The Star Formation Rate and Dense Molecular Gas in Galaxies

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ABSTRACT

HCN luminosity is a tracer of dense molecular gas, $n(H_2) \gtrsim 3 \times 10^4 \text{cm}^{-3}$, associated with star-forming giant molecular cloud (GMC) cores. We present the results and analysis of our survey of HCN emission from 65 infrared galaxies including 9 ultraluminous infrared galaxies (ULIGs, $L_{\text{IR}} \gtrsim 10^{12} L_\odot$), 22 luminous infrared galaxies (LIGs, $10^{11} L_\odot < L_{\text{IR}} < 10^{12} L_\odot$) and 34 normal spiral galaxies with lower IR luminosity (most are large spiral galaxies). We have measured the global HCN line luminosity and the observations are reported in Gao & Solomon (2003, Paper I). This paper analyzes the relationships between the total far-IR luminosity a tracer of the star formation rate, the global HCN line luminosity a measure of the total dense molecular gas content, and the CO luminosity a measure of the total molecular content. We find a tight linear correlation between the IR and HCN luminosities $L_{\text{IR}}$ and $L_{\text{HCN}}$ (in the log-log plot) with a correlation coefficient $R = 0.94$, and an almost constant average ratio $L_{\text{IR}}/L_{\text{HCN}} = 900L_\odot/\text{Kkm s}^{-1}\text{pc}^2$. The IR–HCN linear correlation is valid over 3 orders of magnitude including ULIGs, the most luminous objects in the local universe. The direct consequence of the linear IR–HCN correlation is that the star formation law in terms of dense molecular gas content has a power law index of 1.0. The global star formation rate is linearly proportional to the mass of dense molecular gas in normal spiral galaxies, LIGs, and ULIGs. This is strong evidence in favor of star formation as the power source in ultraluminous galaxies since the star formation in these galaxies appears to be normal and expected given their high mass of dense star-forming molecular gas.

The HCN–CO correlation is also much tighter than the IR–CO correlation. We suggest that the non-linear correlation between $L_{\text{IR}}$ and $L_{\text{CO}}$ may be a consequence of the stronger and perhaps more physical correlations between $L_{\text{IR}}$ and $L_{\text{HCN}}$ and between $L_{\text{HCN}}$ and $L_{\text{CO}}$. Thus, the star formation rate indicated by $L_{\text{IR}}$ depends on the amount of dense molecular gas traced by HCN emission, not the total molecular gas traced by CO emission. One of the main arguments in favor of an AGN as the power source in ULIGs is the anomalously high ratio $L_{\text{IR}}/L_{\text{CO}}$ or $L_{\text{IR}}/M(H_2)$ or high star formation rate per $M_\odot$ of gas, compared with that from normal spiral galaxies. This has been interpreted as indicating that a dust enshrouded AGN is required to produce the very high luminosity. Viewed in terms of the dense gas mass the situation is completely different. The ratio $L_{\text{IR}}/L_{\text{HCN}}$ or $L_{\text{IR}}/M_{\text{dense}}$ a measure of the star formation rate per solar mass of dense gas is essentially the same in all galaxies including ULIGs. $L_{\text{IR}}/M_{\text{dense}}$ is virtually independent of galaxy luminosity and on average $L_{\text{IR}}/M_{\text{dense}} \approx 90L_\odot/M_\odot$, about the same as in GMC cores but much higher than in GMCs. We find that ULIGs simply have a large quantity of dense molecular gas and thus produce a prodigious starburst which heats the dust, produces the IR, and blocks all or most optical radiation. The HCN global luminosity may be used as an indicator of the star formation rate in high redshift objects including hyperluminous galaxies.
The HCN/CO ratio is an indicator of the dense molecular gas fraction and gauges the globally averaged molecular gas density. We find that the HCN/CO ratio is a powerful starburst indicator. All galaxies in our sample with a high dense gas mass fraction indicated by \( \frac{L_{\text{HCN}}}{L_{\text{CO}}} > 0.06 \) are LIGs or ULIGs. Normal spirals all have similar and low dense gas fractions \( \frac{L_{\text{HCN}}}{L_{\text{CO}}} = 0.02 \) to 0.05. The global star formation efficiency depends on the fraction of the molecular gas in a dense phase.

Subject headings: galaxies: infrared — galaxies: ISM — galaxies: starburst — infrared: galaxies — ISM: molecules — radio lines: galaxies

1. INTRODUCTION

Stars are born in the molecular interstellar medium, the raw material for star formation. In the Milky Way, all star formation essentially takes place in molecular clouds and most star formation takes place in giant molecular clouds (GMCs, Solomon, Sanders, & Scoville 1979) with mass \( M > 10^5 M_\odot \), and not the diffuse neutral ISM dominated by atomic hydrogen (extended HI gas disk). The star formation rate (SFR) of molecular clouds can be estimated from the far-infrared (FIR) luminosity emitted by the warm dust heated by embedded high mass OB stars (e.g., Mooney & Solomon 1988). The mass of molecular gas can be determined from the CO luminosity calibrated by \( \gamma \) ray flux from the interaction of cosmic rays with hydrogen molecules (e.g., Bloemen et al. 1986) or by dynamical cloud masses determined from CO kinematics for virialized individual molecular clouds (Solomon et al. 1987; Young & Scoville 1991). These methods are in good agreement (see Solomon & Barrett 1991). All strong high mass star formation regions are associated with GMCs, especially the cores of GMCs. The ratio of FIR luminosity to the CO luminosity, or to the cloud mass, a measure of the SFR per solar mass of the cloud and an indicator of star formation efficiency (SFE), ranges over a factor of 100 for different clouds, and over a factor of 1000 from clouds to the cores of GMCs (e.g., Mooney & Solomon 1988; Plume et al. 1997). An understanding of the physical conditions in GMCs and their relation to galactic dynamics is a prerequisite to the understanding of the star formation process, the SFR in galaxies and starbursts.

Star formation in galaxies is closely tied up with the local gas density, as formulated in the Schmidt (1959) law although the important component is the molecular gas. Globally, the SFR correlates with the molecular gas content in galaxies, as traced by CO emission, including luminous and ultraluminous infrared galaxies (LIGs and ULIGs\(^1\)). In Galactic star-forming regions, active high mass star formation is intimately related to the very dense molecular gas in the cores. While the canonical molecular gas tracers CO shows strong emission in cloud cores, it is not specific enough to reveal their star formation potential. The bulk of the cloud material traced by CO observations is in the GMC envelopes and is at a much lower density. The physical conditions of active star-forming GMC cores are better revealed by emission from very high transition CO lines (in the submillimeter regime) and high dipole-moment molecules like CS and HCN.

HCN is one of the most abundant high dipole-moment molecules which traces molecular gas at densities \( n(H_2) \approx 3 \times 10^4 \text{cm}^{-3} \), more than two orders of magnitude higher than that traced by CO \( (\approx 300 \text{cm}^{-3}) \).

\(^1\)LIGs: \( 10^{12} L_\odot \geq L_{\text{IR}} > 10^{11} L_\odot \), ULIGs: \( L_{\text{IR}} \approx 10^{12} L_\odot \) (to be exactly, \( 10^{11.9} L_\odot \) in this paper). For a definition of the total IR (8 to 1000\( \mu \)m) luminosity \( L_{\text{IR}} \) and FIR luminosity \( L_{\text{FIR}} \), see Sanders & Mirabel (1996). \( L_{\text{IR}} \) is generally larger, by up to \( \sim 20\% \) as it includes both 12 and 25\( \mu \)m emission, than \( L_{\text{FIR}} \). However, we often simply refer the total IR emission as FIR in this paper.
Many HCN(1-0) observations have already been conducted by different groups (Nguyen-Q-Rieu et al. 1989, 1992; Henkel et al. 1990; Solomon, Downes, & Radford 1992; Israel 1992; Helfer & Blitz 1993; Aalto et al. 1995; Curran, Aalto, & Booth 2000) in a variety of nearby galaxies. These previous observations contain small samples with frequent overlap in the sample selection. The total number of galaxies detected in HCN is still small (∼30), and for many galaxies only the central position was observed. Gao & Solomon (2003, hereafter Paper I), in the companion paper, have presented a systematic survey of global HCN luminosity that more than doubles the number of galaxies (∼60) observed in HCN.

Although the dense molecular gas is strongly concentrated in the central regions (∼1 kpc), HCN mapping of a dozen nearby galaxies out to a diameter of ∼D_{25}/4 (Gao 1996, 1997; Gao & Solomon in preparation) show that a substantial fraction of HCN emission originates from the inner disks outside the central ∼1 kpc. All previous HCN observations in external galaxies including a few HCN maps (e.g., Nguyen-Q-Rieu et al. 1992; Reynaud & Downes 1997; Helfer & Blitz 1997a) have primarily been observations of the galactic central regions. In most cases, the total HCN emission from the entire galaxy, has not been measured. The recent HCN observations in 20 Seyfert galaxies are primarily of the galaxy centers and many are non-detections or marginal detections (Curran et al. 2000). Confusion may occur when the results drawn from the HCN observations of central regions of galaxies are compared with the global properties of galaxies. Aalto et al. (1995) have not found a correlation between the molecular line intensity ratio, I_{CO}/I_{HCN}, and FIR emission, or measures of star-forming activity in their sample of 10 interacting galaxies, which appeared to be in conflict with the original findings of a tight FIR–HCN correlation in another sample of 10 LIGs/ULIGs and spiral galaxies where the total HCN emission was measured (Solomon et al. 1992). The situation has now been clarified, by our new HCN survey of ∼60 galaxies (Paper I), which measured the global HCN emission in a wide range of galaxies. There is indeed a tight FIR–HCN correlation in a statistically significant HCN sample.

It is clear from all previous observations that the molecular gas in the central regions of spiral galaxies, starbursts and LIGs/ULIGs, is much denser than the molecular ISM in the disks of spiral galaxies (e.g., Nguyen-Q-Rieu et al. 1992; Solomon et al. 1992; Helfer & Blitz 1997a; Wild & Eckart 2000). From various observational studies of star-forming regions of GMCs in the Milky Way, it is also clearly shown that the active star-forming regions in the disk are the dense molecular cloud cores (e.g., Mooney & Solomon 1988; Plume et al. 1997; Pirogov 1999; Evans 1999) rather than the entire molecular envelopes of GMCs. The SFE of active star-forming clouds that are associated with IR sources readily apparent on IRAS 60 and 100 µm images can be 100 times higher than the IR-quiet clouds with no apparent IR sources revealed by IRAS (Mooney & Solomon 1988). SFE (L_{IR}/L_{CO}) can vary over a factor of 100 as well in galaxies.

Normal spiral galaxies have a SFE similar to that of Galactic GMCs, whereas the SFE of ULIGs/LIGs and advanced mergers can be more than an order of magnitude larger (e.g., Solomon & Sage 1988; Solomon et al. 1997; Gao & Solomon 1999). These differences in SFE can be understood in terms of the different dense molecular gas content as traced by HCN observations. For 10 galaxies including both LIGs/ULIGs and normal spiral galaxies, Solomon et al. (1992) show that there is a tight correlation between the ratio of the IR and CO luminosities L_{IR}/L_{CO} and the HCN/CO luminosity ratio L_{HCN}/L_{CO} in addition to the excellent correlation between L_{IR} and L_{HCN}. This is now fully confirmed with our current HCN study of a sample of 65 galaxies (§3). These correlations demonstrate a close relationship between the SFR and the dense molecular gas reservoir in galaxies. The SFE depends on the fraction of available molecular gas in a dense phase (L_{HCN}/L_{CO}), and the dense molecular content of even gas-rich spirals is much less than that of LIGs/ULIGs of comparable total molecular gas content (Solomon et al. 1992; Radford 1994). Since CO emission traces most of the molecular gas mass and is not
necessarily a specific tracer of the dense molecular gas (e.g., Mauersberger & Henkel 1993; Evans 1999) or the IR luminosity from star formation (Mooney & Solomon 1988), CO alone can give a misleading picture of the densest molecular gas in a galaxy.

In Milky Way Galactic plane GMCs, for example, essentially all OB star formation occurs in the cores of GMCs with strong CS and HCN emission. The ratios of CO/CS and CO/HCN intensities for Galactic disk GMCs are much larger than for Galactic center clouds (Lee, Snell, & Dickman 1990; Jackson et al. 1996; Plume et al. 1997; Helfer & Blitz 1997b) and an order of magnitude larger than for the archetypal ULIG Arp 220 (Solomon, Radford, & Downes 1990). In some galaxies, the molecular gas in the center is much more prominent in HCN emission than in CO. A good example is the center of the Seyfert 2/starburst hybrid galaxy NGC 1068, where all interferometric maps demonstrate that the nuclear region is more prominent than the rings or spiral arms when viewed in HCN while the opposite is true in CO emission (Tacconi et al. 1997, 1994; Helfer & Blitz 1995; Jackson et al. 1993). Similar trend is also observed in the centers of M51 and NGC 1097 (e.g., Kohno et al. 1996, 2003).

