Within the framework of the covariant formulation of Light-Front Dynamics, we develop a nonperturbative renormalization scheme in the fermion model supposing that the composite fermion is a superposition of the "bare" fermion and a fermion+boson state. We first assume the constituent boson to be spinless. Then we address the case of gauge bosons in the Feynman and in the Light-Cone gauges. For all these cases the fermion state vector and the necessary renormalization counterterms are calculated analytically. It turns out that in Light-Front Dynamics, to restore the rotational invariance, an extra counterterm is needed, having no analogue in Feynman approach. For gauge bosons the results obtained in the two gauges are compared with each other. In general, the number of spin components of the two-body (fermion+boson) wave function depends on the gauge. But due to the two-body Fock space truncation, only one non-zero component survives for each gauge. And moreover, the whole solutions for the state vector, found for the Feynman and Light-Cone gauges, are the same (except for the normalization factor). The counterterms are however different.
The understanding of hadronic systems is at the heart of numerous theoretical studies these last 20 years. While the description of experimental processes at very high energy can rely on perturbation theory, the study of bound states necessitates the use of non-perturbative methods. Concerning the latters, the nonperturbative renormalization procedure and the invariance of the system under gauge transformations are the properties of particular interest. Among the various frameworks used over the past to tackle these problems, we shall concentrate on Light Front Dynamics (LFD) [1] which exists in two field-theoretical forms: standard LFD [2] and explicitly Covariant Light Front Dynamics (CLFD) [3]. Standard LFD deals with the state vector defined on the plane $t+z=0$, while in CLFD this plane is given by the invariant equation $\omega \cdot x = 0$, where $\omega$ is an arbitrary four-vector with $\omega^2 = 0$. The particular choice $\omega = (1, 0, 0, -1)$ turns CLFD into standard LFD. We shall see below that the explicit covariance of CLFD is a very powerful tool to address the above questions in a transparent way.

Following the pioneering work of Wilson [4], the perturbative renormalization in QED has been studied both in LFD [5–7] and CLFD [8]. In these papers the fermion self-energy is investigated, and the necessary counterterms needed to renormalize it are found. To generalize the renormalization technique for nonperturbative processes, it is necessary to consider the same questions in terms of the Fock decomposition of the state vector. This has been already done for scalar models in LFD [9–11] and CLFD [12]. Such toy models are important to settle general equations and test numerical procedures. At the same time they are too simple because of, firstly, their super-renormalizability, and, secondly, the absence of contact interactions appearing for non-zero spin particles in LFD and CLFD. The next step therefore is an investigation of fermionic composite states [9,13–16]. Two important properties have emerged from these calculations.

i) The possible Fock sector dependence of the (mass) counterterm, which has been discussed in Ref. [9] for LFD. The necessity to remove, by hands, the counterterm in the equation which defines the last Fock state component considered in the calculation is now well understood [11,12]. This (infinite) counterterm should be balanced by a (also infinite) contribution arising from higher Fock state components not taken into account due to the truncation of the Fock state expansion.

ii) The appearance of non-local counterterms needed to recover known symmetries and to renormalize the theory [6]. As has already been shown in CLFD [8], the knowledge of the structure of the counterterms as functions of the light front plane orientation is necessary to understand how non-local counterterms appear in LFD. We shall come back extensively to this question in the following.

We consider in the present study a fermion $F$ coupled to bosons $b$ with spin either zero or one. We restrict ourselves to the approximation where only the first two Fock sectors $(|F\rangle + |Fb\rangle)$ in the state vector are retained. Such a model is instructive in many aspects: (i) since we are dealing with fermions, we have to take into account contact interactions. This, also, implies non-trivial contributions arising from the counterterms needed to renormalize the theory; (ii) we can investigate the case of gauge theories; (iii) the procedure can be checked by comparison with results of previous calculations based on perturbative methods, as mentioned above. It can thus be extended with some confidence to calculations involving higher Fock state components.

In a first step, we suppose the boson $b$ to be spinless, that just corresponds to the well-known Yukawa model. This gives us an opportunity to develop our method and show the peculiarities of CLFD. We reproduce the results obtained in this model in Ref. [14] in standard LFD.

We then study the system where the constituent boson is a gauge particle. In order to investigate the influence of the gauge on the state vector, we consider two different gauges: the Feynman and the Light-Cone (LC) ones. We will see that the number of spin components of the two-body wave function depends on the gauge. In the Feynman gauge the wave function is determined, in general, by eight components, whereas in the LC gauge it contains four components. However, in the simple model developed in the present paper where we restrict ourselves to the $|F\rangle + |Fb\rangle$ Tamm-Dancoff truncation, three non-zero components remain for the Feynman gauge and two ones for the LC gauge. Moreover, after imposing a certain condition on the counterterms, only one non-zero component of the two-body wave function survives for each gauge.

The plan of the paper is as follows. In Sec. II we present our general framework. In Sec. III the need for a specific extra counterterm in LFD is justified. In Sec. IV we calculate the state vector and the counterterms for the scalar boson case. In Sec. V we solve the problem for the gauge boson in the Feynman gauge. The same problem, but for the LC gauge, is solved in Sec. VI. The results are summarized in Sec. VII, where we discuss also the questions of gauge invariance and comparison with perturbative calculations. Sec. VIII contains our concluding remarks. Technical derivations are rounded up in Appendices.
II. GENERAL CONSIDERATIONS

A. Exact Lagrangian for scalar bosons

In order to clearly settle the starting point of our study, we shall recall in this section some known results. For simplicity, we detail only the scalar boson case. The case of vector bosons can be deduced very easily.

We shall start with the bare Lagrangian written as

\[ \mathcal{L} = \bar{\Psi}_0 [i \partial - m_0] \Psi_0 + \frac{1}{2} \left[ \partial_\nu \Phi_0 \partial^\nu \Phi_0 - \mu_0^2 \Phi_0^2 \right] + g_0 \bar{\Psi}_0 \Psi_0 \Phi_0, \]  

(1)

where \( \partial = \gamma \cdot \partial \equiv \gamma^\nu \partial_\nu \), \( \Psi_0 (\Phi_0) \) is the bare fermion (boson) field, \( m_0 \) \((\mu_0) \) is the bare fermion (boson) mass, and \( g_0 \) is the bare coupling constant. Going over from bare to dressed quantities by means of the standard substitutions

\[ \Psi_0 = Z_2^{1/2} \Psi, \quad \Phi_0 = Z_3^{1/2} \Phi, \quad gZ_1 = g_0 Z_2 Z_3^{1/2}, \]  

(2)

where \( g \) is the physically observed value of the coupling constant, and introducing also the physical masses \( m \) and \( \mu \), we get

\[ \mathcal{L} = \bar{\Psi} [i \partial - m] \Psi + \frac{1}{2} \left[ \partial_\nu \Phi \partial^\nu \Phi - \mu^2 \Phi^2 \right] + g \bar{\Psi} \Phi \]  

\[ + (Z_2 - 1) \bar{\Psi} [i \partial - m] \Psi + \frac{(Z_3 - 1)}{2} \left[ \partial_\nu \Phi \partial^\nu \Phi - \mu^2 \Phi^2 \right] \]  

\[ + \delta m \bar{\Psi} + \frac{1}{2} \delta \mu^2 \Phi^2 + g(Z_1 - 1) \bar{\Psi} \Phi. \]  

(3)

From Eqs. (1)–(3) it is easy to find that

\[ \delta m = Z_2(m - m_0), \quad \delta \mu^2 = Z_3(\mu^2 - \mu_0^2). \]  

(4)

The counterterms \( Z_{1-3}, \delta m, \) and \( \delta \mu^2 \) can be found from the definitions of the observed quantities \( g, m, \) and \( \mu \). Namely, \( \delta m \) and \( \delta \mu^2 \) are defined by the positions of the one-fermion and one-boson propagator poles. The additional requirement that the residues of these propagators at their poles equal \( i \) allows to fix the constants \( Z_2 \) and \( Z_3 \). Finally, the constant \( Z_1 \) is determined from the condition of the coincidence of the on-shell dressed fermion-fermion-boson vertex at zero boson four-momentum with the physical coupling constant.

B. Perturbative vs Fock state decomposition

As we already mentioned in the Introduction, a natural framework to study composite systems in LFD is the Fock state decomposition. Renormalization procedures, on the other hand, are well under control in perturbative Quantum Field Theory using the 4-Dimensional Feynman (4DF) diagrammatic expansion. In the latter approach, the composite fermion mass is defined as the pole of the fermion 2-Point Green’s Function (2PGF) which is obtained by summing up the chain series of the iterations of the fermion self-energy (a sum of all irreducible Feynman diagrams calculated up to a given order of coupling constant), \( \Sigma_{4DF}(\bar{\mu}, m_0) \), according to

\[ \frac{i}{\bar{\mu} - m_0} + \frac{i}{\bar{\mu} - m_0} \left[ -i \Sigma_{4DF}(\bar{\mu}, m_0) \right] + \frac{i}{\bar{\mu} - m_0} + \ldots = \frac{i}{\bar{\mu} - m_0 - \Sigma_{4DF}(\bar{\mu}, m_0)}. \]  

(5)

In LFD state vectors are defined as eigenstates of the corresponding light-front Hamiltonians. By using the Fock decomposition of the state vector, the eigenstate problem can be easily reduced to that of solving a system of linear homogeneous equations which involve various Fock components. Note that the Fock decomposition is made in terms of the number of Fock components rather than the coupling constant powers. Each Fock component effectively brings irreducible contributions of order \( \alpha \) or higher, with \( \alpha = \frac{g^2}{4\pi} \), to the fermion propagator. The summation of these contributions to all orders in \( \alpha \) is done implicitly when solving the system.
of linear equations mentioned above. The mass of the composite particle is found from the condition that this system has a nontrivial solution. Such a procedure is an analogue, in LFD, of the summation of the chain series for the 2PGF in 4DF approach. Below we shall derive and solve this system of equations, for the Fock decomposition restricted to the components $|F\rangle + |Fb\rangle$, for scalar and gauge bosons in both the Feynman and LC gauges.

C. Renormalization scheme

Before going into more detailed and technical considerations, it is necessary to clarify in which scheme we shall work to renormalize the system under study. A nice exposition of the standard derivation can be found in [17,18]. The following two schemes are usually considered in perturbative calculations:

- **Bare renormalization scheme.** The starting point is the bare Lagrangian with bare masses and coupling constants, as given by Eq. (1). These masses and coupling constants are determined from physical conditions. For instance, the bare fermion mass is chosen so to get a pole of the 2PGF at the physical fermion mass. The bare mass expressed in terms of the physical one is in general infinite and must be regularized.

- **Dressed renormalization scheme.** The starting point here is a renormalized Lagrangian (3) defined in terms of (finite) physical masses and coupling constants, and including the counterterms which are thus infinite and must be regularized.

The two regularization schemes should of course give the same physical observables provided no approximations have been made. The latter never happens in practice: one either makes a perturbative expansion, or truncate Fock space in LFD, or both. The equivalence between the two schemes can be directly checked in the lowest order of perturbation theory, as explained in any text book on Quantum Field Theory. In LFD, this question was partially addressed in Refs. [4,14]. We have also tackled it within CLFD using perturbative expansions [8].

Nonperturbative calculations however hardly admit such a check. In the spirit of the Fock state decomposition, one should make sure that the one-particle states are as close as possible to the physical ones. In our case this implies that the basic one-fermion state $|F\rangle$ and the full state $|F\rangle + |Fb\rangle$ correspond to the same physical mass $m$, and we thus should consider the mass counterterm $\delta m$.

The counterterm proportional to $Z_2 - 1$ in Eq. (3) renormalizes the fermion field. However, since we are working here with the state vector rather than with particle fields, it is more appropriate to deal directly with the state vector normalization factor $N$ instead of $Z_2$. For this reason, we will exclude the counterterm with $Z_2$ from our consideration. The normalization factor $N$ does not enter the eigenstate equation for the state vector and can be calculated after the solution is found.

Our task is simplified for the Fock decomposition truncated to the components $|F\rangle + |Fb\rangle$. In this case, the boson state vector is still free, therefore, $Z_3 = 1$ and $\delta \mu^2 = 0$.

Finally, the counterterm with $Z_1$ is related to the coupling constant renormalization. Since in the present paper we do not calculate physical observables like electromagnetic form factors or scattering amplitudes, we may deal with the unrenormalized coupling constant and put $Z_1 = 1$. For convenience, we shall call this coupling constant $g$.

So, as far as the counterterms in the Lagrangian are concerned [the second and the third lines in Eq. (3)], it is enough for solving the nonperturbative problem in the two-body approximation, to retain explicitly the term with $\delta m$ only, while those containing $Z_1$, $Z_2$, $Z_3$, and $\delta \mu^2$ can be dropped out. The quantity $\delta m$ defined now as

$$\delta m = m - m_0, \quad (6)$$

that is a shift of mass caused by the interaction.

III. THE NEED FOR A NEW COUNTERTERM IN LFD

As we shall see explicitly in the following applications, it turns out that one has to introduce in LFD another counterterm which is not reduced to any of those considered above.
To explain its role, let us calculate the light-front 2PGF in the dressed renormalization scheme, using
the two-body approximation. Below in this section we obtain a formal expression for the 2PGF, then find the
perturbative (up to terms of order $g^4$) fermion-boson on-shell scattering amplitude $M^{(4)}$ and demonstrate that
without an additional counterterm such an amplitude would be dependent on the light front orientation (i.e.,
on the four-vector $\omega$). The dependence of approximate physical CLFD amplitudes on $\omega$ is equivalent to the
rotational symmetry violation in standard LFD.

Since we are not dealing here with an eigenstate problem, we should retain the constant $Z_2$ in the
Lagrangian (3), while putting $Z_1 = Z_3 = 1, \delta \mu^2 = 0$, as explained above. So, the Lagrangian we consider in
this section is

$$\mathcal{L} = \bar{\Psi} [i \not\partial - m] \Psi + \frac{1}{2} \left[ \partial_{\nu} \Phi \partial^{\nu} \Phi - \mu^2 \Phi^2 \right] + g \bar{\Psi} \Psi \Phi + (Z_2 - 1) \bar{\Psi} [i \not\partial - m] \Psi + \delta m \bar{\Psi} \Psi. \tag{7}$$

In the following we will use the fact that perturbative expansions of the quantities $Z_2 - 1$ and $\delta m$ start with
terms of order $g^2$.

Denote by $p_1$ ($p_1'$) the initial (final) fermion four-momentum; the corresponding boson four-momenta are $k_1$ ($k_1'$). In the lowest (second) order in $g$ the on-energy-shell scattering amplitude $M^{(2)}$ is defined by the
sum of the two CLFD diagrams shown in Fig. 1. The rules of the CLFD diagram technique are exposed in
detail in Ref. [3]. As usual, straight solid, wavy, and dashed lines correspond to fermions, bosons, and spurions,
respectively. The internal fermion line with a large full blob in the second diagram is the so-called contact
term\(^1\). The amplitude $M^{(2)}$ was calculated in Ref. [3]:

$$M^{(2)} = g^2 \bar{u}(p_1') \left[ \frac{\not p' + \omega \tau + m}{m^2 - p'^2} - \frac{\omega}{2(\omega p)} \right] u(p_1), \tag{8}$$

where $p = p_1 + k_1 = p_1' + k_1'$, $\tau = (m^2 - p'^2)/2(\omega p)$. The two addendums in the square brackets on the r.h.s.
of Eq. (8) just correspond to the contributions from the two diagrams shown in Fig. 1. It is easy to see that the
$\omega$-dependent terms in the sum cancel each other, the whole amplitude $M^{(2)}$ being $\omega$-independent:

$$M^{(2)} = ig^2 \bar{u}(p_1') D(p) u(p_1), \tag{9}$$

where

$$D(p) = i \frac{\not p + m}{p^2 - m^2} \tag{10}$$

is the 2PGF in the lowest (zeroth) order of perturbation theory. Note that it has the same form as in the 4DF
approach, as it should.

The exact light-front 2PGF can be obtained by formal summing up the perturbation series. Such a
procedure is quite similar to that used in standard QED and leading to the Dyson equation. The difference is
that LFD calculations involve contact terms mentioned above.

Let us define the mass operator $-\mathcal{M}(p)$ (the sign minus is introduced for convenience) which is a sum of
all irreducible light-front diagrams containing no one-fermion intermediate states or, in other words, including
only intermediate states with bosons. The equation for the exact 2PGF $D(p)$ is obtained by infinite iterations
of the kernel being a sum of the three terms: the vertices $(Z_2 - 1)(\not p - m)$, $\delta m$, and the mass operator $-\mathcal{M}(p)$.
This equation reads

$$D(p) = D(p) + D(p)[i(Z_2 - 1)(\not p - m) + i\delta m - i\mathcal{M}(p)]D(p). \tag{11}$$

It has the formal solution

$$D(p) = D(p) \frac{1}{1 - i[(Z_2 - 1)(\not p - m) + \delta m - \mathcal{M}(p)]D(p)}$$

$$= \frac{\not p + m}{p^2 - m^2 + (Z_2 - 1)(p^2 - m^2) + [\delta m - \mathcal{M}(p)][\not p + m]}. \tag{12}$$

\(^1\)In Ref. [3] the contact terms were represented by lines with crosses. Here we changed this notation, reserving the cross
symbol for counterterms (see below).
In the $g^2$ (or two-body) approximation the mass operator is given by the sum of the two diagrams shown in Fig. 2. The diagrams (a) and (b) taken with the minus sign each are the light-front fermion self-energy, $-\Sigma(p)$, and a contact term on the fermion line, $-\Sigma_{f,c}(p)$, respectively. Note that the amplitudes of these diagrams include integrals over bosonic momentum. Because of different phase space volumes, it is not judicious to combine the contact term with the usual light-front fermion propagator in the self-energy diagram into the unique term $D(p)$, like it has been done in Eqs. (8)–(10). So, the diagrams (a) and (b) in Fig. 2 must be treated separately. The mass operator $M(p)$ is a light-front analogue of the 4DF fermion self-energy $\Sigma_{4DF}(p)$ calculated in the dressed renormalization scheme.

