Azimuthal single spin asymmetries in SIDIS in the light of chiral symmetry breaking

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Abstract

An attempt is made to understand the $z$-dependence of the azimuthal single spin asymmetries observed by the HERMES collaboration in terms of chiral models based on effective quark and Goldstone boson degrees of freedom. The effects of respectively neglecting and considering Gaussian intrinsic parton transverse momenta and the Sivers effect are explored. Predictions for the transverse target polarization experiment at HERMES are presented.

1 Introduction

The HERMES \cite{1, 2, 3, 4}, CLAS \cite{5} and SMC \cite{6} collaborations reported the observation of nonzero single spin azimuthal asymmetries (SSA) in semi-inclusive deep-inelastic scattering (SIDIS). Single spin asymmetries in hard processes cannot be explained by means of perturbative QCD \cite{7}, rather they signal the appearance of nonperturbative effects described in terms of so far unexplored distribution and fragmentation functions and effects associated with parton transverse momenta and quark orbital angular momenta.

In a factorized picture \cite{8, 9}, the SSA in SIDIS can be explained in terms of the chirally odd twist-2 and twist-3 distribution functions $h_1$, $h_L$ and $e$ \cite{10}, which appear in connection with the Collins fragmentation function $H_{1L}^{+}$ \cite{11, 12}, and the chirally even Sivers distribution function $f_{1T}^{+}$ \cite{13, 14, 15, 16, 17}, $H_{1}^{+}$ describes the left–right asymmetry in the fragmentation of a transversely polarized quark into an unpolarized hadron (Collins effect), while $f_{1T}^{+}$ quantifies the distribution of unpolarized quarks in a transversely polarized nucleon (Sivers effect). Both are referred to as T-odd, i.e. they would vanish by time reversal invariance in the absence of final-state interactions.

The HERMES data on SSA from a longitudinally polarized target \cite{1, 2, 3, 4} were studied in Refs. \cite{18, 19, 20, 21, 22, 23} in terms of the Collins effect only. In these approaches, the Sivers effect was neglected. Different models and assumptions were explored in these works in order to describe the $x$-dependence of the HERMES data, however, only one model has been employed so far, namely the Collins ansatz \cite{11}, for the description of the $z$-dependence of the HERMES data.

In this note we shall attempt to describe the $z$-dependence of the HERMES data using a different model for the Collins fragmentation function based on a chirally invariant approach suggested in Ref. \cite{24}. The required information on the involved distribution functions will be taken from the chiral quark-soliton model \cite{25} in which – as well as in a large class of other chiral models – the Sivers function vanishes \cite{26}. The combination of the two models is justified in the sense that both models describe the dynamics of strong interactions at low energies in terms of effective chiral quark and Goldstone boson degrees of freedom. Thus both models are essentially based on chiral symmetry breaking, which is known to play an important role in non-perturbative QCD in general, and in the T-odd fragmentation process in particular \cite{11}.

The note is organized as follows. In Sec. 2 we will review the relevant details of the HERMES single-spin asymmetry measurement. In Sec. 3 we will compare the results of our model to the HERMES data from the longitudinal target polarization experiment, assuming vanishing (3.1) and Gaussian (3.2) intrinsic parton transverse momenta in the target. In Sec. 4 we shall make a rough estimate of how big the Sivers effect could be to still be compatible with the HERMES data within our approach. Finally, in Sec. 5 we will make predictions for the HERMES transverse target polarization experiment. In Sec. 7 we summarize our work and conclude.
2 The HERMES measurement of the $A_{UL}^{\sin \phi}$ asymmetry

In the HERMES experiments [2, 3, 4] the cross section for the process $lp \rightarrow l'hX$ was measured in dependence of the azimuthal angle $\phi$ between lepton scattering plane and the production plane of the hadron, see Fig. 1.

Let $P$, $l$ and $l'$ denote the momenta of target, incoming and outgoing lepton, respectively. The kinematic variables – center of mass energy $s$, four momentum transfer $q = l - l'$, invariant mass of the photon-target system $W$, $x$, $y$ and $z$ – are defined as

\[ s = (P + l)^2, \quad W^2 = (P + q)^2, \quad Q^2 = -q^2, \quad x = \frac{Q^2}{2Pq}, \quad y = \frac{Pq}{Pq}, \quad z = \frac{Pq}{Pq}. \tag{1} \]

In this notation the azimuthal asymmetry $A_{UL}^{\sin \phi}(z)$ studied by HERMES in the range $0.2 < z < 0.7$ reads

\[ A_{UL}^{\sin \phi}(z) = \frac{1}{2} \int dx \, dy \, d^2P_{h\perp} \, \sin \phi \left( \frac{1}{S^+} \frac{d^3\sigma^+}{dx \, dy \, dz \, d^2P_{h\perp}} - \frac{1}{S^-} \frac{d^3\sigma^-}{dx \, dy \, dz \, d^2P_{h\perp}} \right) \]
\[ \cdot \frac{1}{2} \int dx \, dy \, d^2P_{h\perp} \left( \frac{d^3\sigma^+}{dx \, dy \, dz \, d^2P_{h\perp}} + \frac{d^3\sigma^-}{dx \, dy \, dz \, d^2P_{h\perp}} \right). \tag{2} \]

The subscript “$U$” reminds of the unpolarized beam, and “$L$” reminds of the longitudinally (with respect to the beam direction) polarized proton target. $S^\pm$ denotes the modulus of target polarization vector where “$+$” means polarization opposite to the beam direction. When integrating over $x$ and $y$ one has to consider the experimental cuts [2, 3, 4]

\[ W > 2 \text{ GeV}, \quad Q^2 > 1 \text{ GeV}^2, \quad 0.023 < x < 0.4, \quad 0.2 < y < 0.85. \tag{3} \]