Here, we utilize a large, statistically significant sample with observations of global HCN emission from 65 spiral galaxies, LIGs and ULIGs — 53 from the systematic HCN survey of Paper I, 10 from Solomon, Downes, & Radford (1992) plus 2 from the literature. These galaxies range over 3 orders of magnitude in FIR luminosity. We analyze the various relationships among the global HCN, CO, and FIR luminosities. We further discuss the physical relationship between the dense molecular gas content and the rate of high mass star formation in galaxies.

The HCN sample and observations are briefly reviewed in §2. §3 presents the results. §3.1 is a comparison of the IR–HCN and IR–CO correlations. §3.2 concentrates on the importance of the dense gas mass fraction and the star formation efficiency. §3.3 and the appendix present multi-parameter fits to the data and the effect of dust temperature. §3.4 has a brief summary of all results. In §4.1 to 4.3, we discuss the importance of HCN as a tracer of star-forming molecular gas, the star formation rate as a function of the dense gas mass, and the global star formation law. §4.4 discusses the HCN/CO ratio as a starburst indicator. §4.5 discusses the origin of FIR emission from spirals and ULIGs. §4.6 briefly speculates on the implications for hyperluminous infrared galaxies at high-z. Finally, we summarize the main points of our study in §5.

2. THE HCN SURVEY

The detailed descriptions of our survey sample and HCN (and CO) observations were given in the companion paper (Paper I). Our HCN survey sample is drawn from samples in recent CO observations of galaxies showing strong CO emission (e.g., the CO antenna brightness temperature much larger than 100 mK for normal spiral galaxies, and larger than 20 mK for LIGs/ULIGs). All truly IR-bright galaxies with 60 or 100µm emission larger than 50 or 100 Jy respectively have also been included. Essentially all galaxies with strong CO and IR emission in northern sky have been chosen for the HCN survey.

We carried out several observing runs mainly with the former NRAO2 12m telescope at Kitt Peak, the IRAM 30m telescope at Pico Veleta near Granada, Spain, and the FCRAO 14m telescope for most of our observations. We here only mention our observing strategy of using different telescopes. The IRAM

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2The National Radio Astronomy Observatory is operated by Associated Universities, Inc., under cooperative agreement with the National Science Foundation.
30m was mostly used to observe ULIGs (rather distant) and to map out some nearby starburst galaxies of smaller optical diameters, given the small matching beamsize as compared with the source extent for effective mapping. Essentially all other observations were conducted with the NRAO 12m so that one beam measurement can cover almost all HCN emission from relatively distant galaxies including some merging galaxy pairs. Also almost all nearby large spiral galaxies have been at least mapped along the major axes with the NRAO 12m, and only a few of them were initially tried mapping with then the QUARRY receivers at the FCRAO 14m (due to its limited sensitivity).

The goal of the HCN survey of Paper I is to determine the total HCN emission from the whole inner disks or the entire galaxies in a large sample of galaxies with a wide range of IR luminosity. Combined with the HCN data of 12 galaxies, 10 from Solomon et al. (1992) including half-dozen LIGs/ULIGs, and the other two galaxies, M51 (Nguyen-Q-Rieu et al. 1992) and NGC 4945 (Henkel, Whiteoak, & Mauersberger 1994) that were mapped extensively in HCN, we have a large, statistically significant sample of 65 galaxies with globally measured HCN emission. In addition, there are more than a dozen nearby large spiral galaxies with HCN detections towards the galactic nuclei (Nguyen-Q-Rieu et al. 1992; Helfer & Blitz 1993; Aalto et al. 1995; Curran et al. 2000) including LIGs (e.g., NGC 3256, Casoli, Dupraz & Combes 1992) that do not overlap with our HCN survey sample. In principle, we could further enlarge our HCN sample to a total of about 80 galaxies. These observations of nearby galaxies did not measure the total HCN emission, however, with only the central/nuclear HCN emission observed. Therefore, most HCN data in these nearby galaxies can thus only be used to set lower limits to the total HCN luminosities. Further HCN observations at least along the major axes are required to map out the total HCN emission. For the sake of completeness, consistency, and uniformity of our sample, these galaxies with lower HCN limits in the literature are not included in our sample.

The derived global properties of the line luminosities and various luminosity ratios of galaxies in the HCN survey sample are listed in Table 1 (cf. Table 5 in Paper I), together with a dozen galaxies where the total HCN luminosities are available or can be estimated from the literature. There are also several other fairly distant galaxies in Curran et al. (2000) and Aalto et al. (1995) in which the global HCN luminosities were supposedly measured, but we did not include them in our sample as their data seem to have low signal-to-noise ratio. Mrk273, one of the galaxies that overlaps in both of our sample and that of Curran et al. (2000), for example, was claimed to be detected with an extremely large HCN/CO luminosity ratio of one. Our high quality IRAM 30m spectra (Paper I), however, clearly indicate an HCN/CO intensity ratio of $\sim 1/6$ and a HCN/CO luminosity ratio of 0.23, which is consistent with all other ULIGs that have been significantly detected in HCN so far, rather than an unrealistic ratio of $\sim 1$.

We also have some limits to the total HCN line emission owing to insufficient mapping of HCN (about 10 % of the galaxies in our sample), but our HCN lower limits are probably close to the true values as we have some off-nucleus HCN measurements besides the central beams on the nuclei in most cases. We keep these sensitive HCN limits in Table 1 and include these data for the various analyses.

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3However, Curran et al. (2001) submitted and published several HCN maps (though mostly still very limited spatial coverage of only central 1"–2" regions) of nearby Seyfert galaxies after we submitted our paper.
3. RESULTS AND ANALYSIS

3.1. Comparison of the IR–HCN and IR–CO Correlations

The principal observational result from this survey is the tight linear relation observed between far-infrared luminosity, $L_{\text{IR}}$ and HCN line luminosity $L_{\text{HCN}}$ shown in Fig. 1a. This very good correlation extends over 3 orders of magnitude in luminosity and includes normal spiral galaxies, luminous and ultraluminous infrared galaxies (LIGs/ULIGs). Fig. 1b shows the correlation between $L_{\text{IR}}$ and $L_{\text{CO}}$ of the same sample which has a larger scatter than the FIR–HCN relation and most importantly steepens with higher IR luminosity showing the well-known result that LIGs/ULIGs, although molecular gas rich, have a substantially higher IR luminosity per unit CO luminosity or per solar mass of molecular gas H$_2$ (e.g., Solomon et al. 1997).

A least–square fit using all the data, but excluding HCN limits yields a power law slope of $1.00 \pm 0.05$ and $1.25 \pm 0.08$ for $L_{\text{IR}}$–$L_{\text{HCN}}$ and $L_{\text{IR}}$–$L_{\text{CO}}$ respectively. The corresponding correlation coefficients (squared) are $R^2 = 0.88$ and 0.77. The slopes change to $1.05 \pm 0.05$ and $1.44 \pm 0.08$ for $L_{\text{IR}}$–$L_{\text{HCN}}$ and $L_{\text{IR}}$–$L_{\text{CO}}$ respectively, if an orthogonal fit is used. Including the sources with HCN limits in the fit has little effect on the slope or the fit. The best fit (logarithmic) relation between HCN and IR luminosities excluding galaxies with upper or lower limits is

$$\log L_{\text{IR}} = 1.00(\pm0.05)\log L_{\text{HCN}} + 2.9, \text{ or } L_{\text{IR}}/L_{\text{HCN}} = 900L_\odot/\text{Kkm} \text{ s}^{-1}\text{pc}^2,$$

indeed a linear relation. When we use all galaxies including HCN limits the correlation remains almost the same with $\log L_{\text{IR}} = 0.97\log L_{\text{HCN}} + 3.1$ and with same correlation coefficient (R=0.94).

At first glance of Fig. 1, the correlation between $L_{\text{CO}}$ and $L_{\text{IR}}$ may appear to be nearly as good as that of the $L_{\text{HCN}}$–$L_{\text{IR}}$ correlation even though there is significant difference in the correlation coefficients and the dispersion from the fit. The most obvious difference occurs at the high $L_{\text{IR}}$ end. In Fig. 1b, a line of constant $L_{\text{IR}}/L_{\text{CO}} = 33L_\odot/\text{Kkm} \text{ s}^{-1}\text{pc}^2$ fits the lower luminosity galaxies but lies well below almost all (28/31) of the high luminosity galaxies (LIGs/ULIGs). The correlation between $L_{\text{IR}}$ and $L_{\text{HCN}}$ still fits the ratio determined from lower IR luminosities ($L_{\text{IR}} < 10^{11}L_\odot$), but the correlation between $L_{\text{CO}}$ and $L_{\text{IR}}$ does not. (The lines shown in Fig. 1 are the fits of a fixed slope of 1 to low luminosity galaxies.) The FIR–HCN relation is linear all the way up to $L_{\text{IR}} = 10^{12.5}L_\odot$, but the FIR/CO ratio is systematically higher for high luminosity galaxies than for those of normal luminosity.

One obvious difference between the FIR–CO and FIR–HCN relationships is that the range of CO luminosity is only two orders of magnitude whereas HCN luminosity covers 3 orders of magnitude, almost the same as the IR luminosity. For LIGs/ULIGs the CO is systematically weak compared to the IR. Measurements of CO luminosity of a large sample of ULIGs (Solomon et al. 1997; most of which are not in Fig. 1b since there are no corresponding HCN observations available) show that they all lie above the fitted line in Fig. 1b.

Another way to demonstrate these points is to compare the luminosity ratios of $L_{\text{IR}}/L_{\text{HCN}}$ and $L_{\text{IR}}/L_{\text{CO}}$ with $L_{\text{IR}}$ (Fig. 2). $L_{\text{IR}}/L_{\text{HCN}}$ appears to be nearly independent of $L_{\text{IR}}$, whereas the $L_{\text{IR}}/L_{\text{CO}}$ ratio increases substantially with increasing $L_{\text{IR}}$ as established by previous work (e.g., Young et al. 1989; Sanders, Scoville, & Soifer 1991). These results are also revealed in Table 2, which summarizes the average IR, CO and HCN luminosities among ULIGs, LIGs, normal spiral galaxies, and all galaxies of the entire HCN sample.

In summary, the IR–HCN correlation is linear and is extremely tight over 3 orders of magnitude in
luminosity, when compared to the non-linear IR–CO correlation. While the high luminosity of LIGs/ULIGs requires an elevated SFE of the total molecular gas indicated by $L_{IR}/L_{CO}$, the SFR per unit of dense molecular gas — the SFE of the dense molecular gas indicated by $(L_{IR}/L_{HCN})$ is almost constant and independent of the IR luminosity or total SFR. The tight relationship between $L_{IR}$ and $L_{HCN}$ will be further illustrated by comparing it with other correlations in the following two sections (§3.2 & §3.3).

### 3.2. Dense Molecular Gas Fraction ($L_{HCN}/L_{CO}$) and Star Formation Efficiency (SFE, $L_{IR}/L_{CO}$)

Figure 3 shows a significant correlation between HCN and CO luminosities with a correlation coefficient $R=0.92$ ($R^2 = 0.85$). In all figures, except where $L_{IR}$ is itself explicitly plotted, we distinguish LIGs/ULIGs ($L_{IR} > 10^{11}L_{\odot}$, filled circles) from the less luminous normal spiral galaxies (open circles) so that any systematic difference between these two subsamples can easily be seen. This suggests that the more gas-rich galaxies tend to have more dense molecular gas (and vice versa), and thus are more luminous (Fig. 1). The correlation is much better for normal galaxies (open circles) than for LIGs/ULIGs (filled circles). For normal spiral galaxies there is a tight relationship between the HCN and CO luminosities with a slope of 1.0, shown as the fit line in Fig. 3. There is also a very small dispersion about the fit with $\sigma(\log L_{HCN}) = 0.14$. This good linear HCN–CO correlation is the underlying reason that the FIR–CO correlation is linear for normal galaxies and CO luminosity is reasonably good at predicting their SFR (FIR).

All but 3 of the 31 LIGs have $L_{HCN}$ above the linear fit (Fig. 3) determined from normal spirals, clearly showing that excess HCN emission compared to CO emission is a characteristic of LIGs/ULIGs. An orthogonal fit to all galaxies gives a non-linear relation:

$$\log L_{HCN} = 1.38\log L_{CO} - 4.79, \quad \frac{L_{HCN}}{L_{CO}} = 0.1 \left(\frac{L_{CO}}{10^{16}K\text{ km s}^{-1}\text{pc}^2}\right)^{0.38}.$$  (2)

It is the high ratio of HCN/CO for LIGs/ULIGs which causes the breakdown in the IR–CO relation.

