Abstract

The solution of DGLAP evolution equation for the twist-3 gluon operators is obtained in the Double Logarithmic Approximation of QCD perturbation theory. The method used for the solution is similar to the reggeon field theory. The asymptotics of the twist-3 parton correlation function for the small Bjorken variables $x_B$ is found.

1. The growth of the structure function $f(x_B, Q^2)$ in the region of small Bjorken variables $x_B$ makes it necessary to take into account the high twist contribution. According to the operator product expansion the local operator of spin $J$ contributes to the number $J$ Mellin moment of the structure function $f(x_B, Q^2)$. The $Q^2$-dependence of the operators is perturbatively determined by evolution equation collecting in the leading order (LLA) the terms of the form $(\alpha_S \ln Q^2/\mu^2)^n$. The asymptotic behavior for $x_B \rightarrow 0$ is governed by the rightmost singularity of the anomalous dimension in the variable $J$ continued to the complex plane. However the calculation of the anomalous dimension becomes more complicated for twists $N \geq 3$ because of the large number of the local operators.

There is another approach proposed in Ref.[1] and based on the relation of small-$x_B$ behavior of the structure function and BFKL equation summing up the powers $(\alpha_S \ln 1/x_B)^n$. It enables to find twist-2 anomalous dimension near the singularity position $J \rightarrow 1$ for the leading and next to leadings orders in terms of $\ln Q^2$. The situation is also more involved for the higher twists $N \geq 3$ since one has to solve the equation for $N$ reggeized gluons [2] and then to extract from it the anomalous dimension.

2. Here we consider the evolution equation for the twist-3 quasipartonic operators in double logarithmic approximation (DLA), which collects the
powers of the product $\alpha_s \ln Q^2 / \mu^2 \ln 1/x$. Quasipartonic operator form the closed set of high twist operators allowing for the interpretation in terms of the parton model [5]. They are responsible for the small $x_B$ asymptotics of the structure function [3, 4]. The matrix elements of quasipartonic operators depend only on the fraction $x_i$ of the partons momenta along the hadron momentum $p$ ($p^2 \simeq 0$). The pure gluon channel will be studied below as dominating in the small $x_B$ region. We shall take the quasipartonic operators, which in the axial gauge $n_\mu A_\mu = 0$ with a light-like vector $n$ dual to the hadron momentum $p$, have a general form

$$O_{\mu_1,\mu_2,\mu_3}^{m_1,m_2,m_3} = \Gamma_{\mu_1 \mu_1', \mu_2 \mu_2', \mu_3 \mu_3'} ((i\partial)^{m_1} A^{a_1}_{\mu_1}) ((i\partial)^{m_2} A^{a_2}_{\mu_2}) ((i\partial)^{m_3} A^{a_3}_{\mu_3}),$$

where $m_i$ are positive integers, $\Gamma$ is a color and Lorentz tensor, the particular form of which will not be important in what follows, $\partial = n_\mu \partial_\mu$. The matrix element over a hadron state can be expressed as

$$\langle h | O_{\mu_1,\mu_2,\mu_3}^{m_1,m_2,m_3} | h \rangle = \int d\beta_1 d\beta_2 d\beta_3 N_{\lambda_1,\lambda_2,\lambda_3}^{abc} (\beta_1, \beta_2, \beta_3) \times \frac{O_{\lambda_1,\lambda_2,\lambda_3}^{abc}}{\epsilon_1^{\lambda_1} \epsilon_2^{\lambda_2} \epsilon_3^{\lambda_3}},$$

where $\epsilon_\lambda^{\mu}$ is the gluon polarization vector. The parton correlation function $N_{\lambda_1,\lambda_2,\lambda_3}^{abc} (x_1, x_2, x_3)$ has a meaning of a hadron wavefunction integrated over partons transverse momenta, the greatest transverse momentum being of the order $Q^2$.

