, the real part of the self-energy is incorporated into the calculations, the strength is split into two components and the single collective state picture breaks down. If we focus on the global shape of the strength, we find a width of the order of 3.6 MeV.

From a number of applications that have been carried out [68], it appears that the fragmentation of the GQR observed in the ^{40}Ca due to incoherent effect is reduced when the mass of the system increases. Furthermore, when no additional fragmentation appears, a small increase of the width associated to a small decrease of the mean-energy is systematically obtained. Both effects will be discussed quantitatively in section 6.

5.2 Coherent damping mechanism

The incoherent mechanism alone is generally unable to account for the experimental strength distributions of collective excitations in nuclei. Indeed, the coherent dissipation mechanism due to coupling with low-lying collective states, which is referred to as the $p - h \otimes \text{phonon}$ coupling, plays an important role in describing the response functions of giant resonance excitations [32, 9]. In this section, we discuss the physical ingredients of the self-energy arising from the coherent mechanism and illustrate the effects on the nuclear response in ^{40}Ca .

5.2.1 Numerical implementation

In order to calculate the coherent self-energy, we need to determine a set of collective RPA modes those exhaust large fraction of the EWSR. In figure (12), we show a set of collective modes together with the associated EWSR, that are used in calculations of the coherent self-energies for collective vibrations in ^{40}Ca . Also, we note that since the coherent self-energy

Figure 11: Comparison of the monopole (left), dipole (middle) and quadrupole (right) RPA responses (thin line) obtained for a ^{40}Ca at zero temperature with the extended RPA results with only the incoherent damping (thick line).

requires calculation of coupling matrix elements that involves only low momentum transfer, we can calculate it by employing effective Skyrme-type interactions. In the applications, we calculate matrix elements using the full SLy4 interaction for the coherent part.

5.2.2 Coupling constant and decay states

In figure (13), we report energies of the $p-h$ states (noted E_{ph}) that contribute to coupling between dipole (left) and quadrupole (right) collective states with the low lying octupole states. At zero temperature, from denominator of eq. (57) and (58), we see that a collective states $|\lambda\rangle$ can be coupled to a collective state $|\mu\rangle$ if there exist $p-h$ states with energy in the vicinity of $\hbar(\omega_\lambda - \omega_\mu)$. In addition, the product state $|p-h\rangle \otimes |\mu\rangle$ should allow a coupling to the same quantum numbers as $|\lambda\rangle$. Since we consider doubly magic spherical nuclei, the minimum $p-h$ energy is of the order of level spacing between major shells. Consequently, at zero temperature, only low-energy collective states contribute to damping of giant resonances. We also note that, the phase-space of $p-h$ plus phonon decay channel is generally reduced and only few states contribute to the coherent dissipation mechanism. At finite temperature, on the other hand, not only $p-h$ states but also $p-p$ and $h-h$ states should be incorporated into decay channels, and, since these excitations have no threshold anymore, high-energy collective states can also contribute to damping.

In figure (14), we present the imaginary and the real part of the coherent self-energy by thick lines for the dipole (left) and quadrupole (right) excitations in ^{40}Ca . For comparison, we indicate the imaginary and the real part of the incoherent self-energy by thin lines. We also present in figure (15), the associated strength distributions by incorporating the coherent (top) and incoherent (bottom) dissipation mechanism. From these figures, we see that, no general rule exists for dominance of one dissipative mechanism over the other. Indeed, for the

Figure 12: The set of collective RPA states that are used for calculations of the coherent self-energies in ^{40}Ca . The Sly4 interaction is used in the calculations. The percentage of the EWSR exhausted for each state is indicated in parenthesis.

Figure 13: Energies of the $p-h$ states involved respectively in coupling between dipole (left) or quadrupole (right) collective states with the low-lying octupole states.

Figure 14: Real (top) and imaginary (bottom) part of the coherent self-energy (thick line) at zero temperature for the GDR (left) and GQR (right) excitations in ^{40}Ca as a function of energy. In all cases, the vertical lines indicate the mean energy of the collective response and the corresponding incoherent contribution are indicated by thin lines.

GDR, both the incoherent and the coherent $\Gamma_\lambda(\omega)$ are equal to zero at the mean collective energy, while $\Delta_\lambda(\omega)$ is dominated by the incoherent contribution. As a consequence, the coherent mechanism does not modify the RPA strength appreciably. On the other hand, the damping in the GQR is largely dominated by the coherent mechanism. This observation is related to the strong coupling matrix elements between the collective mode and low-lying states. However, the incoherent contribution in the GQR is not negligible due to the large value of $\Delta_\lambda(\omega)$, which is comparable to the shift obtained in the coherent mechanism. Note that the variation of the energy shift with ω is completely different in the two cases. In particular, the energy shift due to coherent mechanism $\Delta_\lambda^C(\omega)$ changes sign in the vicinity of the mean-energy. Consequently, a shift toward lower energies is observed for the low energy part of the strength, while the high energy part is pushed towards higher energies. This comes from the fact that, as seen in the imaginary part, the majority of states, which couples to the GQR, are lying around or above 20 MeV. The coupling being weak, it can be treated in perturbation theory. At the second order it induces a level repulsion which explains the sign of the energy shift $\Delta_\lambda^C(\omega)$.

In all cases, the effect of the incoherent mechanism cannot be neglected. This is largely in contrast to the general belief that incoherent mechanism due to coupling with $2p - 2h$ states does not affect the nuclear response at zero temperature [36], and may have important issue as far as the comparison to experimental data is concerned.

5.3 Systematic effect of coherent and incoherent damping

The importance of both nucleon-nucleon collisions and coupling to low-lying states has been illustrated in the previous section. In this chapter, we present a systematic investigation of nuclear collective response on the basis of calculations, which incorporate both the coherent mechanism and the collisional damping in a consistent basis. In figure (16), we show the calculated strength functions for giant monopole, dipole and quadrupole excitations in ^{40}Ca , ^{90}Zr , ^{120}Sn and ^{208}Pb at zero temperature. In this figure, thick lines are the results with the incoherent self-energy alone, thin lines are the results with the coherent self-energy alone and the shaded areas correspond to the pure RPA calculations. In figure (17), we represent the strength functions obtained by including both the coherent and the incoherent mechanisms, which are shown by thick lines, and the shaded areas correspond to the pure RPA calculations. For the coherent calculation, the set of collective states that are used for ^{90}Zr , ^{120}Sn and ^{208}Pb are displayed respectively in figure (18), (19) and (20) (see also ref. [68]).

In top panel of figure (21), we show the mean-energy of giant resonances $\langle E \rangle$ associated to each strength function as a function of mass number. The mean-energy of each strength function is calculated according to eq. (63) where moments are calculated in a limited energy interval of $[0, 40]$ MeV. From this figure, we observe that the coherent and the incoherent contributions shift the energy in opposite directions. In fact, the mean-energy calculated with the coherent $p - h \otimes \text{phonon}$ coupling mechanism is always larger than the RPA one, while the one calculated with the incoherent $2p - 2h$ coupling mechanism is always smaller. While the main peak is systematically shifted towards lower energy (also true for the coherent

Figure 15: Top: The RPA strength (thin line) for a monopole (left), dipole (middle) and quadrupole (right) excitations in ^{40}Ca at zero temperature is compared to the extended RPA results when only the coherent mechanism is included (thick lines). Bottom: we also report the case when only the incoherent mechanism is included (thick line) and is compared with the RPA results (thin lines).

