We present preliminary results for the $D_s$ meson spectrum and decay constants in unquenched lattice QCD. Simulations are carried out with $2+1$ dynamical quarks using gauge configurations generated by the MILC collaboration. We use the “asqtad” $a^2$ improved staggered action for the light quarks, and the clover heavy quark action with the Fermilab interpretation. We compare our spectrum results with the newly discovered $0^+$ and $1^+$ states in the $D_s$ system.

1. INTRODUCTION

$D_s$ physics is assuming a larger importance in lattice QCD and in CKM phenomenology with the arrival of the CLEO-c charm factory [1]. CLEO-c will measure the leptonic decay constants $f_D$ and $f_{D_s}$ to an accuracy of around $2\%$. It will measure the amplitudes of the semileptonic decays $D \rightarrow \pi \ell \nu$ and $D \rightarrow K \ell \nu$ to an accuracy of around $1\%$. This will produce new determinations of the CKM matrix elements $V_{cd}$ and $V_{cs}$ to the accuracy that can be achieved in the required lattice calculations, and new checks of the unitarity triangle. New, precise tests of lattice QCD will come from the amplitude ratios $f_D/D \rightarrow \pi \ell \nu$ and $f_{D_s}/D \rightarrow K \ell \nu$. These ratios, which are independent of the CKM matrix, will provide the most precise tests in existence of the types of lattice QCD calculations required to extract CKM matrix elements from $B$ physics and $D$ physics.

The recent discovery of the positive parity partners of the $D_s$ and the $D_s^*$ [2] has added new interest to the spectrum of the $D_s$ system. The new states lie significantly below quark model predictions, and below the $DK$ threshold, so the states are quite narrow.

2. METHODS

We use the “asqtad” order $a^2$ improved staggered light quark action, and the order $a$ improved Fermilab heavy quark action. In unquenched calculations, chiral extrapolation is the least well controlled remaining source of error. Staggered fermions are the only method currently able to reach the region $m_l \sim m_s/5$ that seems to be required to control this error. Using unquenched calculations that reach this region, a number of simple quantities that disagree with each other at the 10% level in the quenched approximation come into good agreement [3]. We use the public MILC unquenched configurations,

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with two light and one strange sea quarks, with properties

- \( 20^3 \times 64, a \sim 1/8 \text{ fm}, \)
- \( (m_l, m_s) = (0.007, 0.05), (0.01, 0.05), (0.02, 0.05), (0.03, 0.05), (\text{True } m_s \approx 0.041), \)
- \( \sim 500 \) configurations at each mass, 4 time sources each,

as described in [4].

\[ \text{Figure 1. The } D_s \text{ correlation function.} \]

With staggered and naive light quarks, the \( 0^+ \) and \( 1^+ \) are in the \( D_s \) and \( D_s^* \) correlators automatically and unavoidably because of fermion doubling. Naive fermions possess a doubling symmetry, \( \psi \to (i\gamma_\mu \gamma_5)(-1)^\mu \psi \), that implies that quarks come in multiples with identical properties. Local fermion operators connect to all of these doubled modes. Therefore, because of the doubling symmetry, an pseudoscalar current couples to a positive parity scalar state as well: \( \Psi_h \gamma_5 \Psi \to \Psi_h \gamma_5 \Psi \to \Psi h_\mu \gamma_5 (-1)^\mu \psi = i \Psi h_\mu \gamma_0 \psi (-1)^\mu \). Oscillating signals for the positive parity states are present in the correlation functions for the \( D_s \) and \( D_s^* \) mesons, but they are small and hard to see, as can be seen in fig. 1, Fig. 2 shows the same correlation function with the leading exponential scaled out. The exponential decay of the oscillating signal gives the mass of the \( 0^+ \) state.

\[ \text{Figure 2. the } D_s \text{ correlation function with the leading exponential scaled out. The exponential decay of the oscillating signal gives the mass of the } 0^+ \text{ state.} \]

\[ \text{Figure 3. the } D_s^* \text{ correlation function with the leading exponential scaled out. The decay of the oscillating signal gives the mass of the } 0^+ \text{ state. Because it is an excited state, the presence of a plateau is more difficult to ascertain than for a ground state.} \]

3. SPECTRUM

The \( D_s \) spectrum on the \( m_l, m_s = 0.01, 0.05 \) lattices is shown in fig. 3. Very little dependence on the sea quark mass is observed. The \( 0^+ \) and \( 1^+ \) states lie somewhat above experiment. However, they are excited states and are somewhat more sensitive to the choices of the priors for the higher states in the Bayes fits than are the ground states. We obtain \( 0.86 (2) \) for the hyperfine splitting divided by its experimental value. This may be compared with \( 0.82 (2) \) for the hyperfine splitting in charmonium. The most likely source for these errors is the one-loop correction to \( O(a) \bar{\psi} \Sigma \cdot B \psi \) operator in the clover action. Since this operator contributes twice to the charmonium hyperfine splitting and once to the the \( D_s \) hyperfine splitting, it is reasonable to expect a larger effect in charmonium.
4. LEPTONIC DECAY AMPLITUDE

This work is part of a larger project with the MILC collaboration. Preliminary results for the amplitude for leptonic decay of the $D_s$ are shown in fig. 4 as a function of the sea quark mass (using $m_s = 0.041$). There is little dependence on the sea quark mass, as expected. The one-loop Fermilab heavy-staggered light axial current renormalization is in progress [5]. To make an estimate of the one-loop correction, we use the formula $Z_A^{nl} = \rho_A \sqrt{Z_{hh}^{hh} Z_V^{ll}}$ [6], using $Z_{hh}^{hh} = 1.33(2)$ and $Z_V^{ll} = 0.86(5)$ [7]. This leads to a current result for the decay constant of $f_{D_s} = 240$ MeV $\pm/\alpha(\alpha)$. However the order $\alpha$ correction could be large, potentially as large as 30%. This is to be compared with Ryan’s unquenched world average of 250 (30) MeV [8] and the unquenched NRQCD heavy staggered light result of 289(\alpha^2) [9]. The CKM independent quantity $f_{D_s}/f_{D^+ \to K}$ can be formed by combining this result with the result for the semileptonic amplitude in ref. [7]. Since high precision is the ultimate goal, we will not quote a result in this early stage of the error analysis.

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