ABSTRACT

We consider a subset of the physical processes that determine the spin $j \equiv a/M$ of astrophysical black holes. These include: (1) Initial conditions. Recent models suggest that the collapse of supermassive stars are likely to produce black holes with $j \sim 0.7$. (2) Major mergers. The outcome of a nearly equal mass black hole-black hole merger is not yet known, but we review the current best guesses and analytic bounds. (3) Minor mergers. We recover the result of Blandford & Hughes that accretion of small companions with isotropically distributed orbital angular momenta results in spindown, with $j \sim M^{-7/3}$. (4) Accretion. We present new results from fully relativistic magnetohydrodynamic accretion simulations. These show that, at least for one sequence of flow models, spin equilibrium ($dj/dt = 0$) is reached for $j \sim 0.9$, far less than the canonical value 0.998 of Thorne that was derived in the absence of MHD effects. This equilibrium value may not apply to all accretion flows, particularly thin disks. Nevertheless, it opens the possibility that black holes that have grown primarily through accretion are not maximally rotating.

Subject headings: accretion, accretion disks, black hole physics, Magnetohydrodynamics: MHD, Methods: Numerical

1. Introduction

The massive, dark objects observed in the centers of galaxies (e.g. Miyoshi et al. 1995; Magorrian et al. 1998) and the stellar-mass compact objects observed in binary systems

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systems (McClintock & Remillard 2003) are most readily interpreted as black holes. Alternative models require the introduction of exotic physics (Bahcall et al. 1990) or modification of Einstein’s equations for the gravitational field (recently DeDeo & Psaltis 2003). More conventional models such as clusters of compact stars are strongly constrained by observations. Most remarkably, proper motion and radial velocity studies of stars near the putative black hole in Sgr A∗ (Schödel et al. 2002; Gezari et al. 2002) require that approximately $3 \times 10^6 \, M_\odot$ be concentrated within a region 120 AU in radius. There is no stable configuration of normal matter with such a large mass in such a small volume; cluster lifetimes are too short (Maoz 1998). Black holes are therefore the “most conservative” model for massive dark objects and galactic black hole candidates (hereafter GBHCs).

Black hole solutions of Einstein’s equations have three parameters: mass $M$, spin $J$, and charge $Q$ (by the “no-hair,” or uniqueness, theorem; see Wald 1984). Of these, $Q$ is likely to be negligible in astrophysical contexts because electric charge is shorted out by the surrounding plasma (Blandford & Znajek 1977). Thus while much of the variation in the observational appearance of black holes is likely due to variation in external parameters such as the angle between black hole spin vector and line of sight, the gas accretion flow geometry and accretion rate $\dot{M}$, and other environmental factors, some might also be due to variation in black hole spin $j \equiv J/M^2 = a/M$.

Several features of supermassive black holes (hereafter SMBHs) and GBHCs have been interpreted as evidence for black hole spin:

(1) Some SMBHs and GBHCs show broad, skewed Fe Kα lines, for example in MCG-6-30-15 (Tanaka et al. 1995; Fabian et al. 2002), Cyg X-1 (Miller et al. 2002), and XTE J1650-500 (Miller et al. 2002; for a review see Reynolds & Nowak 2002). If one assumes that these lines originate in plasma on nearly circular, equatorial geodesics within a few $M$ of the black hole, then the line shape is sensitive to the spin of the hole (e.g. Laor 1991). Within the context of this model rotating holes are required to explain the observed red wing of the line.

(2) The ratio $R$ of observed quasar radiative energy per unit comoving volume to the current mass density of black holes is directly related to the mean radiative efficiency $\epsilon$ of accretion onto black holes (Soltan 1982): $\epsilon > R$. Estimates suggest $R > 0.1$ (Yu & Tremaine 2002; Elvis, Risaliti, & Zamorani 2002). If one assumes that accretion occurred through a classical thin disk in which the binding energy of the innermost stable circular orbit (hereafter ISCO) determines $\epsilon$ (Bardeen 1970), then $R > 0.1$ requires $j > 0.67$.