Now let us expand Eq. (12) in powers of $g$ up to terms of order $g^2$. We have

$$D(p) \approx D^{(2)}(p) \equiv \frac{1}{p^2 - m^2} \left[ (Z_2 - 1)(p - m) + \delta m - M^{(2)}(p) \right] \frac{p + m}{p^2 - m^2}$$

with

$$M^{(2)}(p) = \Sigma(p) + \Sigma_{f,c}(p).$$

Applying the CLFD graph technique rules [3] to the diagrams shown in Fig. 2, we get

$$\Sigma(p) = -\frac{g^2}{(2\pi)^3} \int (\not{q} + m)\theta(\omega - \not{q})\delta(q^2 - m^2)d^4q \frac{\theta(\omega + \not{k})\delta((k^2 - \mu^2)d^4k \delta(\tau - \omega - k)}{2\varepsilon_k} \left( p + \omega \tau - k - q \right) \frac{d\tau}{\tau - i0}$$

$$= -\frac{g^2}{(2\pi)^3} \int \frac{(\not{p} - \not{k} + \not{\omega} + \not{\tau} + \not{m})\theta(\omega(\not{p} - \not{k}))}{2\omega(\not{p} - \not{k})\tau} \frac{d^4k}{2\varepsilon_k} \equiv g^2 \left[ A(p^2) + B(p^2) \frac{\not{p}}{m} + C(p^2) \frac{m \not{\omega}}{\omega \not{p}} \right]$$

and

$$\Sigma_{f,c}(p) = \frac{g^2}{(2\pi)^3} \int \frac{1}{2\omega(\not{p} - \not{k})} \frac{d^4k}{2\varepsilon_k} \equiv g^2 C_{f,c} \frac{m \not{\omega}}{\omega \not{p}},$$

where

$$\tau = \frac{m^2 - (\not{p} - \not{k})^2}{2\omega(\not{p} - \not{k})}$$

and $\varepsilon_k = \sqrt{k^2 + \mu^2}$. The functions $A(p^2)$, $B(p^2)$, $C(p^2)$, and the constant $C_{f,c}$ are given in Appendix B1. Note that the self-energy $\Sigma(p)$ depends on the position of the light front [8], since it contains the contribution proportional to $\not{\omega}$. The same contribution was found earlier in Ref. [6]. In Eq. (B9) from Ref. [6] it looks as $m\not{\omega}/p\not{p}$ for $\omega = (1, 0, 0, -1)$. Substituting the decompositions of $\Sigma(p)$ and $\Sigma_{f,c}(p)$ in terms of $A(p^2)$, $B(p^2)$, $C(p^2)$, and $C_{f,c}$, as defined by the r. h. s. of Eqs. (15) and (16), into Eq. (14) we can write

$$M^{(2)}(p) = g^2 \left\{ A(p^2) + B(p^2) \frac{\not{p}}{m} + C(p^2) + C_{f,c} \right\} \frac{m \not{\omega}}{\omega \not{p}}.$$  

The fermion-boson scattering amplitude calculated in the two-body approximation discussed here can be obtained by sandwiching the 2PGF (12) between the bispinors $\bar{u}(p'_1)$ and $u(p_1)$. Up to terms of order $g^4$ inclusive it is

$$M^{(4)} = ig^2\bar{u}(p'_1)D^{(2)}(p)u(p_1).$$

Substituting Eq. (18) into Eq. (13) and then into Eq. (19), we find after some algebra:

$$M^{(4)} = M^{(4)}_0 + M^{(4)}_\omega,$$

where
\[ M^{(4)}_0 = g^2 \bar{u}(p'_1) \left\{ \frac{2m(\not{p} + m)(\delta m - g^2[A(p^2) + B(p^2) + C(p^2) + C_{fc}])}{(p^2 - m^2)^2} \right. \\
\left. + \frac{[Z_2 - 2 - g^2B(p^2)/(\not{p} + m) + \delta m - g^2[A(p^2) + B(p^2)]]}{p^2 - m^2} \right\} u(p_1), \] (21)

\[ M^{(4)}_ω = g^4 \bar{C}(p^2) + C_{fc} \bar{u}(p'_1) \frac{m \not{\phi}}{ωp} u(p_1). \] (22)

The constants \( Z_2 \) and \( \delta m \) are found in a standard way (we will specify the corresponding procedure below) and do not include any dependence on the orientation of the light front \( ω \). As a result, the quantity \( M^{(4)}_0 \) is also \( ω \)-independent. On the contrary, \( M^{(4)}_ω \) explicitly depends on \( ω \) through the term \( \bar{u}(p'_1) \not{\phi}u(p_1) \). So, the whole amplitude \( M^{(4)} \) depends on \( ω \) as well, that evidently calls for a certain additional counterterm (except for the previously introduced \( Z_2 \) and \( \delta m \)) to eliminate this nonphysical \( ω \)-dependence.

On can also come to the same conclusion from the requirement that the 2PGF (13) has a simple pole at \( \not{p} = m \) with the residue equal to \( i \). In other words, there must be

\[ D^{(2)}(p) \bigg|_{\not{p} = m} = \frac{i}{\not{p} - m} + \ldots = \frac{\not{p} + m}{p^2 - m^2} + \ldots, \] (23)

where the dots designate finite terms. Comparing Eqs. (23) and (13), we have to demand that

\[ \lim_{p^2 \to m^2} \left\{ \frac{(\not{p} + m)[(Z_2 - 1)(\not{p} - m) + \delta m - M^{(2)}(p)\not{p} + m)]}{p^2 - m^2} \right\} = 0. \] (24)

The expression in the curled brackets in Eq. (24) is a matrix. The condition (24) is imposed on any of its matrix element. Substituting Eq. (18) into Eq. (24) and using the Taylor expansions \( A(p^2) \equiv A + A' \cdot (p^2 - m^2) + \ldots, \ B(p^2) \equiv B + B' \cdot (p^2 - m^2) + \ldots, \ C(p^2) \equiv C + C' \cdot (p^2 - m^2) + \ldots \) near the point \( p^2 = m^2 \), we can rewrite Eq. (24) in the form

\[ \left. \left. \frac{2m(\not{p} + m)}{p^2 - m^2} \delta m - g^2(A + B + C + C_{fc}) \right|_{p^2 \to m^2} \right. \\
\left. + \left[ m(Z_2 - 1) - g^2B - 2g^2m^2(A' + B' + C') + \delta m - g^2(A + B) \right] \right|_{p^2 \to m^2} \\
\left. + \left[ m(Z_2 - 1) - g^2B - 2g^2m^2(A' + B' + C') \right] \frac{\not{p}}{m} + g^2(C + C_{fc}) \frac{\not{\phi}}{ωp} \right|_{p^2 \to m^2} = 0. \] (25)

The absence of the pole term in Eq. (25) leads to

\[ \delta m = g^2(A + B + C + C_{fc}) = \bar{u}(p)M^{(2)}(p)u(p) \bigg|_{p^2 = m^2}, \] (26)

that coincides with the standard perturbative expression. The last equality immediately follows from Eq. (18) and the identities \( \bar{u}(p)u(p) = 2m \) and \( \bar{u}(p)\not{µ}u(p) = p^2\bar{u}(p)u(p)/m \) valid at \( p^2 = m^2 \). However, it is impossible to choose \( Z_2 \) in Eq. (25) so that all the other coefficients at the independent matrices \( \not{p}, \not{φ} \) and \( \not{\phi} \) would be zero, unless \( C + C_{fc} = 0 \). As is seen from Eqs. (B4c) and (B4d) in Appendix B 1, the sum \( C + C_{fc} \) generally speaking, differs from zero. Hence, Eq. (25) can not be satisfied. This again indicates the need for another counterterm in the Lagrangian (7).

It is easy to see that the counterterm required must have the following structure (in momentum representation):

\[ Z_ω \left( \frac{m \not{φ}}{ωp} - 1 \right), \] (27)

where \( Z_ω \) is a constant to be determined. We have supplied this constant with the subscript "ω" in order to point out its origin, but it does not in fact depend on ω. The precise form of Eq. (27) is chosen so that this counterterm does not contribute when sandwiched between two bispinors of equal momenta.
After introducing the new counterterm, the quantity $\delta m - M^{(2)}_\omega(p)$ in Eqs. (13) and (24) must be replaced by

$$\delta m - M^{(2)}_\omega(p) + Z_\omega \left( \frac{m \phi}{\omega p} - 1 \right),$$

that is equivalent to the following two substitutions:

$$\delta m \to \delta m - Z_\omega, \quad g^2[C(p^2) + C_{fc}] \to g^2[C(p^2) + C_{fc}] - Z_\omega,$$  \hspace{1cm} (28)

made in all equations obtained above in this section. Now, instead of Eq. (25) we get

$$2m(\not{p} + m) \frac{\delta m - g^2(A + B + C + C_{fc})}{p^2 - m^2} \bigg|_{p^2 = m^2} + [m(Z_2 - 1) - g^2 B - 2g^2 m^2(A' + B' + C') + \delta m - Z_\omega - g^2(A + B)]$$

$$+ [m(Z_2 - 1) - g^2 B - 2g^2 m^2(A' + B' + C')] \frac{\not{p}}{m} + [g^2(C + C_{fc}) - Z_\omega] \frac{\phi}{\omega p} = 0.$$  \hspace{1cm} (29)

From here we reproduce for $\delta m$ the same formula (26), while for $Z_2$ and $Z_\omega$ we find

$$Z_2 = 1 + \frac{g^2}{m} B + 2mg^2(A' + B' + C') = 1 + \frac{1}{2m} \left\{ \bar{u}(p) \frac{\partial M^{(2)}_\omega(p)}{\partial \not{p}} u(p) \right\} \bigg|_{p^2 = m^2},$$  \hspace{1cm} (30)

$$Z_\omega = g^2(C + C_{fc}).$$  \hspace{1cm} (31)

In differentiating, $M^{(2)}_\omega(p)$ is implied to depend on $\not{p}$ both explicitly and through $p^2 = \not{p} \cdot \not{p}$. Note also that $Z_2$ has the same form as in the standard renormalization technique. So, after introducing the additional counterterm (27) the behaviour of the fermion propagator near its pole is indeed governed by Eq. (23).

One can easily check that the counterterm (27) eliminates the $\omega$-dependence of the scattering amplitude (20)–(22). This nonphysical $\omega$-dependence is concentrated in the term $M^{(4)}_\omega$ given by Eq. (22). The substitutions (28) with $Z_\omega$ taken from Eq. (31) transform Eq. (22) into

$$M^{(4)}_\omega = \frac{g^4[C(p^2) + C_{fc}] - g^2 Z_\omega \bar{u}(p_1) m \phi}{p^2 - m^2} \frac{\omega p}{u(p_1)} = g^4 \frac{C(p^2) - C(m^2)}{p^2 - m^2} \bar{u}(p_1) m \phi \frac{\omega p}{u(p_1)},$$  \hspace{1cm} (32)

since, by definition, $C = C(m^2)$. As proved in Appendix B1, $C(p^2)$ does not depend on $p^2$ [in contrast to the functions $A(p^2)$ and $B(p^2)$]. Therefore $M^{(4)}_\omega = 0$ and the whole scattering amplitude (20) turns out to be $\omega$-independent, as it should.

We have constructed the additional counterterm (27) from perturbative considerations. However, as will be shown below, it retains its form for the nonperturbative problem discussed in the present paper. Moreover, its structure is the same for both the scalar and gauge boson cases. In the next section and in Appendix A we will explain how to introduce it in the light-front Hamiltonian.

### IV. SCALAR BOSON

In this section we give a solution for the state vector of a system composed of a spin 1/2 fermion coupled to a scalar boson, in the Fock decomposition restricted to the $|F\rangle + |Fb\rangle$ approximation. A general derivation of the CLFD fermion-boson Hamiltonian for the scalar boson case is presented in Appendix A1.

#### A. Contact terms

As already mentioned in Sec. II, a peculiarity of LFD is the appearance, for particles with spin, of the so-called contact interactions [2,3]. These interactions arise from the elimination of two non-dynamical degrees
of freedom for the fermion field, as recalled in Appendix A. In the case of spinor-scalar interactions, with the initial interaction Lagrangian of the form $L^{int}(x) = g \bar{\psi} \psi \varphi$, where $\psi$ and $\varphi$ are, respectively, the free fermion and boson fields, this contact interaction generates a $\mathcal{F}Fb \propto g^2$ vertex in addition to the usual $\mathcal{F}Fb$ vertex. In the presence of counterterms needed to renormalize the theory, like the standard mass counterterm $\delta m $, the elimination of the non-dynamical degrees of freedom generates in addition two new contributions to the light-front Hamiltonian. The first one proportional to $\delta m^2$ originates from two self-interaction vertices \(^2\), the second one proportional to $g \delta m$ appears from the product of the self-interaction times the ordinary $\mathcal{F}Fb$ vertex. Taking into account the specific counterterm discussed in Sec. III and introducing, for convenience, a shifted mass counterterm $\delta m'$ defined by

$$\delta m' = \delta m - Z_\omega,$$

we obtain the final form of the light-front interaction Hamiltonian. The details of the derivation are given in Appendix A 1. We thus get

$$H^{int}(x) = H_1(x) + H_{1c}(x) + H_2(x) + H_{2c}(x) + H_{3}(x)$$

with

$$H_1(x) = -g \bar{\psi} \psi \varphi,$$

$$H_{1c}(x) = g^2 \bar{\psi} \varphi \frac{\varphi}{2i(\omega \cdot \partial)} \psi,$$

$$H_2(x) = -\delta m' \bar{\psi} \psi,$$

$$H_{2c}(x) = g \delta m' \bar{\psi} \left[ \varphi \frac{\varphi}{2i(\omega \cdot \partial)} + \frac{\varphi}{2i(\omega \cdot \partial)^2} \right] \psi,$$

$$H'_{2c}(x) = \delta m'^2 \bar{\psi} \frac{\varphi}{2i(\omega \cdot \partial)} \psi,$$

$$H_3(x) = -Z_\omega \bar{\psi} \frac{m \varphi}{i(\omega \cdot \partial)} \psi.$$

The Hermitian operator $\frac{1}{i(\omega \cdot \partial)}$, which acts on any coordinate space function $f(x)$ standing to the right of it, turns, in momentum space, to the factor $\frac{1}{i\omega \cdot \vec{k}}$, where $k$ is the momentum conjugated to the coordinate $x$. The formal definition of $\frac{1}{i(\omega \cdot \partial)}$ in coordinate space is given in Appendix A 1. The physical mass of the constituent fermion (boson) is $m$ ($\mu$). The terms $H_1(x)$ and $H_{1c}(x)$ in Eq. (34) are the tree level interactions: the usual elementary $\mathcal{F}Fb$ fermion-boson interaction and the contact term on the fermion line between two elementary interaction vertices. The terms $H_2(x)$, $H_{2c}(x)$, and $H'_{2c}(x)$ are associated with the mass counterterm we mentioned above.

The last, $\omega$-dependent, term $H_3(x)$ plays a particular role. Actually, we postulate its form on the basis of the observations made in Sec. III. It is needed to eliminate nonphysical dependence of calculated observables on the light front position and may come both from the Fock space truncation and from cutting off divergent integrals. A strict derivation of $H_3(x)$ from the “first principles” is however beyond the scope of the present article and should be a subject of another work.

We emphasize that $\delta m$ (but not $\delta m'$ !) is the difference between the dressed and the bare masses [see Eq. (6) above]. Therefore, it represents the shift of mass (from the bare to the dressed one) due to interaction. Since, in gauge theory, the corresponding equations of motion are gauge invariant, the value of $\delta m$ defined in this way is gauge invariant too, though, generally speaking, the counterterms are not obligatory gauge invariant. The gauge invariance of $\delta m$ will be analyzed in our calculations.

We shall apply the Hamiltonian (34) in order to find the composite fermion state vector in the two-body approximation. For this purpose, we must know the matrix elements of $H^{int}(x)$ between $|F\rangle$ and $|Fb\rangle$ states. In practice, it is convenient to represent these matrix elements graphically by means of CLFD diagrams, using for calculation of their amplitudes the graph technique rules [3]. According to the latters, matrix elements of an

\(^2\)The notation $\delta m^2$ means $(\delta m)^2$ and not $\delta(m^2)$
interaction Hamiltonian are given by amplitudes of the corresponding diagrams taken with the opposite sign. In the following we will refer to these amplitudes as interaction vertices.

The interaction vertices for the Hamiltonian (34) are shown graphically in Figs. 3–8. In each diagram the four-vector \( p \) is the total four-momentum of the initial state. As previously, the solid and wavy lines correspond to fermion and boson states, respectively. The dashed lines correspond to spurious — fictitious particles ensuring four-momentum conservation at each vertex [3]. There exist three different "elementary" vertices: a three-point vertex proportional to \( g \) and two two-point ones proportional to \( \delta m \) and \( Z \). We will denote these vertices by a small full blob, a cross and a cross in an open circle, respectively. The contact interaction between two elementary vertices, proportional to \( g \), is given by a small full blob, a cross and a cross in an open circle, respectively. The contact interaction between two elementary vertices, proportional to \( -\frac{\varphi}{2(|\omega|k)} \) in momentum space, is denoted by a fermion line with a large full blob (here \( k \) is the four-momentum corresponding to the line going through the large full blob).

The interaction vertices of \( H_1(x) \), Eq. (35a), are shown in Fig. 3. They describe transitions \( |F\rangle \rightarrow |Fb\rangle \) and \( |Fb\rangle \rightarrow |F\rangle \). The term \( H_{1c}(x) \), Eq. (35b), in our truncated Fock space, conserves the number of particles. It gives rise to transitions \( |F\rangle \rightarrow |F\rangle \) and \( |Fb\rangle \rightarrow |Fb\rangle \) only. The corresponding interaction vertices are shown in Fig. 4. Note that the amplitude of the diagram (c) is infinite since it is proportional to a divergent integral over internal boson momentum. The term \( H_2(x) \), Eq. (35c), produces the only matrix element between one-fermion states, as shown in Fig. 5. The term \( H_{2c}(x) \), Eq. (35d), has four matrix elements indicated in Fig. 6, for transitions \( |Fb\rangle \rightarrow |F\rangle \) and \( |F\rangle \rightarrow |Fb\rangle \). The terms \( H'_{2c}(x) \), Eq. (35e), and \( H_3(x) \), Eq. (35f), do not contain bosonic field operators and produce only one matrix element each (Figs. 7 and 8, respectively).

To avoid any misunderstanding, we emphasize that, in agreement with the rules of the graph technique for CLFD diagrams [3], the lines with large full blobs representing contact interaction do not correspond to propagators, but should be considered as some "complex" vertices, being the products of the factors standing at "elementary" vertices. For example, such a line in Fig. 4(c) is given by

\[
g \left[ -\frac{\varphi}{2\omega(p - k)} \right] g,
\]

where \( p - k \) is just the momentum going through the blob. This momentum is completely defined by the energy-momentum conservation at the vertices, including the spurion four-momentum.