The denominator in the asymmetry $A_{UL}^{\sin \phi}$ in Eq. (2) is the cross section for pion production from scattering of an unpolarized beam off an unpolarized target which – after integrating out the transverse momenta of the produced pions – is given by

\[ \frac{1}{2} \int d^2P_{h\perp} \left( \frac{d^3\sigma^+}{dx \, dy \, dz \, d^2P_{h\perp}} + \frac{d^3\sigma^-}{dx \, dy \, dz \, d^2P_{h\perp}} \right) = \frac{d^3\sigma_{UL}}{dx \, dy \, dz} = \frac{4\pi a^2 s}{Q^4} \left( 1 - y + \frac{y^2}{2} \right) \sum_a e_a^2 x f_1^a(x) D^a_q(z). \tag{4} \]

The cross sections entering the numerator in Eq. (2) were computed in Refs. [8, 9] assuming that the process factorizes for $P_{h\perp} \ll Q^2$. Arguments in favour of this assumption have been given [11], however, a strict proof of a factorization theorem has not been presented yet.

The numerator in Eq. (2) consists of two parts – originating respectively from the longitudinal and the transverse component of the target polarization vector with respect to the photon momentum $q$

\[ \int d^2P_{h\perp} \, \sin \phi \left( \frac{1}{S^+} \frac{d^3\sigma^+}{dx \, dy \, dz \, d^2P_{h\perp}} - \frac{1}{S^-} \frac{d^3\sigma^-}{dx \, dy \, dz \, d^2P_{h\perp}} \right) = \frac{2}{S} \frac{d^3\sigma_{UL}}{dx \, dy \, dz} - \frac{2}{S} \frac{d^3\sigma_{UT}}{dx \, dy \, dz}. \tag{5} \]
with

\[ \frac{d^3\sigma_{UL}}{dx dy dz} = -S_L \frac{\alpha_s}{Q^4} \frac{M_N}{Q} 2(2-y)\sqrt{1-y} \sum_a e_a^2 \frac{x^2}{2} \mathcal{I} \left[ \frac{k_T \cdot \hat{P}_{h\perp}}{2M_h} \hat{h}_L^a(x, p_T^2) H_{1}^{1,a}(z, z^2 k_T^2) \right] 
+ \mathcal{O}\left( \hat{h}_{1\perp}^2 \hat{H} \right) + \mathcal{O}\left( \frac{m_q}{M_N} g_1 H_{1} \right), \tag{6} \]

\[ \frac{d^4\sigma_{UT}}{dx dy dz} = S_T \frac{\alpha_s}{Q^4} \sum_a e_a^2 x \left\{ -(1-y) \mathcal{I} \left[ \frac{k_T \cdot \hat{P}_{h\perp}}{M_h} \hat{h}_L^q(x, p_T^2) H_{1}^{1,q}(z, z^2 k_T^2) \right] 
- (1-y + \frac{y^2}{2}) \mathcal{I} \left[ \frac{p_T \cdot \hat{P}_{h\perp}}{M_N} f_{1\perp}^q(x, p_T^2) D_{1}(z, z^2 k_T^2) \right] \right\}, \tag{7} \]

where \( \hat{P}_{h\perp} \) is a unit vector and we used the shorthand notation

\[ \mathcal{I}[\ldots] \equiv \int d^2P_{h\perp} d^2p_T d^2k_T \delta^{(2)} \left( p_T - \frac{P_{h\perp}}{z} - k_T \right) \left[ \ldots \right], \tag{8} \]

The weight “\( \sin \phi \)” in Eq. (2) has the drawback of leaving the unintegrated distribution and fragmentation functions inside a convolution. The weight “\( \sin \phi |P_{h\perp}| \)” would allow a model-independent deconvolution [9].

3 Model calculation of the \( A_{UL}^{\sin \phi} \) asymmetry

In order to describe the HERMES data we will use three ingredients: Information on the Collins fragmentation function from the model calculation of Ref. [24]. Information on the involved (integrated) distribution functions from the chiral quark-soliton model and the instanton vacuum model [25, 27]. Models for the distribution of transverse quark momenta in the nucleon.

Collins fragmentation function \( H_{1}^{1} \). For the Collins fragmentation function we shall use the results presented in Ref. [24]. In that work, the Collins function has been estimated in a chiral invariant approach à la Manohar and Georgi [28], where the effective degrees of freedom are constituent quarks and pions, coupled via a pseudovector interaction. In order to generate the required phases, one-loop corrections to the quark propagator and vertex have been included [24]. In this approach, the unfavoured Collins function vanishes. (It would appear only if one took into account two- and higher-loop corrections.)

In what follows we shall use the notation

\[ H_{1}^{1} \equiv H_{1}^{1,u/\pi^+} = H_{1}^{1,d/\pi^+} = H_{1}^{1,d/\pi^-} = H_{1}^{1,u/\pi^-} = 2H_{1}^{1,u/\pi^0} = 2H_{1}^{1,d/\pi^0} = 2H_{1}^{1,u/\pi^0} = 2H_{1}^{1,d/\pi^0} \]

\[ \gg H_{1}^{1,d/\pi^+} = H_{1}^{1,u/\pi^+} = H_{1}^{1,u/\pi^-} = H_{1}^{1,d/\pi^-}. \tag{9} \]

The first line of Eq. (9) defines the favoured Collins fragmentation function \( H_{1}^{1} \) in terms of the fragmentation functions for different flavours and pions charge conjugation and isospin symmetry relations. The second line of Eq. (9) expresses the expectation that the unfavoured fragmentation is suppressed with respect to the favoured fragmentation as it has been conjectured on the basis of the Schäfer-Teryaev sum rule [29]. This conjecture remains, however, to be tested.