The evolution equation generalizing twist-2 DGLAP equation is derived in paper [5]. It has the form of $N$-particle one dimensional Schrödinger-type equation with pairwise interaction between gluons,

$$Q^2 \frac{\partial}{\partial Q^2} N_{\lambda_1,\lambda_2,\lambda_3}^{a_1,a_2,a_3,a_4,a_5,a_6} (Q^2, x_1, \ldots, x_N) = \sum_{i<j} \int d\beta_i d\beta_j \delta(x_i + x_j - \beta_i - \beta_j)$$

$$\times \Phi_{\lambda_1 \lambda_j, \lambda_2 \lambda_j'} (x_i, x_j; \beta_i, \beta_j) N_{\lambda_1,\lambda_2,\lambda_3}^{a_1,a_2,a_3,a_4,a_5,a_6} (Q^2, x_1, \ldots, x_N)$$

(1)

This equation sums up in the leading $\ln Q^2$ order the ladder-type diagrams. For the twist $N$ case they comprise the local operator vertex and $N$ gluons in $t$-channel interacting through all possible $s$-channel gluons rungs. The integrals in each ladder cell are ordered in LLA such that transverse momentum in the above cell plays the role of ultraviolet cut-off for the below one. The $Q^2$ value, being the greatest momentum in the upper loop attached to the local operator vertex, is the ultraviolet cut-off for the whole diagram. The evolution equation is obtained by taking the derivative of the diagrams with respect to $\ln Q^2$. The kernel $\Phi_{\lambda_1 \lambda_j, \lambda_2 \lambda_j'} (x_i, x_j; \beta_i, \beta_j)$ is determined by the logarithmic part of the one-loop integral over the transverse parton momentum $k_\bot$. 
Generally the longitudinal momenta $x_i$ are not ordered in LLA, but they have to be ordered in DLA to provide a large logarithm for each ladder cell. The $x_i$ variables increase from the smallest values at the local operator vertex to the order of unity ones in the lower part of a diagram. In such a kinematics the logarithmic divergencies that occur in every loop when $\beta_i \to 0$ is cut from below by the longitudinal momentum in the upper cell. Thus DLA implies the loop integrals in the evolution equation (1) to be limited by the condition

$$\beta_i, \beta_j \ll x_i, x_j,$$

which means that the momenta below the $s$-channel rung ($x$) and above it ($\beta$) are of different order of magnitude. The most singular contribution comes in Eq.(1) from the region where both $\beta_i$ and $\beta_j$ tend to zero. The momentum conservation allows it only if

$$x_i + x_j \ll \beta_i, \beta_j,$$

that is $x_i \approx -x_j$ with logarithmic accuracy. Hereafter it is convenient to assume the momenta directed upward to be positive, directed downward to be negative.

In the logarithmic domain the kernel of the evolution equation in DLA can be easily obtained by keeping the most singular in $\beta$ terms in the gluon-gluon kernel,

$$\Phi_{\lambda_1, \lambda_2, \lambda'_1, \lambda'_2}(x_1, x_2; \beta_1, \beta_2) = 2\delta_{\lambda_1 \lambda_2} \delta_{\lambda'_1 \lambda'_2} i f^{acg} i f^{bqd} x_1 \delta(x_1 - x_2) \frac{1}{\beta_1 \beta_2}. \quad (4)$$

Here $\lambda_i, \lambda'_j$ are the two dimensional transverse helicity indices, $f^{abc}$ are the structure constants of the $SU(N_c)$ group. The momenta $x_1, x_2$, are equal in DLA. They are positive but have the opposite directions, one of them is incoming from below the other is outgoing. The low-scale momenta $\beta_{1,2}$ are not supposed to be equal in DLA since the momentum transfer from below $x_1 - x_2$ is small only compared to the large momenta $x_{1,2}$ but is of the same order as the low-scale ones.

All $x_i, \beta_i$ momenta are supposed to be positive in the DLA kernel (4), the sign is specified with an additional index $\sigma = \{+, -\}$. Thus each gluon in the structure function is characterized by the color index $a$, helicity $\lambda$, momentum value $\beta$ and momentum direction $\sigma$. The interaction occurs only between the gluons with the opposite $\sigma$.

