Simulation of Wilson fermion at finite isospin density

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For QCD with Wilson fermions at isospin chemical potential we study the finite phase transition on an $8^3 \times 4$ lattice at $\kappa = 0.15$. We use two gauge actions: Wilson action and DBW2 action. Both actions give the same results. The phase diagram is qualitatively similar to the one obtained for QCD at small baryon chemical potential. We also calculate the number density for various isospin chemical potentials.

1. INTRODUCTION

Lattice simulations of finite density QCD are difficult due to the sign problem. Namely the action is complex and the complex phase fluctuates, which makes the simulations difficult. Recently considerable progress has been made for small baryon chemical potential $\mu_B$\textsuperscript{1}. There exist several approaches to small $\mu_B$: Taylor expansion method\textsuperscript{2}, reweighting method\textsuperscript{3} and imaginary chemical potential method\textsuperscript{4}.

Apart from the complex action, a model with a positive measure like isospin model can be used to obtain insights to QCD at finite $\mu_B$. Moreover it might be expected that the phase diagram of QCD at small $\mu_B$ is similar to that at small isospin chemical potential $\mu_I$\textsuperscript{5}.

In this study we use Wilson fermions with $\mu_I$ and study the phase diagram and the number density.

2. ISOSPIN DENSITY

Lattice QCD partition function with $N_f$ flavors is given by

$$ Z = \int [dU] \prod_{i=1}^{N_f} \det D(\mu_i) \exp(S_g[U]) $$

where

$$ S_g[U] = \frac{\beta}{3}((1-8c_1)\sum \text{ReTr} W_{11} + c_1 \sum \text{ReTr} W_{12}) $$

and $D(\mu_i)$ is the Wilson fermion matrix at $\mu_i$. $W_{11}$ and $W_{12}$ stand for $1 \times 1$ and $1 \times 2$ loops respectively. For $N_f = 2$ with $\mu_u = -\mu_d \equiv \mu_I$ we obtain

$$ \det D(\mu_u) \det D(\mu_d) = \det D(\mu_I) \det D(-\mu_I) = |\det D(\mu_I)|^2 $$

where a relation $D(\mu_i) = \gamma_5 D^\dagger(-\mu_i)\gamma_5$ is used. We call $\mu_I$ isospin chemical potential. For this $\mu_I$ the measure is positive definite and the standard Monte Carlo technique can be applied.

The parameter $c_1$ in eq.(2) is the one which specifies the gauge action. For example $c_1 = 0$ for the Wilson gauge action, and $c_1 = -1.4089$ for the DBW2 action\textsuperscript{6}.

3. SIMULATIONS

We generate configurations on an $8^3 \times 4$ lattice at $\kappa = 0.15$ by the hybrid Monte Carlo (HMC) algorithm. We use the Wilson gauge action and the DBW2 action\textsuperscript{6}. Simulation parameters are summarized in Tables 1 and 2. We choose a step size so that the acceptance of the HMC algorithm becomes $60 \sim 70\%$ which gives the maximum performance of the HMC algorithm with the 2nd order leapfrog integrator\textsuperscript{7}. This optimum acceptance $\approx 60 \sim 70\%$ does not depend on the details of the action we take. It depends on the order of the leapfrog integrator.
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<th>Traj.</th>
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Table 1
Simulation parameters for the Wilson gauge action. $\Delta t$ is the step size and the trajectory length is set to 1. "Acc." and "Traj." stand for Acceptance and # of trajectories respectively.

<table>
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<tr>
<th>$\mu_I$</th>
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<th>$\Delta t$</th>
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Table 2
Same as in Table 1 but for the DBW2 action.

4. RESULTS

In order to locate the phase transition point we measure susceptibility for various observables. Since for each $\mu_I$ we performed a single simulation at one $\beta$, the reweighting method[8] was used to investigate a region in the vicinity of $\beta$. The expectation value of an observable $O$ at $\beta'$ can be obtained through a single simulation at $\beta$ by

$$\langle O \rangle_{\beta'} = \frac{\langle O \exp(\Delta \beta) \rangle_\beta}{\langle \exp(\Delta \beta) \rangle_\beta}$$

where $\Delta \beta = \beta' - \beta$.

Measurements are done for $1 \times 1$, $1 \times 2$, $2 \times 2$ Wilson and Polyakov loops. Figure 1 shows the $1 \times 1$ loop from the Wilson gauge action for various $\mu_I$. The critical coupling constant $\beta_c$ decreases as $\mu_I$ increases. The precise value of $\beta_c$ is estimated by measuring susceptibilities of the observables.

Typical examples of susceptibilities from various observables are shown in Figures 2-3. All susceptibilities from different observables give similar $\beta_c$. From $\beta_c$ (Wilson gauge action) evaluated from the $1 \times 1$ loop susceptibility we obtain

$$\beta_c(\mu_I) = 5.38 - 0.54\mu_I^2.$$  \hspace{1cm} (5)

The coefficient of $O(\mu_I^2)$ term is similar to those of [4]. Figure 4 shows the phase diagram in the $T - \mu_I$ plane. $\beta$ is converted to $T$ by the 2 loop $\beta$ function. Two phase diagrams from the Wilson and DBW2 gauge actions are in good agreement. The phase diagrams are qualitatively similar to the one obtained with KS fermions at small $\mu_B$[3, 4].

We also calculate the number density $n_d$ defined by $\frac{1}{V} Tr \frac{1}{D} \frac{\partial D}{\partial \mu_I}$. Figure 5 shows $n_d$ for different $\mu_I$. $n_d$ seems to increases with $\beta$.

5. DISCUSSION

We have studied QCD at finite isospin density with Wilson fermions. The results show that the
Figure 3. Same as in Figure 2 but from the DBW2 action at $\mu_I = 0.0$.

Figure 4. Phase diagram in the $T - \mu_I$ plane. $T$ is normalized by $T_C$ at $\mu_I = 0.0$.

The phase diagram is qualitatively similar to the one obtained at small $\mu_B$. The Wilson and DBW2 gauge actions give the same phase diagram. More quantitative analysis is needed to confirm the present results.

ACKNOWLEDGEMENTS

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REFERENCES