It is worth mentioning that the spin structures of the matrix elements corresponding to the diagrams shown in Figs. 4(c) and 8 are exactly the same. Indeed, these matrix elements (denote them by \( \langle H_{1c} \rangle \) and \( \langle H_3 \rangle \), respectively) are given by the following analytical expressions:

\[
\langle H_{1c} \rangle = g^2 C_{fc} \bar{u}' \frac{m}{\omega} \frac{\varphi}{\omega p} u, \quad \langle H_3 \rangle = -Z \bar{u} \frac{m}{\omega} \frac{\varphi}{\omega p} u,
\]

where

\[
C_{fc} = \frac{1}{16\pi^3 m^2} \int \frac{1}{1 - \frac{2\omega k}{\omega p}} \frac{d^3k}{2\epsilon_k},
\]

as follows from Eq. (16), and \( u \) (\( u' \)) is the initial (final) fermion bispinor. The constant \( C_{fc} \) is calculated in an explicit form in Appendix B. Since \( H_{1c}(x) \) and \( H_3(x) \) come into the full Hamiltonian (34) as a sum, the same property takes place for their matrix elements shown in Figs. 4(c) and 8. If we define a new constant

\[
Z = Z_\omega - g^2 C_{fc},
\]

the total contribution of \( \langle H_{1c} \rangle \) and \( \langle H_3 \rangle \) becomes

\[
\langle H_{1c} \rangle + \langle H_3 \rangle = -Z \bar{u}' \frac{m}{\omega} \frac{\varphi}{\omega p} u.
\]

So, the divergent bosonic loop in Fig. 4(c) is effectively "swallowed" by the counterterm proportional to \( Z \).

\[\text{Note that two additional contributions should be added in Fig. 4 if higher Fock states are considered. They correspond to the case where the both boson lines originate either from the initial or final state.}\]
B. Equations in the $|F\rangle + |Fb\rangle$ approximation

In truncated Fock space we consider in this study the state vector $\phi_\sigma(p)$ includes two sectors only. Its Fock decomposition has thus the form

$$\phi_\sigma(p) = \frac{(2\pi)^{3/2}}{\sqrt{N}} \frac{1}{\sqrt{2\varepsilon_{k_1}}} \phi_1,\sigma\sigma'(p_1, p, \omega\tau_1) a^\dagger_{\sigma'}(p_1)|0\rangle$$

$$\times \delta^{(4)}(p_1 - p - \omega\tau_1) 2(\omega p) \frac{d\tau_1 d^3p_1}{(2\pi)^{3/2} \sqrt{2\varepsilon_{p_1}}}$$

$$+ \frac{(2\pi)^{3/2}}{\sqrt{N}} \frac{1}{\sqrt{2\varepsilon_{k_2}}} \phi_2,\sigma\sigma'(k_1, k_2, p, \omega\tau_2) a^\dagger_{\sigma'}(k_1) c^\dagger(k_2)|0\rangle$$

$$\times \delta^{(4)}(k_1 + k_2 - p - \omega\tau_2) 2(\omega p) \frac{d\tau_2 d^3k_1 d^3k_2}{(2\pi)^{3/2} \sqrt{2\varepsilon_{k_1}} (2\pi)^{3/2} \sqrt{2\varepsilon_{k_2}}},$$

(41)

where $a^\dagger$ and $c^\dagger$ are the creation operators of the free fermion and boson, respectively, and $\varepsilon_{p_1} = \sqrt{p_1^2 + m_1^2}$, $\varepsilon_{k_i} = \sqrt{k_i^2 + m_i^2}$, with the appropriate masses. The quantities $\phi_1$ and $\phi_2$ are just the light-front one- and two-body Fock components (wave functions), $N$ is the normalization factor.

The normalization condition for the state vector has the form

$$\phi^\dagger_{\sigma'}(p')\phi_{\sigma}(p) = (2\pi)^3 \delta_{\sigma\sigma'} \delta^{(3)}(p - p'),$$

(42)

that gives for the wave functions [3]:

$$\sum_{\tilde{\sigma}} \phi^\dagger_{1,\sigma\tilde{\sigma}}(p_1, p, \omega\tau_1) \phi_{1,\sigma\tilde{\sigma}}(p_1, p, \omega\tau_1) + \frac{1}{(2\pi)^3} \sum_{\tilde{\sigma}} \int \phi^\dagger_{2,\sigma\tilde{\sigma}}(k_1, k_2, p, \omega\tau_2) \phi_{2,\sigma\tilde{\sigma}}(k_1, k_2, p, \omega\tau_2)$$

$$\times \delta^{(4)}(k_1 + k_2 - p - \omega\tau_2) 2(\omega p) d\tau_2 \frac{d^3k_1 d^3k_2}{2\varepsilon_{k_1} 2\varepsilon_{k_2}} = N \delta_{\sigma\sigma'}.$$  

(43)

The state vector corresponds to a total spin of 1/2 and satisfies the following equation [12]:

$$2(\omega p) \int \tilde{H}^{int}(\omega\tau) \frac{d\tau}{2\pi} \phi_{\sigma}(p) = - \left[ (\tilde{P}^0)^2 - M^2 \right] \phi_{\sigma}(p),$$

(44)

where $\tilde{P}^0$ is the free momentum operator, and

$$\tilde{H}^{int}(\omega\tau) = \int H^{int}(x)e^{-i\omega x} d^4x.$$  

(45)

The interaction Hamiltonian $H^{int}(x)$ is given by Eq. (34). The mass of the composite fermion is denoted by $M$. In the end of the calculation, we shall take the limit $M \to m$.

Following Ref. [12], we define the one- and two-body vertex functions $\Gamma_1$ and $\Gamma_2$ by

$$\tilde{u}_{\sigma'}(p_1)\Gamma_1 u_{\sigma}(p) = 2(\omega p) \tau_1 \phi_{1,\sigma\sigma'}(p_1, p, \omega\tau_1),$$

$$\tilde{u}_{\sigma'}(k_1)\Gamma_2 u_{\sigma}(p) = 2(\omega p) \tau_2 \phi_{2,\sigma\sigma'}(k_1, k_2, p, \omega\tau_2).$$

(46a)

(46b)

Due to the delta-functions coming into the decomposition (41), we have $p_1 = p + \omega\tau_1$ and $k_1 + k_2 = p + \omega\tau_2$, that gives

$$\tau_1 = \frac{m^2 - M^2}{2\omega p}, \quad \tau_2 = \frac{(k_1 + k_2)^2 - M^2}{2\omega p}.$$  

(47)

Now Eq. (44) can be rewritten as

$$\mathcal{G}(p) = - \int \tilde{H}^{int}(\omega\tau) \tau^{-1} \mathcal{G}(p) \frac{d\tau}{2\pi},$$

(48)
where $\mathcal{G}(p)$ is defined by a decomposition analogous to Eq. (41), with the wave functions replaced by the vertex functions according to Eqs. (46a), (46b). The operator $\hat{\tau}^{-1}$ acting on $\mathcal{G}(p)$ multiplies its one-body part by $1/\tau_1$ and its two-body part by $1/\tau_2$.

The system of equations for the vertex functions $\Gamma_1$ and $\Gamma_2$ is represented graphically in Fig. 9. The loop integrals contain $\Gamma_2$ in the integrands. As we shall check in the end of the calculation, $\Gamma_2$ is a constant for our Fock space truncation, so that we can extract it from the integrals. The latter are thus reduced to the loop integrals containing $\Gamma_1$ but it does not arise when the self-energy is given by Eq. (15), i.e., when it is linear in the Dirac matrices. The $\not{Z}m$ shown in Figs. 4(c) and 8 into a single term proportional to $\not{Z}m$. Out of the two-body approximation, we should not take into account the graphs in Figs. 6(c,d). In the system of equations (49) and (50) we also added the contributions from graphs including the interaction vertices shown in Figs. 4(c) and 8 into a single term proportional to $Zm \not{\varphi}/(\omega p)$, according to the discussion in the end of Sec. IV A. For simplicity we kept for this term the same graphical notation (a cross in an open circle) as for the initial counterterm with $Z_m$.

Using Eq. (15), the fermion on-mass-shell self-energy $\Sigma$ can be represented as

$$\Sigma = \mathcal{G}(p)|_{p^2=m^2} = g^2 A + g^2 B \frac{\not{\varphi}}{m} + g^2 C \frac{\not{\varphi}}{\omega p},$$

where, as previously, $A = \mathcal{A}(m^2)$, $B = \mathcal{B}(m^2)$, and $C = \mathcal{C}(m^2)$. We display explicitly the coupling constant in the decomposition of the self-energy. Note that in the general decomposition (52) the term $\not{\varphi} \not{\varphi}$ is also possible, but it does not arise when the self-energy is given by Eq. (15), i.e., when it is linear in the Dirac matrices. The coefficients $A$, $B$, and $C$ are calculated in Appendix B1.
C. Solution

The one-body vertex function $\Gamma_1$ is proportional to a unit matrix. To avoid the pole terms in Eqs. (49) and (50), it is more appropriate to introduce, instead of $\Gamma_1$, the quantity $a_1$ defined by

$$a_1 = \frac{\Gamma_1}{m^2 - M^2}$$

and related to the one-body wave function by $a_1 \bar{\varphi}_\sigma \left( p_1 \right) \varphi_\sigma \left( p \right) = \phi_{1,\sigma\sigma} \left( p_1, p, \omega \tau_1 \right)$.

Substituting $\Gamma_1$ from Eq. (53) into Eqs. (49) and (50) and taking the limit $M \to m$, we get (the fermion spin indices are omitted for shortness)

$$0 = \bar{u} \left( p_1 \right) g \left( 1 - \frac{\phi'}{2\omega^\prime p} \delta m' \right) \left( -\Sigma \right) \Gamma_2 \left( p \right)$$

$$+ \bar{u} \left( p_1 \right) \delta m' \left( 1 - \frac{\phi'}{2\omega^\prime p} \delta m' \right) 2ma_1 u \left( p \right)$$

$$+ \bar{u} \left( p_1 \right) \left( Z \frac{\phi'}{\omega^\prime p} \right) 2ma_1 u \left( p \right),$$

$$\bar{u} \left( k_1 \right) \Gamma_2 u \left( p \right) = \bar{u} \left( k_1 \right) g \left( 1 - \frac{\phi'}{2\omega^\prime p} \delta m' \right) 2ma_1 u \left( p \right)$$

$$+ \bar{u} \left( k_1 \right) g' \left( -2\omega^\prime \right) \left( -\Sigma \right) \Gamma_2 u \left( p \right).$$

From these homogeneous equations we should find $\delta m', Z$, and (up to a common factor) $a_1$ and $\Gamma_2$. In contrast to $\Gamma_1$, the two-body vertex function $\Gamma_2$ has non-trivial spin structure. We represent it in terms of scalar components:

$$\bar{u} \left( k_1 \right) \Gamma_2 u \left( p \right) = b_1 \bar{u} \left( k_1 \right) u \left( p \right) + b_2 \frac{m}{\omega^\prime p} \bar{u} \left( k_1 \right) \varphi u \left( p \right),$$

where $b_{1,2}$ are some constants to be determined. All other possible spin structures are reduced to those listed above. The counting rule which gives the total number $N$ of independent scalar components in the two-body vertex is $N = (2s_1 + 1)(2s_2 + 1)/2 = 2 \times 2/2 = 2$. The structure $\bar{u} \left( p_1 \right) \varphi u \left( p \right)$ for the one-body Fock component is not independent since

$$\bar{u} \left( p_1 \right) \varphi u \left( p \right) = \frac{1}{\tau_1} \bar{u} \left( p_1 \right) \varphi \tau_1 u \left( p \right) = \frac{1}{\tau_1} \bar{u} \left( p_1 \right) \left( \tau_1 - \tau_1 \right) u \left( p \right) = \frac{2\omega^\prime p}{m + M} \bar{u} \left( p_1 \right) u \left( p \right).$$

We can easily eliminate the spinors in Eqs. (54) and (55) by the replacement $\bar{u} \left( k \right) \ldots u \left( l \right) \to \left( k + m \right) \ldots / l + m)$, for any momenta $k$ and $l$. This amounts to multiply both sides by $u \left( k \right)$ to the left and by $\bar{u} \left( l \right)$ to the right, and then sum over spin projections. After that we take the trace of each equation. In order to get the third equation, we multiply the second one by $\delta m' / (\omega^\prime p)$ and calculate the trace again. The results are expressed through scalar products of the four-vectors entering these equations. In the limit $M \to m$, the scalar products are given by

$$pp_1 = m^2, \quad pk_1 = m^2 - \mu^2 / 2 + (1 - x)(s - m^2) / 2,$$

with

$$s = (k_1 + k_2)^2 = (p + \omega \tau_2)^2 = m^2 + 2(\omega^\prime p)\tau_2, \quad x = \frac{\omega^\prime k_1}{\omega^\prime p},$$

where $k_1$ and $k_2$ are the momenta of the two-body system constituents, i.e. the arguments of $\Gamma_2 = \Gamma_2 \left( k_1, k_2, p, \omega \tau_2 \right)$. Note that because of the different conservations laws the fermion momenta $p_1$ (which comes into the one-body Fock component) and $k_1$ (which comes into the two-body Fock component) have different scalar products with $p$. 

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The system of three homogeneous equations for the three unknown constants \( a_1, b_1, \) and \( b_2 \) we thus get is adduced in Appendix C1. The determinant of this system is

\[
\det = -4m^2(m^2 - sx)(1 - x) \left\{ \delta m''^2 - 2(m + g^2 A)\delta m' - g^2[B(g^2 B + 2g^2 C - 2Z) - g^2 A^2] + 2m(g^2 A + g^2 B + g^2 C - Z) \right\}. \tag{59}
\]

In order the system possesses nontrivial solutions, its determinant must be zero. Equating \( \det \) to zero, we obtain a quadratic equation for \( \delta m' \). It has the following solution:

\[
\delta m' = g^2 A + m - \sqrt{(m - g^2 B)[m - g^2 B - 2(g^2 C - Z)]}. \tag{60}
\]

For the other solution (with \(+\sqrt{\ldots}\)) \( \delta m' \) does not tend to zero when \( g \to 0 \). We choose therefore the solution (60). Substituting this \( \delta m' \) into the system of equations (C1)–(C3), we can express \( b_1 \) and \( b_2 \) through \( a_1 \):

\[
b_1 = 2gma_1, \quad b_2 = -gma_1 \left( 1 - \sqrt{1 - \frac{2(g^2 C - Z)}{m - g^2 B}} \right). \tag{61}
\]

We see that both \( b_1 \) and \( b_2 \) do not depend on \( s \) and \( x \). This justifies the extraction of \( \Gamma' \) from the integrals when we derived Eqs. (54) and (55). The solutions (60) and (61) are nonperturbative, since their expansions in powers of the coupling constant \( g \) contain an infinite number of terms. Eqs. (54) and (55) are valid for any \( g \) (for the given Fock space truncation) provided the expressions under the square roots are non-negative.

According to Eqs. (46b) and (56), the components which determine the two-body wave function are obtained by dividing \( b_1 \) and \( b_2 \) by the factor \( 2(\omega p)\tau_2 = (s - m^2) \). As a result, the wave function depends on \( \omega \) through the term

\[
\frac{b_2}{s - m^2} \frac{m}{\omega p} u(k_1) \phi(p).
\]

This dependence must disappear in any physical observable, in particular, in the residue of the wave function at the pole \( s = m^2 \). Since, as follows from the second of Eqs. (61), \( b_2 \) is a constant, the only way to ensure this property is to require \( b_2 = 0 \). From here we find the constant \( Z = g^2 C \) and

\[
\delta m' = g^2(A + B). \tag{62}
\]

Taking into account Eqs. (33) and (39) yields the final solution

\[
b_1 = 2gma_1, \quad b_2 = 0, \tag{63}
\]

\[
\delta m = g^2(A + B + C + C_{f_e}), \quad Z_\omega = g^2(C + C_{f_e}). \tag{64}
\]

The three constants \( a_1, b_1, b_2 \) determine the one- and two-body vertex functions \( \Gamma_1 \) and \( \Gamma_2 \). The components \( \phi_1, \phi_2 \) of the state vector (41) are related to \( \Gamma_1, \Gamma_2 \) by Eqs. (46a) and (46b). Explicit expressions for \( \delta m \) and \( Z_\omega \) are given in Appendix B1.

Substituting the solution for the wave functions into the normalization condition (43), we find the normalization factor \( N \):

\[
N = 4m^2a_1^2 \left\{ 1 + \frac{g^2}{2(2\pi)^3} \int \frac{Tr[(k_1 + m)(\not p + m)]}{(s - m^2)^2} \delta^4(k_1 + k_2 - p - \omega \tau_2)2(\omega p)d\tau_2 \frac{d^3k_1}{2\epsilon k_1} \frac{d^3k_2}{2\epsilon k_2} \right\}. \tag{65}
\]

The integral in Eq. (65) diverges logarithmically. Explicit formulas for \( N \) are given in Appendix B1. Note that since the factor \( \sqrt{N} \) is proportional to \( a_1 \), as well as each of the wave functions, the state vector (41) does not contain the constant \( a_1 \) at all. So, the final solution for the state vector is completely determined by the values of the coupling constant and particle masses.
We remind that we did not renormalize the fermion field by introducing the factor $Z_2$ as in the first of Eqs. (2). The fermion field renormalization is not needed in our approach, because the factor $N$ does the same job for the whole state vector. The two factors $N$ and $Z_2$ are tightly related to each other. Namely, let us define the constant $Z_2$ as

$$Z_2 = \frac{4m^2a_f^2}{N}. \quad (66)$$

It turns out that it is expressed through the self-energy in the standard way:

$$Z_2^{-1} - 1 = -\frac{\partial}{\partial \vec{p}} \Sigma(p) \bigg|_{\vec{p}=m}. \quad (67)$$

Indeed, using the representation (15) of the self-energy, we get

$$\frac{\partial}{\partial \vec{p}} \Sigma(p) \bigg|_{\vec{p}=m} = g^2 \left[ 2m \frac{\partial A(p^2)}{\partial p^2} + 2m \frac{\partial B(p^2)}{\partial p^2} + \frac{1}{m} B(p^2) \right]_{p^2=m^2}. \quad (68)$$

We used the fact (proved in Appendix B1) that $C(p^2)$ does not depend on $p^2$ and that $\frac{\partial}{\partial \vec{p}} \frac{\vec{p}}{m} = -\frac{\vec{p}}{(m^2)} = 0$. Substituting the expressions (B2a) and (B2b) for $A(p^2)$ and $B(p^2)$ into Eq. (68), calculating the corresponding integrals and comparing the result with Eq. (B7) for $N$, we find the relation

$$N \frac{4m^2a_f^2}{N} = 1 - \frac{\partial}{\partial \vec{p}} \Sigma(p) \bigg|_{\vec{p}=m}. \quad (69)$$

From Eqs. (66) and (69) immediately follows Eq. (67). Note that the latter is just a standard expression for the traditional counterterm $Z_2$ (see, e. g., Ref. [18]).