There are two possible color structures for twist-3 operators - the odderon-like $d^{abc}$ and gluon-like $f^{abc}$. Both of them go through the equation resulting into $N_c/2$ factor. This simplifies the color structure of the correlation function, $N^{ab}_{\lambda_1, \lambda_2, \lambda_3} = (d^{abc} F_{\lambda_1, \lambda_2, \lambda_3} + f^{abc} F_{\lambda_1, \lambda_2, \lambda_3})$ or $N^{abc}_{\lambda_1, \lambda_2, \lambda_3} = f^{abc} F_{\lambda_1, \lambda_2, \lambda_3}$, and the action of the kernel can be written as

$$H_{12} F_{\lambda_1, \sigma_1, \lambda_2, \sigma_2, \lambda_3, \sigma_3}(x_1, x_2, x_3) = \frac{1}{4} \frac{1}{\pi} \delta_{\sigma_1, -\sigma_2} \delta_{\lambda_1, \lambda_2} \delta_{\lambda'_1, \lambda'_2} x_1 \delta(x_1 - x_2) \quad (5)$$
\[ \times \int_0^{\beta_1} \frac{d\beta_1}{\beta_1} \int_0^{\beta_2} \frac{d\beta_2}{\beta_2} \left[ F_{\lambda_1,\sigma_1,\lambda_2,\sigma_2,\lambda_3,\sigma_3}(\beta_1, \beta_2, x_3) \pm F_{\lambda_2,\sigma_2,\lambda_1,\sigma_1,\lambda_3,\sigma_3}(\beta_2, \beta_1, x_3) \right], \]

and similarly for \( H_{23}, H_{13} \). Two terms in the r.h.s. (5) represent the sum of the \( s- \) and \( u- \) channel diagrams (only even momentum operators survive in this sum for the twist-2 case).

3. Instead of direct solving of evolution equation we adopt here an other approach similar to reggeon calculus and more suitable to find the asymptotics of the structure function. To this end we rewrite the formal solution of the evolution equation with a given initial condition \( F_0 \),

\[ F(Q^2) = e^{H \log Q^2/\mu^2} F_0, \]

through the Mellin transform as

\[ F(Q^2) = \int \frac{d\nu}{2\pi i} \left( \frac{Q^2}{\mu^2} \right)^{\nu} \frac{1}{\nu} \frac{1}{1 - \frac{1}{\nu} H} F_0, \]

where the integral runs along the imaginary axis to the right from all singularities. The equation

\[ F(\nu) = F_0 + \frac{1}{\nu} H F(\nu) \quad (6) \]

can be treated as the Bethe-Solpeter equation in the theory described by effective action

\[ S = \int dx \Phi^*_{\lambda\sigma}(x) \Phi_{\lambda\sigma}(x) + \frac{1}{4} \frac{\pi}{\nu} \int dx \frac{d\beta_1}{\beta_1} \frac{d\beta_2}{\beta_2} \Phi^*_{\lambda_1,\sigma_1}(x) \theta(x - \beta_1) \Phi_{\lambda_2,\sigma_2}(\beta_1) \]

\[ \times \delta_{\lambda_1,\lambda_2} \delta_{\lambda_3,\lambda_4} \delta_{\sigma_1,\sigma_2} \delta_{\sigma_3,\sigma_4} \delta_{\sigma_2,\sigma_3} \Phi^*_{\lambda_3,\sigma_3}(x) \theta(x - \beta_2) \Phi_{\lambda_4,\sigma_4}(\beta_2). \]

The solution to eq.(6) is given by the convolution with the Green function

\[ F_{\lambda_1,\sigma_1}(\nu, \beta_i) = \int dx_i \varphi_{\lambda_1,\sigma_1}(x_i) G_{\lambda_1,\sigma_1; \lambda_1,\sigma_1}(\nu; x_i, \beta_i) \quad (7) \]

calculated in the effective theory,

\[ G_{\lambda_1,\sigma_1; \lambda_1,\sigma_1}(\nu; x_i, \beta_i) = \prod_{i=1}^3 \langle \Phi^*_{\lambda_1,\sigma_1}(x_i) \Phi^*_{\lambda_1,\sigma_1}(\beta_i) \rangle. \]