A high ratio of $L_{HCN}/L_{CO}$ is a distinguishing feature of LIGs/ULIGs. As can be seen from Table 2, the average ratio systematically increases as IR luminosity increases. Fig. 4 shows the IR luminosity of all 65 galaxies in the sample as a function of the ratio $L_{HCN}/L_{CO}$. It is immediately apparent that LIGs/ULIGs ($L_{IR} > 10^{11}L_{\odot}$) have systematically higher HCN/CO ratios than normal galaxies. But, for normal spirals only, although there is some scatter with $L_{HCN}/L_{CO} = 0.02 - 0.06$, yet there is no correlation between IR and $L_{HCN}/L_{CO}$. All of the most luminous galaxies, i.e., ULIGs, have high $L_{HCN}/L_{CO} > 0.09$. The most striking result is that all (27/27) galaxies with $L_{HCN}/L_{CO} > 0.06$ are luminous (or ultraluminous). The 2 most luminous galaxies in our sample Mrk 231 and IRAS 17208-0014 have HCN/CO = 0.23 and 0.25 respectively. The molecular ratio $L_{HCN}/L_{CO}$ appears to be 100 percent successful as an indicator of galaxies with infrared starbursts. In § 4 we further discuss the contributions of the disk and nuclear regions to these global values of $L_{HCN}$ and $L_{CO}$ as well as the $L_{HCN}/L_{CO}$ ratio.

While all galaxies with a high HCN/CO ratio are IR-luminous, it is not true that all LIGs have a high dense gas fraction, $L_{HCN}/L_{CO}$. There are a group of 7 LIGs in our sample which are IR luminous and have high HCN luminosities but normal ratios of HCN/CO. These are gas-rich (CO-luminous) galaxies with normal SFE. They are the filled circles on the normal galaxy fit line in Fig. 3 and in the upper left of Fig. 4. Their high $L_{IR}$ is simply the result of a tremendous amount of available molecular gas and a normal fraction of dense molecular gas. In this sense they are not starbursts globally. They are globally using their molecular gas at a normal SFR. Some of the notable examples in our sample are Mrk 1027, NGC 1144.
(Gao et al. 1997), NGC 6701, and even Arp 55 (Table 1). Luminous pre-starburst galaxies like Arp 302 (Gao 1996; Lo, Gao, & Gruendl 1997), and even early stage galaxy mergers in the initial starburst phase with extended CO emission (e.g., NGC 6670, Wang et al. 2001), belong to this category as well.

Fig. 5a shows a correlation between $L_{\text{IR}}/L_{\text{CO}}$ and $L_{\text{HCN}}/L_{\text{CO}}$ with a correlation coefficient $R = 0.74$ ($R^2 = 0.55$, there is little effect on the fit whether those HCN limits are used or not). This suggests that the star formation efficiency, SFE ($L_{\text{IR}}/L_{\text{CO}}$) depends on the fraction of molecular gas in a dense phase indicated by ($L_{\text{HCN}}/L_{\text{CO}}$). This correlation demonstrates the direct connection between the IR and HCN luminosities (as shown in Fig. 1a). Both $L_{\text{IR}}$ and $L_{\text{HCN}}$ have been normalized by $L_{\text{CO}}$ to show the physical relationship between $L_{\text{IR}}$ and $L_{\text{HCN}}$ after removing the dependence upon distance, galaxy size, and other possible selection effects. Of course the range of the ratio ($L_{\text{HCN}}/L_{\text{CO}}$) is less than that for $L_{\text{HCN}}$ and the correlation is not as strong as in Fig. 1a although the dispersion about the fit is almost the same. This normalization is dust extinction-free, unlike the use of, for example, the blue luminosity in the normalization.

Similarly, we can show the correlation between $L_{\text{IR}}$ and $L_{\text{CO}}$ divided by $L_{\text{HCN}}$ for normalization. Surprisingly, the strong correlation observed in Fig. 1b has completely disappeared, and there is no any correlation left at all ($R^2 = 0.01$, Fig. 5b). This certainly reflects the tight correlation between IR and HCN and the fact that $L_{\text{IR}}/L_{\text{HCN}}$ is almost independent of galaxy luminosity (Fig. 2a). The absence of a correlation in Fig. 5b also implies that the apparent well known correlation between $L_{\text{IR}}$ and $L_{\text{HCN}}$. The IR–CO correlation may be due to a combination of the tighter IR–HCN and HCN–CO correlations. Nevertheless, $L_{\text{IR}}/L_{\text{HCN}}$ is not completely independent of the CO luminosity (see §3.3 and the appendix) as its inverse $L_{\text{HCN}}/L_{\text{IR}}$ is weakly correlated with $L_{\text{CO}}/L_{\text{IR}}$ with a correlation coefficient $R = 0.56$ ($R^2 = 0.31$, still a meaningful correlation as a consequence of the tight HCN–CO correlation normalized by $L_{\text{IR}}$).

### 3.3. The Model Parameter Fits

Table 3 in the appendix summarizes almost all the parameter fits in the IR, CO and HCN data sets including the warm dust temperature deduced from the 60 and 100 μm fluxes. Here we discuss the three-parameter fits involving the IR, HCN and CO luminosities and the warm dust temperature $T_{\text{dust}}$ dependence of the luminosity ratios. Further details of various other correlation fits are also discussed in the appendix. These fits to the data demonstrate that the two important independent parameters are the IR and HCN luminosities.

#### 3.3.1. Multi-parameter Fits

The IR luminosity from a model fit to the HCN and CO [the IR($L_{\text{HCN}}$, $L_{\text{CO}}$) model], yields

$$\log L_{\text{IR}}(L_{\text{HCN}}, L_{\text{CO}}) = 2.28 + 0.88 \log L_{\text{HCN}} + 0.16 \log L_{\text{CO}}.$$  (3)

The weak dependence on $L_{\text{CO}}$ shows that the IR luminosity is determined principally from the HCN luminosity. The dispersion about the fit is almost the same as the two parameter fit of $L_{\text{IR}}-L_{\text{HCN}}$. The t ratio for $L_{\text{CO}}$ (0.98) is very small as compared to that for $L_{\text{HCN}}$ (6.95) demonstrating that $L_{\text{CO}}$ is almost random after fitting $L_{\text{HCN}}$ for $L_{\text{IR}}$. 

An IR and CO luminosity model fit for $L_{HCN}$ [the HCN($L_{IR}$, $L_{CO}$) model] gives:

$$\log L_{HCN}(L_{IR}, L_{CO}) = -2.62 + 0.56 \log L_{IR} + 0.49 \log L_{CO}. \quad (4)$$

Here, $L_{IR}$ and $L_{CO}$ seem to be almost equally important. This relation produces a tighter fit than either of the 2 parameter fits HCN–CO (Fig. 3) or HCN–IR (Fig. 1a).

Therefore, although the HCN luminosity depends on both the CO and IR luminosity, the IR is predicted almost entirely by the HCN with almost no effect from the CO luminosity. The reason for this surprising result is probably that, of the three separate two parameter (IR–HCN, HCN–CO, & IR–CO) fits, the weakest and the one that has the largest dispersion of the data from the fit is the IR–CO correlation.

The corresponding fit for the CO luminosity is

$$\log L_{CO}(L_{IR}, L_{HCN}) = 3.60 + 0.55 \log L_{HCN} + 0.12 \log L_{IR}. \quad (5)$$

It is clear that the CO is more closely related to the HCN than to the IR luminosity. In another word, the dependence of the CO luminosity on the IR is much weaker than on the HCN luminosity. This result is probably due to the very tight correlation between CO and HCN for normal galaxies (Fig. 3).

### 3.3.2. Warm Dust Temperature Dependence

Here we examine the influence of the warm dust temperature on the luminosity ratios. It is well known that the warm dust temperature is an important parameter in the IR–CO correlation (e.g., Young et al. 1986; Young & scoville 1991). Calculating the warm dust temperature from the $60/100 \, \mu m$ flux ratio, we find the $L_{IR}/L_{CO}$ luminosity ratio depends strongly on $T_{dust}$ (see figures in the appendix)

$$L_{IR}/L_{CO} = 61(T_{dust}/35K)^{5.7}. \quad (6)$$

for optically thin dust with emissivity $\sim \nu^\beta$ ($\beta = 1.5$).

As we can see from Fig. 2a, the ratio $L_{IR}/L_{HCN}$ is almost independent of IR luminosity. If we fit this ratio to the dust temperature (poor correlation), we find a weak correlation with

$$L_{IR}/L_{HCN} \sim 980(T_{dust}/35K)^{1.8}. \quad (7)$$

The IR–HCN relation changes only by a factor of 2 on average across the entire temperature range of the sample galaxies ($T = 29$ to 46K) while the IR–CO ratio changes by a factor of 13. The HCN/CO ratio changes with the dust temperature between these two extremes since (using Equations 6 & 7)

$$L_{HCN}/L_{CO} = (L_{IR}/L_{CO})/(L_{IR}/L_{HCN}) \sim T_{dust}^{3.9}. \quad (8)$$

Indeed, a direct fit to the data (still reasonable correlation with $R^2 \sim 0.5$) gives $L_{HCN}/L_{CO} \sim T_{dust}^{3.6}$.

### 3.4. Summary of the Results

Our results presented in Figs. 1–5 show that dense molecular gas mass indicated by HCN luminosity is a much better predictor of infrared luminosity and star formation than total H$_2$ content indicated by
CO luminosity. We identify 2 types of LIGs according to their total molecular gas content. Both types have high total $L_{\text{HCN}}$ in addition to high $L_{\text{IR}}$. Most LIGs have a high dense molecular gas fraction but a second group has a normal ratio of HCN/CO yet a very high CO luminosity. Are both types of LIGs starbursts? It may well be that LIGs with a normal $L_{\text{HCN}}/L_{\text{CO}}$ are not necessarily genuine starbursts since the infrared luminosity can be produced by a large amount of molecular gas forming stars at a normal rate (efficiency). SFE can remain almost unchanged, whether galaxies are molecular gas-rich (more luminous) or relatively gas-poor (less luminous, cf. Young 1999). However, SFE increases dramatically from dense molecular gas-poor galaxies to dense molecular gas-rich galaxies. All galaxies with a dense gas mass fraction $L_{\text{HCN}}/L_{\text{CO}} > 0.06$ are LIGs or ULIGs in our sample. In essence, HCN traces the molecular gas at high density and at high warm dust temperature that is tightly linked to the active star formation.

4. DISCUSSION

4.1. HCN — Tracer of Active Star-forming Dense Molecular Gas

All the parameter fits essentially show the same thing. CO luminosity by itself leads to a rough prediction for IR luminosity which breaks down for luminous infrared galaxies (LIGs), especially for ultraluminous infrared galaxies (ULIGs), whereas HCN luminosity is much better at predicting the IR luminosity for all galaxies including ULIGs. Therefore, the star formation rate (SFR) indicated by $L_{\text{IR}}$ in galaxies depends on the amount of dense molecular gas traced by HCN, not the total molecular gas content measured by CO. In particular, the IR–CO correlation may not have a solid physical basis as it can be readily related to the stronger and perhaps more physical IR–HCN and HCN–CO correlations, which may be the origin of the correlation between IR and CO. This is reminiscent of the poor IR–HI correlation as compared to the better IR–CO correlation that became apparent two decades ago, when systematic CO observations of significant numbers of galaxies became available.

The HCN radiation is associated with the warm dust traced by the FIR radiation, whereas the total molecular gas traced by CO originates in diverse dust components at different temperatures. The temperature dependence of $L_{\text{IR}}$ (or $L_{60\mu m}$ or $L_{100\mu m}$) is straight forward to understand since it is just the Planck law plus the dust emissivity. In this perspective, it is easy to see why the $L_{\text{IR}}$–$L_{\text{CO}}$ relation has a strong dependence upon the dust temperature term since $L_{\text{CO}}$ at most is proportional to the first power of the temperature while $L_{\text{IR}}$ (or $L_{100\mu m}$), depending on the emissivity law, is proportional to at least the 5th power of the dust temperature. Therefore, many correlations that involve $T_{\text{dust}}$ and $L_{\text{CO}}$ [see appendix, for example, in one case as represented in the CO($L_{100\mu m}$, $T_{\text{dust}}$) model, $\log L_{\text{CO}} = 5.07 + 0.88 \log L_{100\mu m} - 3.0 \log T_{\text{dust}}$] can be easily explained. The more complex question is why this $T_{\text{dust}}$ dependence almost entirely goes away for the $L_{\text{HCN}}$, e.g., the HCN($L_{100\mu m}$, $T_{\text{dust}}$) model, $\log L_{\text{HCN}} = -1.12 + 0.95 \log L_{100\mu m} - 0.21 \log T_{\text{dust}}$, and similarly others involving $L_{\text{HCN}}$ (Table 3). The simple answer to this is that the higher molecular gas density produces more active star formation which raises $T_{\text{dust}}$ owing to heating of the newborn stars. High molecular gas density, strong HCN emission, and warm dust temperature go together. HCN traces the active star-forming molecular gas where both the molecular gas density and dust temperature are high.

We have already shown in Paper I that HCN emission in galaxies is primarily due to the collisional excitation by high density molecular hydrogen, not radiative excitation through the mid-IR pumping (see also Stutzki et al. 1988; Paglione et al. 1997). Even though the mid-IR pumping is not a significant source to excite the rotational transition of HCN emission, there are still some other possibilities that may help
excite HCN, e.g., collisions with electrons (Aalto et al. 1995), the possibly enhanced HCN abundance and
shock excitation, owing to excess supernovae occurred in starburst galaxies and LIGs/ULIGs. However, all
these, even collectively, are only of the secondary effects on a global scale in galaxies, although significant
contribution in a particularly favorable environment in some small localized regions cannot be excluded.