(3) In GBHCs, quasi-periodic oscillations (QPOs) are observed in X-ray light curves at frequencies ranging from a fraction of a Hz to 450 Hz in GRO J1655-40 (Strohmayer 2001).
Assuming that these QPO frequencies are bounded above by the rotation frequency of the ISCO and that the QPO is not an overtone, one can place a limit on the mass and spin of the black hole. In GRO J1655-40, 95% confidence limits on the mass (Shahbaz et al. 1999) require \( M > 5.5 M_\odot \), or \( j > 0.15 \) (Strohmayer 2001). A physical or phenomenological model for the QPO can provide more stringent constraints, but requires additional assumptions.

(4) The shape of the X-ray continuum from an accreting black hole may depend on the spin. Calculating an expected continuum requires the black hole mass, spin, flow geometry (usually, but not always, a thin disk) and a model for the accretion flow atmosphere. Models have been applied to a number of objects by, e.g., Zhang, Cui, & Chen (1997) and Gierliński, Maciołek-Niedźwiecki, & Ebisawa (2001), and usually suggest \( j \sim 1 \).

This list is necessarily incomplete, and in each case the evidence for black hole spin is open to debate. Models for the dynamics of the plasma surrounding the black hole and its radiative properties must be invoked. These models describe an intrinsically complex physical system and use approximations of unknown accuracy. Future calculations, particularly numerical models of the accretion flows, may help reduce the uncertainties. Analysis of gravitational waveforms emitted by perturbed black holes undergoing mergers can reveal their masses and spins and may prove less ambiguous once these signals can be measured reliably (Thorne 1995; Flanagan & Hughes 1998; Dreyer et al. 2003).

Given the existing evidence for black hole spin, it is useful to consider the physical processes governing spin evolution. In this paper we consider initial conditions (§2), mergers with black holes of comparable mass (§3), mergers with smaller objects (§4), and accretion (§5), then summarize our results in §6. Throughout the paper we adopt geometrized units and set \( G = c = 1 \).

2. Initial Conditions

Nonprimordial black holes form from gravitational collapse and in general are born with nonzero spin. If subsequent accretion is negligible, the initial spin state will be preserved and the black hole spin will be determined by the dynamics of the initial collapse.

If the initial collapse occurs from a massive star, the spin depends on the angular momentum profile of the progenitor star and the (magneto)hydrodynamics of core collapse in evolved, spinning stars. The dependence is not fully understood at this time, although detailed Newtonian simulations of the collapse of spinning stars with \( M \lesssim 300M_\odot \) have been performed and suggest how spinning black holes may arise during core collapse (see, e.g. Heger et al. 2002 for a review and references). The simulations show that the fate of the
collapse depends critically on the mass, spin, metallicity and magnetic field of the progenitor, as well as details of the equation of state and neutrino transport, so it is not surprising that the issue is not resolved.

There are also results for idealized versions of the general relativistic collapse problem. Shibata & Shapiro (2002) followed the collapse of a marginally unstable supermassive star in full general relativity. They considered the case of a uniformly rotating star supported by radiation pressure and spinning at the mass-shedding (maximal spin) limit. (Mass-shedding is the likely situation by the time the star has cooled and contracted quasistatically to the point of onset of collapse (Baumgarte & Shapiro 1999), provided that the star can sustain solid body rotation during the contraction phase.) They found that the final object is a Kerr–like black hole surrounded by a disk of orbiting gaseous debris. The final black hole mass and spin were determined to be $M_h/M \approx 0.9$ and $J_h/M^2_h = j = a/M \approx 0.75$, for an arbitrary progenitor star mass $M$. The remaining mass goes into the disk of mass $M_{\text{disk}}/M \approx 0.1$. In fact, the final black hole and disk parameters can be calculated analytically from the initial stellar density and angular momentum distribution (Shapiro & Shibata 2002). The results obtained here apply to the collapse of any marginally unstable $n \approx 3$ polytrope spinning uniformly at mass-shedding. Hence these results may be applicable to core collapse in very massive stars $\gtrsim 300 M_{\odot}$ and to collapsar models of long-duration gamma-ray bursts (MacFadyen & Woosley 1999). This work suggests, therefore, that black holes formed during core collapse of massive stars should be born rapidly rotating, but well below the Kerr limit.