The initial hadron wavefunction \( \varphi_{\lambda_1,\sigma_1}(x_i) \) is to be taken at the low \( Q^2 \) scale. It can’t be find perturbatively, but its precise form do not influence large \( Q^2 \) asymptotics. For definiteness we shall consider below the moments, that is we take

\[ \varphi(x_i) = \prod_{i=1}^3 x_i^{n_i} \quad (8) \]

with the integers \( n_i \geq 0 \).
We shall find the Green function by iterating the Bethe-Salpeter equation. We start from the two gluon ladder, or "reggeon", which will be a main building block in the further proceeding. It is schematically shown in Fig.1, where the solid lines denote gluon. Iterations of the two-particle kernel results into the matrix

$$\tilde{g}_{\lambda_1 \lambda_2, \lambda'_1 \lambda'_2}(x_1, x_2; \beta_1, \beta_2) = \frac{1}{2} \delta_{\lambda_1 \lambda_2} \delta_{\lambda'_1 \lambda'_2} \frac{1}{\beta_1 \beta_2} \tilde{g}_{12}(\nu; x_1, \{\beta_1, \beta_2\})$$

(9)

\[\times x_1 \delta(x_1 - x_2).\]

Here \(\{\beta_1, \beta_2\} \equiv \max\{\beta_1, \beta_2\}\) and matrix \(g_{ik}\) acts on the sign variables \(\sigma_i, \sigma_k\) as

$$g_{ik}(\nu; x, \beta) = g_{\sigma_i \sigma_k, \sigma'_i \sigma'_k}(\nu; x, \beta) = \left( \frac{I \pm P_{ik}}{2} A_{ik} \right)_{\sigma_i \sigma_k, \sigma'_i \sigma'_k} g(\nu; x, \beta),$$

where operator \(P_{ik}\) permutes the indices \(\sigma_i, \sigma_k\), matrix \(A_{ik}\) permits the interaction only between the partons with opposite momentum signs,

$$(A_{ik})_{\sigma_i \sigma_k, \sigma'_i \sigma'_k} = \delta_{\sigma_i \sigma'_i} \delta_{\sigma_k \sigma'_k} \delta_{\sigma_i \sigma'_i} \delta_{\sigma_k \sigma'_k}.$$

and

$$g(\nu; x, \beta) = \int \frac{dj}{2\pi i} \left( \frac{\beta}{x} \right)^{-j} \frac{\overline{\alpha}}{2\nu j - \overline{\alpha}}.$$

(10)

4. The expression (9) leads to the usual structure function in the twist-2 case. Indeed, the general form of twist-2 spin \(J\) gluon operator \((F_1\) structure function) convoluted with the gauge fixing vector \(n\) is

$$n_{\mu_1} \cdots n_{\mu_J} O_{\mu_1, \ldots, \mu_J} = (i\partial) A_\nu (i\partial)^{J-1} A_\nu.$$
for the color singlet). The integration has to be done with account of both
signs of the \( \beta_{1,2} \) variables, which implies the sum over \((+-)\) and
\((-+)\) initial sign states. For the \((+-)\) final state, that is for the positive \( x_1 \), we get

\[
M_2(J) = -[1 + (-1)^J] \delta_{\lambda_1 \lambda_2} x_1' \delta(x_1 - x_2) \int \frac{dj}{2\pi i} \frac{\pi}{\nu(J-1) - \alpha}
\]

and the same for the \((-+)\) state. Thus we have reproduced the double
logarithmic twist-2 anomalous dimension

\[
\gamma_2(J) = \frac{\alpha}{J-1}
\]

together with the selection rule allowing only for the even \( J \) values.