In perturbation theory, taking into account that $\Sigma(p)$ is of order $g^2$, we can transform Eq. (67) to

$$Z_2 = 1 + \frac{\partial}{\partial \vec{p}} \Sigma(p) \bigg|_{\vec{p}=m} = 1 + \frac{1}{2m} \left\{ \bar{u}(p) \frac{\partial M^{(2)}(p)}{\partial \vec{p}} u(p) \right\} \bigg|_{p^2=m^2}, \quad (70)$$

where $M^{(2)}(p)$ is defined by Eq. (14). The last equality follows from the fact that $\partial \Sigma_f(p)/\partial \vec{p} = 0$. The r. h. s. of Eq. (70) coincides with that of Eq. (30) obtained previously in perturbative renormalization technique.

The solution (60), before specifying the value of $Z$, is true nonperturbative, whereas the final solutions (62) and (64) coincide with the perturbative ones. It is not a surprise since the self-consistent calculation in the $|F \rangle + |Fb \rangle$ approximation generates all contributions to $\delta m$ (or $\delta m'$) of order $g^2$, and nothing else. A real difference would appear when higher Fock sectors are taken into account.

The value (62) of $\delta m'$ is in agreement with the solution given in Ref. [14].

V. GAUGE BOSON IN THE FEYNMAN GAUGE

A. Contact terms

We shall follow here very closely the procedure detailed in the previous section. The light-front interaction Hamiltonian for a system of spin 1/2 fermion coupled to gauge bosons is discussed in Appendix A2. It has the general form

$$H^{int}(x) = -g \bar{\psi} \frac{d}{dx} \psi + g^2 \bar{\psi} \frac{d}{dx} \frac{\bar{\psi}}{2i(\omega - \partial)} \phi \psi$$

$$- \delta m' \bar{\psi} \psi + g \delta m' \bar{\psi} \left[ \frac{\phi}{2i(\omega - \partial)} + \frac{\bar{\phi}}{2i(\omega - \partial)} \right] \psi + \delta m'^2 \bar{\psi} \frac{\bar{\psi}}{2i(\omega - \partial)}$$

$$- Z \omega \bar{\psi} \phi \frac{d}{dx} \psi$$

$$+ H'(x). \quad (71)$$
and differs from the Hamiltonian (34) by the replacement \( \varphi \to \tilde{\varphi} \) and by the additional contribution \( H'(x) \) which represents contact terms associated to the gauge field. An explicit form of \( H'(x) \) depends on the gauge. Generally speaking, \( H'(x) \) taken in an arbitrary gauge and expanded in powers of \( g \) gives rise to an infinite number of contact terms, even in truncated Fock space.

For the Feynman gauge, \( H'(x) \) acquires the form

\[
H'_F(x) = H'_{1F}(x) + H'_{2F}(x) + H'_{3F}(x)
\]

with

\[
H'_{1F}(x) = g^2 \bar{\psi} \frac{\tilde{\varphi}}{2i(\omega - D)} \varphi \psi, \\
H'_{2F}(x) = g \delta m' \bar{\psi} \left[ \frac{\varphi \tilde{\varphi}}{2i(\omega - D)} + \frac{\tilde{\varphi} \varphi}{2i(\omega - D)} \right] \psi, \\
H'_{3F}(x) = \delta m'^2 \bar{\psi} \frac{\tilde{\varphi}}{2i(\omega - D)} \psi,
\]

where the Hermitian operator \( \frac{1}{i(\omega - D)} \) is defined in Appendix A 2.a. An expansion of \( H'_F(x) \) in powers of \( g \) starts with a term of order \( g^3 \) (we took into account that the corresponding expansions of \( \delta m' \) and of the operator \( \frac{1}{i(\omega - D)} \) begin with terms of order \( g^2 \) and \( g \), respectively). In full (non-truncated) Fock space this series does not break and the number of contact terms is infinite. However, it becomes finite in truncated Fock space. Since we are working within one- and two-body sectors, we have to know the non-zero matrix elements of \( H'_F(x) \) between these sectors. Let us consider first the term \( H'_{1F}(x) \). We can represent \( H'_{1F}(x) \) as the following series in powers of \( g \):

\[
H'_{1F}(x) = \frac{g^2}{2} \sum_{n_1=0, n_2=0}^{\infty} \frac{(-1)^{n_1}}{n_1! n_2!} g^{n_1 + n_2} H_{n_1 n_2},
\]

where

\[
H_{n_1 n_2} = \bar{\psi} \frac{1}{i \omega - D} \omega \varphi \left( \frac{1}{i \omega - D} \omega \varphi \right)^{n_1} \frac{\tilde{\varphi}}{i \omega - D} \left[ \frac{1}{i \omega - D} \omega \varphi \right]^{n_2} \varphi \psi.
\]

Each term \( H_{n_1 n_2} \) contains \( n_1 + n_2 + 2 \) boson field operators \( \varphi_\nu \). When we calculate matrix elements between states with the fixed number of particles, these operators must be contracted with external boson operators or among themselves. It is easy to see that contractions between two different operators \( \omega \varphi \) or between \( \omega \varphi \) and \( \tilde{\varphi} \) give zero in the Feynman gauge. Indeed, since \( \langle 0 | \varphi_\mu, \varphi_\nu | 0 \rangle \sim -g_{\mu \nu} \), the “internal” contractions between the operators \( \omega \varphi \) in Eq. (75) are proportional to \( \omega \varphi g_{\mu \nu} \), that is zero. Contractions between \( \omega \varphi \) and \( \tilde{\varphi} \) also disappear because they are proportional to \( \gamma^\mu \tilde{\varphi} \omega \varphi g_{\mu \nu} = \tilde{\varphi} \varphi = \omega^2 = 0 \). The only remaining possibility is to contract the operators \( \omega \varphi \) with the external boson field operators. The number of the latters is fixed and does not exceed two in our approximation. Hence, we must have \( n_1 + n_2 \leq 2 \). This just means a truncation of the series (74) to a polynomial of the fourth order in \( g \). Evidently, the same arguments hold true for each of the terms \( H'_{2F}(x) \) and \( H'_{3F}(x) \), and, hence, for the operator \( H'_F(x) \) as a whole. Note that \( H'_F(x) \) does not have non-zero matrix elements within the one-body sector, because it contains at least one operator \( \omega \varphi \).

Considering matrix elements of the operator \( H'_F(x) \) between the states \( |F \rangle \) and \( |F' \rangle \), we find that each piece \( H'_{1F}(x) \), \( H'_{2F}(x) \), or \( H'_{3F}(x) \) produces eight different interaction vertices. For example, in Fig. 10 all possible interaction vertices generated by the piece \( H'_{1F}(x) \) are shown. Each ”elementary” vertex (small black blob) corresponds to the factor \( g \gamma^\mu \), the propagator of boson with four-momentum \( q \) being \( -g_{\mu \nu} \delta(q^2) \theta(q \cdot w) \). The other notations are the same as those adopted for the scalar boson case, except for new factors indicated by large open circles with two fermion and one boson legs. These factors proceed from different powers of the operators \( \frac{1}{i \omega - D} \omega \varphi \) in Eq. (75). A large open circle corresponds to the factor \( \pm g_{\omega}^{k} \), where \( k \) is the four-momentum of the boson line connected with this open circle. The sign plus (minus) is taken for an outgoing (incoming) boson line. Additionally, each matrix element must be multiplied by a combinatorial factor \( 1/(n_1!n_2!) \), where \( n_1 \) (\( n_2 \)) is the number of large open circles standing to the left (to the right) from the large full blob. We have omitted two diagrams which differ from those shown in Figs. 10(g) and (h) by the interchange of the initial and final boson lines because such an interchange does not affect analytical expressions for the corresponding amplitudes.
So, to incorporate the contributions from the omitted diagrams it is enough to multiply the amplitudes of the diagrams in Figs. 10(g), (h) by a factor of two, just cancelling the combinatorial factor 1/2!

As far as the interaction vertices produced by the terms $H_{2F}^{\rho}(x)$ and $H_{3F}^{\rho}(x)$ are concerned, they can be easily obtained from the diagrams of Fig. 10. Indeed, to get the vertices for $H_{2F}^{\rho}(x)$ one should substitute in all these diagrams one of small full blobs by a cross and transform the boson loop to a free boson line incoming to (or outgoing from) the remaining small full blob. In such a way each of the diagrams in Figs. 10(a)–(d) transforms into two new ones, while the diagrams shown in Figs. 10(e)–(h) give nothing because all of them contain after transformation three boson legs. The interaction vertices for $H_{3F}^{\rho}(x)$ are obtained from those in Fig. 10 simply by substituting all small full blobs by crosses and by removing the boson loops.

Note that similarly to the scalar boson case, any line with a large (full or open) blob, or with two of them, is not a propagator, but only a product of the factors corresponding to the elementary vertices (large and small full blobs, and large open circles). For example, the internal solid line in Fig. 10(a) corresponds to the analytical expression

$$ g\gamma^\nu \left[ -\frac{\omega}{2\omega(p-q)} \right] \left( -g\frac{\omega^\rho}{\omega k_1} \right) g\gamma^\mu, \quad (76) $$

where $p = p_1 + k_1 - \omega \tau_1 = p_1' - \omega \tau_1'$ is the total momentum of the initial or final state. To calculate the full matrix element, one should sandwich Eq. (76) with bispinors, then multiply it by the boson propagator $-g_{\mu\nu}\delta(q^2)\theta(\omega q)$ and by the polarization vector $e_\mu^\lambda(k_1)$, integrate over $dq/(2\pi)^3$ and multiply the result by $(-1)$.

It is important to note that although we have calculated matrix elements of $H_{\rho}^{F}(x)$ between the states $|F\rangle$ and $|Fb\rangle$, any of the interaction vertices obtained from $H_{\rho}^{F}(x)$ includes intermediate states with at least two bosons or with one boson and a cross. Repeating the same arguments as those exposed for the scalar boson case, we have thus to conclude that the whole contribution from $H_{\rho}^{F}(x)$ is beyond our two-body approximation.

**B. Equations in the $|F\rangle + |Fb\rangle$ approximation**

The Fock decomposition of the state vector is similar to the one for scalar bosons, given by Eq. (41). The only difference is the dependence of the boson creation operator and the Fock components on the boson spin index $A$. As usual, we define the one- and two-body vertex functions $\Gamma_1$ and $\Gamma_2$ by [cf. with Eqs. (46a) and (46b)]

$$ \bar{u}_{\sigma'}(p_1)\Gamma_1u_{\sigma}(p) = 2(\omega p)\tau_1\phi_{1,\sigma\sigma'}(p_1, p, \omega \tau_1), \quad (77a) $$

$$ \bar{u}_{\sigma'}(k_1)\Gamma_2^\mu u_{\sigma}(p) = 2(\omega p)\tau_2\phi_{2,\sigma\sigma'}(k_1, k_2, p, \omega \tau_2). \quad (77b) $$

We should now establish, in the two-body approximation, the whole set of matrix elements of the Hamiltonian (71) between the one- and two-body sectors. Since the Hamiltonian (71), excepting for the term $H_{\rho}^{F}(x)$, looks quite similar to that for the scalar boson case, Eq. (34), we can make use of this analogy. So, we can take directly the graphs shown in Figs. 3–5, 6(a,b), 7, 8, substituting $g \rightarrow g\gamma^\mu$ everywhere, and add to this set the contributions from $H_{\rho}^{F}(x)$, indicated in Fig. 10, which are compatible with our Fock space truncation. However, as was noticed in the end of Sec. V A, these contributions correspond in fact to many-body sectors and should therefore be disregarded.

For further calculations we will need the fermion self-energy. In the Feynman gauge it is

$$ \Sigma^F = -\frac{g^2}{(2\pi)} \int \frac{\delta(q^\rho)\gamma^\mu(p^\rho - k^\rho + \omega \tau + m)\gamma^\nu \theta(\omega(p-k)) \mu \nu \omega p}{2\omega(p-k)\tau} d^4k \frac{d\tau k}{2\varepsilon_k}, $$

$$ \equiv \gamma^\mu \left( \Sigma^F_{\mu\nu} \right)^\nu = g^2A + g^2B\frac{\phi}{m} + g^2C\frac{m \phi}{\omega p} \quad (78) $$

with $\tau$ defined by Eq. (17). Note that the constants $A$, $B$, and $C$ here do not coincide with those for the scalar boson case, defined by Eq. (52). From Eq. (78) follows

$$ \Sigma^F_{\mu\nu} = -g_{\mu\nu}(g^2A_1 + g^2B_1\frac{\phi}{m} + g^2C_1\frac{m \phi}{\omega p}) \equiv g^2\Sigma^F_{\mu\nu}, \quad (79) $$
As explained in Sec. IV A, we absorbed the (divergent) term proportional to \( \frac{2}{4} \) vector boson wave function has
\[
Z \propto \text{the diagram shown in Fig. 4(c) (with the scalar boson replaced by the gauge one) in the unique counterterm spinor-scalar interaction [cf. with Eq. (53)]:}
\]

where
\[
A = -4A_1, \quad B = 2B_1, \quad C = 2C_1.
\]
The constants \( A, B, \) and \( C \) are given by Eqs. (B12a)–(B12c) in Appendix B2.

We can now write down a system of equations for the vertex functions. The graphical representation of these equations is shown in Fig. 11. It is very similar to the one for scalar bosons, as shown in Fig. 9. The only formal difference consists in the appearance of four-dimensional indices in boson propagators and vertices. Generally speaking, the vertices \( \Gamma_1 \) and \( \Gamma_2^{\nu} \) do have momentum dependence. Because of our Fock space truncation, they are however constants. At the present stage we take this property as an ansatz, and it will be justified by the final results.

In an analytical form, this system of equations reads
\[
\begin{align*}
\bar{u}(p_1)\Gamma_1 u(p) &= \bar{u}(p_1)g\gamma^\nu(-\Sigma_{\nu\rho}^F)\Gamma_2^{\nu} u(p) \\
&+ \bar{u}(p_1)\delta m^I\left(\ \frac{\dot{\phi} + \dot{\phi}_1 + m}{m^2 - M^2}\right)\Gamma_1 u(p) \\
&+ \bar{u}(p_1)\left(Z_{m} \bar{\phi}_p\right) \left(\ \frac{\dot{\phi} + \phi_1 + m}{m^2 - M^2}\right)\Gamma_1 u(p) \\
&+ \bar{u}(p_1)\delta m^I \left(\ \frac{\dot{\phi}}{2\omega_p}\right) \left(\ \frac{\dot{\phi} + \phi_1 + m}{m^2 - M^2}\right)\Gamma_1 u(p),
\end{align*}
\]

\[
\bar{u}(k_1)\Gamma_2^{\nu} e_{\mu}^{\lambda}(k_2) u(p) = \bar{u}(k_1)g\gamma^\mu e_{\mu}^{\lambda}(k_2) \left(\ \frac{\dot{\phi} + \phi_1 + m}{m^2 - M^2}\right)\Gamma_1 u(p) \\
&+ \bar{u}(k_1)g^2 \gamma^\mu e_{\mu}^{\lambda}(k_2) \left(\ \frac{\dot{\phi}}{2\omega_p}\right) \left(\ 2\omega_p\right) \left(\ \frac{\dot{\phi} + \phi_1 + m}{m^2 - M^2}\right)\Gamma_1 u(p). \tag{82}
\]

As explained in Sec. IV A, we absorbed the (divergent) term proportional to \( Z_\omega \) and the contribution from the diagram shown in Fig. 4(c) (with the scalar boson replaced by the gauge one) in the unique counterterm proportional to \( Z \), defined by Eq. (39), where \( C_{Fc} \) is now given by
\[
g^2 C_{Fc} m_{\omega p} = \frac{g^2}{2\pi^3} \int \left(\frac{\gamma^\mu}{2\omega_p}\right) \gamma^\nu d^3 k \left(\frac{1}{2\omega(p - k)}\right).
\]

The constant \( C_{Fc} \) is calculated in Appendix B2 [see Eq. (B12d)].