Figure 16: Strength distributions calculated for the monopole (left), dipole (middle) and quadrupole (right) excitations in ^{40}Ca , ^{90}Zr , ^{120}Sn and ^{208}Pb at zero temperature. The RPA result (shaded areas) is systematically compared to the extended RPA calculations when only coherent mechanism (thin lines) or only incoherent mechanism (thick lines) are included. Results are calculated with a smoothing parameter $\eta = 0.5$ MeV.

Figure 17: Strength distributions calculated for the monopole (left), dipole (middle) and quadrupole (right) excitations in ^{40}Ca , ^{90}Zr , ^{120}Sn and ^{208}Pb at zero temperature. The RPA result (shaded areas) is systematically compared to the extended RPA calculations (thick lines) in which both the coherent and the incoherent mechanisms are included. Results are calculated with a smoothing parameter $\eta = 0.5$ MeV.

Figure 18: Collective RPA states in ^{90}Zr , which are employed in calculation of the coherent collision term. For each state, the percentage EWSR is indicated in parenthesis.

Figure 19: Same as figure (18) for ^{120}Sn . When several states are used with very close energies, these states are indicated by shaded area. In this case, the EWSR indicates the sum of EWSR for the individual states in the interval.

Figure 20: Same as figure (18) for ^{208}Pb

mechanism), this is often counterbalanced by an increase of the strength at higher energy, leading to an apparent increase of the mean-energy. As illustrated in figure (14), this increase is associated to the positive branch of the real part of the self-energy. The differences between the coherent and incoherent mechanisms are related to the fact that $p-h \otimes \text{phonon}$ states lie in general below the $2p-2h$ states because of the collectivity of the phonon states. Indeed, the residual interactions lower the isoscalar phonons compared to the energy of the $p-h$ from which it is constructed. Interestingly enough, the opposite behavior of the two effects tends to cancel each other. As a result, the mean-energy obtained in the full extended RPA is very close to the one obtained in the RPA calculations.

In contrast to the mean energy, which is little affected, the strength appears in figure (17) more fragmented when collisions terms are accounted for. Indeed, aside from the GMR in ^{40}Ca , which is already largely Landau fragmented, we always observe a large increase of the width of the strength function as the number of decay channels increases. In most of the cases, contributions from both the coherent and the incoherent mechanisms are of same order of magnitude, indicating that none of the two effects can be neglected. While the coherent and the incoherent contributions cancel each other for the mean-energy, they add in the case of the spreading width. The width calculated with both the coherent and the incoherent dissipation mechanisms is always much larger than the one obtained in the RPA description. The increase of the width reflects, in general, an additional fragmentation of the strength function (see discussion below).

Figure 21: Mean energy $\langle E \rangle$ calculated in the interval $[0, 40]$ MeV as a function of the mass of the system for the GMR (left), GDR (middle) and GQR (right). The RPA results (thin line) are systematically compared to the extended mean-field results when only the coherent (dashed line), incoherent (dot-dashed line) and both (thick line) part are included.

6 Physical issues of correlated response

6.1 Isoscalar GMR: correlation effect and incompressibility

In last section, we have seen that the mean-energies of strength functions are slightly affected by correlations beyond the RPA. In particular, the monopole vibration in finite nuclei is of special interest since it provides an information on nuclear incompressibility [2]. An effective incompressibility K_A can be defined in finite nuclei assuming a single collective state at the GMR collective energy E_0

$$E_0 = \sqrt{\frac{\hbar^2 K_A}{m \langle r^2 \rangle}}$$

The parameters of the force is adjusted to reproduce the monopole response in finite nuclei, which also determines the value of the incompressibility in infinite nuclear matter (noted K_∞). The interplay between K_A and K_∞ has been recently studied in ref. [83]. The correlations beyond the RPA give further uncertainties in nuclear incompressibility as already noted in [84]. In the formalism we follow, we linearized the extended mean-field equation around Hartree-Fock equilibrium. Hence, the infinite nuclear matter reference remains to be the K_∞ of the mean-field approximation. However, as we see from figure (17), the main peak position of the GMR is slightly shifted to lower energy. In figure (22), we show by circles the prediction of the extended RPA for the main peak energy E_{peak} as a function of the mass number of the system. In same figure, we also show the result of RPA calculations and experimental data by diamonds and squares, respectively. We observe from this figure that inclusion of correlations gives a better description of the experimental observation. Assuming a single peak approximation, we obtain a difference of the peak energy $\Delta E_0 = 0.8$ MeV in ^{90}Zr and $\Delta E_0 = 1.0$ MeV in ^{208}Pb between RPA and extended RPA calculations. For lead nucleus, using the rms value of table (1), we find for the compressibility a value $K_A = 156$ MeV in the RPA and $K_A = 135$ MeV in the extended RPA approach.

Our calculations point out a modification of finite nucleus incompressibility due to correlations beyond mean-field approximation. Indeed, since dissipative processes modify the response, one should in principle re-adjust the parameters of the effective interactions in order to fit the experimental response. As a consequence, the infinite matter compressibility K_∞ , which is extracted from finite nuclei may also be modified. However, the better agreement of the extended RPA with data shows that the compressibility of the considered force, i.e. Sly4 is already good, leading to a compressibility of $K_\infty = 230$ MeV [71].

6.2 The isovector GDR

In a macroscopic picture, the GDR collective mode gives access to the asymmetry term in the liquid drop model. In figure (23), we present main peak energies of the isovector GDR in the extended RPA as a function of the mass number, and we compare those with the RPA results and experimental energies. In these calculations, we used the new Sly4 force. We note that the RPA results (diamonds) although underestimating experimental GDR energies (squares), give a better agreement with data than the extended RPA results (circles). Note

Figure 22: The main peak energy of the isoscalar GMR response obtained in the extended RPA is indicated by circles as a function of the nuclear mass number. For comparison, the experimental mean-energy values for the GMR excitations and the RPA results are shown by squares and by diamonds, respectively. When the response is highly fragmented, the mean-energy around the resonance is taken for the peak value. The data points are taken from ^{40}Ca [75], ^{90}Zr [76], ^{120}Sn [76] and ^{208}Pb [5, 76].

however, that effective forces are generally optimized for nuclei close to the stability line where isospin effects are small compared to other components of the force. As a matter of fact, different effective interactions differ strongly from one another in the regions far from stability. Note also, that for stable nuclei, the isovector RPA response is very sensitive to the effective force used [2]

6.3 The Isoscalar GQR : fragmentation and structures

In previous section, we discussed the average properties of the collective nuclear strength functions, i.e. mean-energy and width. These quantities give a partial characterization for dissipation of collective excitations into internal degrees of freedom. Apart from the average properties, experiments often uncover the existence of a large degree of fragmentation in the nuclear response functions, which cannot be understood in a one-body picture. Furthermore, in high-resolution experiments, fine structure in strength functions may also appear on top of the fragmentation. Recently, various scales associated with these structures have been extracted, which appear to be robust against the type of experiments that are investigated [8]. The diversity of scales, that are present in the strength functions, illustrates complexity of damping mechanism of nuclear collective vibrations. High precision experiments offer a possibility to understand the interplay between one-body and two-body effects in the dynamics of nuclear collective motion. We believe that understanding of fine structure of

Figure 23: Main peak energy of the isovector GDR response obtained in the extended RPA are indicated by circles as a function of the nuclear mass number. For comparison, the experimental mean-energy values for the isovector GDR excitations and the RPA results are shown by squares and by diamonds, respectively. When the response is highly fragmented, the mean-energy around the resonance is taken for the peak value. The data points are taken from ^{40}Ca [77], ^{90}Zr [3], ^{120}Sn [3] and ^{208}Pb [3].

nuclear response is one of the main challenges for theoreticians as well as experimentalists working in the field of giant resonance excitations. In this chapter, we present specific examples, which illustrate different effects arising from correlations beyond the RPA.