Black holes may also arise in other dynamical scenarios, like the coalescence of binary neutron stars. The most detailed calculations of binary neutron star mergers in full general relativity are the hydrodynamic simulations of Shibata & Uryu (2002). They considered the coalescence of irrotational binaries (physically the most likely case), modeled as equal-mass polytropes with adiabatic indices $\Gamma = 2$ and 2.25, and considered a range of initial masses below the maximum mass limit. They followed the merger from the ISCO to coalescence. For intermediate mass stars, the merged remnant is a differentially rotating, “hypermassive” neutron star. For high mass stars, the remnant is a rotating black hole. For the binaries which formed black holes, the precollapse $J/M^2$ ranged from 0.9 – 1.0, with the higher values associated with the smaller masses. The black hole products have $J/M^2 \sim 0.8 – 0.9$ due to the loss of $\sim 10\%$ of the initial angular momentum through gravitational radiation. In these calculations most of the mass is conserved and goes into the black hole, and no disk forms about the hole.
3. Spin-up By Major Mergers

Black hole spin will also change when a black hole merges with a black hole of comparable mass (a “major merger”). The outcome of this merger and the partitioning of energy and angular momentum between internal and radiative degrees of freedom is an area of active research. Here we summarize some current estimates.

Consider binary black holes inspiralling due to gravitational radiation from initially circular orbits (gravitational radiation reduces the eccentricity of the orbits on a timescale short compared to the orbital evolution timescale). Once the black holes reach the ISCO, they will plunge together and merge on an orbital timescale. The location of the ISCO, as well as the global parameters characterizing the binary at this critical separation, are not known to high precision except for black holes with test-particle companions. Several different approaches have been formulated and have yielded approximate solutions (for a recent review and references, see Baumgarte & Shapiro 2003). These range from high-order post-Newtonian calculations to fully nonlinear numerical solutions of the initial value vacuum Einstein equations. We compare some of the available results for nonspinning black holes in Table 1, adapted from Baumgarte & Shapiro (2003).

In constructing this table, we identify the mass of each black hole with the irreducible mass

\[ M_{\text{BH}} = M_{\text{irr}} = \left( \frac{A}{16\pi} \right)^{1/2}, \]

where \( A \) is the proper area of the black hole’s event horizon (Christodoulou 1970). The binding energy can then be defined as

\[ E_b = M - 2M_{\text{BH}}, \]

where \( M \) is the total (ADM) mass of the system measured at large distance from the holes (see, e.g., ÓMurchadha and York 1974). Values for the nondimensional binding energy \( \bar{E}_b \equiv E_b/\mu \), the orbital angular velocity \( \bar{\Omega} \equiv m\Omega \) and the angular momentum \( \bar{J} \equiv J/(\mu m) \) at the ISCO are listed. Here \( \mu \) is the reduced mass, \( \mu = M_{\text{BH}}/2 \), and \( m \) is the sum of the black hole masses, \( m = 2M_{\text{BH}} \). The fractional losses from the system of mass and angular momentum due to gravitational radiation emission during the plunge are expected to be small (see, e.g., Khanna et al. 1999), but precise values await more reliable relativistic calculations. Meanwhile, a reasonable first approximation is to assume that the final black hole will have a mass and angular momentum nearly equal to the binary system at the ISCO. With this assumption, the spin parameter of the final black hole is given by

\[ \frac{J}{M^2} = \frac{\bar{J}}{4 \left[ 1 + \frac{\bar{E}_b}{4} \right]^2}. \]
Results of the numerical calculations of Cook (1994), Baumgarte (2000), and Grandclément et al. (2002) are tabulated, as well as the third-order Post-Newtonian results of Damour et al. (2000). The final black hole spin parameter computed according to equation (3) is listed in the fifth column in the table. A strict upper limit to the final spin parameter, \( (J/M^2)_{\text{max}} \), is provided by the black hole area theorem and is listed in the sixth column for comparison (see equation (4) and discussion below.) In the table we also include the analytical values for a test particle orbiting a Schwarzschild black hole, \( \bar{E}_b = \sqrt{8/9} - 1 = -0.0572, \bar{J} = 2\sqrt{3} = 3.464, \bar{\Omega} = 1/6^{3/2} = 0.0680 \), with \( J/M^2 \) evaluated according to equation (3) and \( (J/M^2)_{\text{max}} \) calculated according to equation (4).