One should note that the assumption of favoured fragmentation cannot be expected to work equally well for all pions. E.g., unfavoured fragmentation effects have been shown to play an important role for \( \pi^- \) production from a proton target [21], while in the case of \( \pi^+ \) production \( u \)-quark dominance in the proton amplifies the effect of favoured fragmentation.

Pioneering steps towards an understanding of the scale dependence of the Collins function have been done in [30]. The results obtained there, however, need to be carefully reexamined in the light of recent theoretical developments [31, 32]. Therefore, for the ratio \( H_{1}^{1}/D_{1} \) we shall use the result from [24], which refers to a low scale below 1 GeV\(^2\) and assume that evolution to the average scale of the HERMES experiment can be neglected. While \( D_{1} \) and possibly \( H_{1}^{1} \) depend on the scale strongly, one may hope that their ratio is less scale dependent.
Chirally odd distribution functions $h_1$ and $h_L$. For the transversity distribution function $h_1^q$ we shall use predictions [25] from the chiral quark-soliton model ($\chi$QSM). The $\chi$QSM is a quantum field-theoretical relativistic model which was derived from the instanton model of the QCD vacuum. The quark and antiquark distribution functions obtained in this model satisfy all general QCD requirements and agree, as far as they are known, to within (10-30)% with phenomenological parameterizations [33]. We shall assume that this model predicts $h_1^q(x)$ with a similar uncertainty [25].

The twist-3 chirally odd distribution $h_L^q(x)$ is given by $h_L^q(x) = 2x \int_0^1 dy \frac{h_1^q(y)}{y^2} + \tilde{h}_L^q(x) + O(m_q/M_N)$. In Ref. [27] it was shown that in the instanton vacuum model the (actual “pure”) twist-3 term $\tilde{h}_L^q(x)$ is strongly suppressed with respect to the twist-2 part in the above-mentioned relation. Thus, we can well approximate $h_L^q(x) = 2x \int_0^1 dy \frac{h_1^q(y)}{y^2}$ by consistently neglecting $\tilde{h}_L^q(x)$ and quark mass terms.

We will also need the deuteron transversity distribution function which we shall estimate, e.g. for the $u$-quark, as $h_1^{1/D} \approx h_1^{u/p} + h_1^{u/n} = h_1^u + h_4^u$, where isospin symmetry was used in the last step (with $h_1^q \equiv h_1^{q/p}$, etc.). Corrections due to the D-state admixture [34] are smaller than other theoretical uncertainties in our approach and we disregard them here.

The results for the chirally odd distribution functions are LO-evolved from the low scale of the model to the average scale of the HERMES experiment of 2.5 GeV$^2$. For the unpolarized distribution functions $f_1^q(x)$ we shall use the parameterization of Ref. [35] at the corresponding scale.

In the $\chi$QSM – as well as in a large class of other chiral soliton models – $T$-odd distribution functions vanish [26]. Therefore in our approach it is consistent to neglect the Sivers function, cf. below Section 3.2.

Transverse momentum distributions. While the transverse momentum distribution of the fragmenting quarks is known from the model calculation of Ref. [24], we have no information about “unintegrated” transverse momentum distributions. While the transverse momentum distribution of the fragmenting quark is known from the model calculation of Ref. [24], we have no information about “unintegrated” transverse momentum distributions. Therefore in our approach it is consistent to neglect the Sivers function, cf. below Section 3.2.

3.1 Neglect of intrinsic $p_T$ in distribution functions

In this Section, we shall use a simple and extreme model. Let $f(x, p_T^2)$ be a generic distribution function, then we assume

$$f(x, p_T^2) = f(x) \delta^{(2)}(p_T) .$$

This ansatz amounts essentially to the disregard of intrinsic quark transverse momenta, abscising the transverse momentum of the outgoing hadron entirely to the fragmentation process. The ansatz (10) immediately “kills” the Sivers effect, however, it is not a too restrictive assumption in our approach, where the Sivers distribution is zero anyway. The cross sections of Eqs. (6) and (7) become

$$\frac{d^3\sigma_{UL}}{dx\,dy\,dz} = S_L \frac{4\pi a^2 s}{Q^4} \frac{M_N}{Q} \frac{2(2 - y)\sqrt{1 - y}}{\sum_a} e_a^2 x^2 h_1^a(x) H_1^{(1/2)a}(z) + O(h_1^L \tilde{H}) + O\left(\frac{m_q}{M_N}\right) ,$$

$$\frac{d^4\sigma_{UT}}{dx\,dy\,dz} = S_T \frac{4\pi a^2 s}{Q^4} (1 - y) \sum_a e_a^2 x h_1^a(x) H_1^{(1/2)a}(z) .$$

$S_L = S \cos \Theta_S$ and $S_T = S \sin \Theta_S$ are respectively the longitudinal and transverse component of the target polarization $S$ with respect to the 3-momentum of the virtual photon, and $\cos \Theta_S \approx 1 - 2M_N^2 x(1 - y)/(s\gamma)$. The transverse moment of the Collins fragmentation function in Eqs. (11, 12) is defined as

$$H_1^{(1/2)a}(z) = z^2 \int d^2k_T \frac{|k_T|}{2m_\pi} H_1^{L,a}(z, z^2k_T^2) .$$

The term $\propto h_1^L \tilde{H}_1^L$ neglected in Eq. (11) was estimated [22] to be small compared to the first term in Eq. (11) in the kinematics of the HERMES experiment. We also neglect quark mass effects.