6.3.1 Microscopic origin of fragmentation in the GQR of ^{40}Ca

As seen in figure (17), while quadrupole response exhibits a single peak at the RPA level, it is largely fragmented in the extended RPA. Experimentally, the splitting of the ^{40}Ca GQR [85] into two main peaks is well established. In addition, the importance of $p - h \otimes \text{phonon}$ coupling has been demonstrated in different investigations as summarized in [86].

In order to illustrate the importance of the incoherent damping in the ^{40}Ca GQR response, in table (2), we report the fraction of EWSR exhausted for several energy intervals, which are calculated by including the incoherent or the coherent mechanism or both mechanisms. In the same table, we also report the measured EWSR. For this nucleus, the strength is known to split into two components with energies around 14 MeV and 18.0 MeV, and with almost equal fraction of the EWSR, around 30 % to 40 % respectively. Only recently [72, 86], it was possible to reproduce this global splitting in a calculations assuming ground state correlations and coupling to low-lying states. However, this calculation describes the global trend of the response, but does not provide an explanation for the equal partition of the EWSR.

$^{40}\text{Ca} / 2^+$	RPA	coherent only	incoherent only	total	Experiment
% EWSR	%	%	%	%	%
[0 – 40]	92.4	86.6	84.7	87.6	33 ± 7 ($e, e'x$) [78, 85] 60 ± 15 ($\alpha, \alpha'\alpha_0$) [79] 28.6 ± 7 (p,p') [87] ~ 40 ($\alpha, \alpha'\alpha_0$) [79] 44 (p,p') [88]
[10 – 16]	0.0	26.6	39.1	31.0	
[16 – 22]	72.5	51.6	34.7	33.6	

Table 2: Percent of Energy-Weighted Sum-Rule calculated in different energy intervals for the GQR in ^{40}Ca in different approximations. From left to right, the RPA only, RPA + coherent damping only, RPA + incoherent damping only, total extended RPA results are presented. In right side, experimental data are also reported

In our calculations, ground state correlations are not included. When only the coherent self-energy is taken into account, we already see that the response splits into two peaks with expected energies. However the EWSR exhausted by the lower energy peak is predicted too high as in ref. [72]. Looking at table (2), only when coherent and incoherent mechanisms are included, both splitting of the strength into two main components and equal EWSR around the peaks are found (31 % in the interval 10-16 MeV and 33.6 % in the interval 16-22 MeV), which are consistent with experimental data. This particular example demonstrates the necessity of taking both coherent and incoherent damping mechanisms at the same time and illustrates complementarity of the two effects. Indeed, without the coherent mechanism splitting of the strength does not occur, while without the incoherent damping the percentages of EWSR are not reproduced.

We can gain a better understanding of the fragmentation in strength functions of the extended RPA by investigating microscopic properties of decay channels. Decay channels associated with the coherent mechanism are $p-h$ states plus low-lying collective modes $|\nu\rangle$, noted $|ph\rangle \otimes |\nu\rangle$. A set of $p-h$ states, which may form a decay channel of the isoscalar quadrupole excitations in ^{40}Ca , is given in left panel figure (13). However, only a few of these $p-h$ states, which have large coupling matrix elements with the collective state, make sizable contributions to dissipation. This is illustrated in figure (24). In the left part of this figure, we present the complete set of $p-h$ states associated with two low-lying octupole states $|3^- \rangle$ (upper panel) and with the lowest $|5^- \rangle$ state (lower panel). In the right part of figure, we also show the magnitude of coupling matrix elements, $\overline{V}_{\lambda,ph\nu}^2 = \left| \langle p|[Q_\lambda, h_\nu^\dagger]|h \rangle \right|^2$, between the quadrupole state and decay channels $|ph\rangle \otimes |\nu\rangle$. It is seen that among a large number of channels only a few of them have large coupling matrix elements.

In figure (25), solid line shows the quadrupole strength function in ^{40}Ca calculated in the extended RPA by including only the coherent mechanism. Vertical lines in upper panel indicate the RPA strength at each collective energy. In order to see influence of the coher-

Figure 24: Left: particle-hole energies E_{ph} of states involved in the coupling between the quadrupole state $|\lambda\rangle$ and channel states $|ph\rangle \otimes |\nu\rangle$ those correspond to two low-lying octupole states 3^- (upper panel) and the lowest 5^- state (lower panel). Right: magnitude of coupling matrix elements $\overline{V}^2_{\lambda,ph\nu} = \left| \langle p|[Q_\lambda, h_\nu^\dagger]|h \rangle \right|^2$ between the quadrupole state and the 3^- decay channels (upper panel) and the 5^- decay channels (lower panel), which are indicated in logarithmic scale. The notation $(2^+)^{17.1}/(3^-)^{5.3}$ indicates that we have selected matrix elements between the 2^+ state at 17.1 MeV energy and the $|ph\rangle \otimes |\nu\rangle$ state where $|\nu\rangle$ is the octupole state at 5.3 MeV energy.

ent coupling on the fragmentation pattern, we superimpose coupling strengths between the quadrupole state and decay channels in middle panel of figure (25). These coupling strengths are taken from figure (24) and positioned at energies $E_{\nu,ph} = \hbar\omega_\nu + \varepsilon_p - \varepsilon_h$ of channel states $|ph\rangle \otimes |\nu\rangle$. In this figure, we can distinguish three energy regions: [14, 16] MeV, [16, 19.5] MeV and [19.5, 22] MeV. In the first and the last energy regions, structures are closely related to energies of decay channels $E_{\nu,ph}$. In the intermediate region, although a large number of $|ph\rangle \otimes |\nu\rangle$ states are present with rather strong coupling matrix elements, correspondence between the strength and the $E_{\nu,ph}$ does not appear. On the contrary, as seen from top panel of figure (25), there is one-to-one correspondence between RPA energies and peak positions of the strength in the extended RPA. The peak located around 17 MeV is not correlated with properties of decay channels, but it provides a signature of the mean-field effect in the collective response while the others originate from the coupling with $|ph\rangle \otimes |\nu\rangle$ states. In bottom panel of figure (25), we also show, by thin line, the strength function obtained in the extended RPA including both the coherent and the incoherent mechanisms in order to emphasize that most of the structure persists. Only in low-energy region, positions of peaks are modified by the incoherent mechanism, and correlation between peak energies and channel energies $E_{\nu,ph}$ is not visible anymore.

6.3.2 Systematic energy dependence of the GQR

In figure (26), we show energies of the main peaks in quadrupole response as a function mass number. The RPA and the extended RPA results are presented by circles in upper panel and lower panel, respectively, and compared with experimental data indicated by squares [68]). As we indicated earlier, the RPA calculations fail to reproduce peak positions as well as fragmentation of the strength. The strength calculated in the extended RPA usually splits into two or three peaks in energy region between 10 and 20 MeV. In this region, the lowest energy peak (or the two lowest energies for ^{40}Ca) is in good agreement with experimental data. We also remark that an additional peak is systematically present at higher energy. Such a peak also exists in the second RPA calculations for quadrupole excitations in ^{208}Pb [10, 64] but no experimental evidence of these peaks exists so far.