Despite the differences in the ISCO calculations (for a recent detailed comparison, see Cook 2003), the values obtained for the expected final spin parameter based on mass and angular momentum conservation are all comparable and high, \( J/M^2 \gtrsim 0.8 \). Note that the range of these estimated final spin parameters is far narrower than the range of calculated ISCO orbital frequencies. Note also that these results are for nonspinning holes.

For spinning black holes one knows from test particle orbits that the location of the ISCO depends strongly on spin. Most numerical treatments of this problem use the conformal flatness approximation in constructing the metric. This is problematic because isolated Kerr black holes are not conformally flat, and forcing them to be so is tantamount to adding in a compensating gravitational radiation field. Conformally flat treatments of spinning binary black holes are therefore contaminated with spurious gravitational radiation, and are probably not as reliable as treatments of zero spin black holes.

Nevertheless, the calculations of Pfeiffer et al. (2000), who generalized the numerical calculation of Cook (1994) by allowing for spin, but again assumed conformal flatness, are revealing. They considered binaries consisting of holes of equal mass and equal spin magnitude in circular orbits. Their study was restricted to binaries in the range \(-0.50\) to \(++0.17\). where the + or − sign denotes whether each hole is co- or counter-rotating, respectively, and the numerical coefficient indicates the magnitude of the spin parameter \( J/M^2_{\text{BH}} \) of each hole. Assuming that the total mass and angular momentum are conserved during the plunge from the ISCO, one finds from their numerical data that the spin \( J/M^2 \) of the final, merged hole varies from 0.63 for the \(-0.50\) binary to 0.82 for the \(++0.17\) binary.

These values can be compared with the strict upper limit, \( (J/M^2)_{\text{max}} \), provided by the area theorem, combined with the fact that the final angular momentum cannot exceed the total angular momentum of the system at the ISCO:

\[
\left( \frac{J}{M^2} \right)_{\text{max}} = \frac{2}{x_{\text{max}}} \left( 1 - \frac{1}{x_{\text{max}}^2} \right)^{1/2}
\]  

(4)
where
\[
x_{\text{max}}^2 = 1 + \frac{\tilde{J}^2}{4 \left(1 + \sqrt{1 - (J/M_{\text{BH}}^2)}\right)^2}
\] (Pfeiffer, Cook, & Teukolsky 2002). The values of \((J/M^2)_{\text{max}}\) reach 0.92 for the −−0.50 binary and 0.97 for the ++0.17 binary. We expect that the values given by assuming that mass and angular momentum at the ISCO are conserved during the plunge provide more realistic estimates. The reason is that the amount of radiation allowed by the area theorem is far larger than actually found by numerical computations, where such computations are available. For example, the area theorem allows 29% of the mass-energy to be radiated in a head-on collision of identical, nonspinning black holes falling from rest at large separation, while the numerical calculations yield 0.1% (Smarr 1979; Anninos 1993).

Typically, therefore, the merger of two black holes of comparable mass will immediately drive the spin parameter of the merged hole to \(\gtrsim 0.8\).

4. Spin-Down by Minor Mergers

Recently, Hughes & Blandford (2003) have considered the spin evolution of a black hole due to mergers with smaller companions. Here we briefly revisit the problem. Our approach is slightly simplified, does not use the Fokker-Planck formalism, and evaluates the power law for spin decay exactly in the limit of small \(j\). Along the way, we provide a small \(j\) expansion for the radius and specific energy of the ISCO.

Consider a merger between a large black hole of mass \(M\) and a small black hole of mass \(m\), with \(q \equiv m/M \ll 1\). The large black hole has spin angular momentum \(J\). The change in the total spin angular momentum of the black hole is
\[
\Delta(J^2) = (J + \Delta J)^2 - J^2 = 2J \cdot \Delta J + (\Delta J)^2.
\] (6)

We are interested in how the spin evolves due to a large number of mergers, so we take an ensemble average:
\[
\langle \Delta(J^2) \rangle = \langle 2J \cdot \Delta J \rangle + \langle (\Delta J)^2 \rangle.
\] (7)

The first term on the right is the “resistive” or “dynamical friction” term, due to correlations between the black hole spin and its change in angular momentum due to the accreted object. The second term on the right is the “random walk” term. What is perhaps surprising is that the dynamical friction term does not vanish.