5. We consider the diagrams for the Green function occurring in the eq.(7).
The general sum of the three-gluon ladder diagrams can be equivalently pre-
sented as a sum of two-gluon ladders ("reggeons") developing between each
gluon pairs accompanied with a third single gluon as is shown in Fig.2 – 4.
By employing this representation all diagrams can be summed up in a closed
form. The Green function reads

\[
G_{d,f}^{\text{tot}}(x_1, \sigma_1, \lambda_1 | \beta_{i'}, \sigma_{i'}, \lambda_{i'})
\]

where the sum is taken over independent permutations of the incoming and
outgoing particles while the functions \( G_{d,f} \) stand for the diagrams with a fixed
order of the external lines. The symbols \( d \) and \( f \) label the Green functions for
the \( d^{abc} \) and \( f^{abc} \) color structures. The formula (11) implies the simultaneous
permutations of all quantum numbers, that is momenta, helicities and colors,
which means symmetrization with respect \( \{x_i, \sigma_i, \lambda_i\} \) (or \( \{\beta_j, \sigma'_j, \lambda'_j\} \)) pair for
\( d^{abc} \) tensor and antisymmetrization for \( f^{abc} \) tensor.

![Figure 2](image_url)

The effective diagrams constructed from the two-gluon ladder and gluon
line turns out to be rather simple to calculate the Green function by the
direct summation. The result is presented by the sum of three contributions.
Two of them are degenerate in the sense that they comes from the finite
number of iterations. The first one includes the "reggeon" only once. There
are 3 diagrams of this type for 3 various gluon pairs combined into the ladder.
One of them for the pair 12 is shown in Fig. 2,
\[
\overline{G}_{d,f}^{(I)}(x_i, \beta_i)_{(12)} = \frac{1}{2} \delta_{\lambda_1 \lambda_2} \delta_{\lambda_3 \lambda'_3} \delta_{\lambda_1' \lambda_2'} \frac{1}{\beta_1 \beta_2 \beta_3} \hat{g}_{12}(\nu; x_1, \{\beta_1, \beta_2\}) \times x_1 \delta(x_1 - x_2) \delta(x_3 - \beta_3) \delta_{\sigma_3 \sigma'_3}. 
\]
The others can be obtained by permutations of the indices (123) → (231) and (123) → (132).

The second contribution arises from the diagram with the two "reggeons".

Fig. 3 presents the diagram where the gluon pair 12 switches to the pair 23,
\[
\overline{G}_{d,f}^{(II)}(x_i, \beta_i)_{(23)(12)} = \frac{1}{4} \delta_{\lambda_1 \lambda_2} \delta_{\lambda_3 \lambda'_3} \delta_{\lambda_1' \lambda_2'} \frac{1}{\beta_1 \beta_2 \beta_3} \hat{g}_{23}(\nu; x_1, \{\beta_1, \beta_2\}) \times x_2 \delta(x_2 - x_3) \delta_{\sigma_3 \sigma'_3}. 
\]
The other 5 terms result in this case from (12) after independent permutations (123) → (231) and (123) → (132) of the upper and lower (in the sense of Fig.3) indices but excluding the equal ones. In other words the sum is taken over various ways to combine the incoming and outgoing gluons into different two-particle ladders.

The contributions starting with the three "reggeons" develop a regular series which can be written as
\[
\overline{G}_{d,f}^{(III)}(x_i, \beta_i)_{(12)(12)} = \frac{1}{4} \delta_{\lambda_1 \lambda_2} \delta_{\lambda_3 \lambda'_3} \delta_{\lambda_1' \lambda_2'} \frac{1}{\beta_1 \beta_2 \beta_3} \hat{g}_{12}(\nu; x_1, x_3) 
\times \int \frac{d\beta}{\beta} W_{d,f}(x_3, \nu; \{\beta_3, \beta\}) \hat{g}_{12}(\nu; \beta, \{\beta_1, \beta_2\}),
\]
\[
W_{d,f}(\nu; x, \beta) = \frac{1}{2} \hat{g}(\nu; x, \beta) + \frac{1}{4} \int \frac{d\beta'}{\beta'} \hat{g}(\nu; x, \beta') \hat{g}(\nu; \beta', \beta) + \cdots. 
\]