6.3.3 Fine structure effects

In view of the success of the extended RPA for describing the global shape of strength functions, we can go a step further and compare the fine structure of giant resonances. For this purpose, we choose isoscalar GQR in ^{208}Pb . The quadrupole response of ^{208}Pb appears to be a special case, since existence of fine structure has been reported already a long time ago from (e, e') experiment [15, 6] and it has been confirmed by other experimental probes, for example (p, p') experiments [14], which can access a fine resolution of the order of 50 keV. In top panel of figure (27), we compare the calculated strength distribution of isoscalar GQR in ^{208}Pb with the experimental strength function. Calculations are performed in the extended RPA by including both the coherent and the incoherent mechanisms with a smoothing parameter corresponding to the experimental resolution in (p, p') experiment. The energies of peaks extracted from experimental data in the energy region 8 – 12 MeV

Figure 25: The GQR strength calculated in ^{40}Ca in the extended RPA by including only the coherent mechanism with a smoothing parameter $\eta = 25$ keV is presented by thin line in top and middle panel. The set of vertical lines in upper panel is the strength distribution in the RPA approach. Vertical lines in middle panel correspond, in log scale, to the coupling strengths between the quadrupole state and the dominant decay channels. These coupling strengths are taken from figure (24) and positioned at energies $E_{\nu,ph} = \hbar\omega_{\nu} + \varepsilon_p - \varepsilon_h$ of channel states $|ph\rangle \otimes |\nu\rangle$. In bottom part, we also display the strength obtained by including both the coherent and the incoherent mechanisms (thick line) compared to the strength obtained with coherent collision term only. This figure illustrates the persistence of the fine structure with incoherent effect.

Figure 26: The main peak energies in GQR excitations as a function mass number. The RPA and the extended RPA results are presented by circles in upper panel and lower panel, respectively, and compared with experimental data indicated by squares. Data are taken from references: ^{40}Ca [78, 79], ^{90}Zr [80], ^{120}Sn [80]and ^{208}Pb [80].

Figure 27: Top: Comparison of the strength function for isoscalar GQR in ^{208}Pb calculated in the extended RPA and the experimental strength deduced from inelastic proton scattering [14]. A small smoothing parameter $\eta = 25$ keV is used to match with the experimental resolution. Bottom: comparison of the strength function for isovector GDR in ^{208}Pb calculated in the extended RPA with a smoothing $\eta = 25$ keV and the same experimental strength as top panel.

(e,e') [6]	(p,p') [90]	(p,p') [7]	incoherent and coherent	coherent only	incoherent only
8.9	8.9	8.9			
9.4	9.3	9.4	9.3	9.3	9.3
9.6		9.6	9.9	9.9	
10.1		10.1	10.2	10.3	
10.7	10.6	10.7	10.7	10.8	
11.5		11.0	11.0	11.3	
					11.9

Table 3: Left: experimental peak energies of GQR strength function in ^{208}Pb excitations in (p,p') and (e,e') experiments. Right: calculated peak energies in the extended RPA by including only the coherent mechanism, only the incoherent mechanism or both (taken from [68]).

are reported in table (3) and compared with calculations. We see from this table that fine structure appear as soon as the coherent mechanism are taken into account, while the incoherent mechanism alone is not enough to generate them. When both mechanisms are included, the peak positions in this energy interval compare well with the recent (p,p') experiment. It has been pointed out that peaks occurring outside this interval maybe due to dipole excitations [89]. In bottom panel of figure (27), we compare the experimental strength with the calculated GDR strength distribution in ^{208}Pb with the same smoothing parameter $\eta = 25$ keV. As seen from lower panel, the fine structure in the GDR response in energy region 8-9 MeV matches to the experimental structure. We also point out that missing peaks may be due to higher order correlations those are neglected in the present calculations.

6.4 Summary

In this section, we illustrated many facets of applications of the extended RPA to the giant resonance excitations at zero temperature. The extended RPA approach by including two-body correlations due to coupling with incoherent 2p-2h states and due to coupling with low-lying collective states greatly improves description of giant resonance excitations and associated strength functions. The comparison with experimental data indicates that the extended RPA approach provides a powerful tool not only for understanding the global structure but also for describing the fine structure of strength distributions. Note that, for isovector modes, the agreement is not yet satisfactory. This might be either due to the fact that physical effects are still missing in the extended RPA calculations or due to the isovector part of the force which is not completely under control.

It should also be noted that extension of mean-field requires a complete comprehension of the two-body interaction. In order to perform the applications presented in this section we use a modification of the effective interaction to cut the unphysical high momentum

components. Similar problem arises when mean-field theories are extended to include pairing. Although our results reproduce correctly giant resonances in nuclei, it would be more satisfactory to remove the uncertainty in the force (see discussion in appendix C) by using interactions that properly account for finite-range effects.

7 Finite temperature response

During the last two decades, a large effort has been made to study hot GDR, i.e. the GDR built on excited states. These investigations indicate that, as a function of excitation energy or temperature, the mean resonance energy does not change much but the resonance becomes broader as seen in ^{120}Sn [91, 92] and ^{208}Pb nuclei [93]. At high temperature, possible saturation of the hot GDR width has been reported, but this effect is still under debate. On theoretical side, much work has been done to describe properties of collective vibrations at finite temperature [47]. For this purpose, the extended RPA approach presented in chapter 3 of this report provides a suitable basis not only at zero temperature but also at finite temperature. In this section, we present some selected applications of the extended RPA formalism at finite temperature.

7.1 Incoherent damping at finite temperature

For increasing temperature, since Pauli blocking becomes less effective, the phase-space of $2p$ - $2h$ states in the collision term becomes larger. As an example, figure (28) illustrates the number of decay channels contributing to incoherent damping at zero temperature and 4 MeV. As a result, magnitude of the incoherent damping becomes larger for increasing temperature. Furthermore, at finite temperature, in addition to $2p - 2h$ states, $3p - 1h$, $1p - 3h...$ states can also couple to the RPA states. In fact, already at the RPA level, the Landau damping is slightly increased because of the presence of $p-p$ and $h-h$ configurations on top of the usual $p-h$ ones (see figure (29)). We note, however, that there are two factors contributing to the incoherent damping: (i) the number of decay channels and (ii) the average value of the coupling matrix elements.

Figure (29) presents the calculated strength functions for the GMR, GDR and GQR excitations in ^{40}Ca at temperatures $T = 0$ MeV and $T = 4$ MeV [62, 63]. Thick lines show the result of the extended RPA calculations including only the incoherent mechanism. For comparison, we also indicate the result of the RPA calculations by thin lines. In all of these strength functions, due to the incoherent mechanism, we observe a large increase of the spreading width with increasing temperature.

7.1.1 GDR in ^{120}Sn and ^{208}Pb at finite temperature

Extensively studied collective vibrations at finite temperature are the GDR excitations in ^{120}Sn [4] and ^{208}Pb [93]. In these experiments, interpretation of the observed strength distributions; in particular, the apparent width, has been difficult due to different mechanisms involved in damping of the GDR excitations [47, 9]. New experiments based on in elastic

Figure 28: Left: Density of $2p - 2h$ states contributing to the incoherent damping of the GQR in ^{40}Ca at zero temperature. Right: Density of $2p - 2h$ states contributing to the incoherent damping at $T = 4$ MeV.

Figure 29: Calculated strength functions for the GMR, GDR and GQR excitations in ^{40}Ca at temperatures $T = 0$ MeV and $T = 4$ MeV [62, 63]. Thick lines: the extended RPA calculations by including only the incoherent mechanism. Thin lines: the RPA calculations.

alpha scattering are now available [20, 91, 92, 94]. These experiments make it possible to investigate thermal effect more clearly without mixing with the angular momentum effect.

Figure 30: Calculated GDR strength in ^{120}Sn and ^{208}Pb at temperatures $T = 0$ MeV (thin lines), 2 MeV (dashed lines) and 4 MeV (thick lines). The result of RPA and extended RPA calculations are shown in top and bottom panels, respectively.