In any case, the physical explanation for the tight correlation between the HCN and IR is star
formation in dense molecular gas. The active high mass star-forming sites are the cores of GMCs where the
molecular gas is warmer and denser than in the GMC envelopes where most of the CO emission originates.
Currently, detailed statistical study examining the relationships among FIR, HCN, and CO on the scale of
GMCs cores is not yet available. There are extensive observations (Plume et al. 1997) of another dense gas
tracer, CS emission, of high mass star formation cores in the Milky Way. All of these regions have an H₂
density more than sufficient to produce strong HCN emission.

4.2. Dense Molecular Gas and Star Formation Rates

For an initial mass function (IMF), typically taken to be the Salpeter IMF, \( L_{\text{IR}}/M(\text{gas}) \) can be
interpreted as a measure of star formation efficiency (SFE), i.e., SFR per unit gas mass. This is because the
SFR is related to \( L_{\text{IR}} \) by

\[
\dot{M}_{\text{SFR}} \approx 2 \times 10^{-10} \left( L_{\text{IR}} / L_{\odot} \right) M_{\odot} \text{yr}^{-1},
\]

assuming that the observed FIR emission is produced primarily from dust heating by O, B, and A stars
(e.g., Scoville & Young 1983; cf. Gallagher & Hunter 1987; Kennicutt 1998b). Although \( L_{\text{IR}} \) correlates with
\( L_{\text{CO}} \), the correlation is non-linear, with a higher \( L_{\text{IR}}/L_{\text{CO}} \) ratio for higher \( L_{\text{IR}} \) (Fig. 2b). On the other
hand, \( L_{\text{IR}} \) linearly correlates with \( L_{\text{HCN}} \) implying an almost constant SFR per unit of dense molecular gas
mass for all galaxies.

The HCN luminosity can be related to the mass of dense gas, \( M_{\text{dense}} = \alpha_{\text{HCN}} L_{\text{HCN}} \), if we assume the
emission originates in the gravitationally bound cloud cores (see Paper I). For a volume averaged core density
\( n(\text{H}_2) \sim 3 \times 10^4 \text{cm}^{-3} \) and brightness temperature \( T_b = 35 \text{ K} \), \( \alpha_{\text{HCN}} = 2.1 \sqrt{n(\text{H}_2)/T_b} = 10 M_{\odot}/\text{Kkm s}^{-1}\text{pc}^2 \).
Substituting in Equation 1 gives a luminosity to dense gas mass ratio

\[
L_{\text{IR}}/M_{\text{dense}} = 90(\alpha_{\text{HCN}}/10)L_{\odot}/M_{\odot}
\]

for all galaxies, although the mean is actually slightly higher (\( \sim 120 L_{\odot}/M_{\odot} \)) for the most luminous galaxies
(see Table 2).

Combining Equation 1 with Equation 9, the SFR in terms of the HCN luminosity is:

\[
\dot{M}_{\text{SFR}} \approx 1.8 \times 10^{-7} (L_{\text{HCN}} / \text{Kkm s}^{-1}\text{pc}^2) M_{\odot} \text{yr}^{-1}.
\]

In terms of the dense gas mass, the star formation rate becomes

\[
\dot{M}_{\text{SFR}} \approx 1.8 \frac{M_{\text{dense}}}{10^8 M_{\odot}} \left( \frac{10}{\alpha_{\text{HCN}}} \right) M_{\odot} \text{yr}^{-1}.
\]

Since this is a linear relation, the HCN emission is a direct tracer of the SFR in all galaxies. The dense gas
characteristic depletion time (half life) is

\[
\tau_{1/2} = 0.5 M_{\text{dense}} / \dot{M}_{\text{SFR}} \approx 2.7 \alpha_{\text{HCN}} \text{Myrs}.
\]
Although we adopted $\alpha_{\text{HCN}} \sim 10 M_\odot / \text{Kkm} s^{-1} \text{pc}^2$ for normal spirals (Paper I), this conversion factor might be smaller for ULIGs as the $T_b$ can be quite high (Downes & Solomon 1998), leading to shorter dense gas depletion time scales. HCN observations could potentially become one of the best SFR tracers in galaxies in the nearby and distant universe given the high-sensitivity and the high spatial resolution achievable at millimeter wavelength with the next generation of the millimeter telescopes.

There also appears to be some correlation between HCN emission and HCO$^+$, CS, and other tracers of star formation, e.g., C[II] line emission and the FIR and 20cm continuum emission (Nguyen-Q-Rieu et al. 1992), although their sample is very limited. However, it has become clear that C[II] is underluminous in ULIGs (e.g., Luhman et al. 2003) and is not a consistent star formation indicator. In addition, in Equation 9, it was assumed that most of the $L_{\text{IR}}$ originates from star formation with little contribution from AGN and/or from the general interstellar radiation field. It will be interesting to examine correlations between HCN and other indicators of star formation in a large sample of galaxies to assess which best indicates the SFR.

The tight correlation between the IR and HCN emission also implies that the dominant IR emission originates from the HCN emission region, especially in LIGs/ULIGs with concentrated molecular gas distribution. We know little of the size scales of the FIR emission regions in LIGs/ULIGs. The dominant contribution of the radio continuum and mid-IR emission in most LIGs appears to be from the inner regions (Condon et al. 1991; Telesco, Dressel, & Wolstencroft 1993; Hwang et al. 1999; Xu et al. 2000; Soifer et al. 2001). CO emission is usually concentrated in the inner regions, typically within a kpc of the center for ULIGs/LIGs (e.g., Scoville, Yun, & Bryant 1997; Downes & Solomon 1998; Sakamoto et al. 1999; Bryant & Scoville 1999; cf. Gao et al. 1999). HCN emission originates from the dense cores of the CO emitting regions, the sites of star formation and the source of the FIR emission. Thus, we may predict the size scales and location of the FIR emission by determining the HCN source sizes from the HCN mapping.

### 4.3. The Global Star Formation Law

The IR–HCN linear correlation is valid over 3 orders of magnitude from low IR luminosity to the most luminous galaxies in the local universe. The direct consequence of the linear IR–HCN correlation is that the star formation law in terms of dense molecular gas content has a power law index of 1.0. The global SFR is linearly proportional to the mass of the dense molecular gas (Equation 11 and Fig. 6). Parametrization in terms of observable mean surface densities of the dense molecular gas and the SFR will not change the slope of 1 in the IR–HCN correlation (SFR–$M_{\text{dense}}$ correlation), or the linear power index of the star formation law, as both quantities are simply normalized by the same galaxy disk area. Our finding of an SFR proportional to the first power of the dense gas mass is different from the widely used star formation law with a slope of 1.4, derived by Kennicutt (1998a) for the disk averaged SFR as a function of the total (HI and H$_2$) or just molecular gas surface density traced by CO emission. As we show below that this law is not valid for normal spiral galaxies and results only by combining normal galaxies with starburst galaxies and ULIGs.

#### 4.3.1. Normal Spiral Galaxies

The IR–CO correlation (SFR–$M_{\text{H}_2}$ correlation) is essentially linear up to luminosity $L_{\text{IR}} = 10^{11}$ L$_\odot$ (ULIGs and most LIGs excluded, see Fig. 1b). This seems to also be true in terms of the mean surface
densities of the SFR and molecular gas mass for the nearest galaxies with spatially resolved observations (e.g., Wong & Blitz 2002; Rownd & Young 1999). The linear IR–CO correlation at low to moderate IR luminosity is expected since we find that the HCN–CO correlation is extremely tight and linear for normal spiral galaxies (Fig. 3). Thus the linear form of the global star formation law in terms of total molecular gas density as traced by CO for normal galaxies is due to the constant dense gas mass fraction indicated by the HCN/CO ratio (discussed in the next section). For normal star-forming spirals, the star formation law is linear in terms of both the total molecular gas and the dense molecular gas. Then, how did Kennicutt (1998a) obtain a slope of 1.4 in the fitting of the star formation law? In our sample, a fit for the normal galaxies in the IR–CO correlation gives a slope of 1.0 (Fig. 1b), but this is not the case in Kennicutt’s normal galaxy sample, where there is a poor correlation between the SFR and the gas surface density. Thus no reasonable slope can be derived from his normal galaxy sample alone.

### 4.3.2 All Galaxies: Normal Spiral, Starburst, and Ultraluminous Galaxies

A direct orthogonal regression fit of the IR–CO correlation for all galaxies in our sample (Fig. 1b) leads to a slope of 1.44. But the best-fitting least-squares slope (errors in $L_{\text{IR}}$ only) is 1.27 (1.25 if galaxies with HCN limits are excluded, see §3.1). These fit slopes of the IR–CO correlation are almost identical to the star formation power law index in Kennicutt’s (1998a) 36 circumnuclear starbursts and ULIG sample. It is obvious from Fig. 1b that only galaxies with $L_{\text{IR}} > 10^{11}L_\odot$ (ULIGs and most LIGs) lie above the fixed line of slope 1. This combination of normal and very luminous galaxies leads to a fit with a power law index of 1.4. Therefore, this slope is not a universal slope at all as it changes according to the sample selection. The 1.4 slope of the composite Schmidt law (Kennicutt’s Fig. 6, 1998a) is determined almost entirely from the starburst sample. The circumnuclear starbursts have some of the characteristics of ULIGs/LIGs. In particular, they must have a high dense gas fraction indicated by HCN/CO ratios (see section §4.4).

Indeed, when we add more LIGs/ULIGs into the sample for the IR–CO correlation, the slope becomes steeper. In Fig. 7, we present an SFR–$M(H_2)$ (IR–CO) correlation diagram with an additional 40 galaxies, mostly ULIGs, with CO data from the literature (Solomon et al. 1997; Gao & Solomon 1999; Mirabel et al. 1990; Sanders et al. 1991), in addition to our HCN sample shown in Fig. 1b. The least-squares fit gives a steep slope of 1.53, and the orthogonal regression fit leads to a much steeper slope of 1.73. In this sample luminous galaxies and normal galaxies have equal weight. It is clear that the ULIGs steepen the slope of the sample. There also appears to be a trend that some normal spirals with the lowest $\Sigma_{\text{SFR}}$ and $\Sigma_{H_2}$ in Kennicutt’s sample tend to lie below the 1.4 power fit line. Adding more extreme galaxies, both luminous ULIGs and low luminosity spirals tends to steepen the slope further towards 2. Therefore, it is difficult to derive a unique 1.4 power law based upon the total molecular gas or the total gas content.

The star formation rate in a galaxy depends linearly on the dense molecular gas content as traced by HCN, regardless of the galaxy luminosity or the presence of a “starburst”, and not the total molecular gas and/or atomic gas traced by CO and/or HI observations respectively. Since dense molecular cloud cores are the sites of high mass star formation, it is the physical properties, location and mass of these cores that set the star formation rate. A detailed star formation law can be determined from observations directly probing the Milky Way cloud cores, particularly in the Milky Way molecular ring with spatially resolved measurements. The molecular tracers which best quantitatively indicate the presence of a starburst are primarily abundant molecules with high dipole moments such as HCN and CS requiring high molecular hydrogen density for excitation. The molecular property which best characterizes the star formation rate of a galaxy is the mass of dense gas. The gas density traced by HCN emission is apparently at or near the
4.4. HCN/CO Ratio — A Better Indicator of Starbursts

Although HCN luminosity is a better indicator of star formation than CO in galaxies, it is very useful to compare the HCN with CO to obtain the HCN/CO ratio. The HCN/CO ratio is an indicator of the fraction of dense molecular gas available for vigorous star formation and gauges the globally averaged molecular gas density. The HCN/CO ratio is also a very successful predictor of starbursts (Fig. 4) and directly correlates with the SFE \( \frac{L_{\text{IR}}}{L_{\text{CO}}} \) (Fig. 5a). The SFE increases dramatically from dense molecular gas-poor galaxies to dense molecular gas-rich galaxies. The global luminosity ratio \( \frac{L_{\text{HCN}}}{L_{\text{CO}}} \) differs among galaxies of different luminosities (see Tables 1 & 2) with an average ratio of 1/6 for ULIGs and only 1/25 for normal spirals. All galaxies in our sample with a global dense gas mass fraction \( \frac{L_{\text{HCN}}}{L_{\text{CO}}} > 0.06 \) are LIGs or ULIGs (Fig. 4). We note that Curran et al. (2000) also find an average global ratio of 1/6 in luminous IR Seyfert galaxies which they attribute to star formation.

The HCN/CO surface brightness ratio is potentially a better and more practical indicator than the IR/CO ratio (the standard SFE diagnostic) of the starburst strength. Using the ratio IR/CO as a diagnostic, the IR emission is presumed to entirely originate from star formation. Other possible sources such as AGNs and the general interstellar radiation field are assumed to be negligible. The HCN/CO ratio instead, is directly related to the molecular gas properties, particularly the local molecular gas density which is tied to star formation. Moreover the projection and confusion of different velocities along the line-of-sight is inevitably present in IR maps, whereas the different velocity components can be distinguished from the kinematics obtained in the detailed CO and HCN maps. In addition, the low spatial resolution available in the FIR, even with SIRTF and the upcoming SOFIA, and other future far-IR space missions, is incompatible with the high resolution available from millimeter interferometers. Therefore, it is important to map the HCN/CO ratio in galaxies in order to fully explore the star formation properties and SFE.