To evaluate \(\Delta J\) we will assume that the merger occurs through the slow, gravity-wave driven inspiral of the smaller hole onto the larger hole. The larger hole is assumed to be
slowly rotating, \( j \ll 1 \), and the smaller hole is assumed to have isotropically distributed orbital angular momentum \( \mathbf{L} \). The inspiral may be regarded as progressing through a series of nearly circular orbits of fixed inclination (Hughes 2001). The change in total angular momentum of the larger black hole is then approximately \( \mathbf{L} \) evaluated at the ISCO (we assume that the radiation of energy and angular momentum during the plunge is negligible).

With this picture in mind, the random walk term is

\[
\langle (\Delta \mathbf{J})^2 \rangle \approx \mathbf{L}^2 = (mlM)^2,
\]

where \( l \approx 2\sqrt{3} + \mathcal{O}(j) \) is the specific orbital angular momentum of a particle on the ISCO around a hole of unit mass.

Now consider the dynamical friction term. One might naively expect \( \mathbf{L} \) to be uncorrelated with \( \mathbf{J} \) if \( \mathbf{L} \) is isotropically distributed, as we have assumed. But consider a particle with orbital inclination angle \( i \), \( \mu = \cos(i) \). By expanding the fundamental equations for the ISCO to lowest order in \( j \) (see Hughes & Blandford 2003 equations 1 and 2), one can show that

\[
\frac{L_z}{mM} = 2\sqrt{3}\mu - \frac{2\sqrt{2}}{3} j\mu^2 + \mathcal{O}(j^2),
\]

\[
\mathcal{E} = \frac{2\sqrt{2}}{3} - \frac{1}{18\sqrt{3}} j\mu + \mathcal{O}(j^2),
\]

and

\[
\frac{r}{M} = 6 - \frac{4\sqrt{2}}{\sqrt{3}} j\mu + \mathcal{O}(j^2).
\]

Here \( L_z \) is the component of orbital specific angular momentum parallel to the black hole spin axis, \( \mathcal{E} \) is the particle specific energy, and \( r \) is the ISCO orbital radius measured in Boyer-Lindquist coordinates. Evidently prograde (\( \mu = 1 \) orbits have orbital angular momentum of smaller magnitude than retrograde (\( \mu = -1 \) orbits; their ISCO lies “closer” to the black hole. Thus \( \mathbf{J} \) is correlated with \( \Delta \mathbf{J} \).

Using this result,

\[
2\mathbf{J} \cdot \Delta \mathbf{J} = 2jM^2 \Delta \mathbf{J}_z \approx 2jM^3m(2\sqrt{3}\mu - \frac{2\sqrt{2}}{3}\mu^2j).
\]

Averaging over \( \mu \),

\[
\langle 2\mathbf{J} \cdot \Delta \mathbf{J} \rangle \approx \frac{1}{2} \int_{-1}^{1} d\mu \left( 2jM^3m(2\sqrt{3}\mu - \frac{2\sqrt{2}}{3}\mu^2j) \right) = -\frac{4\sqrt{2}M^3mj^2}{9},
\]

since the first term vanishes under integration.
We are now in a position to develop an expression for the evolution of \( j \):

\[
\frac{M(\Delta(J^2))}{\mathcal{E}m} \approx \frac{Md(j^2M^4)}{dM} = \frac{d(j^2M^4)}{d\ln M}
\]  

(14)

where \( \mathcal{E}m \) is the change in mass of the large hole in each event, and we can pass to the continuum limit only if \( q \ll 1 \). Using equation (8),

\[
\frac{d(j^2M^4)}{d\ln M} = \frac{M}{\mathcal{E}m} \left( -\frac{4\sqrt{2}}{9} j^2M^3m + l^2m^2M^2 \right),
\]  

(15)

and solving for \( d\ln j/d\ln M \),

\[
\frac{d\ln j}{d\ln M} = -2 - \frac{2\sqrt{2}}{9\mathcal{E}} + \frac{l^2m}{2\mathcal{E}Mj^2}.
\]  

(16)

