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Towards a Determination of g_8 and g_{27}
from Twisted Mass Lattice QCD

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Towards a determination of g_8 and g_{27} from twisted mass lattice QCD*

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Abstract

A first step towards solving the $\Delta I = 1/2$ puzzle consists in a determination of the lowest order coupling constants g_8 and g_{27} in the effective chiral theory. This could be achieved by matching to lattice results for unphysical $K \rightarrow \pi$ transitions, provided light meson masses can be reached at reasonable computational costs. Using the framework of twisted mass QCD with an active charm quark, we show that the renormalization of $K \rightarrow \pi$ matrix elements requires at most the subtraction of a linear divergence. In the (partially) quenched approximation one may eliminate the linear divergence by a suitable choice of the twist angles and the interpolating meson fields.

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1 Introduction

Experimentally, non-leptonic kaon decays with isospin change $\Delta I = 1/2$ are enhanced over those with $\Delta I = 3/2$. For K^0 -decays into a pion pair of isospin I ,

$$\mathcal{A}(K^0 \rightarrow (\pi\pi)_I) = A_I e^{i\delta_I}, \quad (1)$$

this so-called $\Delta I = 1/2$ rule reads

$$|A_0/A_2| \approx 22. \quad (2)$$

A quantitative understanding of this large ratio within the Standard Model has remained elusive, as it requires a good quantitative control of non-perturbative QCD. While lattice QCD provides a useful framework to address this question, a direct evaluation of physical $K \rightarrow \pi\pi$ amplitudes, though possible in principle [1, 2], is still beyond the reach of current algorithms and hardware resources. We therefore follow a less ambitious approach and consider unphysical $K \rightarrow \pi$ matrix elements, which may then be related to the physical amplitudes by lowest order chiral perturbation theory (χ PT) [3]. In χ PT the relevant weak interaction vertex to lowest order is given by (see ref.[4] for notation)

$$\begin{aligned} \mathcal{H}_w(x) = & 4\frac{G_F}{\sqrt{2}}V_{ud}V_{us}^*\left\{\frac{3}{5}g_{27}\mathcal{O}_{27}(x) + 2g_8\mathcal{O}_8(x) \right. \\ & \left. + 2g'_8\mathcal{O}'_8(x)\right\}, \end{aligned} \quad (3)$$

and the $\Delta I = 1/2$ ratio in this approximation becomes

$$|A_0/A_2| = \frac{9}{5\sqrt{2}}\{g_8/g_{27} + 1/9\}. \quad (4)$$

The physically relevant couplings g_8 and g_{27} also appear in $K \rightarrow \pi$ matrix elements, e.g. for charged mesons

$$\begin{aligned} \langle \pi^+, \mathbf{q} | \mathcal{H}_w(0) | K^+, \mathbf{p} \rangle \propto & \left(\frac{2}{3}g_{27} + g_8\right)p \cdot q \\ & + 2m_K^2 g'_8, \end{aligned} \quad (5)$$

and different combinations appear for neutral mesons. To the extent that higher order contributions in χ PT can be neglected, the matching of lattice results to matrix elements like (5) may solve the problem. Unfortunately, even at the physical values of kaon masses, the lowest order relations of χ PT are expected to hold only at the 20-30 per cent level, and even larger deviations must be expected at unphysically large meson masses. As there is a proliferation of unknown parameters at the next order of χ PT (see refs. [5, 6] for attempts in this direction), we prefer to stay at lowest order and require that the lattice formulation

- is computationally cheap, so that meson masses even lighter than the physical kaon can be reached, while keeping the finite-volume effects small,
- does not require a complicated renormalization procedure for the $\Delta S = 1$ four-quark operators in the effective weak Hamiltonian.

These criteria exclude Ginsparg-Wilson quarks for their huge computational costs, while standard Wilson quarks fail to satisfy the second point. This motivates us to look at Wilson quarks with non-standard, chirally twisted mass terms, which might help circumvent some lattice specific renormalization problems and at the same time avoid unphysical fermion zero modes [7, 8, 9].