In figure (30), we present the calculated GDR strength functions in ^{120}Sn and ^{208}Pb at temperatures $T = 0, 2$ and 4 MeV. Top and bottom panels show results of the RPA calculations and the extended RPA calculations, respectively. The RPA calculations give a Full Width at Half Maximum (FWHM) of 2.0 MeV for ^{120}Sn and 2.2 MeV for ^{208}Pb and the shape of the strength distributions are almost independent of temperature, as discussed in previous chapter [70, 62]. At $T = 0$ MeV, as seen in lower panel, the incoherent dissipation mechanism induces a small shift towards lower energy and a small increase of the width. As a result, the extended RPA calculations with the incoherent mechanism slightly underestimate the experimental FWHM of 5 MeV for GDR excitations in ^{120}Sn and 4 MeV in ^{208}Pb at zero temperature. In figure (30), we observe a strong increase of the FWHM accompanied by a relatively large shift of the mean-energy towards lower energies in ^{120}Sn . On the other hand, in ^{208}Pb , the FWHM weakly depends on temperature and the shift in the mean-energy is negligible.

The qualitative difference between the magnitude of the incoherent dissipation in two nuclei, ^{120}Sn and ^{208}Pb , arises from different behavior of the incoherent self-energy as a function of frequency. In particular, the shift in the mean resonance energy is related to asymmetry of the real part of the self-energy with respect to the collective frequency. This asymmetry strongly depends on the density of $2p - 2h$ states around the collective frequency and on the magnitude of their coupling with the collective state. Combination of these two effects yields a distribution of the coupling matrix elements, which may differ strongly from one nucleus to another and hence gives rise to a different strength and temperature dependence of the internal mixing for each nucleus. In the ^{120}Sn , distribution of coupling matrix elements with $2p - 2h$ states rapidly increases around the collective energy, hence it produces a strong downward shift of the mean resonance energy. On the other hand, in the ^{208}Pb , the coupling matrix elements have a smooth distribution around the collective frequency, hence, it produces a weaker shift of the mean GDR energy.

7.2 Coherent damping at finite Temperature

In this section, we investigate the effect of temperature on the coherent damping mechanism. As seen in the previous sections, the RPA wave-functions are not affected much by temperature. In order to reduce the numerical effort for calculating coherent self-energies, we take advantage of this property and assume that the RPA wave-functions at finite temperature have the same $p - h$ components as those at zero temperature.

For the coherent dissipation mechanism, we expect a modification of the decay channels properties for increasing temperature. Indeed, while for cold nuclei, phonons involved are limited to the low energy region due to the denominator of eq. (57) and (58), this constraint is relaxed at finite temperature. This is illustrated in figure (31), where the coupling strength between a dipole state in ^{120}Sn and different $|ph\rangle \otimes |\nu\rangle$ states as a function of the energy of the $|ph\rangle$ component are displayed. In this figure, we observe an evolution of large coupling matrix elements towards lower values of E_{ph} . As a consequence, collective states $|\nu\rangle$, which couples the GDR, may explore higher energies.

Figure 31: Magnitude of the coupling strength between a collective dipole state in ^{120}Sn and $|ph\rangle \otimes |\nu\rangle$ states as the function of the E_{ph} energy for temperatures, from left to right, $T = 0$, $T = 2$ and $T = 4$ MeV.

In upper panel of figure (32), we present the GDR strength distributions associated with the coherent self-energy in ^{120}Sn (left) and ^{208}Pb (right) for temperatures $T=0$ and $T=2$ MeV. We observe that, although decay channel properties are largely modified by the increase of

temperature, the strength is only weakly affected. Similar conclusion have been drawn in ref. [95]. Only at rather large temperature around $T = 4$ MeV, the strength distribution spreads more than those at $T = 0$ MeV. In bottom panel of figure (32), we also display the strength distributions resulting from both the coherent and incoherent damping. As we mention in previous section, the incoherent damping has a strong effect in ^{120}Sn and we see that the evolution of the strength is largely dominated by the incoherent mechanism. On the other hand, the incoherent effect is weak in ^{208}Pb and is of the same order of magnitude as the coherent effect. This is illustrated in figure (33), where evolution of the width and shift in the mean energy of the strength relative to the zero temperature case are plotted as a function of temperature for the coherent (thin lines) and the incoherent (dashed lines) mechanisms, and also by including both mechanisms (thick lines).

In a recent experiments [94], it was observed that the mean resonance energy in ^{208}Pb remains almost constant when temperature increases from 1.34 to 2.05 MeV, while in ^{120}Sn , it decreases by 1.5 MeV when temperature varies from 1.24 to 3.12 MeV. Our calculations are able to reproduce qualitatively this behavior. In the same temperature interval, calculations give a 1.4 MeV shift of the mean resonance energy in ^{120}Sn , while, in ^{208}Pb , the mean resonance energy almost remains constant. Experiment shows that, the FWHM in ^{120}Sn increases about 4.5 MeV as temperature moves up from $T=0$ MeV to $T=2.5$ MeV. The extended RPA calculation is able to describe this behavior, but at higher temperature, the calculation slightly underestimates the observed width. On the other hand, the extended RPA calculation is not able to explain the increase of the FWHM in ^{208}Pb as a function of temperature. This indicates that large amplitude shape fluctuations, besides the incoherent dissipation, may play an important role in the GDR excitations in ^{208}Pb at finite temperature.

Indeed, applications of models which includes shape fluctuations are able to reproduce the increase of the width with temperature in ^{208}Pb [96, 97, 98] (see figure (3)). These fluctuations, which correspond to large amplitude motion, are not included in the linear version of extended mean-field theory. We also note that, thermal shape fluctuations are known to differ between ^{120}Sn and ^{208}Pb (see discussion in ref. [20]). On the other hand, our approach is particularly suitable in the low temperature regime. Recent experiments indicate that width of the GDR ^{120}Sn have a nearly a constant value in the interval $T = 0$ to 1 MeV. In the calculations presented here, we observe that the coherent coupling mechanism induces a reduction of the width between $T = 0$ and 2 MeV. This may be a possible explanation of the observed constant width. Indeed, at very low temperature, coherent effects play a significant role and may counterbalance the increase of the width due to both incoherent damping and shape fluctuations.

8 Summary and Conclusions

The small amplitude limit of the TDHF theory is equivalent to the random phase approximation, and hence, it constitutes a suitable basis for describing mean energies of nuclear collective vibrations and the fragmentation of strength functions due to Landau damping.

Figure 32: Top : Strength distributions associated with the coherent self energy in ^{120}Sn (left) and ^{208}Pb (right) nuclei at $T=0$ MeV (thin lines), $T=2$ MeV (dashed lines) and $T=4$ MeV (thick line). Bottom : Strength distributions obtained when both the coherent and the incoherent mechanisms are accounted for.

Figure 33: The shift in the FWHM $\Delta\Gamma$ (upper panel) and in the mean resonance energy ΔE (lower panel) relative to their ground state values of the GDR excitations in ^{120}Sn (left) and in ^{208}Pb (right) are plotted as a function of temperature. The values of energy and width are obtained in all cases using a fitting procedure of the strength by a Breit-Wigner distribution. The results for the coherent mechanism, for the incoherent mechanism and for incorporating both mechanisms are indicated by thin lines, dashed lines and thick lines, respectively.