The first term describes conservation of spin angular momentum (Hughes & Blandford’s “doctrine of original spin”), the second term dynamical friction, and the third term the random walk. Substituting for \( \mathcal{E} \) and \( l \),

\[
\frac{d\ln j}{d\ln M} = -\frac{7}{3} + \frac{9m}{\sqrt{2}Mj^2}.
\]  

(17)

The final term can be ignored whenever \( j^2/q \gg 27/(7\sqrt{2}) \); then \( j \sim M^{-7/3} \), in agreement with the Hughes & Blandford (2003) result \( j \sim M^{-2.4} \). At late times, when the final term becomes comparable to the first term, the hole will fluctuate around \( j \sim q^{1/2} \). Thus minor mergers with smaller objects with isotropically distributed orbital angular momentum will spin down a hole.

5. Spin-up By Gas Accretion

Once formed, black holes may grow through accretion of the surrounding plasma. Accretion onto GBHCs in X-ray binaries is well established. In the case of SMBHs, appreciable growth of black hole seeds by gas accretion is supported by the consistency between the total energy density in QSO light and the BH mass density in local galaxies, adopting a reasonable accretion rest-mass–to–energy conversion efficiency (Soltan 1982; Yu & Tremaine 2002; Elvis, Risaliti, & Zamorani 2002). But quasars have been discovered out to redshift \( z \sim 6 \), so it follows that the first SMBHs must have formed by \( z_{\text{BH}} \gtrsim 6 \) or within \( t_{\text{BH}} \lesssim 10^9 \) yrs after the Big Bang. This timescale provides a tight constraint on SMBH seed formation scenarios. For example, it has been argued that if they indeed grew by accretion, seeds of mass \( \gtrsim 10^5 \, M_\odot \) must have formed by \( z \sim 9 \) to have sufficient time to reach a mass of
\( \sim 10^6 \, M_\odot \) (Gnedin 2001). For a discussion of plausible scenarios for forming these black hole seeds, see Shapiro (2003).

Accretion will cause the spin \( j = a/M = J/M^2 \) of a black hole to evolve in both magnitude and direction. In this section we will assume that the orientation of the spin vector is fixed. Adopting Kerr-Schild coordinates \( t, r, \theta, \phi \), the angular momentum accretion rate is

\[
\dot{J} = \int d\theta d\phi \sqrt{-g} T^r_{\phi}
\]

where the integral is taken on the horizon, \( g \) is the metric determinant, and \( T^{\mu\nu} \) is the stress-energy tensor of the accreting material.\(^4\) We will consistently use Kerr-Schild coordinates here and below, but notice that Kerr-Schild \( r \) and \( \theta \) are identical to the more familiar Boyer-Lindquist \( r \) and \( \theta \). Mass-energy is accreted at a rate

\[
\dot{M} = \dot{E} = \int d\theta d\phi \sqrt{-g} T^{r}_t,
\]

and finally spin evolution is governed by

\[
\frac{dj}{dt} = \frac{\dot{J}}{M^2} - \frac{2j \dot{E}}{M}.
\]

It is useful to define the dimensionless spinup parameter \( s \),

\[
s = \frac{dj}{dt} \frac{M}{M_0},
\]

where

\[
M_0 = \int d\theta d\phi \sqrt{-g} \rho_0 u^r
\]

is the rest-mass accretion rate, \( \rho_0 \) is the rest-mass density, and \( u^r \) is the radial component of the plasma four-velocity. If \( s < 0 \) the black hole is spinning down.

It was first noted by Bardeen (1970) that a black hole accreting through a cold disk of constant orientation could achieve maximal rotation \( j = 1 \) in finite time. Bardeen assumed that \( \dot{J} = l \dot{M}_0 M \) and \( \dot{E} = \mathcal{E} \dot{M}_0 \) where \( l = u_\phi \) and \( \mathcal{E} = -u_t \) are the mean angular momentum and energy per unit rest mass of a particle on the ISCO, respectively. This is equivalent to the “no torque boundary condition”: no torque is exerted on the disk by the plasma in the plunging region.

\(^4\)We assume that the mass and angular momentum of the disk is small compared to that of the black hole.
Thorne (1974) noted that a thin disk inevitably radiates, and some of this radiation will be accreted by the hole. Preferential accretion of low angular momentum photons then limits the spin of the hole to a maximum value \( j = 0.998 \).