2 $N_f = 4$ twisted mass QCD

Twisted mass QCD with $N_f = 4$ quark flavours is described by the fermionic continuum action

$$S_f = \int d^4x \sum_{i=u,d,s,c} \bar{\psi}_i (D_\mu \gamma_\mu + m_i + i\gamma_5 \mu_i) \psi_i. \quad (6)$$

The 8 mass parameters obey two constraints

$$\mu_u/m_u = -\mu_d/m_d = \tan \alpha, \quad (7)$$

$$\mu_s/m_s = -\mu_c/m_c = \tan \beta, \quad (8)$$

so that there are only six independent parameters, the 2 twist angles α, β , and the 4 radial masses

$$M_i^2 = m_i^2 + \mu_i^2, \quad i = u, d, s, c. \quad (9)$$

These constraints ensure that 4-flavour QCD with a standard mass term can be recovered by the non-singlet axial field rotation

$$\psi \rightarrow \psi' = R(\alpha, \beta) \psi, \quad \bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi} R(\alpha, \beta), \quad (10)$$

$$R(\alpha, \beta) = \exp \left\{ \frac{i}{2} \gamma_5 (\alpha \tau_l^3 + \beta \tau_h^3) \right\}, \quad (11)$$

$$\tau_l^3 = \text{diag}(1, -1, 0, 0), \quad (12)$$

$$\tau_h^3 = \text{diag}(0, 0, 1, -1), \quad (13)$$

$$\Rightarrow S_f = \int d^4x \sum_{i=u,d,s,c} \bar{\psi}'_i (D_\mu \gamma_\mu + M_i) \psi'_i. \quad (14)$$

Performing a change of variables in the functional integral yields identities between correlation functions in twisted mass QCD and standard QCD:

$$\begin{aligned} \langle O[\psi, \bar{\psi}] \rangle_{(\alpha, \beta)} &= \\ \langle O[R(-\alpha, -\beta)\psi, \bar{\psi}R(-\alpha, -\beta)] \rangle_{(0,0)}, \end{aligned} \quad (15)$$

with the same physical masses M_i on both sides. For instance, considering the effective weak Hamiltonian

$$\begin{aligned} \mathcal{H}_{\text{eff}} &= V_{ud}V_{us}^* \frac{G_F}{\sqrt{2}} [C_+ O^+ + C_- O^-] + \text{h.c.}, \\ O^\pm &= (\bar{s}\gamma_\mu^L d)(\bar{u}\gamma_\mu^L u) \pm (\bar{s}\gamma_\mu^L u)(\bar{u}\gamma_\mu^L d) \\ &\quad - [u \leftrightarrow c], \end{aligned} \quad (16)$$

with the parity even and odd components

$$O^\pm = O_{VV+AA}^\pm - O_{VA+AV}^\pm, \quad (17)$$

the operator mapping goes as follows,

$$\begin{aligned} [O_{VA+AV}^\pm]_{(\alpha, \beta)} &= \left[\cos\left(\frac{\alpha+\beta}{2}\right) O_{VA+AV}^\pm \right. \\ &\quad \left. + i \sin\left(\frac{\alpha+\beta}{2}\right) O_{VV+AA}^\pm \right]_{(0,0)}. \end{aligned} \quad (18)$$

Here, the brackets $[\dots]_{(\alpha, \beta)}$ indicate operator insertion in correlation functions labelled by the same twist angles. In these identities we want to compute the l.h.s. on the lattice with their physical *interpretation* determined by the r.h.s.

3 Lattice regularization

On the lattice, we take the same action, where the kinetic term is given by the standard Wilson-Dirac operator. Eq. (15) is then only expected to hold for the renormalized theory, where the angles are defined through ratios of renormalized current quark masses. We will assume mass degenerate light quarks, $m_u = m_d = m_l$, $\mu_u = -\mu_d = \mu_l$ and concentrate on the special cases

- $\alpha = \beta = \pi/2$: “fully twisted” case; all renormalized standard masses $m_{R,i}$ vanish; the quark determinant for strange and charm quarks is complex in general, so that strange and charm quarks should remain in the quenched approximation,

- $\alpha = \pi/2, \beta = 0$: twisted light quarks and standard Wilson quarks for strange and charm; the quark determinant is real and completely unquenched simulations should be possible.

The renormalized four-quark operator has the following structure,

$$(O_R)_{VA+AV}^\pm = Z^\pm (O_{VA+AV}^\pm + c_P^\pm \bar{s}\gamma_5 d + c_S^\pm \bar{s}d), \quad (19)$$

where, for generic α and β , both coefficients, $c_P^\pm$ and $c_S^\pm$, are linearly divergent.