Fig. 4 shows the diagram corresponding either to the first term in eq.(14) or to all terms in (14) if the middle "reggeon" is replaced with the full function $W_{d,f}$. 
There are 8 other terms besides (13), which can be obtained from it by independent permutations \((123) \rightarrow (231)\) and \((123) \rightarrow (132)\) of the upper and lower indices in the Fig. 4.

The Mellin transform (10) turns the convolutions over \(\beta'\) into the usual products, making the series (14) to be equivalent to the purely matrix problem,

\[
W = \frac{1}{2 \nu j} H + \left( \frac{1}{2 \nu j} H \right)^2 + \cdots = H \frac{1}{2^2} \frac{1}{2^2} = H, \tag{15}
\]

where the "reggeons" "dissociate" into two-particle interaction,

\[
H = \sum H_{i,i+1}, \quad H_{i,k} = \frac{1}{2} \delta_{\lambda_i \lambda_k} \delta_{\lambda'_i \lambda'_k} \frac{1}{2} \pm \frac{P_{ik}}{2} A_{ik},
\]

and the problem is reduced to the inversion of the finite matrix.

The above iterations exhibit that at each step two momenta with opposite directions have the same value much larger than the value of the third momentum \((x_{2,3} \gg x_1\) in Fig. 3 and \(x_{1,2} \gg x_3\) in Fig. 4). This property expresses the longitudinal momentum conservation within the DLA accuracy – the sum of all momenta is small compared to their natural scale.

6. The Green function is convoluted over variables \(\beta_i\) and helicities indices \(\lambda'_i\) with the operator vertex given by the expression

\[
O_{\lambda'_1,\mu_1,\lambda'_2,\mu_2,\lambda'_3,\mu_3}(\beta_1, \beta_2, \beta_3) = \Gamma_{\lambda'_1,\mu_1,\lambda'_2,\mu_2,\lambda'_3,\mu_3} \beta_1^{m_1} \beta_2^{m_2} \beta_3^{m_3} \delta(\beta_1 + \beta_2 + \beta_3), \tag{16}
\]

where the longitudinal \(\delta\)-function corresponds to forward kinematics without momentum transfer. Taking then the moments with respect variables \(x_i\) (8) we get the Green function in the moments representation, \(G_{ij}^{tot}(m_i, n_i)\).

Note, that the integrals over \(x_i, \beta_i\) imply the positive as well as the negative values of the momenta. The negative values are described through the sign variables \(\sigma_i = \pm\), for example, the configuration where \(\beta_1 < 0, \beta_{2,3} > 0\) is associated with the state \((-+, +, +)\) and similarly for \(x_i\). The integral over
all sign configurations is recovered by the sum over all initial \( \sigma'_i \) and final \( \sigma_i \) values. As a result the Green function written in terms of the moments takes into account both signs of \( \beta_i \) and \( x_i \) and does not contain the auxiliary variables \( \sigma_i \),

\[
G_{d,f}^{\text{tot}}(m'_i, n_i) = \sum_{\{i':\{i\}} \delta_{\lambda'_{i_1} \lambda_{i_2}} \delta_{\lambda'_{i_1}} \delta_{\lambda'_{i_3} \lambda_{i_3}} G_{d,f}(m'_{i_1}, m'_{i_2}, m'_{i_3}; n_{i_1}, n_{i_2}, n_{i_3}).
\]

(17)

The sum here means the independent symmetrization with respect the pairs \( \{m'_i, \lambda'_i\} \) and \( \{n_i, \lambda_i\} \) for \( d^{abc} \) and antisymmetrization for \( f^{abc} \) structures. The helicities \( \lambda'_i \) should be convoluted with the tensor \( \Gamma_{\lambda'_i, \mu} \) specifying the operator vertex (16).