**C. Solution**

In order to solve Eqs. (81) and (82), it is convenient to decompose the vertex functions in invariant amplitudes, similarly to what was done in Sec. IV. The structure of \( \bar{u}(p_1)\Gamma_1 u(p) \) coincides with the one for the spinor-scalar interaction [cf. with Eq. (53)]:
\[
\frac{\bar{u}(p_1)\Gamma_1 u(p)}{m^2 - M^2} = a_1 \bar{u}(p_1) u(p),
\]
where \( a_1 \) is a constant and the factor \( (m^2 - M^2)^{-1} \) reflects the behaviour of \( \Gamma_1 \) in the limit \( M \rightarrow m \). The number of independent invariant amplitudes for the vertex function \( \bar{u}(k_1)\Gamma_2^{\nu} e_{\mu}^{\lambda}(k_2) u(p) \) coincides with that for the reaction \( 1/2 + 0 \rightarrow 1/2 + 1 \). However, one should take into account that in the Feynman gauge the vector boson wave function has four independent components. So, the total number of invariant amplitudes is \((2 \times 2 \times 4)/2 = 8\). We choose the following set of invariant amplitudes:
After these transformations we take the trace from Eq. (81). Then we multiply the transformed Eq. (82) in the following form:}

\[
\tilde{u}(k_1)\left[\Gamma_2 e^\mu_\mu(k_2)\right]u(p) = \tilde{u}(k_1) \left[ b_1 \gamma^\mu + b_2 \frac{m \omega^\mu}{\omega p} + b_3 \frac{\omega \gamma^\mu}{\omega p} + b_4 \frac{m^2 \omega \gamma^\mu}{(\omega p)^2} + b_5 \frac{\omega_p^\mu}{m} + b_6 \frac{k_1^\mu}{m} + b_7 \frac{m \omega \gamma^\mu}{\omega p} + b_8 \frac{m \omega k_1^\mu}{\omega p} \right] e^\mu_\mu(k_2)u(p). \tag{85}
\]

One can easily check that in our approximation the structures proportional to \(\gamma^\mu\) and \(k_1^\mu\) do not contribute at all. Indeed, whatever term from Eq. (85) we substitute into Eq. (82), the result will always include structures proportional to \(\gamma^\mu, \omega^\mu, \omega \gamma^\mu\), or \(\omega \omega^\mu\) only, i.e. the four structures proportional to \(b_{5-8}\) must be absent. Although all the four coefficients \(b_{5-8}\) can be obtained automatically, as a solution of a system of four linear equations, with the result \(b_5 = b_6 = b_7 = b_8 = 0\), we may set \(b_{5-8} = 0\) from the very beginning in order to simplify the calculations, and reduce Eq. (85) to

\[
\tilde{u}(k_1)\left[\Gamma_2 e^\mu_\mu(k_2)\right]u(p) = \tilde{u}(k_1) \left[ b_1 \gamma^\mu + b_2 \frac{m \omega^\mu}{\omega p} + b_3 \frac{\omega \gamma^\mu}{\omega p} + b_4 \frac{m^2 \omega \gamma^\mu}{(\omega p)^2} \right] e^\mu_\mu(k_2)u(p). \tag{86}
\]

To solve the equations (81) and (82), we repeat the same steps as done in Sec. IV C for scalar bosons. We insert Eqs. (84), (86) into Eqs. (81), (82), take the limit \(M \to m\), and substitute the spinors and polarization vectors according to \(\tilde{u}(p_1) \to (\gamma p_1 + m)\), \(\tilde{u}(k_1) \to (\gamma k_1 + m)\). After these transformations we take the trace from Eq. (81). Then we multiply the transformed Eq. (82) in turn by \(\gamma^\mu, m \omega^\mu/(\omega p), m \gamma^\mu, \omega/(\omega p)\), and \(\omega_p^\mu/m\), and take traces again. As a result, we get a system of five homogeneous linear equations for the five coefficients \(a_1, b_{1-4}\), which is given in Appendix C 2. For finding nontrivial solutions of this system, we have to demand its determinant to be zero:

\[
Det = 4m^4(2m^2 - 4g^2 A)\delta m' - 4g^2(2g^2 B + 2g^2 C_1 - 4g^2 A_1)^2 - 4m(2g^2 A_1 - 2g^2 B_1 - 2g^2 C_1 + Z) = 0. \tag{87}
\]

Solving Eq. (87) with respect to \(\delta m'\), we find two solutions. Evidently, we should choose the solution which gives \(\delta m' \to 0\) as \(g \to 0\). Expressing \(A_1, B_1, \) and \(C_1\) through the constants \(A, B, \) and \(C\) by means of Eqs. (80), we get

\[
\delta m' = -4g^2 A_1 + m - \sqrt{(m - 2g^2 B_1)[m - 2g^2 B_1 - 2(2g^2 C_1 - Z)]} = g^2 A + m - \sqrt{(m - g^2 B)[m - g^2 B - 2(g^2 C - Z)]}. \tag{88}
\]

Substituting Eq. (88) into Eqs. (C4)–(C8), we obtain

\[
b_1 = 2gma_1, \tag{89}
\]

\[
b_2 = 2b_3 = -2gma_1 \left(1 - \sqrt{1 - \frac{2(g^2 C - Z)}{m - g^2 B}}\right), \quad b_4 = 0. \tag{90}
\]

Like to the scalar boson case, we require the residue of the two-body wave function at the pole \(s = m^2\) to be \(\omega\)-independent. This leads to the conditions \(b_2 = b_3 = b_4 = 0\) which can be satisfied if \(Z = g^2 C\). We thus arrive to the following final solution for the vertex functions and the two counterterms:

\[
b_1 = 2gma_1, \quad b_{2-8} = 0, \tag{91}
\]

\[
\delta m = g^2(A + B + C + C_{fc}), \quad Z_\omega = g^2(C + C_{fc}). \tag{92}
\]

Eqs. (92) formally coincide with Eqs. (64) found for the scalar boson case. However the constants \(A, B, C,\) and \(C_{fc}\) theirselves depend on the boson type. Due to this reason the exact values of the counterterms \(\delta m\) and \(Z_\omega\) here are different from those in Eqs. (64). They are given in Appendix B 2.
VI. GAUGE BOSON IN THE LIGHT-CONE GAUGE

A. Contact terms

The interaction Hamiltonian has the same form (71) as in the Feynman gauge, except for the gauge dependent term $H'$. In the LC gauge where $\omega \cdot \tau = 0$ we have (see Appendix A 2 b)

$$H'_{LC}(x) = g^2 (\bar{\psi} \phi \psi) \frac{1}{2i(i\omega \cdot \partial)^2} (\bar{\psi} \phi \psi).$$ (93)

In contrast to all other gauges, this contribution to the light-front Hamiltonian generates only one contact term, of order $g^2$, associated to the propagator of the gauge field.

As far as the non-zero matrix elements of $H'_{LC}(x)$ are concerned, the situation here is simpler than in the case of the Feynman gauge. This part of the Hamiltonian conserves the number of particles in truncated Fock space which we consider here. It generates two interaction vertices in the one-body sector, as shown diagrammatically in Fig. 12.

The fermion contact term with the boson loop, shown in Fig. 4(c) (with the scalar boson replaced by the gauge one) is now

$$g^2 C_{fc} \frac{m \phi}{\omega p} = \frac{g^2}{(2\pi)^3} \int \frac{d\mu \nu \gamma^\mu \gamma^\nu}{2\omega(p-k)^2} \frac{d^3k}{2\varepsilon_k},$$ (94)

where

$$d_{\mu \nu} = -g_{\mu \nu} + \frac{k_\mu \omega_\nu + k_\nu \omega_\mu}{\omega \cdot k}$$ (95)

results from the vector boson propagator in the LC gauge.

The sum of the two (divergent) fermionic loops shown in Fig. 12 represents a contact correction, on the boson line, to the self-energy contribution. It can be written as

$$g^2 C_{bc} \frac{m \phi}{\omega p} \equiv \frac{g^2}{(2\pi)^3} \int \left( \frac{\phi \cdot k \cdot \phi}{2[\omega(p-k)^2]} - \frac{\phi \cdot k \cdot \phi}{2[\omega(p+k)^2]} \right) \frac{d^3k}{2\varepsilon_k}.$$ (96)

The expressions (94) and (96) have the same matrix structure as the one coming from the $\omega$-dependent counterterm proportional to $Z_\omega$. We can therefore again absorb them, as in Eq. (39), into a unique term which, for simplicity, we also call $Z$:

$$Z = Z_\omega - g^2 C_{fc} - g^2 C_{bc}.$$ (97)

B. Equations in the $|F\rangle + |Fb\rangle$ approximation

The contribution (96) should be excluded from the equation for the last Fock component $\Gamma_2$. The reason for this is that $H'_{LC}(x)$ defined by Eq. (93) describes corrections to bosonic propagators. These corrections to $\Gamma_2$ can be regarded as "foot-prints" of contributions from the three-body sector containing one fermion and two bosons, which is outside of our model space.

The fermion self-energy has now the form

$$\Sigma^{LC} = -\frac{g^2}{(2\pi)^3} \int \frac{d\mu \nu \gamma^\mu (\phi \cdot k + \phi \cdot \tau + m) \gamma^\nu}{2\omega(p-k)^2} \frac{d^3k}{2\varepsilon_k}$$

$$\equiv \gamma^\mu \Sigma^{LC}_{\mu \nu} = g^2 A + g^2 B \frac{\phi}{m} + g^2 C_{m} \frac{\phi}{\omega p},$$ (98)

where $d_{\mu \nu}$ and $\tau$ are given by Eqs. (95) and (17), respectively.
The symmetric tensor $d_{\mu\nu}$ is transversal to both $\omega$ and the boson momentum $k$. As a consequence, the operator $\Sigma_{\mu\nu}^{LC}$ in Eq. (98) is a symmetric tensor transversal to $\omega$. Besides that, it contains only zeroth and first powers of $\gamma$-matrices. Due to all these, the tensor $\Sigma_{\mu\nu}^{LC}$ has the following general structure:

$$\Sigma_{\mu\nu}^{LC} = - \left( g_{\mu\nu} - \frac{p_{\mu}\omega_{\nu} + p_{\nu}\omega_{\mu}}{\omega \cdot p} \right) \left( g^{2}A_{1} + g^{2}B_{1} \frac{p^{\mu}}{m} \right) + g^{2}C_{1} \frac{m}{\omega \cdot p} \left( g_{\mu\nu} \gamma_{\mu} \omega_{\nu} - \gamma_{\mu} \omega_{\mu} \right)$$

$$+ g^{2}C_{2} \frac{m}{\omega \cdot p} \left( \frac{\frac{p_{\mu}\omega_{\nu} + p_{\nu}\omega_{\mu}}{\omega \cdot p} - \gamma_{\mu} \omega_{\nu} - \gamma_{\nu} \omega_{\mu}}{(\omega \cdot p)^{2}} \right) \left( g^{2}A_{2} + g^{2}B_{2} \frac{p}{m} + g^{2}C_{2} \frac{\gamma_{\mu}}{\omega \cdot p} \right),$$

(99)
determined by seven scalar functions $A_{1}$, $B_{1}$, $C_{1}$, $A_{2}$, $B_{2}$, and $C_{2}$. As usual, we define $\Sigma_{\mu\nu}^{LC} = g^{2}\Sigma_{\mu\nu}^{LC}$. Multiplying Eq. (99) by $\gamma^{\mu}$ to the left and by $\gamma^{\nu}$ to the right, contracting over the indices $\mu$ and $\nu$, and comparing the result with the r. h. s. of Eq. (98), we get

$$A = -2A_{1}, \quad B = 2B_{1}, \quad C = 2(B_{1} + B_{2} - 5C_{1} - 4C_{1}').$$

(100)
The values of $A$, $B$, and $C$ are given in Appendix B 3 by Eqs. (B16a)–(B16c).

We thus arrive, in the LC gauge, at a system of equations for the vertex functions, which is quite similar to that for the case of gauge bosons in the Feynman gauge, shown graphically in Fig. 11. Because $H_{\gamma}(x)$ does not produce any matrix elements of new structure as compared with the case of the Feynman gauge, we can get the analytical form of the equations for the functions $\Gamma_{1}$ and $\Gamma_{2}$ directly from Eqs. (81) and (82). In the LC gauge the system of corresponding equations reads

$$\bar{u}(p_{1})\Gamma_{1}(p) = \bar{u}(p_{1})g\gamma^{\mu}(-\bar{\Sigma}_{\mu\nu}^{LC})\Gamma_{1}^{\nu}(p) \tag{101}$$

$$+ \bar{u}(p_{1})\delta m_{1} \left( \gamma^{\mu} - \frac{\gamma_{\mu}}{\omega \cdot p} \right) \frac{m^{2} - M^{2}}{m^{2}} \Gamma_{1}(p)$$

$$+ \bar{u}(p_{1}) \left( Z \frac{m}{\omega \cdot p} \right) \left( \gamma^{\mu} - \frac{\gamma_{\mu}}{\omega \cdot p} \right) \frac{m^{2} - M^{2}}{m^{2}} \Gamma_{1}(p)$$

$$+ \bar{u}(p_{1}) g\delta m_{1} \left( \gamma^{\mu} - \frac{\gamma_{\mu}}{\omega \cdot p} \right) \frac{m^{2} - M^{2}}{m^{2}} \Gamma_{1}(p),$$

$$\bar{u}(k_{1})[\Gamma_{2}^{\mu} e_{\mu}^{\lambda}(k_{2})]u(p) = \bar{u}(k_{1})g\gamma^{\mu} e_{\mu}^{\lambda}(k_{2}) \frac{m^{2} - M^{2}}{m^{2}} \Gamma_{1}(p) \tag{102}$$

$$+ \bar{u}(k_{1}) g^{2} \gamma^{\mu} e_{\mu}^{\lambda}(k_{2}) \left( - \frac{\gamma}{\omega \cdot p} \right) \gamma^{\nu}(-\bar{\Sigma}_{\mu\nu}^{LC})\Gamma_{2}^{\nu}(p)$$

$$+ \bar{u}(k_{1}) g\delta m_{1} \gamma^{\mu} e_{\mu}^{\lambda}(k_{2}) \left( - \frac{\gamma_{\mu}}{\omega \cdot p} \right) \frac{m^{2} - M^{2}}{m^{2}} \Gamma_{1}(p).$$

Note however that the polarization vectors $e_{\mu}^{\lambda}(k_{2})$ here differ from those in the system (81), (82).

C. Solution

As above, the structure of the one-body vertex function $\Gamma_{1}$ is given by Eq. (84). The number of independent invariant amplitudes for the vertex function $\bar{u}(k_{1})[\Gamma_{2}^{\mu} e_{\mu}^{\lambda}(k_{2})]u(p)$ can be calculated analogously to the case of the Feynman gauge, taking into account, however, that in the LC gauge the vector boson polarization vector is orthogonal to both $\omega$ and the boson momentum. The conditions $\bar{e}^{\lambda}(k_{2}) \omega = 0$ and $\bar{e}^{\lambda}(k_{2}) k_{2} = 0$ leave two independent components. The total number of invariant amplitudes is thus $(2 \times 2 \times 2) = 4$. We choose the following set of invariant amplitudes:

$$\bar{u}(k_{1})[\Gamma_{2}^{\mu} e_{\mu}^{\lambda}(k_{2})]u(p) = \bar{u}(k_{1}) \left( b_{1} \gamma^{\mu} + b_{2} \frac{m \gamma^{\mu}}{\omega \cdot p} + b_{3} \frac{p^{\mu}}{m} + b_{4} \frac{m \gamma^{\mu}}{\omega \cdot p} \right) e_{\mu}^{\lambda}(k_{2})u(p).$$

(103)
Again, in our approximation, the structures proportional to $p^\mu$ do not arise at all. We can therefore put $b_3 = b_4 = 0$ and reduce Eq. (103) to

$$\bar{u}(k_1)[\Gamma_2^\mu e_\mu(k_2)]u(p) = \bar{u}(k_1) \left( b_1 \gamma^\mu + b_2 \frac{m \gamma^\mu}{\omega - p} \right) e_\mu(k_2)u(p).$$  \hfill (104)

We insert Eqs. (84), (104) into Eqs. (101), (102) and replace, as before, the spinors $\bar{u}(p_1)$, $\bar{u}(k_1)$, $u(p)$ by $(p_1 + m)$, $(k_1 + m)$, $(p + m)$, respectively, and the polarization vectors $e_\mu(k_2)$ by $-g_{\mu\mu'} - (k_{2\mu}\omega_{\mu'} + k_{2\mu'}\omega_{\mu})/(\omega \cdot k_2)$. After that we take the limit $M \to m$ and calculate the trace from Eq. (101). Then we multiply Eq. (102) by $\gamma^\mu$ and by $m \gamma^\mu/(\omega - p)$ successively and take traces again. In calculating the traces we need to know the scalar products $p \cdot k_2$, $k_1 \cdot k_2$, and $\omega \cdot k_2$ in addition to those given by Eq. (57). These scalar products can be easily expressed through the variables $s$ and $x$ [see Eq. (58)] by using the conservation law $k_1 + k_2 = p + \omega \tau$ and the conditions $k_1^2 = p^2 = m^2$, $k_2^2 = 0$, $\omega^2 = 0$. The result is

$$p \cdot k_2 = \frac{x(s - m^2)}{2}, \quad k_1 \cdot k_2 = \frac{s - m^2}{2}, \quad \frac{\omega \cdot k_2}{\omega - p} = 1 - x.$$  \hfill (105)

Finally, we get a system of three homogeneous linear equations for the coefficients $a_1$, $b_{1,2}$. It is given in Appendix C3.

Equating the determinant of this system to zero, we come to a quadratic equation for $\delta m'$:

$$Det = \frac{2m^2(m^2 - sx)(1 + x^2)}{(1 - x)} \left\{ \delta m'^2 + 2(2g^2A_1 - m)\delta m' + 4g^2[g^2A_1^2 - B_1(3g^2B_1 + 2g^2B_2 - 10g^2C_1 - 8g^2C_1' - Z)] - 2m(2g^2A_1 - 4g^2B_1 - 2g^2B_2 + 10g^2C_1 + 8g^2C_1' + Z) \right\} = 0$$  \hfill (106)

with the solution (we choose the minus sign at the square root, as explained above)

$$\delta m' = -2g^2A_1 + m - \sqrt{(m - 2g^2B_1)[m - 2(3g^2B_1 + 2g^2B_2 - 10g^2C_1 - 8g^2C_1' - Z)]},$$

where we used the relations (100) between the coefficients. The substitution of Eq. (107) into Eqs. (C9)–(C11) allows to express the functions $b_{1,2}$ through $a_1$. In our two-body approximation the functions $b_{1,2}$ are constants:

$$b_1 = 2gma_1,$$  \hfill (108)

$$b_2 = gma_1 \left( 1 - \sqrt{1 - \frac{2(2g^2B_1 + 2g^2B_2 - 10g^2C_1 - 8g^2C_1' - Z)}{m - 2g^2B_1}} \right).$$  \hfill (109)

If we impose the condition $Z - g^2C = 0$ on the renormalization constant $Z$ which is still free, the function $b_2$ turns to zero, as it should. Taking into account Eqs. (33) and (97), we find the following final solution:

$$b_1 = 2gma_1, \quad b_{2-4} = 0,$$  \hfill (110)

$$\delta m = g^2(A + B + C + C_{fe} + C_{be}), \quad Z = g^2(C + C_{fe} + C_{be}).$$  \hfill (111)

It formally differs from the corresponding solution (91)–(92) for the case of the Feynman gauge by the presence of the boson contact term $C_{bc}$. Note however that the values of $A$, $B$, $C$, and $C_{fe}$ are different for the two gauges. Explicit expressions for $\delta m$ and $Z_{\omega}$ are given in Appendix B3.

Though the solution for the state vector in the LC gauge, Eqs. (110), formally coincides with that obtained for the Feynman gauge and given by Eqs. (91), the normalization factors of the corresponding state vectors are different. These normalization factors are calculated fully analogously to how it was done for the scalar boson case.
Let us denote the contact term contribution by $C_c$:

$$C_c = \begin{cases} C_{fc} & \text{for scalar boson and for gauge boson in the Feynman gauge}, \\ C_{fc} + C_{bc} & \text{for gauge boson in the LC gauge}. \end{cases}$$ (112)

Then the expressions for $Z_\omega$ and $\delta m$ in terms of $A$, $B$, $C$, $C_c$ obtain the same form for all the cases discussed above:

$$\delta m = g^2(A + B + C + C_c),$$

$$Z_\omega = g^2(C + C_c).$$ (113) (114)

The constants $A$, $B$, $C$, and $C_c$ are however different for different bosons. For the scalar boson case all the constants are given in Appendix B1. For the cases of gauge boson in the Feynman and LC gauges they are given in Appendices B2 and B3, respectively. Substituting their explicit values into Eqs. (113) and (114), we find the analytical formulas for $Z_\omega$ and $\delta m$.