3.1 The fully twisted case

In the fully twisted case one sets all standard bare mass parameters m_i to the critical mass. One then finds that $c_S^\pm = O(1)$, so that the linear divergence is only contained in $c_P^\pm$. To obtain $K^+ \rightarrow \pi^+$ matrix elements we would like to compute the 3-point function

$$\langle (P_R)_{du}(x)(O_R)_{VV+AA}^\pm(z)(P_R)_{us}(y) \rangle_{(0,0)}. \quad (20)$$

Using the operator mapping

$$[(P_R)_{du}]_{(\alpha,\beta)} = [(P_R)_{du}]_{(0,0)}, \quad (21)$$

$$[(S_R)_{us}]_{(\frac{\pi}{2},\frac{\pi}{2})} = [-i(P_R)_{us}]_{(0,0)}, \quad (22)$$

$$[(O_R)_{VA+AV}^\pm]_{(\frac{\pi}{2},\frac{\pi}{2})} = [i(O_R)_{VV+AA}^\pm]_{(0,0)}, \quad (23)$$

the above 3-point function is reproduced by

$$\langle (P_R)_{du}(x)(O_R)_{VA+AV}^\pm(z)(S_R)_{us}(y) \rangle_{(\pi/2,\pi/2)}. \quad (24)$$

The fully twisted case is special in that the operators have a definite physical parity. In fact, parity may be used to subtract the linear divergence explicitly [9]. However, this may not even be necessary, as the following observation shows: the linearly divergent counterterm is proportional to

$$[(P_R)_{sd}]_{(\pi/2,\pi/2)} = [(P_R)_{sd}]_{(0,0)}. \quad (25)$$

Hence, while the renormalized operator has even parity, its counterterm is odd. If the interpolating fields of the kaon and pion are chosen of a definite physical parity up to $O(a)$ (as we did above), then the linearly divergent counterterm only contributes at $O(1)$ to the correlation function. Moreover, this $O(1)$ contribution either adds to $c_S^\pm$ or picks

the $O(a)$ parity violating piece of the kaon source. This generates the scalar partner of the kaon, which is known to be much heavier than the kaon itself. In the asymptotic regime where the matrix elements are extracted one therefore finds that this contribution is exponentially suppressed and thus negligible. Finally, to fix the $O(1)$ counterterm proportional to $c_S^\pm$, one may require that the $K \rightarrow \pi$ matrix element vanishes at zero meson momenta. In the effective theory this determines the unphysical coupling g'_8 .

3.2 Twisted mass QCD at $\beta = 0$

At $\beta = 0$ the operator $(O_R)_{\text{VA}+\text{AV}}^\pm$ is mapped to a linear combination without definite parity:

$$\begin{aligned} \langle (O_R)_{\text{VA}+\text{AV}}^\pm \tilde{S}_{K\pi} \rangle_{(\pi/2,0)} = \\ \langle \frac{1}{\sqrt{2}} [(O_R)_{\text{VA}+\text{AV}}^\pm + i(O_R)_{\text{VV}+\text{AA}}^\pm] S_{K\pi} \rangle_{(0,0)}. \end{aligned} \quad (26)$$

The idea then is to choose the combined kaon-pion source $\tilde{S}_{K\pi}$ with even physical parity so that the unwanted contribution is projected out,

$$\langle (O_R)_{\text{VA}+\text{AV}}^\pm S_{K\pi} \rangle_{(0,0)} = 0. \quad (27)$$

Note, however that this requires the parity of $\tilde{S}_{K\pi}$ to be non-perturbatively restored up to $O(a^2)$. This is possible and will be explained in detail in a forthcoming publication [10].

References

- [1] L. Lellouch and M. Lüscher, Commun. Math. Phys. **219** (2001) 31.
- [2] C. J. Lin et al., Nucl. Phys. B **619** (2001) 467.
- [3] C. W. Bernard et al., Phys. Rev. D **32** (1985) 2343.
- [4] P. Hernández and M. Laine, JHEP **0301** (2003) 063.
- [5] J. Laiho and A. Soni, Phys. Rev. D **65** (2002) 114020; hep-lat/0306035.
- [6] C. J. Lin et al., Nucl. Phys. B **650** (2003) 301.
- [7] R. Frezzotti et al. [Alpha collaboration], JHEP **0108** (2001) 058.

- [8] M. Guagnelli et al., [Alpha collaboration], Nucl. Phys. Proc. Suppl. **106** (2002) 320;
P. Dimopoulos et al., [Alpha collaboration], these proceedings.
- [9] C. Pena, S. Sint and A. Vladikas, hep-lat/0209045.
- [10] C. Pena, S. Sint and A. Vladikas, in preparation