As we show elsewhere from HCN maps of nearby galaxies (Gao 1996; Gao & Solomon in preparation), the ratio of \( I_{\text{HCN}}/I_{\text{CO}} \), i.e., the surface brightness ratio (SBR), in most cases, is high in the centers of spiral galaxies, and typically drops off at large galactic radii. A significant fraction of dense molecular gas is (traced by HCN emission) distributed in the inner disks of galaxies outside the nuclear or inner ring starburst regions, and can be detected to radii as large as a few kpc, perhaps to diameters of \( \sim D_{25}/4 \). We find the highest fraction of dense molecular gas, indicated by the SBR (as measured by single-dish telescopes with beamsizes \( \sim 1 \) kpc)

\[
\text{SBR (cores)} \equiv \frac{I_{\text{HCN}}}{I_{\text{CO}}} \approx 0.1,
\]

nearly comparable to those observed globally from ULIGs, in the centers of most normal spiral galaxies observed (usually the nuclear starburst cores). We attribute this to the presence of a starburst. Helfer & Blitz (1993, 1997a) found similarly high ratios in the centers of normal galaxies which they relate to the high ambient pressure but they do not correlate their data with infrared luminosity or star formation rates.

We also find that the SBR ratio generally falls off in the disks at larger radii (e.g., \( \gtrsim 3 \) kpc), to a very low SBR \( \sim 0.015–0.03 \). This low SBR (disks) \( \lesssim 0.03 \) is the same as that found in the Milky Way’s disk on average, and over the full extent of nearby GMCs in the Milky Way (Helfer & Blitz 1997b), as well as outside the central regions in several normal spiral galaxies mapped with an interferometer (Helfer & Blitz 1997a).
Although the global luminosity ratio $L_{\text{HCN}}/L_{\text{CO}} = <\text{SBR}>$ differs dramatically among galaxies of different luminosities (see Tables 1 & 2), the difference between the SBR in central beam measurements of galaxies is related to the different telescope beam diameters (e.g., Helfer & Blitz 1993), different source sizes (of CO and HCN), and different size scales of the starburst cores in the centers of individual galaxies. The CO/HCN intensity ratio (the inverse of SBR) changes from 20 – 80 for normal spiral galaxies to 4 – 10 for ULIGs. But, if only central beam measurements are used, the ratio seems quite uniform (Aalto et al. 1995). The central beam measurement is not representative of the global measurent (of the entire galaxy) and may not even be an accurate measurement of the central region since it is telescope dependent. This explains why Aalto et al. (1995) did not find the strong $L_{\text{IR}} - L_{\text{HCN}}$ correlation.

The distribution of the HCN emission, particularly HCN/CO ratio maps, can be used directly to locate starburst sites in nearby galaxies. High resolution maps of HCN and HCN/CO have been obtained in the central regions of a few nearby normal galaxies, some with clear central starbursts. Paglione et al. (1995) mapped the central bar in the starburst nucleus of NGC 253 and found a peak HCN/CO=0.2 falling off to 0.04 at a galactocentric radius of 200 pc. The peak SBR in this starburst is similar to the global value in the most luminous galaxies in our sample (Fig. 4). Both the strongest HCN surface brightness and the strongest SBR occurred at the location of strong, extended nonthermal radio continuum and thermal free-free emission associated with the starburst. Using our Fig. 5a, this high SBR (0.15–0.2) indicates that this small central starburst has an $L_{\text{IR}}/L_{\text{CO}}$ ratio comparable to that of ULIGs, $L_{\text{IR}}/L_{\text{CO}} \sim 200 L_\odot / \text{Kkm s}^{-1} \text{pc}^2$. Unfortunately, there are no high resolution IR observations available to test this prediction. Paglione et al. (1995) estimated the $L_{\text{IR}}$ from the radio continuum to be $L_{\text{IR}} \sim 1 \times 10^9 L_\odot$. The $L_{\text{CO}}$ estimated from the HCN and HCN/CO ratios is approximately $4 \times 10^7 \text{Kkm s}^{-1} \text{pc}^2$ yielding a rough $L_{\text{IR}}/L_{\text{CO}} \sim 250 L_\odot / \text{Kkm s}^{-1} \text{pc}^2$, in agreement with the prediction of Fig. 5a.

High HCN/CO ratios (0.1–0.2) are also found in the central $\lesssim 1$ kpc regions of several other galaxies (Downes et al. 1992; Helfer & Blitz 1997a; Reynaud & Downes 1997; Kohno et al. 1996, 1999). Kohno et al. (1999) found that most of the HCN emission, as well as an enhanced HCN/CO ratio, is associated with the circumnuclear star-forming ring in NGC 6951. No significant enhancement of the HCN/CO ratio is observed at the CO peaks. In IC 342 with a resolution of 60 pc (Downes et al. 1992), however, only three of five HCN clouds seem to be actively forming into stars based on the presence of free-free emission. But, the observations of the free-free emission are still of limited sensitivities, either at 2cm and 6cm (Turner & Ho 1983; Turner et al. 1993) or at 3 mm (Meier & Turner 2001). And there are no high-resolution FIR observations available to clearly indicate the star formation activities and star formation rates at this small scale. Judging from all these observations and the $H\alpha$ emission (Turner & Hurt 1992), there is probably star formation in all the HCN clouds (D. Downes 2003 private communication). Are some of these HCN clouds precursors to a starburst? Although the HCN/CO ratios are high ($\sim 0.14$) for these HCN clouds, the average of a several hundred parsec central region has only an HCN/CO ratio of $\lesssim 0.05$ in IC 342, clearly not a strong starburst.

A recent summary of the HCN/CO ratios obtained in 6 central nuclear starbursts (Shibatsuka et al. 2003) lists a typical ratio HCN/CO$ \approx 0.1$–0.25. Sorai et al. (2002) also find the highest HCN/CO ratio of $\sim 0.1$ in central regions of a few nearby galaxies. This is the same as the global ratio we find in many LIGs and all ULIGs which have HCN luminosities several hundred times greater than the central starbursts of normal galaxies.

The only LIGs that have well been imaged in HCN are the AGN/starburst hybrid galaxy NGC 1068 (Tacconi et al. 1997, 1994; Helfer & Blitz 1995; Jackson et al. 1993) which has very high HCN/CO ratio ($\gtrsim 0.3$) within $\sim 100$ pc nuclear region, the merging pair Arp 299 (Aalto et al. 1997; Casoli et al. 1999), and
the archetypal ULIG Arp 220 (Radford et al. 1991; N.Z. Scoville 2001 private communications). Arp 299 may harbor an AGN in the eastern nucleus (e.g., Gehrz et al. 1983), similar to NGC 1068. It appears that some weak Seyferts also have rather high HCN/CO ratio ($\gtrsim 0.2$) in the innermost $\lesssim 100$ pc nuclear region around the AGN (e.g., M51, Kohno et al. 1996). But this contributes little to the average HCN/CO ratios in the circumnuclear ($\sim 0.5$–1 kpc) starburst rings where most of molecular gas is located.

Ultimately, it is the dense molecular gas, rather than the total molecular gas content, that is the raw material for active star formation in galaxies. Our global measurements of 65 galaxies show that the dense molecular gas fraction indicated by $L_{\text{HCN}}/L_{\text{CO}}$ is an important measure of the star formation efficiency; the mass of dense molecular gas indicated by $L_{\text{HCN}}$ is a very good measure of the star formation rate deduced from $L_{\text{IR}}$. The total molecular content indicated by $L_{\text{CO}}$ is an unreliable indicator of star formation rate particularly in starburst galaxies. The location of starburst regions and better characterization of the starbursts in individual galaxies could be better indicated by the local SBR measurements, obtained by the high resolution and high sensitivity HCN and CO observations.

4.5. Starburst Origin of the Far-IR Emission

Our results show that high mass star formation in dense molecular gas is responsible for the infrared luminosity from a wide range of galaxies including normal spirals of moderate IR luminosity ($5 \times 10^9 L_\odot \lesssim L_{\text{IR}} \lesssim 10^{11} L_\odot$), luminous infrared galaxies (LIGs, $10^{11} L_\odot \lesssim L_{\text{IR}} \lesssim 10^{12} L_\odot$), and ultraluminous galaxies (ULIGs, $L_{\text{IR}} \gtrsim 10^{12} L_\odot$). The star formation rate or IR luminosity is proportional to the mass of dense molecular gas in all galaxies.

4.5.1. Luminous and Ultraluminous Infrared Galaxies

Evidence is mounting that the dominant energy source in most ULIGs is from the extreme starbursts rather than the dust-enshrouded AGNs (e.g., Solomon et al. 1997; Downes & Solomon 1998; Genzel et al. 1998; Scoville et al. 2000; Soifer et al. 2001). The debate about the energy source in ULIGs, particularly in “warm” ULIGs, has been going on for over a decade (e.g., Sanders et al. 1988; Veilleux et al. 1999; Sanders 1999; Joseph 1999). Although AGNs are present in many ULIGs (e.g., Nagar et al. 2003; Franceschini et al. 2003), there is little evidence indicating that the AGN contribution is the dominant source of the FIR emission.

Our results summarized in § 3 provide compelling evidence in favor of a star formation origin for the huge infrared luminosity from ultraluminous galaxies. We have shown that the infrared luminosity of all molecular gas rich spiral galaxies including ULIGs is proportional to the dense star forming molecular gas mass traced by $L_{\text{HCN}}$ (see Fig. 1a and Fig. 6).

One of the main arguments in favor of an AGN as the power source in ULIGs is the anomalously high ratio $L_{\text{IR}}/L_{\text{CO}}$ or $L_{\text{IR}}/M(H_2)$. The IR luminosity and thus the required star formation rate per solar mass of molecular gas (as traced by CO emission) is an order of magnitude higher in ultraluminous and most luminous galaxies than in normal spiral galaxies, suggesting that an AGN rather than star formation is required (Sanders et al. 1988; Sanders et al. 1991) in order to produce the high infrared emission. Viewed in terms of the dense gas mass the situation is completely different. The ratio $L_{\text{IR}}/L_{\text{HCN}}$ or $L_{\text{IR}}/M_{\text{dense}}$ is essentially the same in all galaxies including ULIGs. Fig. 8 shows that $L_{\text{IR}}/M_{\text{dense}}$ is virtually independent
of galaxy luminosity and on average \( L_{\text{IR}}/M_{\text{dense}} = 90L_\odot/M_\odot \), about the same as in molecular cloud cores but much higher than in GMCs as a whole (Mooney & Solomon 1988). Ultraluminous galaxies simply have a large quantity of dense molecular gas and thus produce a prodigious starburst which heats the dust. It is not surprising that starbursts of this magnitude are observed in the infrared and never seen in the optical-UV part of the spectrum. The young OB stars are imbedded in very massive and dense regions dwarfing anything found in Milky Way GMCs and all of their optical-UV radiation is absorbed by dust. Although ULIGs are not simply scaled up versions of normal spirals in terms of their total molecular mass they are scaled up version in terms of their dense molecular gas mass, which is exactly what is expected if star formation is the power source.

Ultimately, even for warm ULIGs (e.g., Surace et al. 1998), which might have, in some degree, evolved into the phase of the dust-enshrouded QSOs/AGN (e.g., Sanders et al. 1989; Surace & Sanders 1999; Veilleux et al. 1999; Genzel et al. 1998; Sanders 1999), we may still be able to tell whether starbursts still dominate most of the high \( L_{\text{IR}} \) or not by examining their total dense molecular gas content and the fraction of dense molecular gas. For instance, \textit{IRAS} 05189-2524 and Mrk 231 are warm ULIGs, but they have similar \( L_{\text{IR}}/M_{\text{dense}} \) and HCN/CO ratio as other 7 out of 9 ULIGs in our sample. Clearly more HCN observations are required to judge if warm ULIGs distinguish themselves from other ULIGs.

4.5.2. Normal Spiral Galaxies

Although some fraction of \( L_{\text{IR}} \) originates from outer disks in nearby large spiral galaxies, the dominant contribution is still from the centers (e.g., Rice et al. 1988). Better resolution \textit{IRAS} maps, obtained by improved imaging deconvolution algorithm, tend to show much more centrally concentrated FIR emission in the inner disks (Rice 1993). Devereux & Young (1990) have shown that the global FIR luminosity in spiral galaxies is consistent with that contributed by warm dust heating from the high mass OB stars. Higher resolution (as compared with that of \textit{IRAS} ) measurements of the 160 \( \mu \text{m} \) and Hα emission in nearby galaxies NGC 6946 and M51 suggest that the FIR luminosity is in quantitative agreement with that expected from OB stars throughout the star-forming disks (Devereux & Young 1992, 1993).