For scalar boson:

$$\delta m^{sc} = -\frac{g^2 m}{8\pi^2} \log\frac{L^2}{m^2},$$

$$Z_\omega^{sc} = -\frac{g^2 m}{32\pi^2} \log\frac{L^2}{m^2}.$$ (115) (116)

For gauge boson in the Feynman gauge:

$$\delta m^F = -\frac{g^2 m}{8\pi^2} \log\frac{L^2}{m^2},$$

$$Z_\omega^F = -\frac{g^2 m}{16\pi^2} \log\frac{L^2}{m^2}.$$ (117) (118)

For gauge boson in the LC gauge:

$$\delta m^{LC} = -\frac{g^2 m}{8\pi^2} \log\frac{L^2}{m^2},$$

$$Z_\omega^{LF} = -\frac{g^2 m}{32\pi^2} \log\frac{L^2}{m^2}.$$ (119) (120)

Let us now discuss what happens with fermion-boson scattering amplitudes calculated in CLFD within the two-body approximation. Although this question, strictly speaking, goes beyond the scope of the present paper, we think that our results are rather instructive and may be useful for future investigations. In Sec. III we considered the perturbative expansion (up to terms of order $g^4$) of such an amplitude for the scalar boson case and showed that, after introducing the counterterm $Z_\omega$, it became $\omega$-independent. Below we perform a similar analysis of the scattering amplitudes without making expansions in powers of $g$.

In the two-body approximation the fermion-boson scattering amplitude $M$ can be obtained by sandwiching the fermion 2PGF, $\mathcal{D}(p)$, with the corresponding bispinors. The 2PGF, in its turn, is easily derived from Eq. (12) by putting there $Z_2 = 1$ and by adding the counterterm (27) to $\delta m$. As a result, we find

$$M = \frac{\Gamma'(\not{p} + m)\Gamma}{p^2 - m^2 + \left[\delta m - \mathcal{M}^{(2)}(p) + Z_\omega \left(\frac{\not{p}}{\not{p} - \omega} - 1\right)\right] (\not{p} + m)} u(p_1),$$ (121)

where $\Gamma = \Gamma' = 1$ for scalar bosons, $\Gamma = \ell(k_1)$, $\Gamma' = \ell(k'_1)$ for gauge bosons, $k_1$ and $k'_1$ are the boson four-momenta before and after scattering. The superscript of the mass operator indicates that we are working in the two-body approximation. Using Eq. (112), we can write $\mathcal{M}^{(2)}(p)$ in the form common for all boson types [cf. with Eq. (18)]:

$$\mathcal{M}^{(2)}(p) = g^2 \left\{ A(p^2) + B(p^2) \frac{\not{p}}{m} + C(p^2) + C_c \frac{\not{p}}{\omega p} \right\}.$$ (122)
The coefficients \( A, B, \) and \( C \) in Eqs. (113) and (114) are, respectively, the values of the functions \( A(p^2), B(p^2), \) and \( C(p^2) \) at \( p^2 = m^2 \). In spite of that Eqs. (121) and (122) have the same form for all boson types, these functions, as well as the constants \( \delta m, C, \) and \( Z_\omega \), are different for different bosons.

Substituting Eq. (122) into Eq. (121) and using Eqs. (113) and (114), we see that the \( \omega \)-dependence of the amplitude is determined by the term

\[
(g^2 [C(p^2) + C_c] - Z_\omega) \frac{m}{\omega p} \frac{\dot{\omega}}{\omega} (p + m) = g^2 \left[ C(p^2) - C(m^2) \right] \frac{m}{\omega p} \frac{\dot{\omega}}{\omega} (p + m)
\]

(123)

in the denominator of Eq. (121). In Appendix B 1 we proved that \( C(p^2) - C(m^2) = 0 \) for the scalar boson case. For the case of gauge boson in the Feynman gauge the situation is completely the same, since the function \( C(p^2) \) differs from that for scalar bosons, given by Eq. (B2c), by a factor of 2 only (see the argumentation in Appendix B 2). So, the quantity (123) equals zero for both scalar bosons and gauge bosons in the Feynman gauge, and the scattering amplitude (121) does not depend on \( \omega \) at all.

The latter statement is not true for the case of the LC gauge. The function \( C(p^2) \) can be obtained by substituting the integral from Eq. (98), without putting \( p^2 = m^2 \), into the last of Eqs. (B1). Technical details are completely equivalent to those exposed in Appendix B 1. Thus, we get for the LC gauge:

\[
C(p^2) = -\frac{1}{16\pi^2 m} \int_0^{L^2} dk_\perp^2 \int_\delta^{1-\delta} dx \frac{k^2_\perp (x^2 - 2x + 2) + m^2 x^2 - p^2 x^2 (1 - x)^2}{x^{2}(1 - x)[k^2_\perp + m^2 x - p^2 x (1 - x)]}
\]

(124)

Subtracting from Eq. (124) the value \( C(m^2) \), we arrive at the expression

\[
C(p^2) - C(m^2) = \frac{(p^2 - m^2)}{16\pi^2 m} \int_0^{L^2} dk_\perp^2 \int_\delta^{1-\delta} dx \frac{k^2_\perp (2x^2 - 3x + 2) + m^2 x^3}{x[k^2_\perp + m^2 x^2][k^2_\perp + m^2 x - p^2 x (1 - x)]}
\]

(125)

The latter integral diverges logarithmically at \( x = 0 \) and can not be zero. From here follows that the scattering amplitude (121) for gauge bosons in the LC gauge, generally speaking, depends on \( \omega \). Such an unpleasant feature is however not specific to LFD, but also takes place in 4DF formalism, as explained in Ref. [19]. This is caused by the bad regularization of the singularity in \( \omega \cdot k \) to which the function \( C(p^2) \) is sensitive.

Now we can summarize the results obtained above.

- LFD requires an additional counterterm in the Hamiltonian, depending on the light front orientation (i.e., on the four-vector \( \omega \)). In the two-body approximation this counterterm in momentum space has the form \( -Z_\omega \frac{m}{\omega p} \). In the nonperturbative calculations presented above it eliminates the \( \omega \)-dependence of the two-body wave function residue at the pole \( s = m^2 \).

- We have found, for the truncation \( |F\rangle + |Fb\rangle \), the true nonperturbative solutions for the mass counterterm \( \delta m \) and the composite fermion state vector for the cases of scalar and gauge bosons. Expanded in a series in powers of the coupling constant \( g \), these solutions contain all degrees of the latter. Besides that, the two-body wave functions have a non-zero spin components explicitly depending on \( \omega \). The additional counterterm, \( Z_\omega \), eliminates these components. After that the solutions (115)–(120) coincide with the perturbative ones, as they should if the \( |F\rangle + |Fb\rangle \) Tamm-Dancoff truncation is used. A real difference between perturbative and nonperturbative results must appear when higher Fock sectors are taken into account. In the latter case one can hardly expect the vertex functions to be constants. Consequently, there is no any reason to demand that all their \( \omega \)-dependent components equal zero identically in order to get an \( \omega \)-independent residue of the two-body wave function at its pole.

- The mass shift \( \delta m \) is gauge invariant, as is seen from the coincidence of Eqs. (117) and (119).

- The mass shift \( \delta m \) does not coincide with the corresponding value found in Ref. [6], since it depends on the choice of cutoffs. Our cutoffs are different from those used in Ref. [6]. We show in Appendix D that our formulas with the cutoffs taken from Ref. [6] reproduce \( \delta m \) obtained there.

- The value of the counterterm \( Z_\omega \) depends on gauge, as expected.

- In spite of the fact that intermediate calculations involve strongly divergent integrals [see, e.g., Eqs. (B4c), (B4d), (B12c), (B12d), (B16c)–(B16e)], the final results for \( \delta m \) and \( Z_\omega \) contain logarithmic divergences only.
We considered a spin 1/2 fermion state consisting of the bare fermion $F$ coupled to bosons $b$ with spin either zero or one, in the approximation where the first two Fock sectors ($|F\rangle + |Fb\rangle$) are kept in the state vector. The spin one bosons are the gauge ones.

Although the state vector of the composite fermion is simplified, the study of this model is of utmost importance in order to settle the various pieces of the light-front Hamiltonian we start from. While the light-front Hamiltonian is simple at the tree level, this is not true anymore when higher order processes are considered. Due to the explicit covariance of our approach, we exhibit in the present study the exact operator structure of the counterterms needed for the renormalization. In the two-body approximation two counterterms are required to renormalize the theory: a mass counterterm and a specific one for LFD, which explicitly depends on the orientation of the light front plane (i.e., on the four-vector $\omega$). We calculated them in an explicit form. While the mass counterterm is analogous to that which appears in standard 4DF approach, the specific LFD counterterm has no such an analogue.

These two counterterms generate additional contributions to the light-front Hamiltonian, due to their coupling with contact interactions. The latter also depend on the light front plane orientation. The solutions we found for the couplings of scalar and gauge bosons to a spin 1/2 fermion are in complete agreement with the CLFD perturbation theory, although our calculation does not rely on perturbative expansions. The condition imposed on the mentioned above specific counterterm eliminates the $\omega$-dependence of the two-body wave function residue at its pole.

We explicitly showed that the mass counterterm $\delta m$ is gauge independent. We calculated the state vector in two gauges — the Feynman and the LC ones. Though finally the number of the two-body wave function spin components turned out to be the same in both gauges, this is a peculiarity of the $|F\rangle + |Fb\rangle$ Fock space truncation. Such a property should disappear when higher Fock components are taken into account. At the same time, physical observables (e.g., the electromagnetic form factors) must be the same in any gauge.

The results reported here are encouraging in the perspective of tackling physical composite systems in QED and QCD. This will be a subject of forthcoming researches.

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APPENDIX A: LIGHT-FRONT HAMILTONIANS

1. Scalar boson

For non-zero spin fields, the light-front Hamiltonian should incorporate the so-called contact interactions. Their origin and derivation are explained, e.g., in Ref. [2] (in the LC gauge, for the gluon field). For completeness, we briefly explain this derivation, in the Feynman gauge too.

We start with the case of scalar boson field. The corresponding field theory Lagrangian, at the tree level, is

$$\mathcal{L} = \frac{i}{2} \left[ \bar{\Psi} \gamma^\nu (\partial_\nu \Psi) - (\partial_\nu \bar{\Psi}) \gamma^\nu \Psi \right] - m \bar{\Psi} \Psi + \frac{1}{2} \left[ \partial_\nu \Phi \partial^\nu \Phi - \mu^2 \Phi^2 \right] + g \bar{\Psi} \Phi \Phi, \quad (A1)$$

where $\Psi$ and $\Phi$ are, respectively, the Heisenberg spinor and scalar field operators. In a standard way we obtain the energy-momentum tensor

$$\Theta^{\nu\rho} = \frac{\partial \bar{\Psi}}{\partial (\partial_\nu \Psi)} \frac{\partial \mathcal{L}}{\partial (\partial_\nu \Psi)} \partial^\rho \Psi + \frac{\partial \mathcal{L}}{\partial (\partial_\nu \Psi)} \partial^\rho \Phi - g^{\nu\rho} \mathcal{L} \quad (A2)$$
and the four-momentum operator

\[ P^\rho = \int d\sigma(x) \Theta^\rho. \] (A3)

Integration in Eq. (A3) is performed on the three-dimensional space element orthogonal to the "time" direction. The role of time is played in CLFD by the invariant combination \( \omega \cdot x \). To simplify the subsequent algebraical manipulations it is convenient to introduce temporarily a particular value\(^4\) of the four-vector \( \omega \), namely, \( \omega = (1, 0, 0, -1) \), corresponding to the usual form of noncovariant LFD on the plane \( x^0 + x^3 = 0 \). In the final results we will return to explicitly covariant notations. This is simply achieved by the replacement of the plus-component of any four-vector \( a \) by the contraction \( \omega \cdot a \).

Let us introduce also the light-front coordinates

\[ x^+ = x^0 + x^3, \quad x^- = x^0 - x^3, \quad x^\perp = \{x^1, x^2\}. \] (A4)

The scalar product is defined as

\[ a_\nu b^\nu = \frac{1}{2} a^+ b^- + \frac{1}{2} a^- b^+ - a^\perp \cdot b^\perp. \] (A5)

The metric tensor is

\[ g^{+ -} = g^{- +} = 2, \quad g_{+ -} = g_{- +} = \frac{1}{2}, \quad g^{11} = g^{22} = g_{11} = g_{22} = -1, \] (A6)

all the other components being zeroes. In such a formalism the \( P^+ \) and \( P^\perp \) components of the momentum operator are free, while the \( P^- \) component contains interaction. \( P^- \) is given by the expression

\[ P^- = \frac{1}{2} \int d^2 x^\perp dx^- \Theta^{+ -}. \] (A7)

From Eqs. (A2) and (A3) with the Lagrangian (A1) we get

\[ P^- = \frac{1}{2} \int d^2 x^\perp dx^- \left[ -i \bar{\Psi} \gamma^- \partial^+ \Psi + 2i \bar{\Psi} \gamma^\perp \cdot \partial^+ \Psi + 2m \bar{\Psi} \Psi + (\partial^+ \Phi)^2 + \mu^2 \Phi^2 - 2g \bar{\Psi} \Psi \Phi \right]. \] (A8)

The time evolution of the operators \( \Psi \) and \( \Phi \) is governed by the equations of motion obtained from the Lagrangian (A1). Thus we have the following Dirac-type equation for \( \Psi \):

\[ (i \partial - m) \Psi = -g \Phi \Psi. \] (A9)

The spinor field \( \Psi \) has four components. However, it turns out that Eq. (A9) in the light-front coordinates results in a time-dependent equation only for two components among the four. For the other two components, it gives a time-independent constraint. The elimination of these two components from the Hamiltonian, by using this constraint, generates an extra term in the Hamiltonian, which is just the contact interaction.

To demonstrate this in more detail, let us split the four-component spinor \( \Psi \) into two two-component pieces, \( \Psi^{(\pm)} \) and \( \Psi^{(-)} \), defined as

\[ \Psi^{(\pm)} = \Lambda^{(\pm)} \Psi, \] (A10)

where \( \Lambda^{(\pm)} = \frac{1}{4} \gamma^\pm \gamma^\pm \) with \( \gamma^\pm = \gamma^0 \pm \gamma^3 \) are projection operators with the properties

\[ \Lambda^{(\pm)} = \Lambda^{(\pm)}, \quad \Lambda^{(\pm)} \Lambda^{(-)} = \Lambda^{(-)} \Lambda^{(\pm)} = 0, \quad \Lambda^{(\pm)^\dagger} = \Lambda^{(\pm)}. \]

\(^4\)In principle, one can carry out the derivation without specifying the particular four-vector \( \omega = (1, 0, 0, -1) \), but introducing four orthogonal four-vectors, one of which is \( \omega \).
\[ \Lambda^{(\pm)} \gamma^0 = \gamma^0 \Lambda^{(\mp)}, \quad \Lambda^{(\pm)} \gamma^\perp = \gamma^\perp \Lambda^{(\mp)}. \]

The equation of motion (A9) also splits into two equations
\[ i \partial^- \Psi^+ - (i \alpha^\perp \cdot \partial^\perp + m \gamma^0 - g \Phi \gamma^0) \Psi^- = 0, \]  
\[ i \partial^+ \Psi^- - (i \alpha^\perp \cdot \partial^\perp + m \gamma^0 - g \Phi \gamma^0) \Psi^+ = 0. \]  

It is important that Eq. (A12) does not contain the "time" derivative \( \partial^- \equiv 2 \frac{\partial}{\partial \tau} \) and, hence, represents a constraint connecting the components \( \Psi^+ \) and \( \Psi^- \) at any "time":
\[ \Psi^- = \frac{1}{i \partial^+} (i \alpha^\perp \cdot \partial^\perp + m \gamma^0 - g \Phi \gamma^0) \Psi^+. \]  

The Hermitian integral operator \( \frac{1}{i \partial^+} \) acts on a function \( f \) according to the following rule:
\[ \frac{1}{i \partial^+} f(x^-) = - \frac{i}{4} \int dy^- \epsilon(x^- - y^-) f(y^-), \]  
where \( \epsilon(x) = 1 \) for \( x > 0 \) and \( \epsilon(x) = -1 \) for \( x < 0 \). In momentum space it corresponds to the division by \( p^+ \) (or by \( \omega p \)), where \( p \) is the four-momentum conjugated to the coordinate \( x \).

We represent the state vector in Fock space formed by the action of the usual (free) creation operators on the vacuum. One should also express the Hamiltonian in terms of these free operators, i.e., the operators in Schrödinger representation. The momentum operator \( P^- \) in Schrödinger representation has the same form as \( \mathcal{P}^- \), changing Heisenberg field operators to Schrödinger ones. The Heisenberg operators \( \Psi^+ \) and \( \Phi \) are connected with the corresponding Schrödinger operators \( \psi^+ \) and \( \varphi \) by the relations
\[ \psi^+ = e^{\frac{i}{2} \partial^- x^+} \psi^+ e^{-\frac{i}{2} \partial^- x^+}, \quad \Phi = e^{\frac{i}{2} \partial^- x^+} \varphi e^{-\frac{i}{2} \partial^- x^+}, \]  
where it is supposed that at \( x^+ = 0 \) the Heisenberg and Schrödinger operators coincide. As it can be seen from Eq. (A15), the possibility to express the momentum operator through the free fields is essentially based on the existence of a "time" moment (chosen to be \( x^+ = 0 \)), when all field components entering this operator are free simultaneously. However, in CLFD the constraint (A13) connecting the spinor field components \( \Psi^+ \) and \( \Psi^- \) at any time moment involves interaction terms. By this reason in order to construct the momentum operator one must express the latter through independent field components only and after that substitute the Heisenberg field operators by the Schrödinger ones. In the case considered above the independent components are \( \Psi^+ \) and \( \Phi \).

So, changing in Eq. (A8) \( \Psi^+ \) by \( \psi^+ \) and \( \Phi \) by \( \varphi \), denoting
\[ \psi = \left[ 1 + \frac{i \alpha^\perp \cdot \partial^\perp + \gamma^0 m}{i \partial^+} \right] \psi^+, \]  
where \( \psi \) satisfies the free Dirac equation, and returning to the covariant notations, we find for the interaction part of the four-momentum:
\[ P^\text{int}_\rho = \omega_\rho \int d^4x \left[ -g \bar{\psi} \gamma^\perp \varphi + g^2 \bar{\psi} \gamma^\perp \varphi \frac{\varphi}{2i(\omega \cdot \partial)} \right]. \]  

The integrand in Eq. (A16) is nothing else than the interaction CLFD Hamiltonian.