\footnote{For a careful discussion of the precise meaning of “unintegrated” transverse momentum dependent distribution functions in QCD see Ref. [36].}
Table 1: The constants $C_L(\text{pion, target})$ and $C_T(\text{pion, target})$ as defined in Eqs. (14, 16) and (30, 32), respectively.

<table>
<thead>
<tr>
<th>pion π</th>
<th>$C_L(\pi, \text{p})$</th>
<th>$C_L(\pi, \text{d})$</th>
<th>$C_T(\pi, \text{p})$</th>
<th>$C_T(\pi, \text{d})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+$</td>
<td>0.154</td>
<td>0.075</td>
<td>0.781</td>
<td>0.354</td>
</tr>
<tr>
<td>$\pi^0$</td>
<td>0.109</td>
<td>0.063</td>
<td>0.536</td>
<td>0.297</td>
</tr>
<tr>
<td>$\pi^-$</td>
<td>-0.025</td>
<td>0.038</td>
<td>-0.204</td>
<td>0.181</td>
</tr>
</tbody>
</table>

Using charge conjugation and isospin symmetry and neglecting unfavoured fragmentation we obtain for the azimuthal asymmetries

$$A_{UL}^{\sin \phi}(z, \pi, \text{target}) = C_L(\pi, \text{target}) a^{(1/2)}(z).$$  \hspace{1cm} (14)

As a consequence of the favoured flavour fragmentation and the simplified treatment of the deuteron target the $z$-dependence of the azimuthal asymmetries for different pions from different targets is given by a “universal” function

$$a^{(1/2)}(z) = \frac{H^{1(1/2)}_1(z)}{D_1(z)},$$  \hspace{1cm} (15)

while the information on the respective pion produced from the respective target is contained in the constant $C_L(\pi, \text{target}) = 2 \int dx \int dy \sum_n e_n^2 x/Q^4 [2 \cos \Theta_S (2 - y) \sqrt{1 - y(M_N/Q) x} h^{a_{\text{target}}}_L(x) - \sin \Theta_S (1 - y) x h^{a_{\text{target}}}_L(x)] \int dx \int dy (1 - y + y^2/2)/Q^4 \sum_n e_n^2 x f^{a_{\text{target}}}_L(x)$  \hspace{1cm} (16)

where $\sum_n$ denotes the summation over the favoured flavours relevant for the respective pion. Furthermore, the constants $C_L(\pi, \text{target})$ depend on the experimental cuts which enter the integrations over $x$ and $y$ in (16). The results for the constants $C_L(\pi, \text{target})$ are given in Table 1.

The results for the azimuthal asymmetries $A_{UL}^{\sin \phi}(z)$ are shown as dotted lines in Figs. 2a-2f and compared to the HERMES data [2, 3, 4]. We conclude that, under the assumption (10) of vanishing intrinsic quark transverse momenta, the Collins effect computed in a chiral invariant approach can explain the data within the - at present admittedly sizeable - statistical error of the experiment.

3.2 Gaussian model for transverse quark momenta

The results presented in the previous Section rely on the assumption that the intrinsic transverse momentum of partons in the target is zero. In this section we try to estimate what is the effect of introducing a nonzero intrinsic transverse momentum. In order to do this, we will assume a Gaussian distribution of transverse momenta, both for the distribution and fragmentation functions. Such an assumption is in fair agreement with the HERMES data [2].

Let $f(x, p_T^2)$ and $D(z, \pi^2 k_T^2)$ denote respectively a generic unintegrated distribution and fragmentation function. We assume that

$$f(x, p_T^2) = \frac{f(x)}{\pi(p_T^2)} \exp\left(-\frac{p_T^2}{p_T^2}\right) \quad \text{and} \quad D(z, K_T^2) = \frac{D(z)}{\pi z^2(K_T^2)} \exp\left(-\frac{K_T^2}{K_T^2}\right)$$  \hspace{1cm} (17)

holds, where $K_T = -z k_T$ is the transverse momentum the hadron acquires in the fragmentation process in the frame where the fragmenting quark has no transverse momentum. The functions are normalized such that $\int d^2 p_T f(x, p_T^2) = f(x)$ and $\int d^2 K_T D(z, K_T^2) = D(z)$.

Let us remark that the assumption (17) is not consistent with the model result of Ref. [24], which yields a different transverse momentum distribution, nor with the positivity bounds of Ref. [37]. However, we do not address here the issue of the transverse momentum dependence of the asymmetries but merely their $z$-dependence, and here only averages over transverse momenta such as $H^{1(1/2)}_1(z)$ or $\langle K_T^2(z) \rangle$ are of relevance.
At the level of such averages, Eq. (17) is compatible with the results from [24] and can comply with the integrated positivity bounds, which is sufficient for our purposes.

The ansatz (17) is a convenient choice which allows a disentanglement of the distribution and fragmentation functions in the observed asymmetry. In fact, under the assumption (17) the azimuthal asymmetry is given by

\[
\frac{d^3\sigma_{UL}}{dx dy dz} = S_L \frac{4\pi\alpha^2 s}{Q^4} \frac{M_N}{Q} 2(2 - y)\sqrt{1 - y} \sum_a e_a^2 x h_1^a(x) H_1^{L(1/2)\alpha}(z) / \sqrt{1 + z^2 (p_T^2(x))/\langle M_N^2(z) \rangle},
\]

(18)

\[
\frac{d^4\sigma_{UT}}{dx dy dz} = S_T \frac{4\pi\alpha^2 s}{Q^4} \sum_a e_a^2 (1 - y) \frac{x h_1^a(x) H_1^{L(1/2)\alpha}(z)}{\sqrt{1 + z^2 (p_T^2(x))/\langle M_N^2(z) \rangle}}
\]

\[-(1 - y + y^2/2) \frac{x f_2^{L(1/2)\alpha}(x) D^a_2(z)}{\sqrt{1 + \langle K_T(z) \rangle / \langle M_N^2(z) \rangle}}\]

(19)

The Sivers function appears in Eq. (19) because the partons in the target are now allowed to have non-vanishing intrinsic transverse momenta. In our approach the Sivers function vanishes, however, in the next Section we shall make use of the explicit expressions with the Sivers effect in Eq. (19). If we neglected transverse quark momenta in the target, i.e. if we took the limit \(\langle p_T^2 \rangle \to 0\), we would recover the results of the previous Section.