The extended TDHF theory goes beyond the mean field approximation by incorporating damping mechanism due to coupling to incoherent 2p-2h configurations, which is usually referred to as the collisional damping. Hence, the small amplitude limit of the extended TDHF theory provides a basis for including collisional damping mechanism into nuclear response functions. In general, it is rather difficult to carry out applications of the extended TDHF theory to nuclear dynamics. However, for small amplitude motion, we are faced with a relatively easier task. By linearizing the transport equation, it is possible to deduce a closed form expression for the collision term. The linearized collision term includes the essential ingredients of the memory and quantal effects and is well suited for numerical calculations. However, the linear response treatment based on the standard extended TDHF has an important shortcoming: although the collisional damping mechanism due to mixing with the incoherent 2p-2h excitations is important at relatively high temperature, it is weak at low temperature due to long nucleon mean-free-path. For a proper description of the damping properties of giant resonance excitations, the coherence between the p-h pairs should be taken into account. In a recent development, based on a stochastic treatment, the extended TDHF theory is improved by incorporating a coherent collision term arising from the coupling of the single-particle motion with the mean-field fluctuations. In this article, we have presented a review of the extended mean-field theory including both the incoherent and the coherent damping mechanisms. The small amplitude limit of this improved transport theory provides a powerful tool to investigate giant resonance excitations at zero and finite temperature. We carry out fully microscopic calculations using Skyrme forces of strength functions of isoscalar and isovector giant resonance excitations in ^{40}Ca , ^{90}Zr , ^{120}Sn and ^{208}Pb nuclei and investigate the relative importance of the coherent and the incoherent damping mechanisms. In most cases, this extended RPA description provides a good understanding of the strength distributions of giant monopole, dipole and quadrupole excitations at zero and finite temperature. In cold nuclei, the theory is able to reproduce not only the gross properties of giant resonances at zero temperature but also the fragmentation of the strength as well as the fine structure. However, in some cases, especially at finite temperature, the model does not provide a good description for damping of collective modes. For example, while the increase with temperature of the width of the GDR in ^{120}Sn is correctly reproduced, the calculations underestimate the increase in ^{208}Pb . As a possible explanation for this behavior, it has been proposed that the dipole mode might be coupled to large amplitude shape fluctuations. This mechanism is not incorporated in the transport model presented in this review. It will be interesting to include the large amplitude surface fluctuations into the model so that a unified description of strength functions becomes possible.

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Appendix

A Irreversible processes in extended TDHF

Given the density matrix at a time t , its evolution during a time interval Δt may be expressed according to

$$\begin{aligned} \rho(t + \Delta t) = & U(t + \Delta t, t) \rho(t) U^\dagger(t, t + \Delta t) \\ & - \frac{i}{\hbar} \int_t^{t+\Delta t} ds U(t + \Delta t, s) K_I(s) U^\dagger(s, t + \Delta t) \end{aligned} \quad (69)$$

where the first term represents the pure mean-field evolution and the second term is the perturbation caused by the collision term during the time interval Δt . Thus, starting from the natural states at time t , the first terms is diagonal in the propagated basis while the second is not,

$$\rho_{\alpha\alpha'}(t + \Delta t) \simeq n_\alpha(t) \delta_{\alpha\alpha'} - \frac{i}{\hbar} \int_t^{t+\Delta t} ds \langle \bar{\alpha}(s) | K_I(s) | \bar{\alpha}'(s) \rangle (t)$$

Using the weak coupling approximation in combination with the first order perturbation theory, evolution of occupation numbers can be transformed into a generalized master equation which account for the Pauli principle [99]:

$$\frac{d}{dt} n_\alpha(t) = \int_{t_0}^t ds \left\{ (1 - n_\alpha(s)) \mathcal{W}_\alpha^+(t, s) - n_\alpha(s) \mathcal{W}_\alpha^-(t, s) \right\} \quad (70)$$

where the gain $\mathcal{W}_\lambda^+$ and loss $\mathcal{W}_\lambda^-$ kernels are given by

$$\mathcal{W}_\alpha^+(t, s) = \frac{1}{\hbar^2} \sum Re \left\{ \langle \alpha\delta | v_{12} | \lambda\beta \rangle_A |_t \langle \lambda\beta | v_{12} | \alpha\delta \rangle_A |_s \right\} n_\lambda(s) n_\beta(s) (1 - n_\delta(s)) \quad (71)$$

and

$$\mathcal{W}_\alpha^-(t, s) = \frac{1}{\hbar^2} \sum Re \left\{ \langle \alpha\delta | v_{12} | \lambda\beta \rangle_A |_t \langle \alpha\beta | v_{12} | \lambda\delta \rangle_A |_s \right\} (1 - n_\lambda(s)) (1 - n_\beta(s)) n_\delta(s). \quad (72)$$

By assuming correlation of the residual interaction much smaller than the relaxation time of the occupation numbers, we can approximate the master eq. (70) further in the form of a relaxation ansatz as

$$\frac{dn_\alpha}{dt} = (1 - n_\alpha(t)) \mathcal{G}_\alpha(t) - n_\alpha(t) \mathcal{L}_\alpha(t) \quad (73)$$

where $\mathcal{G}_\alpha(t) = \int_{t_0}^t \mathcal{W}_\alpha^+(t, s) ds$ and $\mathcal{L}_\alpha(t) = \int_{t_0}^t \mathcal{W}_\alpha^-(t, s) ds$. Expression (73) helps to understand the effect of the incoherent collision term. While mean-field theories alone are unable to describe the reorganisation of single-particle degrees of freedom leading for instance to thermal equilibrium, the incoherent collision terms drives occupation numbers to a local equilibrium value $n_\alpha^{eq}(t)$ within a time scale τ_α^{rel} , which are given by

$$\begin{cases} \tau_\alpha^{rel}(t) = (\mathcal{L}_\alpha + \mathcal{G}_\alpha)^{-1} \\ n_\alpha^{eq}(t) = \mathcal{L}_\alpha / (\mathcal{L}_\alpha + \mathcal{G}_\alpha) \end{cases} \quad (74)$$

B Discussion on the linearization of mean-field equation with a stochastic term.

In the stochastic transport theory, the RPA amplitudes $z_\lambda^{(n)}(t) = \text{Tr} O_\lambda \delta \rho^{(n)}(t)$ evolves according to the following stochastic equations,

$$i\hbar \frac{d}{dt} z_\lambda^{(n)} + \hbar \omega_\lambda z_\lambda^{(n)} - \text{Tr}[O_\lambda, A(t)] \rho_0 = \int_{t_0}^t \mathcal{K}_\lambda^I(t-s) ds + F_\lambda^{(n)}(t) \quad (75)$$

where

$$\mathcal{K}_\lambda^I(t) = K_{\lambda\lambda}^I(t) z_\mu^{(n)}(s) + \widetilde{K}_{\lambda\lambda}^I(t) z_\mu^{*(n)}(s). \quad (76)$$

Here, the collisional coupling between RPA states is neglected and $F_\lambda^{(n)}$ denotes the projection of the stochastic term on the RPA state λ ,

$$F_\lambda^{(n)}(t) = \text{Tr} \left(O_\lambda \delta K^{(n)}(t) \right) = \text{Tr} \left([v_{12}, O_\lambda] \delta C_{12}^{(n)}(t) \right). \quad (77)$$

From the Langevin eq. (75), it is possible to deduce an equation for the second moment of the RPA amplitudes

$$\sigma_\lambda(t) = \overline{|z_\lambda^{(n)}(t)|^2} - \overline{|z_\lambda^{(n)}(t)|}^2 \quad (78)$$

which evolves as [54]

$$\frac{d}{dt} \sigma_\lambda = -\mathcal{D}_\lambda(t) \sigma_\lambda + 2\mathcal{S}_\lambda(t). \quad (79)$$