To get the complete Hamiltonian incorporating the mass renormalization and the specific CLFD counter-term discussed in Sec. III [see Eq. (27)] we should add to Eq. (A1) the following additional terms:
\[ \delta m \bar{\Psi} \Psi + Z_\omega \bar{\Psi} \left( \frac{m \varphi}{\omega \cdot \partial} - 1 \right) \Psi \equiv \delta m' \bar{\Psi} \Psi + Z_\omega \bar{\Psi} \frac{m \varphi}{\omega \cdot \partial} \Psi, \]  
where \( \delta m' \) is defined by Eq. (33). It is easy to see that the counterterm with \( Z_\omega \) in Eq. (A17) just generates Eq. (27) in momentum representation.
A question may arise whether it makes sense to add the counterterm with \( Z_\omega \), explicitly depending on the light-front position, to the Lagrangian. On the one hand, the Lagrangian as a fundamental quantity describing the physical system considered may "know" nothing about CLFD and truncated Fock space. On the other hand, we can not neglect the fact that the structure of counterterms substantially depends on the method of calculations. Since CLFD is based on Hamiltonian dynamics, and the Hamiltonian is derived from the Lagrangian, we can incorporate necessary counterterms, formally including the latters directly in the initial Lagrangian. At this level such a procedure looks like a formal trick allowing to simplify the construction of the Hamiltonian.

After adding the counterterms (A17) we need not repeat the derivation of the Hamiltonian from the very beginning. In a first step, we put \( Z_\omega = 0 \) on the r. h. s. of Eq. (A17) and take into account the term with \( \delta m' \) only. The additional term \( \delta m' \bar{\Psi}(\omega) \Psi \) in the Lagrangian changes the equation of motion (A9) to

\[
(i \not\! \partial - m)\Psi = -(g\Phi + \delta m')\Psi. \tag{A18}
\]

Note that Eq. (A18) can be obtained from Eq. (A9) by the simple substitution \( \Phi \to \Phi + \delta m'/g \). Hence, making an analogous substitution \( \varphi \to \varphi + \delta m'/g \) in Eq. (A16), we find the mass operator including mass renormalization effects. In a second step, we add the counterterm \( Z_\omega \bar{\Psi}[m \not\! \partial/(\omega \not\! \partial)]\Psi \) to the Lagrangian. Since \( \bar{\Psi} \not\! \partial \Psi = 2\bar{\Psi}^{(+)}\Psi^{(+)} \), this counterterm involves the \( \Psi^{(+)} \) components only. By this reason, it does not affect the constraint (A13) and comes into the Hamiltonian as an addendum, without changing its initial form.

Summarizing, we conclude that the final momentum operator including the whole set of counterterms can be obtained from Eq. (A16) by changing \( \varphi \to \varphi + \delta m'/g \) and by adding to the resulting integrand the quantity \( -Z_\omega \bar{\Psi}[m \not\! \partial/(\omega \not\! \partial)]\Psi \). After simple algebraic transformations we get the regularized momentum operator

\[
P^\text{int}_\rho = \omega \rho \int d^4x \delta(\omega x) H^\text{int}(x), \tag{A19}
\]

with \( H^\text{int}(x) \) given by Eq. (34). Except for the "normal" interaction \(-g\bar{\psi}\gamma^\mu \varphi\), all other terms in the Hamiltonian (34) are contact ones.

2. Vector boson

The Lagrangian of a system of interacting spinor \( \Psi \) and massless vector \( \Phi^\mu \) fields is, also at the tree level,

\[
\mathcal{L} = \frac{i}{2} \left[ \bar{\Psi} \gamma^\mu (\partial_\mu \Psi) - (\partial_\mu \bar{\Psi}) \gamma^\mu \Psi \right] - m \bar{\Psi} \Psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{\lambda}{2} (\not\! \partial \cdot \Phi)^2 + g \bar{\Psi} \Phi \Psi, \tag{A20}
\]

where \( F^{\mu\nu} = \partial^\mu \Phi^\nu - \partial^\nu \Phi^\mu \). We have added to the Lagrangian a term proportional to the square of the vector field divergence in order to have a possibility to choose different gauges. Due to the gauge invariance all physical results must be independent of the choice of \( \lambda \).

The equations of motion read:

\[
(i \not\! \partial - m)\Psi = -g \Phi \Psi, \tag{A21}
\]

\[
\partial^2 \Phi^\nu - (1 - \lambda) \partial^\nu (\partial \cdot \Phi) = -g \bar{\Psi} \gamma^\nu \Psi, \tag{A22}
\]

where \( \partial^2 \equiv \partial \cdot \partial = \partial^+ \partial^- - \partial^\bot^2 \). The momentum operator acquires the form

\[
P^- = \frac{1}{2} \int d^2x^+ dx^- \left\{ -i \bar{\Psi} \gamma^\pm \partial^\pm \Psi + 2i \bar{\Psi} \gamma^\bot \cdot \partial^\bot \Psi + 2m \bar{\Psi} \Psi + \frac{1 - \lambda}{4} \left[ (\partial^+ \Phi^\pm)^2 - (\partial^\mp \Phi^-)^2 \right] + \lambda (\partial^\bot \cdot \Phi^\bot)^2 + [\partial^\bot \times \Phi^\bot]^2 \right. \\
\left. - (\partial^\bot \Phi^-) \cdot (\partial^\bot \Phi^+) + (1 - \lambda)(\partial^\bot \Phi^-) \cdot (\partial^\bot \Phi^+) - 2g \bar{\Psi} \Phi \Psi \right\}. \tag{A23}
\]
Analogously to the scalar boson case, only two spinor components coming into \( \Psi^+ \) are independent, \( \Psi^- \) being expressed through \( \Psi^+ \). To demonstrate it we split, as before, the spinor \( \Psi \) into \( \Psi^+ \) and \( \Psi^- \). Instead of Eq. (A12) we now have

\[
(i\partial^+ + g\Phi^+)\Psi^- - (i\alpha^\perp \cdot \partial^\perp + m\gamma^0 - g\gamma^0 \Lambda^+ \Phi)\Psi^+ = 0.
\]  

(A24)

Note that since \( \Lambda^+ \Phi \Psi^+ = -\gamma^\perp \cdot \Phi^+ \Psi^+ \), the \( \Phi^- \) component does not contribute to Eq. (A24) at all. The solution of this equation is

\[
\Psi^- = \Psi_0^- + \gamma^\perp \gamma^0 \Lambda^+ \Phi \Psi^+,
\]

(A25)

\[
\Psi_0^- = \frac{1}{i\partial^+} (i\alpha_\perp \cdot \partial^\perp + m\gamma^0)\Psi^+,
\]

(A26)

\[
\gamma^\perp = -\frac{g}{i\partial^+} \gamma^0 \Lambda^+ \Phi \Psi^+,
\]

(A27)

\[
\gamma^\perp \gamma^0 \Lambda^+ \Phi \Psi^+ = \frac{1}{i\partial^+} (i\alpha_\perp \cdot \partial^\perp + m\gamma^0 - g\gamma^0 \Lambda^+ \Phi)\Psi^+,
\]

where the Hermitian operator \( \frac{1}{i\partial^+} \) is defined as

\[
\frac{1}{i\partial^+} f(x^-) = \frac{i}{4} \int dy^- \epsilon(x^- - y^-) \left[ \exp \left( \frac{ig}{2} \int_{y^-}^{x^-} \Phi^+ (z^-) dz^- \right) - 1 \right] f(y^-).
\]

(A28)

The operator \( \frac{1}{i\partial^+} \) is constructed so that it turns to zero for any \( f(x^-) \) if \( g = 0 \) or \( \Phi^+ = 0 \).

As far as the vector field \( \Phi^\nu \) is concerned, the situation differs strongly from that for scalar bosons. Indeed, the vector field has not one, but four components satisfying Eq. (A22). In principle, some restrictions can be imposed on these components, reducing the number of independent ones (this just corresponds to a certain choice of gauge). Besides that, the momentum operator (A23) depends on the first order "time" derivative \( \partial^- \Phi^+ \), while Eq. (A8) does not depend on \( \partial^- \Phi^+ \). When constructing the momentum operator in Schrödinger representation, we can not merely substitute \( \partial^- \Phi^+ \) by the corresponding free field derivative \( \partial^- \varphi^+ \). The reason for this is that \( \partial^- \Phi^+ \) is not an independent quantity, because Eq. (A22) taken for \( \nu = + \) allows to express \( \partial^- \Phi^+ \) through the other field components and their "spatial" derivatives:

\[
\partial^- \Phi^+ = \left( \frac{2}{1 + \lambda} \right) \left[ \frac{1}{\partial^+} (\partial^{+2} \Phi^+ - 2g\Psi^{(+1)} \Psi^{(+1)}) + \left( \frac{1 + \lambda}{2} \right) (\partial^+ \Phi^- - 2\partial^\perp \cdot \Phi^+) \right].
\]

(A29)

It is important that this connection includes interaction terms.

Applying the operator \( \partial^- \) to the both sides of Eq. (A22) gives

\[
\lambda \partial^2 (\partial \cdot \Phi) = 0.
\]

(A30)

In deriving Eq. (A30) we took into account the current conservation, \( \partial^- (\Psi^\nu \partial^\nu \Psi) = 0 \), which, in its turn, follows from Eq. (A21). In spite of that, Eq. (A30) contains vector field components only and cannot be regarded as a constraint because it involves "time" derivatives of the second order.

a. Feynman gauge

In the Feynman gauge all the four vector field components are treated as being independent. The light-front free vector boson propagator is \( -g_{\mu\nu} \delta(q^2) \theta(\omega q) \). To meet these requirements one should put \( \lambda = 1 \). Indeed, consider the solution of Eq. (A22) corresponding to a free boson (i.e. at \( g = 0 \)). This solution is a superposition of plane waves \( \varphi^\nu(q)e^{-iqx} \) with \( q^2 = 0 \). Substituting it into Eq. (A22), one gets \( (1 - \lambda)q^\nu [q \cdot \varphi(q)] = 0 \). If \( q \cdot \varphi(q) = 0 \), only three field components are independent, but not four. So, we are forced to put \( \lambda = 1 \). Under this condition, the momentum operator (A23) does not depend on \( \partial^- \Phi^+ \).
Since in the Feynman gauge the independent fields are $\Psi^{(+)}$ and $\Phi^\nu$, the subsequent analysis is quite similar to that made for the scalar boson case. It leads to the following expression for the interaction part of the momentum operator in Schrödinger representation:

$$ F_{\rho}^{\text{int}} = \omega_{\rho} \int d^4 x \delta(\omega x) H_{\rho}^{\text{int}}(x) $$

(A31)

with the interaction Hamiltonian

$$ H_{\rho}^{\text{int}}(x) = -g \bar{\psi} \gamma_{\rho} \gamma_{\mu} \psi + g^2 \bar{\psi} \left( \frac{\phi}{2i(\omega \cdot \partial)} \right) \phi \psi + g^2 \bar{\psi} \left( \frac{\phi}{2i(\omega \cdot D)} \right) \phi \psi. $$

(A32)

The operator $1/(\omega \cdot D)$ is given by the r. h. s. of Eq. (A28) after the replacement $\Phi^+ \rightarrow \varphi^+ \equiv \omega \cdot \varphi$. In an explicitly covariant form it can be rewritten as

$$ \frac{1}{i(\omega \cdot D)} f = \left[ R^* \frac{1}{i(\omega \cdot D)} R - \frac{1}{i(\omega \cdot D)} \right] f, $$

(A33)

where $R = \exp \left[ \frac{\omega}{i(\omega \cdot D)} (\omega \cdot \varphi) \right]$. Note that the operator $\frac{\omega}{i(\omega \cdot D)}$ inside the exponent acts on $\omega \varphi$ only, while the other such operators in Eq. (A33) act on all functions standing to the right of them.

The second term in Eq. (A32) is a fermion contact term. In the Feynman gauge, to order $g^2$, contact terms related to the massless vector field are absent. The decomposition of the last term on the r. h. s. of Eq. (A32) starts with $g^2$ and contains an infinite number of terms. The matrix element of a term of arbitrary order contains the contractions of the vector field operators entering Eq. (A32) between themselves and with the external fields. As is shown in Sec. V A, in the Feynman gauge the internal contractions disappear since they are always proportional to $\omega^2 = 0$. The contractions with external fields only survive. Therefore, in truncated Fock space, the number of contact terms is finite and the Hamiltonian (A32) has polynomial dependence on the coupling constant and field operators. We emphasize that this property is a merit of the Feynman gauge, while another choice of gauge may lead to appearance of an infinite number of contact terms even for truncated Fock space.

Incorporating the self-interaction $\delta m' \bar{\Psi} \Psi$ by means of the substitution $\phi \rightarrow \phi + \delta m'/g$ [with $\delta m'$ given by Eq. (33)] and adding the counterterm $-Z_{\omega} \bar{\psi} [m \delta m'/i(\omega \cdot D)] \psi$, we obtain the Hamiltonian (71) with $H'(x)$ given by Eqs. (72)–(73c).

b. Light-cone gauge

Maximal simplifications are obtained in the LC gauge where $\Phi^+ = 0$ (equivalent to $\omega \cdot \Phi = 0$ in CLFD). In this gauge the bosonic propagator $-[g_{\mu \nu} - (q_{\mu} \omega_{\nu} + q_{\nu} \omega_{\mu})/(\omega \cdot q)] \delta(q^2) \delta(\omega \cdot q)$ is transversal to $\omega$ in both indices. Note that expressions for vector meson propagators in the LC gauge can be found in Refs. [2,3].

For $\Phi^+ = 0$, we get from Eq. (A29):

$$ \Phi^- = \frac{2}{\partial^+} (\theta^+ \cdot \Phi^\perp) + \frac{4g}{1 - \lambda} \frac{1}{\partial^+} \Psi^{(+)^\dagger} \Psi^{(+)}. $$

(A34)

This is a constraint indicating that only two transversal components $\Phi^\perp$ among four initial ones, $\Phi^\nu$, are independent. Under these requirements the vector field, generally speaking, does not satisfy the Lorentz condition:

$$ \partial \cdot \Phi = \frac{1}{2} \partial^+ \Phi^- - \partial^\perp \Phi^\perp = \frac{2g}{1 - \lambda} \frac{1}{\partial^+} \Psi^{(+)^\dagger} \Psi^{(+)} \neq 0. $$

(A35)

The relation (A30) now reads:

$$ \frac{2g\lambda}{1 - \lambda} \partial^2 \left[ \frac{1}{\partial^+} \Psi^{(+)^\dagger} \Psi^{(+)} \right] = 0. $$

(A36)

To meet the last equation one has to put $\lambda = 0$. 

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Now we can take Eq. (A23) and set \( \lambda = 0 \), \( \Phi^+ = 0 \). Then, using Eq. (A34) we get the momentum operator expressed through the independent fields \( \Psi^+ \), \( \Phi^⊥ \). In Schrödinger representation we obtain

\[
\hat{P}^{\text{int}}_\rho = \omega_\rho \int d^4x \delta(\omega x) H^{\text{int}}_{LC}(x), \tag{A37}
\]

\[
H^{\text{int}}_{LC}(x) = -g\bar{\psi} \gamma^\mu \psi + g^2\bar{\psi} \gamma^\mu \frac{\partial}{\partial \omega} \bar{\psi} \gamma^\mu \psi + \bar{\psi} \gamma^\mu \psi \frac{1}{2(i\omega - \partial)^2}(\bar{\psi} \gamma^\mu \psi). \tag{A38}
\]

The first term in Eq. (A38) is the standard spinor-vector interaction Hamiltonian, while the second and the third ones are the contact terms. Adding to this Hamiltonian all necessary counterterms, one gets Eq. (71) with \( H'(x) \) given by Eq. (93).

**APPENDIX B: CALCULATION OF COUNTERTERMS**

1. Scalar boson

Explicit calculation of the fermion self-energy (15) is similar to the one given in Ref. [8]. We will also calculate the contribution (16) of the fermion contact term. From Eq. (15) we can express the functions \( A(p^2) \), \( B(p^2) \), and \( C(p^2) \) through \( \Sigma(p) \):

\[
g^2A(p^2) = \frac{1}{4}\text{Tr}[\Sigma(p)], \quad g^2B(p^2) = \frac{m}{4\omega p}\text{Tr}[\Sigma(p) \cdot \partial], \quad g^2C(p^2) = \frac{1}{4m}\text{Tr}\left[\Sigma(p) \left( \bar{\psi} - \frac{p^2 \gamma^\mu \partial}{\omega p} \right) \right]. \tag{B1}
\]

For our purposes it is enough to know these function in the region \( p^2 > 0 \) only.

We introduce the variables \( k^+ = k_0 + k_z \), \( k_\perp = (k_x, k_y) \) and \( x = k^+/p^+ \). For convenience, we will carry out calculations in the frame where \( \mathbf{p} = 0 \) and \( \omega = (1,0,0,-1) \). Hence,

\[
x = \frac{k^+}{\sqrt{p^2}}, \quad k_z = \frac{1}{2}k^+ - \frac{k_\perp^2 + \mu^2}{2k^+}, \quad k_0 = k^+ - k_z = \frac{1}{2}k^+ + \frac{k_\perp^2 + \mu^2}{2k^+}.
\]

Substituting the integral (15) for \( \Sigma(p) \) in Eq. (B1) [and similarly for Eq. (16)], we get

\[
A(p^2) = -\frac{m}{16\pi^2} \int_0^L dk_\perp^2 \int_0^1 dx \frac{1}{k_\perp^2 + m^2x - p^2x(1-x) + \mu^2(1-x)}; \tag{B2a}
\]

\[
B(p^2) = -\frac{m}{16\pi^2} \int_0^L dk_\perp^2 \int_0^1 dx \frac{1-x}{k_\perp^2 + m^2x - p^2x(1-x) + \mu^2(1-x)}; \tag{B2b}
\]

\[
C(p^2) = -\frac{1}{32\pi^2m} \int_0^L dk_\perp^2 \int_0^{1-\delta} dx \frac{k_\perp^2 + m^2 - p^2(1-x)^2}{(1-x)[k_\perp^2 + m^2x - p^2x(1-x) + \mu^2(1-x)]}; \tag{B2c}
\]

\[
C_{fc} = -\frac{1}{32\pi^2m} \int_0^L dk_\perp^2 \left[ \int_0^{1-\delta} dx \frac{1}{x(1-x)} + \int_{1+\delta}^{x(1-x)} \right]. \tag{B2d}
\]

We introduced in Eqs. (B2a)–(B2d) two cutoffs \( L \) and \( \delta \) in the variables \( k_\perp^2 \) and \( x \), respectively, which restrict the integration region as

\[
\delta < x < 1 - \delta, \quad 1 + \delta < x < +\infty, \quad k_\perp^2 < L^2.
\]

In the following it is implied that \( L \to \infty \) and \( \delta \to 0 \). The two cutoffs do not constrain the integration in a spherically symmetric domain and, in principle, violate rotational invariance. We will see that \( \delta m \) depends on the cutoff \( L \) logarithmically and does not depend on \( \delta \).