An important point is to reproduce the behaviour of \(\langle |P_{h\perp}|(z) \rangle\) observed in the HERMES experiment [4]. If there were no transverse quark momenta in the target then \(\langle |P_{h\perp}|(z) \rangle\) would arise from \(\langle |K_T|\rangle\) only, i.e. one would have \(\langle |P_{h\perp}|(z) \rangle = \langle |K_T|(z) \rangle\). In Fig. 3 we see that the \(\langle |K_T|(z) \rangle\) from [24] alone underestimates the HERMES data on \(\langle |P_{h\perp}|(z) \rangle\) [4] by about 30%.

This discrepancy could, of course, be attributed to the theoretical uncertainty of the model calculation of Ref. [24], in particular because the results of [24] refer to a low scale below 1 GeV\(^2\) while the HERMES

![Figure 2: The azimuthal single spin asymmetries \(A_{UL}^\phi(z)\) from a proton and deuteron target for \(\pi^+\), \(\pi^0\) and \(\pi^-\) in comparison to the HERMES data [2, 3].](image)
data refer to \(Q^2 = 2.5\) GeV\(^2\). We have implicitly taken such a point of view in the previous Section, where transverse quark momenta in the target were manifestly neglected.

Here she shall take an opposite point of view and assume that the fragmentation transverse momentum is correctly described by the model of Ref. [24] and determine the \(p_T^2\) required to achieve a better description of \(\langle |P_{h\perp}(z)| \rangle\) at HERMES [4]. The relation between \(P_{h\perp}\) of the hadron, the intrinsic parton transverse momentum \(p_T\) in the target and the transverse momentum \(K_T\) the hadron acquires in the fragmentation process is given by [24]

\[
\langle P_{h\perp}^2(z) \rangle = z^2 \langle p_T^2 \rangle + \langle K_T^2(z) \rangle .
\]

(20)

It is by no means clear how to use the relation (20) in order to describe \(\langle |P_{h\perp}(z)| \rangle\). If the distribution of the transverse momenta of the produced hadrons were Gaussian, then \(\langle P_{h\perp}^2 \rangle = \frac{z}{2} \langle |P_{h\perp}|^2 \rangle\) and

\[
\langle |P_{h\perp}(z)| \rangle = \langle |K_T(z)| \rangle \sqrt{1 + z^2 \langle p_T^2 \rangle / \langle K_T^2(z) \rangle} .
\]

(21)

If we assume the relation (21) then we find that \(\langle p_T^2 \rangle = 0.5\) GeV\(^2\) allows to better describe the HERMES data, see Fig. 3. Two remarks are in order. Firstly, in general \(\langle p_T^2 \rangle\) could be a function of \(x\) which we disregard here for simplicity. Secondly, a somehow smaller value of, say, \(\langle p_T^2 \rangle = 0.4\) GeV\(^2\) would also allow to describe reasonably the data in Fig. 3. However, we here we prefer to choose the larger value as an opposite to the limiting case \(\langle p_T^2 \rangle \to 0\) considered in the previous Section 3.1.

Our estimate of \(\langle p_T^2 \rangle = 0.5\) GeV\(^2\) lies in the range of the values considered in literature: It somehow overestimates the results from Refs. [38, 39, 40] and underestimates the numbers \(\langle p_T^2 \rangle \sim 0.8\) GeV\(^2\) reported in Ref. [41].

The azimuthal asymmetry is given now by Eq. (14) with \(a^{(1/2)}(z)\) replaced by

\[
a_{\text{Gauss}}^{(1/2)}(z) = \frac{1}{\sqrt{1 + z^2 \langle p_T^2 \rangle / \langle K_T^2(z) \rangle}} \frac{H_1^{(1/2)}(z)}{D_1(z)} .
\]

(22)

Using \(\langle p_T^2 \rangle = 0.5\) GeV\(^2\) we obtain the results for \(A_{UL}^{\text{sin}}(z)\) plotted as solid lines in Figs. 2a-2f. The description of the HERMES data [2, 3, 4] can be viewed as equally satisfactory as in the case discussed in Section 3.1.

We conclude that the two different approaches – the assumptions that the distribution of parton transverse momenta in the target is negligible and that it is Gaussian with a sizeable width – cannot be discriminated experimentally at present. One may expect that an optimized phenomenological description would require a \(\langle p_T^2 \rangle\) somewhere between the limiting case \(\langle p_T^2 \rangle \to 0\) and \(\langle p_T^2 \rangle = 0.5\) GeV\(^2\).

\(^2\)It is worthwhile mentioning that this is the only place where we use an absolute number from the model of [24]. In all other quantities ratios of model results enter, where theoretical uncertainties can, of course, add up but also have a chance to cancel.
4 Is there room for Sivers effect in $A_{UL}^{\sin \phi}$ asymmetries?