The existence of cold dust components of \( \lesssim 30 \) K and \( \lesssim 20 \) K in nearby galaxies has been revealed from both recent ISOPHOT FIR observations (e.g., Haas et al. 1998; Alton et al. 1998a; Davies et al. 1999) and SCUBA submillimeter observations (e.g., Alton et al. 1998b; 2000) respectively. The cold dust distribution usually has a larger radial extent (e.g., Alton et al. 1998a) and the dominant cold dust component may not be closely associated with the active star-forming inner disks. HCN maps of nearby galaxies (e.g., Nguyen-Q-Rieu et al. 1992; Helfer & Blitz 1997a; Gao & Solomon in preparation) show that the HCN emission region is much more compact than that of CO emission region in normal spiral galaxies and/or starburst galaxies, and is thus closely related to the warm dust in the inner disks.

The contribution of the general interstellar radiation field to the total FIR emission in galaxies might be significant at the low \( L_{\text{IR}} \) end, where the general infrared interstellar radiation field is comparable to the IR radiation from the active star formation. This might be testable on the IR—HCN correlation plot (Fig. 1a) for galaxies of the lowest \( L_{\text{IR}} \) (\( \lesssim 5 \times 10^9L_\odot \)) when more observations of the lowest \( L_{\text{IR}} \) sources are available to show a statistically significant trend. Given that the tight correlation between these two quantities might be used to predict one another, one can check if low \( L_{\text{IR}} \) galaxies have a higher \( L_{\text{IR}}/L_{\text{HCN}} \) than expected from the IR—HCN correlation.
AGN may become more important than star formation at the very high IR luminosity end, especially in warm ULIGs (Veilleux et al. 2003), in contributing the bulk of energy output (e.g., Sanders et al. 1988; Veilleux et al. 1999). For extremely luminous galaxies with $L_{\text{IR}} \sim 10^{13}L_\odot$, the so-called hyperluminous infrared galaxies (HLIGs, Sanders & Mirabel 1996; Rowan-Robinson 2000), the implied SFR (Equation 9) would be $\sim 2000 M_\odot yr^{-1}$. Although such super-starburst galaxies likely exist at high redshifts and HLIGs could indeed be such super-starburst systems (if the magnification by a gravitational lens and the AGN contribution to the IR emission are not important), there are no similar systems in the local universe.

If the tight correlations obtained from our local HCN sample are applicable to high-z galaxies, we can roughly estimate their expected molecular gas properties. For the whole sample $L_{\text{IR}}/L_{\text{HCN}}=900L_\odot/K\text{Km s}^{-1}\text{pc}^2$ (Equation 1). But, ULIGs have a slightly higher ratio of $L_{\text{IR}}/L_{\text{HCN}}=1200L_\odot/K\text{Km s}^{-1}\text{pc}^2$ (Table 2). For high-z galaxies with $L_{\text{IR}} \sim 10^{13}L_\odot$, we thus expect to have $L_{\text{HCN}} \sim 0.8 \times 10^{10}K\text{Km s}^{-1}\text{pc}^2$ if they are the analogs of local ULIGs. This is only a factor of 2–3 higher than the highest $L_{\text{HCN}}$ observed in local ULIGs. The first high-z HCN detection is from the Cloverleaf quasar at $z=2.567$, helped by the magnification of the gravitational lens, with the intrinsic, magnification-corrected $L_{\text{HCN}} \sim 0.3 \times 10^{10}K\text{Km s}^{-1}\text{pc}^2$ (Solomon et al. 2003). It appears that, if the hot dust AGN component can be subtracted from the total IR emission, then even the Cloverleaf quasar seems to fit the IR–HCN correlation determined here from the local universe.

At present, more than 20 CO sources at high-z have been detected, all with high CO luminosity (e.g., Cox et al. 2002; Carilli et al. 2002; Papadopoulos et al. 2001; Guilloteau et al. 1999; Downes et al. 1999; Frayer et al. 1998). We can estimate the expected $L_{\text{HCN}}$ for these extraordinarily large $L_{\text{CO}}$ sources detected at high-z using our HCN–CO correlation (Fig. 3). Cox et al. (2002) report the strongest CO emitter detected to date, with $L_{\text{CO}} \sim 10^{11}K\text{Km s}^{-1}\text{pc}^2$ even after the magnification by the gravitational lens is corrected. Using Equation 2, or $L_{\text{HCN}}/L_{\text{CO}} = 0.1 \times (L_{\text{CO}}/10^{10}K\text{Km s}^{-1}\text{pc}^2)^{0.38}$, we obtain $L_{\text{HCN}} \sim 2.4 \times 10^{10}K\text{Km s}^{-1}\text{pc}^2$ and $L_{\text{HCN}}/L_{\text{CO}}=0.24$. This $L_{\text{HCN}}/L_{\text{CO}}$ ratio is same as the very highest found for local ULIGs. For this largest $L_{\text{CO}}$ source, $L_{\text{IR}} \sim 2.0 \times 10^{13}L_\odot$ is estimated by Cox et al. (2002) based on the 250 and 350 GHz measurements (Omont et al. 2001; Isaak et al. 2002). And using $L_{\text{IR}}/L_{\text{HCN}}=1200L_\odot/K\text{Km s}^{-1}\text{pc}^2$, we expect to have $L_{\text{HCN}} = 1.7 \times 10^{10}K\text{Km s}^{-1}\text{pc}^2$. Perhaps better estimate of the expected HCN luminosity can be constrained from both IR and CO luminosities by using the multiparameter fit from Equation 4. Indeed, we also obtain $L_{\text{HCN}} = 1.7 \times 10^{10}K\text{Km s}^{-1}\text{pc}^2$. If such high $L_{\text{HCN}}$ is eventually detected in this source, then star formation rather than AGN must be responsible for most of the high infrared luminosity.

We can also roughly estimate the expected upper limit of $L_{\text{IR}}/L_{\text{CO}}$ for HLIGs and/or high-z galaxies, if they are powered by star formation and the correlation between $L_{\text{IR}}/L_{\text{CO}}$ and $L_{\text{HCN}}/L_{\text{CO}}$ found in Fig. 5a is applicable. Although very large scatters ($\sim 0.5$ dex, 2$\sigma$) in the fit, this should be good within a factor of $\sim 3$. Here, we can take $L_{\text{HCN}}/L_{\text{CO}} \lesssim 1$ as upper limits. Using the orthogonal fit $\log L_{\text{IR}}/L_{\text{CO}} = 1.24\log L_{\text{HCN}}/L_{\text{CO}}+3.24$, we obtain $L_{\text{IR}}/L_{\text{CO}} \lesssim 1700L_\odot/K\text{Km s}^{-1}\text{pc}^2$. Therefore, the expected maximum $L_{\text{IR}}$ from star formation is always less than $\lesssim 1.7 \times 10^{14}(L_{\text{CO}}/10^{11}K\text{Km s}^{-1}\text{pc}^2)L_\odot$.

Although many submillimeter galaxies are HLIGs, few have $L_{\text{IR}} \sim 10^{14}L_\odot$ (Chapman et al. 2003). The highest $L_{\text{CO}}$ detected among submillimeter galaxies is $\sim 0.7 \times 10^{11}K\text{Km s}^{-1}\text{pc}^2$ (Neri et al. 2003; Greve, Ivison, & Papadopoulos 2003). If any sources indeed have high $L_{\text{HCN}} \sim 0.7 \times 10^{11}K\text{Km s}^{-1}\text{pc}^2$, it appears that extreme starbursts from abundant active star-forming dense molecular gas are still possible to power such $L_{\text{IR}} \sim 10^{14}L_\odot$ sources. This corresponds roughly to the maximum possible SFR of $\sim 2 \times 10^5 M_\odot/yr$.
HCN luminosity is a tracer of dense molecular gas, $n(H_2) \gtrsim 3 \times 10^4 \text{cm}^{-3}$, associated with star-forming molecular cloud cores. Here we briefly summarize the principal results found from an analysis of our HCN survey of galaxies:

1. A tight linear correlation between the IR and HCN luminosities $L_{\text{IR}}$ and $L_{\text{HCN}}$ in 65 galaxies is established with a correlation coefficient $R=0.94$ (Fig. 1a). There is also a significant correlation between the normalized luminosities $L_{\text{IR}}/L_{\text{CO}}$ and $L_{\text{HCN}}/L_{\text{CO}}$ which confirms the true physical relationship between $L_{\text{IR}}$ and $L_{\text{HCN}}$. The IR–HCN linear correlation is valid over 3 orders of magnitude including ultraluminous infrared galaxies, the most luminous galaxies in the local universe. The direct consequence of the linear IR–HCN correlation is that the star formation law in terms of dense molecular gas content has a power law index of 1.0. The global star formation rate is linearly proportional to the mass of dense molecular gas in normal spiral galaxies, luminous infrared galaxies and ultraluminous infrared galaxies. This is strong evidence in favor of star formation as the power source in ultraluminous infrared galaxies since the star formation in these galaxies appears to be normal and expected given their high mass of dense star-forming gas.

2. The star formation rate indicated by $L_{\text{IR}}$ depends on the amount of dense molecular gas traced by HCN emission, not the total molecular gas traced by CO emission. One of the main arguments in favor of an AGN (Sanders et al. 1988) as the power source in ultraluminous infrared galaxies is the anomalously high ratio $L_{\text{IR}}/L_{\text{CO}}$ or $L_{\text{IR}}/M(H_2)$. The IR luminosity and thus the required star formation rate per solar mass of molecular gas traced by CO emission, is an order of magnitude higher in ultraluminous infrared galaxies and most luminous infrared galaxies than in normal spiral galaxies. This has been interpreted as indicating that a dust enshrouded AGN rather than star formation is required to produce the very high luminosity (Sanders et al. 1991). Viewed in terms of the dense gas mass the situation is completely different. The ratio $L_{\text{IR}}/L_{\text{HCN}}$ or $L_{\text{IR}}/M_{\text{dense}}$ is the same in all galaxies including ultraluminous infrared galaxies. Fig. 8 shows that $L_{\text{IR}}/M_{\text{dense}}$ is virtually independent of galaxy luminosity and on average $L_{\text{IR}}/M_{\text{dense}} \approx 90L_\odot/M_\odot$, about the same as in molecular cloud cores but much higher than in GMCs as a whole (Mooney & Solomon 1988). Ultraluminous infrared galaxies simply have a large quantity of dense molecular gas and thus produce a prodigious starburst which heats the dust, produces the IR, and blocks all or most optical radiation. Although ultraluminous infrared galaxies are not simply scaled up versions of normal spirals in terms of their total molecular mass they are scaled up version in terms of their dense molecular gas mass, which is exactly what is expected if star formation is the power source. We note that our sample includes 9 ultraluminous infrared galaxies ($L_{\text{IR}} > 0.8 \times 10^{12}L_\odot$) and 23 luminous infrared galaxies ($L_{\text{IR}} > 10^{11}L_\odot$) but only 2 “warm” ultraluminous infrared galaxies (Surace et al. 1998), a subclass representing about 25% of all ultraluminous infrared galaxies. Although these 2 warm ultraluminous infrared galaxies appear normal in terms of the IR/HCN ratio further investigation of warm ultraluminous infrared galaxies is required.

3. The HCN/CO ratio is an indicator of the fraction of dense molecular gas available for vigorous star formation and gauges the globally averaged molecular gas density. It is a powerful starburst indicator.

There is a strong correlation between the HCN and CO luminosities in galaxies although the
correlation is not linear. The ratio $L_{\text{HCN}}/L_{\text{CO}}$ is constant for normal spirals and increases for luminous and ultraluminous IR galaxies. The global ratio $L_{\text{HCN}}/L_{\text{CO}} = <\text{SBR}>$ (see Tables 1 & 2) has an average ratio of 1/6 for ultraluminous infrared galaxies and only 1/25 for normal spirals. The HCN/CO ratio is a very successful predictor of starbursts (Fig. 4) and directly correlates with the star formation efficiency indicator $(L_{\text{IR}}/L_{\text{CO}})$ (Fig. 5a). The SFE increases dramatically from dense molecular gas-poor galaxies to dense molecular gas-rich galaxies. All galaxies in our sample with a high dense gas mass fraction indicated by $L_{\text{HCN}}/L_{\text{CO}} > 0.06$ are luminous or ultraluminous infrared galaxies (Fig. 4).

4. The correlation between $L_{\text{IR}}$ and $L_{\text{CO}}$ may be a consequence of the stronger correlations between $L_{\text{IR}}$ and $L_{\text{HCN}}$ and between $L_{\text{HCN}}$ and $L_{\text{CO}}$. A model two parameter fit for $L_{\text{IR}}(L_{\text{HCN}}, L_{\text{CO}})$ shows almost no dependence on $L_{\text{CO}}$. Much of the CO emission originates from moderate density regions with low to moderate dust temperature and little or no active high mass star formation. A critical molecular parameter that measures star formation rates in galaxies is the amount of dense molecular gas measured by the HCN luminosity. High molecular gas density, strong HCN emission and warm dust heated by the newly formed OB stars go together.

5. A quantitative star formation theory must start with the processes that form dense cloud cores, particularly massive cores. It appears from our survey of 65 IR/CO bright galaxies including normal spirals and very luminous IR starbursts that once the local density is raised from that of a typical GMC $(n(H_2) \sim \text{a few } \times 10^2 \text{ cm}^{-3})$ to $3 \times 10^4 \text{cm}^{-3}$, star formation including high mass star formation is efficient and progresses rapidly. The characteristic time scale for using half of all the dense gas is about $2 \times 10^7$ years.