Exceptions to the Thorne (1974) result have been noted by several authors in succeeding years. Abramowicz, Jaroszinski, & Sikora (1978) pointed out that material accreted from a thick, partially pressure supported disk would have greater specific angular momentum than that accreted from a thin disk, and so one might obtain \( j > 0.998 \). Popham & Gammie (1998) considered the evolution of a relativistic hot flow using a viscous model for angular momentum transport. They typically found spin equilibrium at \( j \approx 0.7 \). Finally, Thorne (1974), quoting Bardeen, noted that magnetic fields could connect material in the disk and the plunging region, and thus sharply reduce the equilibrium spin.

Magnetic fields in the plunging region have received some attention in recent years, beginning with the work of Krolik (1999) and Gammie (1999), based on earlier work by Takahashi et al. (1990) and others. The inflow model of Gammie (1999) considered an inflowing plasma near the equatorial plane. It assumed that a well-ordered magnetic field threaded both the plunging region and the inner edge of the disk, and integrated the resulting one dimensional steady flow equations. Li (2000, 2002) considered a model in which a thin disk is connected to the black hole via high latitude field lines, rather than through the inflow itself. Agol & Krolik (2000) considered the implications of the magnetic accretion torques, including the possible change in surface brightness of the disk.

Recently it has become possible to study the dynamics of nonspherical black hole accretion numerically in full general relativity (Gammie, McKinney, & Tóth 2003; De Villiers & Hawley 2003). Given an appropriate set of initial and boundary conditions, this allows one to calculate \( \frac{dj}{dt} \) directly for a nonradiative flow, within the magnetohydrodynamic (MHD) approximation.

Here we consider a sequence of initial conditions in which \( j \) alone is varied and ask for which model is spin equilibrium, \( s = 0 \), achieved. We use HARM (Gammie, McKinney, & Tóth 2003) to integrate the equations of ideal, general relativistic magnetohydrodynamics (GRMHD) in a stationary, Kerr background spacetime in a variant of Kerr-Schild coordinates. Axisymmetry is assumed. Details of the scheme, the coordinate system, and an extensive series of convergence tests may be found in Gammie, McKinney, & Tóth (2003).

Our initial conditions contain a Fishbone & Moncrief (1976) torus with inner radius at \( r = 6M \) and pressure maximum at \( r = 12M \). The initial magnetic field is purely poloidal and field lines follow isodensity contours. It is derived from a vector potential \( A_\phi \propto \text{MAX}(\rho_0/\rho_{0,\text{min}} - 0.2, 0) \). The field is constrained to have a minimum ratio of gas to
magnetic pressure of 100. Our equation of state is $p = (\gamma - 1)u$, where $p$ is the gas pressure, $u$ is the internal energy, and $\gamma = 4/3$.

We set the inner boundary of the computational domain at $r_{in} = 0.98 r_h$ where $r_h = (1 + \sqrt{1 - j^2})$ is the event horizon radius. Because the inner boundary is inside the event horizon it is causally isolated from the rest of the flow. The outer boundary of the computational domain is located at $r_{out} = 40M$. This is distant enough that the influence of the outer boundary on the inner accretion flow ($r \sim M$) is negligible. Outflow boundary conditions (zero order extrapolation of primitive variables with a switch forbidding inflow) are used at the outer boundary. Tests indicate that all results are independent of $r_{in}$ and $r_{out}$, unless $r_{in}$ is outside the horizon. Our numerical resolution is $256 \times 256$ in zones equally spaced in the coordinates $x_1 = \ln r$ and $x_2$ such that $\theta = \pi x_2 + (1/2)(1 - H)\sin(2\pi x_2)$, where $H$ is a parameter that gradually concentrates zones toward to the equator as $H$ is decreased from 1 to 0. Here we use $H = 0.3$. We find that varying the resolution by a factor of 2 in either direction leaves our results unchanged.