Here,

$$\mathcal{D}_\lambda(t) = -2\text{Re} \int_{t_0}^t \mathcal{K}_\lambda^I(t-s) ds \quad (80)$$

denotes the friction induced by the incoherent mechanism, while

$$\mathcal{S}_\lambda(t) = \text{Re} \int_{t_0}^t \overline{F_\lambda^{(n)}(t)} F_\lambda^{(n)*}(s) ds \quad (81)$$

is the diffusion coefficient associated with the mode. These coefficients $\mathcal{S}_\lambda$ and $\mathcal{D}_\lambda$ are linked through the quantal fluctuation-dissipation theorem at finite temperature,

$$\mathcal{S}_\lambda(t) = \frac{\mathcal{D}_\lambda(t)}{4} \coth \left(\frac{\hbar \omega_\lambda}{2T} \right). \quad (82)$$

Starting from the relation (24), the coherent term takes the form [54],

$$K_C(\rho) = -\frac{i}{\hbar} \int_{t_0}^t ds \sum_{\lambda} [h_{\lambda}^{\dagger}(t), U(t, s) F_{\lambda}(s) U^{\dagger}(t, s)] - h.c. \quad (83)$$

where the sum runs over the time-dependent collective RPA modes and

$$F_{\lambda}(s) = \left(\sigma_{\lambda} + \frac{1}{2} \right) (1 - \rho(s)) h_{\lambda}(s) \rho(s) - \left(\sigma_{\lambda} - \frac{1}{2} \right) \rho(s) h_{\lambda}(s) (1 - \rho(s)). \quad (84)$$

Expression (83) is nothing more than Eq. (25) when the notation $\sigma_{\lambda} = N_{\lambda} + \frac{1}{2}$ as been introduced, $N_{\lambda}(t)$ being referred to the occupation number of RPA phonons.

C Discussion on the residual interaction

The effective Skyrme [100] interaction, which is employed to calculate single-particle wave functions and the RPA states, is parameterized according to [71]),

$$\begin{aligned} V(\mathbf{r}_1, \mathbf{r}_2) = & \quad t_0(1 + x_0 P_{\sigma}) \delta(\mathbf{r}) & \text{central term} \\ & + \frac{1}{2} t_1(1 + x_1 P_{\sigma}) [\mathbf{P}'^2 \delta(\mathbf{r}) + \delta(\mathbf{r}) \mathbf{P}^2] & \text{p dependent term} \\ & + t_2(1 + x_2 P_{\sigma}) \mathbf{P}' \cdot \delta(\mathbf{r}) \mathbf{P} \\ & + \frac{1}{6} t_3(1 + x_3 P_{\sigma}) [\rho(\mathbf{R})]^{\sigma} \delta(\mathbf{r}) & \rho - \text{dependent term} \\ & + i W_0 \sigma \cdot [\mathbf{P}' \times \delta(\mathbf{r}) \mathbf{P}] & \text{spin - orbit term} \end{aligned} \quad (85)$$

Here $\mathbf{r}$ and $\mathbf{R}$ are the relative distance and the center of position of two-particle system, and $\mathbf{P}$ denotes relative momentum operator,

$$\begin{aligned} \mathbf{r} &= \mathbf{r}_1 - \mathbf{r}_2, & \mathbf{R} &= \frac{1}{2} (\mathbf{r}_1 + \mathbf{r}_2) \\ \mathbf{P} &= \frac{1}{2} (\mathbf{p}_1 - \mathbf{p}_2) \\ \sigma &= \sigma_1 + \sigma_2 & P_{\sigma} &= \frac{1}{2} (1 + \sigma_1 \cdot \sigma_2) \end{aligned} \quad (86)$$

where σ are the usual Pauli matrices. There exist different parameterization of the Skyrme interactions, which can be found for instance in refs. [101, 102, 71]. The parameters of the two interactions, *Sly4* and *SGII*, used in the paper, are given in table (4). Skyrme interactions have been employed with great success in the mean-field description of finite and infinite nuclear system [101, 23].

In expression (85), momentum dependent terms, i.e. the non-local part of the interaction, may be understood as the second order expansion of a finite range force [101]. Such an expansion is valid when a typical momentum transfer Δp is much smaller than the inverse of range of the interaction $\hbar/\Delta a$. In mean-field processes at low energy, typical momentum transfer involved is of the order of the Fermi momentum $p_F \simeq 265$ MeV/c and the range of the interaction is around $\Delta a = 0.5$ fm, which leads to $\hbar/\Delta a \simeq 2$. This simple argument

Interaction	t_0 (MeV.fm ³)	t_1 (MeV.fm ⁵)	t_2 (MeV.fm ⁵)	t_3 (MeV.fm ^{3+3σ})	x_0	x_1	x_2	x_3	σ	W_0 (MeV.fm ⁵)
Sly4 [71]	-2488.91	486.82	-546.39	13777	0.834	-0.344	-1	1.354	$\frac{1}{6}$	123
SGII [69]	-2645	340	-41.9	15595	0.09	-0.0588	1.425	0.06044	$\frac{1}{6}$	105

Table 4: Parameters of the Skyrme interactions used in this paper.

Figure 34: Left: GQR strength function of ^{40}Ca calculated in the extended RPA by employing the standard Skyrme interaction in the incoherent collision term. Right: GQR strength function of ^{40}Ca calculated in the extended RPA by employing the modified Skyrme interaction in the incoherent collision term with an effective range $\beta = 1$ fm. In both parts, thin lines represent the RPA strength function.

provides a support for validity of a treatment of the finite range as a quadratic expansion in p of a zero range effective interactions in the mean-field description of nuclear properties at low energy. However, zero range effective interactions are not suitable for calculating the incoherent collision term because they involve high momentum transfer. In left panel of figure (34), we present the calculated GQR strength function in ^{40}Ca . Solid line is the result of the extended RPA result obtained by including only the incoherent mechanism with the standard Skyrme interaction, while thin line is the result of RPA calculations. We see that the strength is shifted down by an unreasonable amount of about 10 MeV as compared to the RPA value. A similar result was reported in [10]. As shown in ref. [63], this may be related to the fact that a zero range interaction with a quadratic momentum dependence does not involve a natural cut-off for large values of the momentum transfer. This problem is less important for the imaginary part of the self-energy, since the high frequency matrix elements are cut off by energy conserving Lorentzian factor in $\Gamma_\lambda(\omega)$. However, the situation is different in the real part of the self-energy. Unrealistic p dependence of zero range interactions give rise to a large shift in strength distributions.

Figure 35: GDR strength function of ^{120}Sn calculated in the extended RPA by including only the incoherent damping mechanism at $T=2.6$ MeV. Different curves correspond to different values of the cut-off parameter $\beta = 1.0$ fm (dashed line), $\beta = 1.4$ fm (thick line), $\beta = 2.0$ fm (thin line). Taken from reference [64]

For a more accurate description of the real and imaginary part of the incoherent self-energy, we need to use a realistic treatment of the finite range interaction in the incoherent collision term. One may think to use a finite range interaction such as Gogny. However, numerical calculations of the collision term with such an interaction require a large amount of numerical effort. For the time being, we propose a practical method for calculation of the collision term [63]. In order to incorporate the finite range effect approximately, we introduce a Gaussian cut-off factor in matrix elements of the central term of the Skyrme interaction in eq. 85), noted v_S , as,

$$\langle ij|v|kl \rangle = \langle ij|v_S|kl \rangle \cdot \exp\left(-\beta^2 \frac{\langle \mathbf{q}^2 \rangle}{\hbar^2}\right)$$

Here, β is the cut-off factor related to the range of the interaction. The quantity $\langle \mathbf{q}^2 \rangle$ defined by the relation $\langle ij|\delta(\mathbf{r})|kl \rangle \langle \mathbf{q}^2 \rangle = \langle ij|\mathbf{q}^2\delta(\mathbf{r}) + \delta(\mathbf{r})\mathbf{q}^2|kl \rangle$, provides a measure for the relative momentum. For low momentum transfer, expansion of the gaussian factors leads to a similar expression as the Skyrme interaction, the matrix elements involved in the cut-off factor being the same as those calculated using in the Skyrme force. As a result, numerical effort with the modified Skyrme interaction are not increased compared to standard Skyrme interaction. In right panel of figure (34), we present the calculated GQR strength function in ^{40}Ca . Solid line is the result of the extended RPA result obtained by including only the incoherent mechanism with the modified Skyrme interaction, while thin line is the result of RPA calculations. The new shift in the strength function is much less than the one obtained with the standard Skyrme interaction. In figure (35), we present calculated GDR strength in ^{120}Sn at a temperature $T = 2.6$ MeV for different β values. We observe that calculations are rather sensitive to the cut-off parameter β . However, taken into account typical range of effective interactions, we find that $\beta = 1.4$ fm is a suitable value for the cut-off parameter.