Separating the common factors the functions \( G_{d,f}(m_i, n_i) \) takes the form

\[
G_{d,f}(m_i, n_i) = \left[ 1 + (-1)^{m+n} \right] \left[ (-1)^{n_3} \pm (-1)^{n_2} \right] \left[ \frac{(-1)^{m_1}}{m_2} \pm \frac{(-1)^{m_2}}{m_1} \right]
\]

\[
\times \frac{1}{m + n + 2} G_{d,f}(m_i, n_i),
\]

(18)

\[ m \equiv m_1 + m_2 + m_3, \quad n \equiv n_1 + n_2 + n_3, \]

where \(+\) and \(-\) stand for \( d \) and \( f \) structures, respectively, and the function \( G_{d,f} \) is expressed in terms of the three contributions considered above,

\[
G_{d,f} = G_{d,f}^{(I)} + G_{d,f}^{(II)} + G_{d,f}^{(III)}.
\]

The first contribution yields

\[
G_{d,f}^{(I)}(m_i, n_i) = \frac{3}{4} \frac{\alpha}{2\nu j - \alpha},
\]

(19)

\[ j = m + n_3, \]

while the second one takes the form:

\[
G_{d}^{(II)}(m_i, n_i) = \frac{3}{8} \frac{\alpha}{2\nu(m - 1) - \alpha} \frac{\alpha}{2\nu j - \alpha},
\]

\[
G_{f}^{(II)} = 0.
\]

(20)

(21)

\( (G_{f}^{(II)} \) vanishes after antisymmetrization over end points). The third contribution with the matrix (15) inverted reads

\[
G_{d}^{(III)} = \frac{3}{8} \frac{\alpha}{2\nu(m - 1) - \alpha} \frac{\alpha}{2\nu j - \alpha} \frac{\alpha}{4\nu(m - 1) - 3\alpha},
\]

\[
G_{f}^{(III)} = \frac{3}{8} \frac{\alpha}{2\nu(m - 1) - \alpha} \frac{\alpha}{2\nu j - \alpha} \frac{\alpha}{4\nu(m - 1) - \alpha}.
\]

(22)

(23)

7. The asymptotic behavior of the structure function for Bjorken variable \( x_B \to 0 \) is determined by the rightmost singularity in the variable \( J \), which
has the meaning of the local operator spin continued to the complex plane. The spin of quasipartonic operator \( J = m \), therefore one needs to continue the function \( \mathcal{G}_{d,f}(m_i, n_i) \) to \( m \rightarrow 1 \) formally keeping the other variables \( m_i, n_i \) to be fixed. Because of the signature-like factors \((-1)^{m+n} \) the terms with even or odd \( m \) are to be treated separately. Note that in a general LLA case this continuation is non trivial since the mixing matrix describing the evolution has a rank depending on \( J \) [8]. The explicit form of the DLA solutions (19) - (23) makes the continuation much more simple and straightforward. The obtained results show that the anomalous dimension for \( d^{abc} \) color structure is

\[
\gamma_d(J) = \frac{3}{4} \frac{\alpha}{J-1}. \tag{24}
\]

The main singularity for \( f^{abc} \) structure is actually given by the pole of 2 gluon state,

\[
\gamma_f(J) = \frac{1}{2} \frac{\alpha}{J-1}. \tag{25}
\]

The contribution of "developed" 3 gluon ladder is more weak in this channel, \( \gamma_f'(J) = \gamma_f(J)/2 \). The contributions of other singularities are strictly speaking beyond the DLA accuracy since they produce the extra positive powers of \( x_B \).

The DLA anomalous dimension (24) is smaller compared to those, which can be derived from the direct solution of BFKL equation obtained in refs.[4, 6, 7, 9]. The possible reason for this is that the known BFKL solutions founded for Odderon do not really correspond to quasipartonic operators of twist 3. In this case the DLA result (24) could indicate to an existence of another solution of quasipartonic type.

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References