The introduction of a Gaussian distribution of transverse momentum allows us to consider in the analysis the contribution of the Sivers effect. In the previous Sections we set the Sivers function to zero based on the results from the $\chi$QSM. However, despite the successes of this model, we have to admit that so far it has not been used and tested in the arena of unintegrated parton distributions. The vanishing of the Sivers function and other T-odd distributions in the $\chi$QSM and a large class of other chiral models demonstrates the limitations of such models [26]. From the point of view of the $\chi$QSM and the instanton model of the QCD vacuum T-odd distribution functions appear to be suppressed with respect to the T-even ones [42]. The instanton vacuum suppression mechanism was demonstrated to be strong in the case of $g_T^b(x)$ [43] which recently was confirmed experimentally [44]. In the case of the Sivers function this mechanism has not yet been studied rigorously but concluded on general grounds and could be less pronounced [42]. It is therefore instructive to investigate whether anything can be concluded on the magnitude of the Sivers effect from the HERMES data on $A_{UL}^{\sin \phi}$.

At first glance Figs. 2a-2f may give the impression that the Collins effect alone is able to explain nicely the data – leaving no or little room for the Sivers effect and implying that the Sivers distribution function is small. However, we shall see in the following that this needs not be the case. For that let us concentrate on the asymmetries that are least sensitive to the assumption of favoured flavour fragmentation, namely $A_{UL}^{\sin \phi}$ for $\pi^+$ and $\pi^0$ from the proton target. Under the assumption of favoured flavour fragmentation, one can expect the total $A_{UL}^{\sin \phi}(z)$ to behave as

$$A_{UL}^{\sin \phi}(z, \pi) = C_L(\pi, \text{target}) e^{(1/2)}_{\text{Gauss}}(z) + B_{\text{Siv}}(\pi) \int \frac{1}{\sqrt{1 + \langle K_T^2(z) \rangle/\langle z^2(P_T^2) \rangle}}. \quad (23)$$

where $B_{\text{Siv}}$ is a constant independent of $z$ due to the Sivers effect.

Figs. 4a and 4b show that reasonable descriptions of $A_{UL}^{\sin \phi}(z)$ for $\pi^+$ and $\pi^0$ is possible with roughly

$$-\frac{1}{80} \lesssim B_{\text{Siv}}(\pi^+) \lesssim \frac{1}{80}, \quad 0 \lesssim B_{\text{Siv}}(\pi^0) \lesssim \frac{1}{40} \quad (24)$$

The different ranges of $B_{\text{Siv}}$ for $\pi^+$ and $\pi^0$ could reflect the flavour dependence of the Sivers function.

From the expression (19) we obtain roughly but with sufficient accuracy for our purposes

$$B_{\text{Siv}} = \frac{2 \int dx \, dy \, \sin \Theta_S (1 - y + y^2/2) Q^{-4} \sum_{a} e_a^2 x f_{1T}^{(1/2)a}(x)}{\int dx \, dy (1 - y + y^2/2) Q^{-4} \sum_{a} e_a^2 x f_{1T}^{(1/2)a}(x)} \approx \frac{2 \langle \sin \Theta_S \rangle \int dx \sum_{a} e_a^2 x f_{1T}^{(1/2)a}(x)}{\int dx \sum_{a} e_a^2 x f_{1T}^{(1/2)a}(x)}. \quad (25)$$

We estimate $\langle \sin \Theta_S \rangle \approx 2M_N^2(x)(1 - (y))/s(y) = 0.05$ (with $\langle x \rangle = 0.09$ and $\langle y \rangle = 0.57$ [2, 3]). This gives

$$\frac{\int dx \sum_{a} e_a^2 x f_{1T}^{(1/2)a}(x)}{\int dx \sum_{a} e_a^2 x f_{1T}^{(1/2)a}(x)} \approx 40 B_{\text{Siv}}, \quad (26)$$
and we obtain the bounds
\[
-\frac{1}{2} \lesssim \frac{\int dx \sum_a e_a^2 x f_{1T}^{l(1/2)a}(x)}{\int dx \sum_b e_b^2 x f_{1T}^b(x)} \lesssim \frac{1}{2}, \quad 0 \lesssim \frac{\int dx \sum_a e_a^2 x f_{1T}^{l(1/2)a}(x)}{\int dx \sum_b e_b^2 x f_{1T}^b(x)} \lesssim 1.
\] (27)

However, the Sivers function obeys the positivity bound \(|f_{1T}^{l(1/2)a}(x)}| \leq 1/2f_1(x) [37]\), such that for any pion
\[
-\frac{1}{2} \leq \frac{\int dx \sum_a e_a^2 x f_{1T}^{l(1/2)a}(x)}{\int dx \sum_b e_b^2 x f_{1T}^b(x)} \leq \frac{1}{2}
\] (28)

Thus, the bounds (27) do not provide any useful information on the Sivers function except for the lower bound in the \(\pi^0\) case. Our procedure to obtain the “bounds” (27) is admittedly crude and model dependent. E.g., by assuming a somehow lower value for \(p_T^0\) we could have obtained a negative lower bound in the \(\pi^0\) case in Eq. (27). Still it allows to learn an important lesson. The HERMES data on \(A_{UL}^{\sin}\) can be well described without the Sivers effect by invoking the Collins effect alone. However, from this observation we by no means can conclude that the Sivers effect is small. Indeed, a Sivers effect as large as allowed by the positivity bound [37] – in particular as large as required to explain [17, 41] the large SSA in \(p^1 p \to \pi X\) [45] – could not be resolved at present within the statistical error bars of the HERMES data [2, 3].