Many thanks are due to Judy Young for helpful discussions and suggestions. We appreciate the generous support and allocation of observing time from the NRAO 12m, IRAM 30m, and FCRAO 14m telescopes. We also thank the anonymous referee for a careful and helpful report. This research has made use of the NASA/IPAC Extragalactic Database (NED) which is operated by the Jet Propulsion Laboratory, Caltech, under contract with the National Aeronautics and Space Administration.
Table 1. Global Properties of Galaxies in the HCN Survey

<table>
<thead>
<tr>
<th>Galaxies</th>
<th>$D_L$ (Mpc)</th>
<th>$L_{IR}$ ($10^{10} L_\odot$)</th>
<th>$L_{CO}$ ($10^8$ Kkm$^{-1}$pc$^{-2}$)</th>
<th>$L_{HCN}$ ($b$)</th>
<th>$L_{HCN}/L_{CO}$</th>
<th>$L_{IR}/L_{HCN}$ ($L_\odot$/$K$km$^{-1}$pc$^{-2}$)</th>
<th>$T_{dust}$ (K)</th>
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REFERENCES

Devereux, N.A., & Young, J.S. 1992, AJ, 103, 1536
Gao, Y. 1996, Ph.D. thesis, SUNY at Stony Brook
Gao, Y. 1997, PASP, 109, 1189
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**Literature HCN Data**

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*a* This table contains all HCN Survey data of Gao & Solomon (2003, Paper I) and includes a dozen galaxies in the literature (almost entirely from Solomon et al. 1992, but M51 and NGC 4945 from Nguyen-Q-Rieu et al. 1992 and Henkel et al. 1994 respectively). LIGs with $L_{IR}$ > $10^{11}$L$_\odot$ are in boldface, and ULIGs with $L_{IR}$ > $10^{11}$L$_\odot$ are further marked with.

*b* The 2σ upper limit (<) is listed for NGC 1055. The lower limits (>) are for nearby galaxies where we only detected HCN in the galaxy centers or more extensive mapping is still required.

*c* The warm dust temperature, $T_{dust}$ and $T_{BB}$ are for optical thin dust with emissivity $\sim \nu^\beta$, $\beta = 1.5$ and black-body dust respectively.
Fig. 1.— (a) The correlation between HCN and IR luminosities in 65 galaxies. Some limits in HCN luminosities are indicated with arrows. (b) The correlation between $L_{\text{CO}}$ and $L_{\text{IR}}$ for the same HCN sample. The sample is divided into luminous infrared galaxies (LIGs) and ultraluminous infrared galaxies (ULIGs) with $L_{\text{IR}} \geq 10^{11}L_{\odot}$ and less luminous “normal” spiral galaxies. The solid lines are the fits to the less luminous galaxies with a slope fixed at unity. A single slope fits HCN data for both low and high IR luminosities, but not for the CO data.
Fig. 2.— (a) The correlation between IR/HCN and IR is almost of nonexistence; (b) whereas the correlation between $L_{IR}/L_{CO}$ and $L_{IR}$ is rather prominent. The different trends are particularly obvious at high IR luminosity for LIGs and ULIGs with $L_{IR} \geq 10^{11}L_\odot$. 
Fig. 3.— Strong correlation exists between $L_{\text{HCN}}$ and $L_{\text{CO}}$ indicating that the more molecular gas-rich galaxies tend to have larger amount of dense molecular gas as well. Similar to Figure 1b, the fit line is for less luminous normal galaxies (open circles) with a fixed slope at unity. Apparently, almost all LIGs and ULIGs (filled circles, $L_{\text{IR}} \geq 10^{11} L_\odot$) lie above the line.

The fit line: $L_{\text{HCN}}/L_{\text{CO}} = 0.039$ for $L_{\text{IR}} < 10^{11} L_\odot$
Fig. 4.— $L_{\text{HCN}}/L_{\text{CO}}$ stays at fairly small values for normal spiral galaxies, but increases dramatically for LIGs and ULIGs ($L_{\text{IR}} \geq 10^{11}L_\odot$). All galaxies with $L_{\text{HCN}}/L_{\text{CO}} > 0.06$ in the sample are luminous and ultraluminous galaxies.
Fig. 5.— (a) Correlation between $L_{\text{HCN}}/L_{\text{CO}}$ and $L_{\text{IR}}/L_{\text{CO}}$ revealing the true relationship between the HCN and IR since both luminosities are normalized by $L_{\text{CO}}$. This removes all dependence on distance and galaxy size and shows that there is a true physical correlation between the IR and HCN luminosities. The best fit has a correlation coefficient of 0.74. (b) No correlation between $L_{\text{IR}}/L_{\text{HCN}}$ and $L_{\text{CO}}/L_{\text{HCN}}$ (a correlation coefficient of 0.1) suggests that the correlation between $L_{\text{IR}}$ and $L_{\text{CO}}$ observed in Figure 1b may not be a truly physical relation. The sample is divided into LIGs and ULIGs with $L_{\text{IR}} \geq 10^{11}L_\odot$ (filled circles) and less luminous normal spiral galaxies (open circles).
Fig. 6.— The global star formation law of dense molecular gas. The star formation rate is linearly proportional to the dense molecular gas mass. The solid circles are for LIGs and ULIGs with $L_{IR} > 10^{11} L_\odot$, whereas the open circles are for the less luminous normal spiral galaxies.
Fig. 7.— The star formation rate vs. total molecular gas mass. The solid circles are for LIGs and ULIGs with $L_{\text{IR}} \gtrsim 10^{11} L_\odot$, whereas the open circles are for the less luminous normal spiral galaxies. More CO data of mostly ULIGs available from literature (stars) are added to our HCN sample (see Fig. 1b). The orthogonal least-squares fit now has a slope of 1.73 with a correlation coefficient $R = 0.89$. The line of fixed slope of 1, a valid fit for normal spirals, is also shown for comparison.
Fig. 8.— The star formation rate (far-IR luminosity) per unit of dense molecular gas mass is essentially independent of the total IR luminosity. The average for the entire sample is $L_{\text{IR}}/M_{\text{dense}} = 90L_\odot/M_\odot$. 
Stutzki, J., Genzel, R., Harris, A.I., Herman, J., & Jaffe, D.T. 1988, 330, L125

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A. Correlations with the Warm Dust Temperature

$T_{\text{dust}}$ can be estimated as a function of $f_{60\mu m}/f_{100\mu m}$ and $\beta$, where the emissivity is the Planck function of a single warm dust temperature times the frequency to the power of $\beta$, i.e., $\nu^\beta$. This is for warm dust $T_{\text{dust}}$ between $\sim 25$ and 60 K and $\beta \sim 1-2$ (e.g., see the appendix of Lonsdale et al. 1985). Although there are increasing evidences for a substantial amount of cold dust in spiral galaxies (e.g., Alton et al. 2000), the warm dust is probably still the most important component, especially in luminous and ultraluminous infrared galaxies (LIGs and ULIGs), and often a single dust temperature fitting of the far-IR spectral energy distribution (SED) including submillimeter measurements is still a good approximation (e.g., Lisenfeld, Isaak, & Hills 2000; Dunne et al. 2000).

The tightness of the IR–HCN correlation can be further examined by comparing the ratio of $L_{\text{IR}}/L_{\text{HCN}}$ with $f_{60\mu m}/f_{100\mu m}$. Fig. 9a shows that $L_{\text{IR}}/L_{\text{HCN}}$ has only a weak dependence upon $f_{60\mu m}/f_{100\mu m}$ or $T_{\text{dust}}$, whereas Fig. 9b indicates that $L_{\text{IR}}/L_{\text{CO}}$ correlates strongly with $f_{60\mu m}/f_{100\mu m}$ ($R^2 = 0.72$) or $T_{\text{dust}}$. The fits in terms of dust temperature have been given in Equations 6 & 7. Unlike Fig. 5b, where $L_{\text{IR}}/L_{\text{HCN}}$ appears to be totally independent of $L_{\text{CO}}/L_{\text{HCN}}$, Fig. 9a indicates that $L_{\text{IR}}/L_{\text{HCN}}$ might still be weakly dependent upon $T_{\text{dust}}$. There is also a meaningful correlation between $L_{\text{HCN}}/L_{\text{CO}}$ and $f_{60\mu m}/f_{100\mu m}$ ($R^2 = 0.46$). An orthogonal fit can be obtained as: $L_{\text{HCN}}/L_{\text{CO}} = 10^{-0.71} \times (f_{60\mu m}/f_{100\mu m})^{2.02 \pm 0.8}$ with a fairly large uncertainty in the slope since the correlation is not tight.

It is not surprising that $L_{\text{IR}}/L_{\text{CO}}$ is a strong function of $T_{\text{dust}}$ since the dust radiates thermally as $T_{\text{dust}}^4$ to $T_{\text{dust}}^6$, depending on the dust emissivity to produce IR emission (e.g., Soifer et al. 1989). Thus, Fig. 9 is not independent of Fig. 2 as the $T_{\text{dust}}$ is closely related to $L_{\text{IR}}$. And $L_{\text{CO}}$ depends only linearly on the temperature, $\sim T_{\text{dust}}^1$, if the dust and gas are coupled and the gas is thermalized with the intrinsic brightness temperature $T_b \sim T_{\text{dust}}^1$ (e.g., black-body model of Solomon et al. 1997), and more weakly in the power of $T_{\text{dust}}$ dependence if not coupled/thermalized. HCN luminosity seems also to depend, at a first glance, only on the first power of $T_{\text{dust}}$ if $T_b \sim T_{\text{dust}}^1$, but is actually not simply proportional to $T_{\text{dust}}^1$, since the HCN emission does not fill the source area probed by the telescope beam, whereas CO has probably an area filling factor close to unity, especially for LIGs/ULIGs. Recent interferometric HCN imaging in some nearby galaxies indeed shows the much smaller HCN source size than that of the CO (e.g., Downes et al. 1992; Helfer & Blitz 1997a; Kohno et al. 1996 Kohno, Kawabe, & Vila-Vilaro 1999). It is possible that the HCN filling factor depends on some power of $T_{\text{dust}}$. In addition, the weak correlation between $L_{\text{IR}}/L_{\text{HCN}}$ and $T_{\text{dust}}$ indicates that there might be other important parameters, probably the molecular gas density, responsible for the high $L_{\text{HCN}}$ in LIGs/ULIGs. Thus, $L_{\text{HCN}}$ depends on a much higher power of $T_{\text{dust}}$, and the ratio of $L_{\text{HCN}}/L_{\text{CO}}$ should depend fairly strongly on some power of $T_{\text{dust}}$ (Equation 8) rather than being independent.

This rather strong $T_{\text{dust}}$ dependence of $L_{\text{HCN}}$ can be understood since $L_{\text{HCN}}/L_{\text{CO}}$ also correlates with $L_{\text{IR}}$ (Fig. 4). Because $L_{\text{HCN}}/L_{\text{CO}}$ ratio is such a good indicator of starburst and most LIGs/ULIGs have the highest $L_{\text{HCN}}/L_{\text{CO}}$ ratio, warmer LIGs/ULIGs of higher $T_{\text{dust}}$ (higher $L_{\text{IR}}$) should have higher fraction of the dense molecular gas (higher $L_{\text{HCN}}/L_{\text{CO}}$), thus very likely high average gas density. Therefore, $L_{\text{IR}}$ somehow correlates with the molecular gas density and $T_{\text{dust}}$ is thus also related to the gas density, which should be expected as the gas and dust are somehow coupled.
B. Other Correlations and Model Fits

B.1. Simple Two-parameter Fits

Although \( L_{\text{HCN}} \) and \( L_{\text{CO}} \) are strongly correlated (Fig. 3), there are significant differences, however, when they are compared with either \( L_{100\mu m} \) or \( L_{\text{IR}} \) (Table B1). The better correlation between \( L_{\text{HCN}} \) and \( L_{\text{IR}} \) (vs \( L_{\text{CO}} \) and \( L_{\text{IR}} \)) can be recognized in both \( R^2 \) (§3.1) and the R.M.S. deviations of the correlation fits (Table 3, 0.24 vs 0.34). \( L_{100\mu m} \) also correlates better with \( L_{\text{HCN}} \) than with \( L_{\text{CO}} \) as the difference in \( R^2 \) is also significant (0.89 vs 0.80). Nevertheless, \( L_{\text{CO}} \) appears to be much better correlated with \( L_{100\mu m} \) than with \( L_{\text{IR}} \) (there is a significant difference in the logarithmic R.M.S. deviations: 0.22 vs 0.34), though the difference in \( R^2 \) (0.80 vs 0.77) is small. In comparison, there is little difference in the correlations between \( L_{100\mu m} \) and \( L_{\text{HCN}} \) vs between \( L_{\text{IR}} \) and \( L_{\text{HCN}} \) \( (R^2=0.89 \text{ vs } 0.87 \text{ and logarithmic R.M.S. } 0.22 \text{ vs } 0.23) \). But the HCN correlations are much tighter than the CO correlations, even though \( L_{100\mu m}-L_{\text{CO}} \) correlation has same R.M.S. deviation.