In order to calculate the constants \( A, B, \) and \( C \) we use Eqs. (B2a)–(B2c) at \( p^2 = m^2 \). We thus find
The integrand has no singularities at $x_0$ or $x = 1$. Integrating over $x$ results in zero for arbitrary $p^2$:

$$\Delta C(p^2) = 0.$$  \hfill (B11)
2. Gauge boson in the Feynman gauge

The fermion self-energy with the gauge boson propagator in the Feynman gauge is given by Eq. (78). From this equation follows that the quantity \( \Sigma^{F}_{\mu\nu} \) differs from the on-mass-shell self-energy for the scalar boson case, \( \Sigma \), defined by Eqs. (52) and (15), by the factor \(-g_{\mu\nu}\) and by that the gauge boson mass is zero. Hence, the coefficients \( A_{1} \), \( B_{1} \), and \( C_{1} \) in Eq. (79) can be obtained directly from Eqs. (B4a)–(B4c) by setting \( \mu = 0 \), that does not make any influence on their asymptotics at \( L \to \infty \). Then, using the relations (80), we can easily find the coefficients \( A \), \( B \), and \( C \) determining the self-energy (78) in the Feynman gauge. Therefore without repeating our previous calculations we obtain

\[
A = \frac{m}{4\pi^{2}} \log \frac{L^{2}}{m^{2}}, \quad (B12a) \\
B = -\frac{m}{16\pi^{2}} \log \frac{L^{2}}{m^{2}}, \quad (B12b) \\
C = \frac{L^{2} \log \delta}{16\pi^{2}m} - \frac{m}{16\pi^{2}} \log \frac{L^{2}}{m^{2}}, \quad (B12c) \\
C_{fc} = \frac{L^{2} \log \delta}{16\pi^{2}m}. \quad (B12d)
\]

This gives

\[
Z_{\omega} = g^{2}(C + C_{fc}) = -\frac{g^{2}m}{16\pi^{2}} \log \frac{L^{2}}{m^{2}}, \quad (B13) \\
\delta m = g^{2}(A + B + C + C_{fc}) = \frac{g^{2}m}{8\pi^{2}} \log \frac{L^{2}}{m^{2}}. \quad (B14)
\]

Like in the scalar boson case, the terms quadratically divergent in \( L^{2} \) and depending on the cutoff \( \delta \) cancel in the sum \( C + C_{fc} \).

3. Gauge boson in the Light-Cone gauge

The self-energy in the LC gauge is given by Eq. (98). Doing the same as in deriving Eqs. (B3a)–(B3c), we obtain for the coefficients \( A \), \( B \), \( C \) [and similarly for the contributions from the fermion and boson contact terms \( C_{fc} \) and \( C_{bc} \) defined by Eqs. (94) and (96), respectively]:

\[
A = \frac{m}{8\pi^{2}} \int_{0}^{L^{2}} dk_{\perp}^{2} \int_{0}^{1} dx \frac{1}{k_{\perp}^{2} + m^{2}x^{2}}, \quad (B15a) \\
B = -\frac{m}{8\pi^{2}} \int_{0}^{L^{2}} dk_{\perp}^{2} \int_{0}^{1} dx \frac{1 - x}{k_{\perp}^{2} + m^{2}x^{2}}, \quad (B15b) \\
C = -\frac{1}{16\pi^{2}m} \int_{0}^{L^{2}} dk_{\perp}^{2} \int_{0}^{1-\frac{1}{\delta}} dx \frac{k_{\perp}^{2}(x^{2} - 2x + 2) + m^{2}x^{3}(2 - x)}{x^{2}(1 - x)(k_{\perp}^{2} + m^{2}x^{2})}, \quad (B15c) \\
C_{fc} = \frac{1}{16\pi^{2}m} \int_{0}^{L^{2}} dk_{\perp}^{2} \left[ \int_{0}^{1-\frac{1}{\delta}} dx \frac{1}{x(1 - x)} + \int_{1+\frac{1}{\delta}}^{\infty} dx \frac{1}{x(1 - x)} \right], \quad (B15d) \\
C_{bc} = \frac{1}{16\pi^{2}m} \int_{0}^{L^{2}} dk_{\perp}^{2} \left[ \int_{0}^{1-\frac{1}{\delta}} dx \frac{1}{(1 - x)^{2}} + \int_{1+\frac{1}{\delta}}^{\infty} dx \frac{1}{(1 - x)^{2}} - \int_{0}^{\infty} dx \frac{1}{(1 + x)^{2}} \right]. \quad (B15e)
\]

Calculating these integrals, taking the limits \( L \to \infty \) and \( \delta \to 0 \) and keeping divergent terms only, we get
\[ A = \frac{m}{8\pi^2} \log \frac{L^2}{m^2}, \]  
\[ B = -\frac{m}{16\pi^2} \log \frac{L^2}{m^2}, \]  
\[ C = -\frac{1}{\delta} \frac{L^2}{8\pi^2 m} + \frac{L^2 \log \delta}{16\pi^2 m} + \frac{L^2}{8\pi^2 m} + \frac{m}{16\pi^2} \log \frac{L^2}{m^2}, \]  
\[ C_{fc} = \frac{L^2 \log \delta}{16\pi^2 m}, \]  
\[ C_{bc} = \frac{1}{\delta} \frac{L^2}{8\pi^2 m} - \frac{L^2}{8\pi^2 m}. \]  

From here we find

\[ Z_\omega = g^2 (C + C_{fc} + C_{bc}) = \frac{m}{16\pi^2} \log \frac{L^2}{m^2} \]  

and

\[ \delta m = g^2 (A + B + C + C_{fc} + C_{bc}) = \frac{g^2 m}{8\pi^2} \log \frac{L^2}{m^2}. \]

The terms \( C, C_{fc}, \) and \( C_{bc} \) are quadratically and logarithmically divergent in the cutoff \( L \) and divergent in the cutoff \( \delta \) like \( 1/\delta \) and \( \log \delta \). However, all the divergences excepting \( \log(L^2/m^2) \) ones cancel in the sum \( C + C_{fc} + C_{bc} \).

**APPENDIX C: EQUATIONS FOR THE SPIN COMPONENTS**

1. Scalar boson

The spin components \( a_1, b_1, \) and \( b_2 \) in the scalar boson case are defined by Eqs. (53) and (56). As explained in Sec. IV C, the equations for the spin components are obtained by substituting these equations into Eqs. (54) and (55) and taking traces. As a result, we get the following system of three homogeneous equations for the three unknown constants \( a_1, b_1, \) and \( b_2 \):

\[ 2a_1 m (\delta m^2 - 2m\delta m' - 2mZ) - b_1 g [(A + B)(\delta m' - 2m) - 2mC] \]
\[ + 2b_2 g [(A + B)m - \delta m'B] = 0, \]  
\[ 2a_1 gm [m^2 (3 + x) - \delta m'(1 + x) + s(1 - x)] + b_1 [g^2 m (A + B)(1 + x) - m^2 (3 + x) - s(1 - x)] \]
\[ + 2b_2 m (g^2 B - m)(1 + x) = 0, \]  
\[ 2a_1 gm [m (1 + x) - \delta m' x] + b_1 [g^2 (A + B)x - m (1 + x)] + 2b_2 (g^2 B - m)x = 0. \]

The condition that the determinant (59) of this system is zero results in the solution (60), (61).

2. Feynman gauge

In the case of gauge boson in the Feynman gauge, the one-body fermion vertex function \( \Gamma_1 \) is determined by one spin component only, as given by Eq. (84), while the two-body vertex function \( \Gamma_2 \) by eight spin components, four of them being zeroes when sectors higher than \( |Fb\rangle \) are omitted in calculations. The derivation of

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the system of equations for the spin components is described in Sec. V C. The system of five equations for \( a_1 \), \( b_1 \)–\( d_1 \) is given by

\[
a_{1m}(2m\delta m' - \delta m'^2 + 2mZ) - b_1 g[(2A_1 - B_1)(\delta m' - 2m) + 2C_{1m}] + b_2 mg(A_1 + B_1) - 2b_3 g[A_1 m + B_1 (\delta m' - 2m)] + 2b_4 g B_1 m = 0, \tag{C4}
\]

\[
2a_1gm \left[ m^2 (3 - x) - s(1 - x) - m\delta m' (2 - x) \right] - b_1 \left[ 2mg^2(2A_1 - B_1)(2 - x) + m^2 (3 - x) - s(1 - x) \right] - b_2 m^2 (1 + x) - 2b_3 m[2g^2 B_1(x - 2) + m(1 - 2x)] - 2b_4 m^2 x = 0, \tag{C5}
\]

\[
(2a_1gm - b_1)(1 + x) = 0, \tag{C6}
\]

\[
2a_1gm[x\delta m' + m(1 - 2x)] + b_1 [2g^2 (2A_1 - B_1)x - m(1 - 2x)] + 2b_3 (2g^2 B_1 - m)x = 0, \tag{C7}
\]

\[
2a_1g \left\{ m^2 (1 + x) + [2m^2 + s(1 - x)](m - \delta m') \right\} - b_1 \left\{ m^2 (1 + x) + \frac{2m^2 + s(1 - x)}{m}[2g^2(2A_1 - B_1) + m] \right\} - b_2 [m^2 (3 + x) + s(1 - x)] - 2b_3 \left[ m^2 (1 + x) + 2g^2 B_1 \frac{2m^2 + s(1 - x)}{m} \right] - 2b_4 m^2 (1 + x) = 0. \tag{C8}
\]

The determinant of this system of equations is given by Eq. (87). The condition \( \text{Det} = 0 \) results in the solution (88)–(90).

3. Light-Cone gauge

The equations for the spin components, derived in Sec. VI C, have the form

\[
a_{1m}[2m\delta m' - \delta m'^2 + 2mZ] + b_1 g[\delta m'(B_1 - A_1) + 2m(A_1 - 2B_1 - B_2 + 5C_1 + 4C_1')] + 2b_2 g[m(B_1 - A_1) - B_1 \delta m'] = 0, \tag{C9}
\]

\[
2a_1gm \left[ \frac{1 + x^2}{1 - x} (s - m^2) - 2m^2 + m\delta m' (1 - x) \right] - b_1 \left[ \frac{1 + x^2}{1 - x} (s - m^2) - 2m^2 - 2g^2 m(A_1 - B_1)(1 - x) \right] + 2b_2 m(2g^2 B_1 - m)(1 - x) = 0, \tag{C10}
\]

\[
2a_1gm[m(1 - x) + x\delta m'] - b_1 [m(1 - x) - 2g^2 (A_1 - B_1)x] + 2b_2 (2g^2 B_1 - m)x = 0. \tag{C11}
\]

The condition that the determinant of this system is zero [see Eq. (106)] results in the solution (107)–(109).
Here we will show how to adjust the cutoffs in Eqs. (B15a)–(B15e) in order to reproduce the perturbative value of $\delta m$ obtained in Ref. [6] for the LC gauge.

The coefficients $A$, $B$, and $C$ proceed from the light-front diagram shown in Fig. 2(a), whereas $C_{fc}$ and $C_{bc}$ originate from the loop diagrams shown in Figs. 2(b) and 12, respectively. Following Ref. [6], we regularize the amplitude of the diagram of Fig. 2(a) by the condition

$$s = \frac{k^2}{x} + \frac{k^2 + m^2}{1 - x} < \Lambda^2$$  \hspace{1cm} (D1)

with the additional requirement

$$k_\perp < L \ll \Lambda.$$  \hspace{1cm} (D2)

Eq. (D1) leads to the restrictions $\alpha < x < 1 - \beta$, where

$$\alpha = \frac{k^2}{\Lambda^2}, \quad \beta = \frac{k^2 + m^2}{\Lambda^2}.$$  \hspace{1cm} (D3)

The integrands in the expressions for $A$ and $B$ [see Eqs. (B15a), (B15b)] are not singular in $x$, hence, $A$ and $B$ are given by the same formulas (B16a), (B16b). The expression (B15c) transforms to

$$C = -\frac{1}{16\pi^2 m} \int_0^{L^2} dk_\perp^2 \int_\alpha^{1 - \beta} dx \frac{k^2 (x^2 - 2x + 2) + m^2 x^3 (2 - x)}{x^2 (1 - x) (k^2 + m^2 x)}$$

$$= \frac{1}{16\pi^2 m} \int_0^{L^2} dk_\perp^2 \left[ \int_0^1 \frac{2m^2 (1 - x) dx}{m^2 x^2 + k^2} - \int_\alpha^1 \frac{2dx}{x^2} - \int_0^{1 - \beta} \frac{dx}{1 - x} \right]$$

$$= \frac{m}{16\pi^2} \log \frac{L^2}{m^2} + \frac{1}{16\pi^2 m} \int_0^{L^2} dk_\perp^2 \left[ \log \beta - \frac{2}{\alpha} + 2 \right].$$  \hspace{1cm} (D4)

As far as the divergent integrals over $x$ for the coefficients $C_{fc}$ and $C_{bc}$ in Eqs. (B15d), (B15e) are concerned, the prescription of Ref. [6] demands to use for their regularization the only cutoff $\alpha$. Therefore, we have

$$C_{fc} = \frac{1}{16\pi^2 m} \int_0^{L^2} dk_\perp^2 \left[ \int_\alpha^{1 - \alpha} \frac{dx}{x (1 - x)} + \int_1^{\infty} \frac{dx}{x (1 - x)} \right] = -\frac{1}{16\pi^2 m} \int_0^{L^2} dk_\perp^2 \log \alpha,$$  \hspace{1cm} (D5)

$$C_{bc} = \frac{1}{16\pi^2 m} \int_0^{L^2} dk_\perp^2 \left[ \int_\alpha^{1 - \alpha} \frac{dx}{(1 - x)^2} + \int_1^{\infty} \frac{dx}{(1 - x)^2} - \int_0^{\infty} \frac{dx}{(1 + x)^2} \right]$$

$$= \frac{1}{16\pi^2 m} \int_0^{L^2} dk_\perp^2 \left[ \frac{2}{\alpha} - 2 \right].$$  \hspace{1cm} (D6)

Now, using the first of Eqs. (111) we get

$$\delta m = \frac{g^2 m}{8\pi^2} \log \frac{L^2}{m^2} + \frac{g^2}{16\pi^2 m} \int_0^{L^2} dk_\perp^2 \log \frac{\beta}{\alpha}.$$  \hspace{1cm} (D7)

Taking into account Eqs. (D3) and performing the integration we finally obtain

$$\delta m = \frac{3g^2 m}{16\pi^2} \log \frac{L^2}{m^2}.$$  \hspace{1cm} (D8)
The same coefficient $3/16$ as in Eq. (D8) appears also for the regularized perturbative $\delta m_{4DF}$ in standard 4DF approach:

$$
\delta m_{4DF} = \frac{3g^2m}{16\pi^2} \log \left( \frac{L_{4DF}^2}{m^2} \right),
$$

where $L_{4DF}$ is a relativistically invariant cutoff restricting the region of integration in four-dimensional space. Note however that in spite of the similarity between Eqs. (D8) and (D9) the cutoffs $L$ and $L_{4DF}$ restrict different integration regions and therefore have different meaning.

So, the only difference between our method of calculation and the method from Ref. [6] consists in that integrating over $x$ we apply the same cutoff $\delta$ for all the divergent integrals in Eqs. (B15c)–(B15e), while the authors of Ref. [6] used two different cutoffs, $\alpha$ and $\beta$ (dependent on $k^2_{\perp}$), as exposed above. Putting $\alpha = \beta$ in Eq. (D7) immediately reproduces our result (B18) for $\delta m$. 

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\[ M^{(2)} = \left\langle p_1, g \mid k_1, g \right\rangle + \left\langle p_1, g \mid k_1', g \right\rangle \]

FIG. 1. Fermion-boson CLFD scattering amplitude in $g^2$-order of perturbation theory.

\[ -M^{(2)}(p) = \left\langle p_1, g \mid k, g \right\rangle + \left\langle p_1, g \mid k', g \right\rangle \]

(a)

(b)

FIG. 2. Mass operator in CLFD. The lowest order contributions are shown: the light-front self-energy, $-\Sigma(p)$ (a), and the contact term, $-\Sigma_{fc}(p)$ (b). Here $p = p_1 - \omega \tau_1$.

\[ (a) \quad (b) \]

FIG. 3. Interaction vertices generated by the Hamiltonian $H_1(x)$, Eq. (35a).
FIG. 4. Interaction vertices generated by the Hamiltonian $H_{1c}(x)$, Eq. (35b).

FIG. 5. Interaction vertex generated by the Hamiltonian $H_2(x)$, Eq. (35c).
FIG. 6. Interaction vertices generated by the Hamiltonian $H_{2c}(x)$, Eq. (35d).

\[
\delta m' \quad \delta m' \quad \frac{i}{2\omega_p}
\]

FIG. 7. Interaction vertex generated by the Hamiltonian $H'_{2c}(x)$, Eq. (35e).

\[
Z \frac{\omega^m}{\omega_p}
\]

FIG. 8. Interaction vertex generated by the Hamiltonian $H_3(x)$, Eq. (35f).
FIG. 9. System of equations for the one- and two-body vertex functions for the scalar boson case.
FIG. 10. Interaction vertices generated in the Feynman gauge by the piece $H_1'(x)$, Eq. (73a), of the light-front Hamiltonian.
\[ \Gamma_1 = \Gamma_2 g\gamma^\nu + \Gamma_1 \delta m' + \Gamma_1 Z_{\omega p} \frac{m^d}{\omega p} - \frac{\delta}{2\omega p} \]

\[ \Gamma_2 = \Gamma_1 g\gamma^\mu + \Gamma_2 g\gamma^\nu g\gamma^\mu - \frac{\delta}{2\omega p} \]

FIG. 11. System of equations for the one- and two-body vertex functions for the case of vector boson in the Feynman gauge.

\[ k g\gamma^\mu g\gamma^\nu - \frac{\omega^\mu \omega^\nu}{(\omega \cdot (p-k))^2} \]

\[ k g\gamma^\mu g\gamma^\nu - \frac{\omega^\mu \omega^\nu}{(\omega \cdot (p+k))^2} \]

FIG. 12. Interaction vertices generated in the LC gauge by the piece \( H'_{LC}(x) \), Eq. (93), of the light-front Hamiltonian.