5 Predictions for the \(A_{UT}^{\sin(\phi+\phi_s)}\) Collins asymmetry

In the previous Sections we have seen that the HERMES data on the \(A_{UL}^{\sin}\) asymmetries can well be described in terms of the Collins effect, however, they are compatible with a sizeable Sivers effect, too. Azimuthal single spin asymmetries from a transversely polarized target are key observables, since they allow to cleanly distinguish the Collins and Sivers effect by the different azimuthal distribution of the produced pions, schematically

\[
A_{UT} \propto \left( \frac{d\sigma^\uparrow}{S_T} - \frac{d\sigma^\downarrow}{S_T} \right) \propto \sin(\phi + \phi_S) \cdot (\text{Collins effect}) + \sin(\phi - \phi_S) \cdot (\text{Sivers effect})
\] (29)

where \(1^{(1)}\) denote the transverse with respect to the lepton beam target polarizations and \(\phi_S\) is the azimuthal angle of the target polarization vector, see Fig. 1. Thus, by considering appropriate weights [9] both effects can be separated. (In the longitudinal target polarization experiments \(\phi_S\) was 0 or \(\pi\) and dropped out from the weighting factor \(\sin \phi\)).

Transverse target polarization experiments are in progress at HERMES [46] and COMPASS [47]. In this Section we shall estimate the transverse target single spin asymmetry due to the Collins effect for the HERMES experiment. Defining \(A_{UT}^{\sin(\phi+\phi_s)}\) in analogy to Eq. (2) and using the same assumptions as in Section 3.1 we obtain

\[
A_{UT}^{\sin(\phi+\phi_s)}(z, \pi, \text{target}) = C_T(\pi, \text{target}) a^{(1/2)}(z)
\] (30)

with \(a^{(1/2)}(z)\) as defined in Eq. (15) while if we adopt a Gaussian ansatz as done in Section 3.2 we obtain

\[
A_{UT}^{\sin(\phi+\phi_s)}(z, \pi, \text{target}) = C_T(\pi, \text{target}) a^{(1/2)}_{\text{Gauss}}(z),
\] (31)

with \(a^{(1/2)}_{\text{Gauss}}(z)\) as defined in Eq. (22). The constant \(C_T\) turns out to be

\[
C_T(\pi, \text{target}) = 2 \frac{\int dy d\phi \sin \phi S(1 - y)/Q^4 \sum_a e_a^2 x h_{a/\text{target}}(x)}{\int dy d\phi (1 - y + y^2/2)/Q^4 \sum_a e_a^2 x f_{1/\text{target}}(x)}.
\] (32)

Taking the cuts as in the longitudinal target polarization experiment, Eq. (3), we obtain for \(C_T(\pi, \text{target})\) the results quoted in Table 1. Figs. 5a-5f show the results for \(A_{UT}^{\sin(\phi+\phi_s)}(z, \pi, \text{target})\) for the proton and deuteron target, for the two different assumptions on the transverse momentum distribution: no intrinsic \(p_T\) (dotted line) and a Gaussian \(p_T\) (solid line).

Finally, we consider the weighted asymmetry \(A_{UT}^{\sin(\phi+\phi_s)P_T/m_x}\) [9]. Assuming favoured flavour and isospin invariance relations among the fragmentation functions, but without any assumptions on the transverse momentum distribution, the asymmetry takes the form

\[
A_{UT}^{\sin(\phi+\phi_s)P_T/m_x}(z, \pi, \text{target}) = C_T(\pi, \text{target}) a^{(1)}(z)
\] (33)
Figure 5: The asymmetries $A_{UL}^\sin(\phi+\phi_S)(z)$ for pion production from a transversely polarized proton target at HERMES for the assumptions that intrinsic transverse parton momenta in the target follow a Gaussian distribution (solid lines) and are negligible (dashed lines).

with the same constant $C_T$ as in $A_{UT}^\sin(\phi+\phi_S)$, cf. Eq. (32), but a different “universal function” $a^{(1)}(z)$ defined as

$$a^{(1)}(z) = \frac{zH_1^{(1)}(z)}{D_1(z)}.$$  \hspace{1cm} (34)

where

$$H_1^{(1)}(z) \equiv \int d^2K_T \frac{K_T^2}{2z^2m_H^2} H_1^+(z, K_T^2).$$  \hspace{1cm} (35)

Figs. 6a and 6b show the predictions of our model for the weighted asymmetry for the proton and deuteron target, respectively.

6 Comment on the $x$-dependence of $A_{UL}^\sin\phi$ and $A_{UT}^\sin(\phi+\phi_S)$

The $x$-dependence of the asymmetries $A_{UL}^\sin\phi(x)$ and $A_{UT}^\sin(\phi+\phi_S)(x)$ is determined by the predictions for the chirally odd distribution functions from the chiral quark-soliton and instanton vacuum model [25, 27], while their overall normalization is fixed by the predictions for the Collins fragmentation function from the calculation in the Georgi-Manohar model [24]. Also in Refs. [22, 23, 49] the chiral quark-soliton and instanton vacuum model predictions were explored to study the $x$-dependence of the HERMES data using, however, different information on the Collins fragmentation function. Let us discuss how much the results presented in Refs. [22, 23, 48, 49] would be altered, if we used instead the Georgi-Manohar model results for the Collins fragmentation function [24].