D Discussion on collisional coupling

In illustrations presented in this article, we neglected the mixing due to coupling between RPA modes of same multipolarity through the off-diagonal part of self-energy ($K_{\lambda\mu} = 0$ if $\lambda \neq \mu$). Such a coupling through the decay channels may lead to some specific effects [11], which can be illustrated with a schematic two-level model [59]. Let us consider a Hilbert space containing three types of states : (i) the ground state $|0\rangle$ at energy 0, (ii) an ensemble of excited states $|\lambda\rangle$ with energies $\hbar\omega_\lambda$ associated with the Hamiltonian H_1 , and (iii) their decay channel $|i\rangle$ with an energies ε_i associated with the Hamiltonian H_2 . The decay process is assumed to proceed via coupling v_{12} , which is supposed to act only between states $|\lambda\rangle$ and $|i\rangle$. The system is excited by an external perturbation $A(t)$ which is assumed to couple only the ground state to the excited states $|\lambda\rangle$. Time evolution of the system, $|\Psi(t)\rangle$, can be determined using the time dependent Schrödinger equation with a total Hamiltonian $H(t) = H_1 + H_2 + v_{12} + A(t)$. Projecting $|\Psi(t)\rangle$ on states $\{|0\rangle, |\lambda\rangle, |i\rangle\}$ and eliminating the $z_i = \langle i|\Psi(t)\rangle$ component, we obtain a transport equation for the amplitude $z_\lambda = \langle \lambda|\Psi(t)\rangle$,

$$i\hbar \frac{\partial}{\partial t} z_\lambda(t) - \hbar\omega_\lambda z_\lambda(t) - A_\lambda = \sum_\mu \int_{t_0}^t dt' K_{\lambda\mu}(t-t') z_\mu(t') \quad (87)$$

The right hand side can be regarded as a non-Markovian collision term,

$$K_{\lambda\mu}(t-t') = -\frac{i}{\hbar} \sum_i V_{\lambda i} V_{\mu i}^* e^{-i(t-t')\varepsilon_i/\hbar}$$

with $A_\lambda = \langle \lambda|A|0\rangle$ and $V_{\lambda i} = \langle \lambda|V_{12}|i\rangle$. In terms of the Fourier transform, this leads to a matrix equation for the amplitudes that is analogous to the one obtained in the extended RPA context (eq. (54)) with a self-energy,

$$K_{\lambda\mu}(w) = \sum_i \frac{\langle \lambda|v_{12}|i\rangle \langle i|v_{12}|\mu\rangle}{w + i\eta - \varepsilon_i} \quad (88)$$

where ε_i play the role of $2p - 2h$ states and the states λ, μ correspond to the RPA states.

We apply the above formalism to a schematic model of only two states $|\lambda\rangle$ called $|1\rangle$ and $|2\rangle$ coupled with a constant interaction $\langle\lambda|v_{12}|i\rangle = v_\lambda$ to a dense spectrum of equally spaced decay channels, with $\Delta\varepsilon = \varepsilon_i - \varepsilon_j$. Two simple limiting cases may appear:

- **Decay towards independent channels :** In this case, we suppose that each state λ decays towards its own decay phase-space that is independent from decay of other states. Then, eq. (88) could be replaced by its diagonal part:

$$K_{\lambda\mu}(w) = \delta_{\lambda\mu} \sum_i \frac{|v_\lambda|^2}{\omega + i\eta - \varepsilon_i} \quad (89)$$

Taking the continuous limit $\Delta\varepsilon \rightarrow 0$ and $v_\lambda \rightarrow 0$ keeping $\Gamma_\lambda = 2\pi v_\lambda^2/\Delta\varepsilon$ constant. Each collective state decay independently from each other and the strength results from the independent decay of each RPA mode with its own width. When more than two states are considered and when Γ_λ is much larger than the collective level spacing, the formalism leads to the well known Ericson fluctuations [103].

- **Decay towards common channels :** A different situation appears when the states λ decay towards the same states $|i\rangle$. For simplicity, let us assume that the interaction is independent of λ $v_\lambda = v$ leading to a constant parameter $\Gamma_\lambda = \Gamma$. Then, we have $K_{\lambda\mu}(w) = -i\Gamma$. Assuming that $\hbar\omega_1 = E_0 + \Delta E$ and $\hbar\omega_2 = E_0 - \Delta E$ the strength function exhibits two poles at energies,

$$\omega_\pm = E_0 \pm \sqrt{\Delta E^2 - \Gamma^2} - i\Gamma \quad \text{for} \quad \Delta E > \Gamma \quad (90)$$

$$\omega_\pm = E_0 \pm i\sqrt{\Gamma^2 - \Delta E^2} - i\Gamma \quad \text{for} \quad \Delta E < \Gamma. \quad (91)$$

In the weak coupling limit, the collision term induces a shift, which reduces the distance between states, and a width, which increases with increasing magnitude of the coupling v . On the other hand, in the strong coupling case, the two states converge towards the same energy. One of the state becomes very broad while the second state becomes very narrow. The width of the narrow state is inversely proportional to the damping width Γ , and is given by $\Delta E/\Gamma$. This effect, which appears as a specific coherent effect in the strong coupling limit is known as the Dicke superradiance in quantum optics [104], and may be considered as an analogous to the motional narrowing [105]. For collective vibration, it corresponds to the transition from zero sound to the first sound, that is a narrow vibration generated through the presence of a strong collision term.

The response of nuclear system is certainly in between these two limiting cases. Indeed, states with same multipolarity select naturally the same $2p - 2h$ decay channels due to quantum number selection rules. However, the decay of the collective states may be regarded as occurring independently, since the magnitude or equivalently the relative phases of coupling matrix elements fluctuates from one collective state to the other. In order to determine in which regime the nucleus dynamics lies, one should evaluate the strength function. In figure

(36), we present examples of strength functions for the monopole and dipole excitation of ^{40}Ca (adapted from ref. [59]). Thick lines show the calculations when the collisional coupling between RPA states is included, while thin lines are the results without the collisional coupling. We observe that the collisional coupling has a weak effect on the collective response. Indeed, while the height of the peak changes slightly, the position and width are almost unaffected. Similar results have been obtained in all medium and heavy nuclei at zero and finite temperature, which indicates that collisional coupling is weak and that the nuclear vibration are not yet in the first sound regime. Therefore, in this report, we do not consider the collisional coupling in any further.

Figure 36: Strength distribution obtained with the extended RPA method including incoherent effect at $T = 0$ MeV (top) and $T = 4$ MeV (bottom) for the GMR (left) and GDR (right) of ^{40}Ca as a function of the energy. In all cases, the thick line presents the results when the incoherent coupling is included while thin line represents results with only the diagonal part of the self-energy.

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