\( L_{100\mu m} \) traces relatively the cooler component of the warm dust emission at \( T_{\text{dust}} \sim 25 \text{ to } 50 \text{ K} \). In this regime the 100 \( \mu m \) emission is a reasonable tracer of the warm dust mass \( M_{\text{dust}} \). In comparison, the total IR luminosity \( L_{\text{IR}} \) contains the mixture of the various dust components including the hot dust radiated at the mid-IR emission. We here estimate the warm dust mass \( M_{\text{dust}} \) from the 100 \( \mu m \) flux density and the warm dust temperature \( T_{\text{dust}} \) and correlate it with \( L_{\text{HCN}} \) and \( L_{\text{CO}} \) in Table 3 as well. It appears that \( L_{\text{CO}} \) is only slightly better correlated with \( M_{\text{dust}} \) than \( L_{\text{HCN}} \) with \( M_{\text{dust}} \). In short, the differences in the above mentioned various correlations suggest that HCN is a much better tracer of both the total IR and 100 \( \mu m \) emission than CO, whereas CO only traces better the 100 \( \mu m \) than the total IR emission. The dense molecular gas rather than the total molecular gas is more intimately related to both the total IR and 100 \( \mu m \) emission.

B.2. Multi-parameter Fits

The predicted \( L_{\text{IR}} \) from the CO and \( T_{\text{dust}} \) [the IR(\( L_{\text{CO}}, T_{\text{dust}} \)) model, \( \log L_{\text{IR}}(L_{\text{CO}}, T_{\text{dust}}, \text{model fit}) = -6.34 + 0.97 \log L_{\text{CO}} + 5.41 \log T_{\text{dust}} \)] turns out to be the tightest correlation \( (R^2 = 0.94) \) among all. The prediction of \( L_{\text{IR}} \) from CO and \( T_{\text{dust}} \) is more accurate than from HCN alone and also slightly better than from HCN plus \( T_{\text{dust}} \). This surely suggests the importance of \( T_{\text{dust}} \) when the total molecular gas content, rather than the dense molecular gas, is concerned in predicting \( L_{\text{IR}} \). This also implies that \( T_{\text{dust}} \) is an important parameter in regulating star formation of the total molecular gas, but not necessarily the dense (active star-forming) molecular gas.

We also use \( T_{\text{dust}} \) as well as \( L_{100\mu m} \) and \( L_{60\mu m} \), though they are not independent, in the three parameter model fits as listed in Table 3, we find:

1. \( L_{\text{CO}} \) decreases with \( T_{\text{dust}} \) for given \( L_{100\mu m} \) or \( L_{\text{IR}} \). \( L_{\text{HCN}} \) is, however, linearly proportional to \( L_{100\mu m} \) or \( L_{\text{IR}} \) with no correction or only marginal correction of \( T_{\text{dust}} \). Thus, \( L_{\text{HCN}} \) decreases little with \( T_{\text{dust}} \) for given \( L_{100\mu m} \) or \( L_{\text{IR}} \). To put in a different perspective, \( L_{\text{IR}} \) is only slightly better predicted when \( T_{\text{dust}} \) is considered together with \( L_{\text{HCN}} \), whereas \( L_{\text{IR}} \) is much better predicted when both \( L_{\text{CO}} \) and \( T_{\text{dust}} \) are in the fit.

2. The HCN(\( L_{\text{CO}}, T_{\text{dust}} \)) model implies that HCN correlates with \( T_{\text{dust}} \) to the 3.3 power as well as linearly (the first power) with \( L_{\text{CO}} \) (cf. Equation 8). Thus, HCN responds sensitively to \( T_{\text{dust}} \) with a
power of $\sim 4.3$. The IR($L_{\text{HCN}}, T_{\text{dust}}$) model gives a 2.9 power correlation with $T_{\text{dust}}$ and a 0.86 power with $L_{\text{HCN}}$, again roughly consistent with the high ($\sim 6$) power $T_{\text{dust}}$ dependence (cf. Equation 6) when the $T_{\text{dust}}$ dependence of $L_{\text{HCN}}$ (Equation 8) has been considered. In short, the difference in the dependences of the HCN and IR upon the warm dust temperature is not dramatic. Thus, the ratio of $L_{\text{IR}}/L_{\text{HCN}}$ has only a weak dependence upon the $T_{\text{dust}}$ (Equation 7 and Fig. 9a).

3. The HCN($L_{60\mu m}, L_{100\mu m}$) model fit and CO($L_{60\mu m}, L_{100\mu m}$) 3 parameter correlation model reveal that HCN correlates primarily with $L_{100\mu m}$, essentially none with $L_{60\mu m}$, whereas CO correlates with both $L_{60\mu m}$ and $L_{100\mu m}$. This again implies the much tighter dependence of CO upon the $T_{\text{dust}}$, and is basically reiterating the first point described above since there are only two independent parameters among $T_{\text{dust}}, L_{60\mu m}, L_{100\mu m}$, and $L_{\text{IR}}$.

### B.3. Fits Between the Ratios

Figs. 4–5 showed already some correlations between the parameter ratios. We here just list the weak correlation between $L_{\text{IR}}/L_{\text{HCN}}$ and $f_{60\mu m}/f_{100\mu m}$ ($R^2 = 0.17$, Fig. 9a), as directly obtained from the orthogonal fit, $L_{\text{IR}}/L_{\text{HCN}} = 10^{0.31} \times (f_{60\mu m}/f_{100\mu m})^{0.63}$ with a logarithmic R.M.S. of 0.21. The $T_{\text{dust}}$ power and the fit have large scatters owing to the poor correlation. The scatter is about the same or only slightly better than the R.M.S. of the observed mean luminosity ratio (in logarithm) of $\log \langle L_{\text{IR}}/L_{\text{HCN}} \rangle$ for the entire sample which is $\sim 0.23$. The correlation fit for $L_{\text{IR}}/L_{\text{CO}}$ vs $f_{60\mu m}/f_{100\mu m}$ or $T_{\text{dust}}$ (Fig. 9b, Equation 6) is significantly tighter and has a logarithmic R.M.S. of 0.17. This small R.M.S. is obviously much better than the R.M.S. of the observed mean $\log \langle L_{\text{IR}}/L_{\text{CO}} \rangle$ of the entire sample which is $\sim 0.37$. 
Table 2. Summary of the Average IR, CO and HCN Luminosities

<table>
<thead>
<tr>
<th>$L_{IR}/L_\odot$</th>
<th>No. 1 of galaxies</th>
<th>$L_{HCN}$ $10^8$ Kkm s$^{-1}$pc$^2$</th>
<th>$L_{CO}$ $10^8$ Kkm s$^{-1}$pc$^2$</th>
<th>$L_{IR}/L_{CO}$</th>
<th>$L_{IR}/L_{HCN}$</th>
<th>$L_{CO}/L_{HCN}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal $&lt; 10^{11}$</td>
<td>26</td>
<td>0.35</td>
<td>8.7</td>
<td>30</td>
<td>740</td>
<td>25</td>
</tr>
<tr>
<td>$10^{11} &lt; L_{IGS} \lesssim 0.8 \times 10^{12}$</td>
<td>22</td>
<td>3.1</td>
<td>47.6</td>
<td>60</td>
<td>890</td>
<td>14</td>
</tr>
<tr>
<td>ULIGs $\gtrsim 10^{11.9}$</td>
<td>9</td>
<td>9.0</td>
<td>69.0</td>
<td>170</td>
<td>1200</td>
<td>6</td>
</tr>
<tr>
<td>All</td>
<td>57</td>
<td>1.5</td>
<td>20.0</td>
<td>50</td>
<td>870</td>
<td>17</td>
</tr>
</tbody>
</table>

1 Galaxies with only limits in HCN luminosity are excluded.

2 The inverse of average surface brightness ratio $<SBR> \equiv L_{HCN}/L_{CO}$ indicating the fraction of dense molecular gas in galaxies.
Table 3. Model Fit Parameters

<table>
<thead>
<tr>
<th>Para.</th>
<th>$L_{60\mu m}$</th>
<th>$L_{100\mu m}$</th>
<th>$L_{IR}$</th>
<th>$T_{dust}$</th>
<th>$L_{HCN}$</th>
<th>$L_{CO}$</th>
<th>Const.</th>
<th>RMS(^a)</th>
<th>$R^2(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{IR}$</td>
<td>0.88</td>
<td>0.16</td>
<td>2.29</td>
<td>0.24</td>
<td>0.87</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{IR}$</td>
<td>5.4</td>
<td>0.97</td>
<td>6.34</td>
<td>0.17</td>
<td>0.94</td>
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<td></td>
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</tr>
<tr>
<td>$L_{IR}$</td>
<td>2.9</td>
<td>0.86</td>
<td>0.48</td>
<td>0.20</td>
<td>0.92</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{HCN}$</td>
<td>3.3</td>
<td>1.00</td>
<td>2.22</td>
<td>0.21</td>
<td>0.89</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{HCN}$</td>
<td>0.56</td>
<td>0.49</td>
<td>2.62</td>
<td>0.19</td>
<td>0.90</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{HCN}$</td>
<td>0.96</td>
<td>(−1.7)</td>
<td>0.17</td>
<td>0.21</td>
<td>0.89</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{CO}$</td>
<td>0.89</td>
<td>−4.4</td>
<td>6.27</td>
<td>0.17</td>
<td>0.88</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$L_{HCN}$</td>
<td>0.95</td>
<td>(−0.2)</td>
<td>−1.12</td>
<td>0.21</td>
<td>0.89</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{CO}$</td>
<td>0.88</td>
<td>−3.0</td>
<td>5.07</td>
<td>0.17</td>
<td>0.88</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$L_{HCN}$</td>
<td>(−0.06)</td>
<td>1.01</td>
<td>−1.41</td>
<td>0.21</td>
<td>0.89</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_{CO}$</td>
<td>−1.13</td>
<td>2.02</td>
<td>0.47</td>
<td>0.17</td>
<td>0.88</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3 parameter correlations

| $L_{IR}$ | 1.25          | 0.72           | 0.34     | 0.77        |          |        |        |          |          |
| $L_{IR}$ | 1.02          | 2.58           | 0.24     | 0.88        |          |        |        |          |          |
| $L_{HCN}$ | 1.26         | −3.73          | 0.28     | 0.85        |          |        |        |          |          |
| $L_{HCN}$ | 0.91          | −1.93          | 0.23     | 0.87        |          |        |        |          |          |
| $L_{HCN}$ | 0.96          | −1.52          | 0.22     | 0.89        |          |        |        |          |          |
| $L_{CO}$  | 0.72          | −2.12          | 0.22     | 0.80        |          |        |        |          |          |
| $L_{CO}$  | 0.62          | 2.49           | 0.30     | 0.75        |          |        |        |          |          |
| $L_{CO}$  | 0.67          | 3.92           | 0.27     | 0.85        |          |        |        |          |          |
| $M_{dust}$ | 0.84          | 0.41           | 0.25     | 0.84        |          |        |        |          |          |
| $M_{dust}$ | 1.13          | −3.35          | 0.21     | 0.87        |          |        |        |          |          |

simple 2 parameter correlations

\(^a\)R.M.S. = Root Mean Square deviation (logarithmic) between the data (observed) and the predicted from the (correlation) model fits.

\(^b\)The squared correlation coefficient $R^2$.

Note. — All quantities are logarithmic in the correlation fits. The line luminosities are in units of $K\text{km s}^{-1}\text{pc}^2$ and the total IR, 60, and 100 $\mu$m luminosities are in units of $L_\odot$ while $T_{dust}$ in Kelvin. The warm dust mass $M_{dust}$ was estimated using only the 100 $\mu$m flux and warm dust temperature. The model fit for $L_{IR}$ from $L_{HCN}$ and $L_{CO}$ (the IR($L_{HCN}$, $L_{CO}$) model), for example, is $\log L_{IR}(predicted) = 0.88 \log L_{HCN} + 0.16 \log L_{CO} + 2.3$ with the correlation coefficient $R = 0.93$ ($R^2 = 0.87$) and the R.M.S. error $(\Sigma[\log L_{IR}(predicted) − \log L_{IR}(obs.))]^2/N)^{1/2} = 0.24$. Numbers with brackets () indicate only marginal significance.
Fig. 9.— (a) $L_{\text{IR}}/L_{\text{HCN}}$ as a function of the far-infrared (FIR) color (the 60$\mu$m-to-100$\mu$m flux ratio) which indicates the warm dust temperature $T_{\text{dust}}$. (b) $L_{\text{IR}}/L_{\text{CO}}$ as a function of the FIR color (or $T_{\text{dust}}$). There is only weak correlation between $L_{\text{IR}}/L_{\text{HCN}}$ and $T_{\text{dust}}$, whereas strong correlation exists between $L_{\text{IR}}/L_{\text{CO}}$ and $T_{\text{dust}}$. For optically thin dust with emissivity $\propto \nu^{1.5}$, $T_{\text{dust}}$ is approximately in a range of 25 to 50 K for galaxies in our sample. The sample is divided into luminous and ultraluminous infrared galaxies (LIGs/ULIGs) with $L_{\text{IR}} \geq 10^{11}L_{\odot}$ (filled circles) and less luminous galaxies (open circles).