In Refs. [22, 23, 48, 49] a different treatment of transverse parton momenta was employed\(^3\) and a different

\(^3\)In Refs. [49] a Gaussian distribution of parton transverse momenta in the target was assumed with a width $\langle p_T \rangle = 0.4\ GeV$ from [39, 40]. However, Eq. (20) was not used to relate the involved transverse momenta, but $zK_T$ was directly identified with the transverse momentum $P_{h\perp}$ of the produced hadron.
normalization of the Collins fragmentation function was used. In Refs. \[22, 23, 48, 49\] the quantity

\[
\left[\frac{1}{2(z)\sqrt{1 + (p_T^2)/m^2} - (D_1)}\right]^{1/2} \left[H_1^+(z)\right]^{[22, 23, 48, 49]} = \begin{cases} 
0.12 & \text{with } |\langle H_1^+ \rangle| \text{ from DELPHI} [50], \\
0.14 & \text{with } \langle H_1^- \rangle \text{ “from HERMES”} [22]
\end{cases}
\] (36)

corresponds to the following $z$-averaged quantities in our approach (the integration goes over $0.2 \leq z \leq 0.7$)

\[
\langle a^{1/2}(z) \rangle = \frac{\int dz H_1^{(1/2)}(z)}{\int dz D_1(z)} = 0.140,
\]

\[
\langle a^{1/2}_{\text{Gauss}}(z) \rangle = \frac{\int dz H_1^{(1/2)}(z)\sqrt{1 + z^2(p_T^2)/m_D^2}}{\int dz D_1(z)} = 0.104.
\] (37)

The first number in (36) follows from using the value from the DELPHI experiment on $e^+e^-$ annihilation at the $Z^0$ peak [50], which provided the first experimental indication for $H_1^+$. The second number follows from the value extracted in Ref. [22] from the HERMES (SIDIS) data [2, 3] assuming the chiral quark-soliton model results for the unknown distribution functions. These numbers are numerically close to each other and to the values in (37) from the model calculation of [24]. This means that the approach considered in this work describes the $z$-dependence of the azimuthal single spin asymmetries measured at HERMES with a longitudinally polarized target similarly well as the approach of Refs. [22, 23], and predicts numerically similar effects for CLAS longitudinal and HERMES transverse target polarization experiments [48, 49].

In this context it should be mentioned that using the DELPHI ($e^+e^-$ annihilation) result [22] in the description of the HERMES (SIDIS) experiment has – apart from disregarding possible Sudakov suppression effects [51] – the drawback of presuming universality of the Collins fragmentation function which has recently been questioned [32]. (Though, no indications for process dependence have been observed in leading order perturbation theory [52]. Also in the model calculations [24, 53, 54] the Collins function is universal.) It is remarkable that the numbers in (36, 37) from different experiments and model calculations referring to different scales are numerically close to each other.

7 Summary and conclusions

One aim of this work was to study how much of the HERMES data on azimuthal single spin asymmetries $A_{UL}^{\sin \phi}$ from a longitudinally polarized target can be explained by chiral physics, relying on effective chiral quark and Goldstone boson degrees of freedom. The only T-odd effect which is needed to explain single spin asymmetries and can be modelled in terms of the chiral degrees of freedom is the Collins effect. The Sivers effect requires to take into account explicitly gluonic degrees of freedom\(^4\) as, e.g., it is done in models with one-gluon exchange [56, 57, 58, 59].

\(^4\)This statement is based on the observation that one cannot simply model T-odd distribution functions directly in chiral models [26]. However, it cannot be excluded that it is possible to find an effective representation of the gauge link in terms of chiral degrees of freedom on the basis of a formalism which is able to relate gluonic degrees of freedom to effective quark degrees of freedom – such as the instanton formalism developed in Ref. [55].
In particular, we focused on the $z$-dependence of the asymmetries $A_{UL}^{\sin \phi}$ since the Collins and Sivers effect have different dependence on $z$ and could – at least in principle – be distinguished in this way. For the Collins fragmentation function we took the prediction from the calculation in the Georgi-Manohar model [24]. The required information on the integrated chirally odd distribution functions, which provides the overall normalization, we took from the chiral quark-soliton and instanton vacuum model [25, 27]. We used two different assumptions on intrinsic transverse parton momenta in the target, namely that they vanish or are Gaussian distributed, respectively. Both assumptions yield a reasonable description of the HERMES data [2, 3, 4].

The good description of the HERMES data in terms of the Collins effect only, as observed also previously in other analyses [18, 19, 20, 21, 22, 23], could give rise to the suspicion that the Sivers effect is small. However, by making a crude qualitative estimate, we have shown that within the present experimental error bars even a sizeable Sivers effect could be hidden – as observed in [42] on the basis of independent arguments.

Whether or not chiral degrees of freedom – as considered in our models – are able to model realistically the Collins effect can cleanly be tested in the transverse target polarization experiments which are in progress at HERMES [46] and COMPASS [47]. For that we have predicted the asymmetry $A_{UT}^{\sin (\phi + \phi_S)}(z)$ for the HERMES kinematics which appears to be large, of order 20% for $\pi^+$ from the proton target (to be compared with $A_{UL}^{\sin \phi} \sim (2 - 4\%)$). In the kinematics of the COMPASS experiment the effect is similarly large.

A measurement of $A_{UT}^{\sin (\phi + \phi_S)}$ asymmetries in the HERMES and COMPASS experiments of comparable magnitude to what predicted here would suggest that chiral symmetry breaking – in the way it is considered in our approach – can be used as a guideline to estimate the Collins effect, thus offering a valuable contribution to the understanding of the theory and phenomenology of single spin asymmetries.

Comment added. After this work was basically completed the first results from the HERMES transverse polarization proton target experiment have been released [60]. The preliminary data for $\pi^+$ are compatible with our predictions. (Note that the definition of $A_{UT}$ used here includes an extra factor of 2 compared to the definition used by the HERMES collaboration [60].) The asymmetries for $\pi^0$ and more clearly $\pi^-$ seem not to be compatible with our estimates. This probably indicates that taking only favoured fragmentation into account is not sufficient to describe the Collins effect.

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References


\footnote{It will be interesting to see whether this possibility can be practically explored in the CLAS and Hall-A experiments at Jefferson Lab – where the high luminosity promises a large statistics and high precision data.}