Figure 1 illustrates the outcome of a typical evolution of this configuration around a black hole with $j = 0.75$. It shows the logarithm of the density field in the $R = r\cos(\theta), Z = r\sin(\theta)$ plane. The disk has become turbulent due to the magnetorotational instability (Balbus & Hawley 1991), and angular momentum transport by MHD turbulence leads to gradual inflow along the equator. The overall structure of the flow is similar to that observed by de Villiers et al. (2003), with an evacuated funnel near the poles, outflow at intermediate latitudes, a nearly-Keplerian equatorial torus, and a plunging region between the torus and the event horizon.

Figure 2 shows the evolution of $\dot{M}_0, \dot{E}/\dot{M}_0$, and $\dot{J}/(\dot{M}_0 M)$ for a model with $j = 0.9375$. The cyan lines show the values expected for the classical thin disk in which the specific energy and angular momentum of accreted material is equal to that of a particle on the ISCO, as in Bardeen (1970). While the specific energy of accreted material is accurately predicted by the thin disk model, the specific angular momentum is substantially lower. This is a result of ordered magnetic fields in the plunging region which transport angular momentum outward into the bulk of the disk.

A similar suppression of accreted angular momentum has been observed in fully relativistic MHD simulations of accretion onto black holes by de Villiers & Hawley (2003) and de Villiers et al. (2003). In particular, Table 2 of the latter can be used to estimate $s = 1.64$ for their model with $j = 0.5$. This is in very close agreement with $s = 1.66$ for our model. Given that different initial conditions and completely different numerical methods were used for these two calculations, and that our calculation is two dimensional while de Villiers et al.'s calculation is three dimensional, the agreement is remarkable. We also note that the
$j = 0.998$ calculation by de Villiers et al. (2003) shows $s < 0$.

Figure 3 shows the dimensionless spin evolution factor $s = (dj/dt)(M/\dot{M}_0)$ for four separate models with $j = 0.5, 0.75, 0.88, 0.97$. At $j = 0.97$ the hole is spinning down ($s < 0$). Figure 4 shows $s(j)$, including models that were excluded from Figure 3 for clarity. This sequence of accretion models reaches equilibrium ($s = 0$) for $j \approx 0.93$. This suggests that magnetic interactions can lead to spin equilibration at lower $j$ than the canonical $j = 0.998$ of Thorne (1974).

We are not suggesting that spin equilibrium is always reached at $j \approx 0.93$. Models with different initial conditions may produce different results. For example, the models we have considered here have a ratio of scale height to local radius $H/r \sim 0.2-0.3$ at pressure maximum. Thinner disks are likely to produce different results; indeed, in a later publication we will present results from the self-consistent evolution of a thin disk that closely match the predictions of a classical thin disk model. But thinner disks imply lower accretion rates, so thin disk accretion will have lower weight in determining the black hole spin than thick disk accretion over a comparable timescale. To sum up, what we have shown is that there exist self-consistent GRMHD models with $j \gtrsim 0.93$ that are unambiguously spinning down the black hole: accretion need not necessarily lead to near-maximal rotation.

6. Conclusions

In this paper, we have considered astrophysical processes that influence the spin evolution of black holes. Fully relativistic collapse calculations suggest that the initial spin of a newborn black hole $j \lesssim 0.75-0.9$, where the upper limit applies to the collapse of maximally and uniformly rotating massive stars. The outcome of subsequent mergers with black holes of comparable mass is not yet fully understood, but current best estimates suggest a final spin $j \sim 0.8-0.9$ (see Table 1). Mergers with black holes of much smaller mass can be treated in the test-particle approximation and, following Hughes & Blandford (2003), we have presented an argument showing that such mergers tend to spin down the black hole to $j \ll 1$, provided that the small black holes have isotropically distributed orbital angular momentum.

We have also presented results from fully relativistic magnetohydrodynamic models of accretion onto a rotating hole. These show that, at least for the particular series of thick disk models we consider, spin equilibrium is reached at $j \approx 0.93$. This demonstrates that accretion need not lead to near-maximal rotation. Our models have a ratio of scale height to local radius $H/r \sim 0.2-0.3$, and thus correspond to near-Eddington accretion rates. Accretion at
lower rates (through thinner disks) may be capable of producing higher spin, provided that the orbital angular momentum of the accreting material remains aligned with the black hole spin. If the orbital plane of the accreting material varies, as seems likely (see the discussion of Natarajan & Pringle 1998), even thin disk accretion may be unable to produce $j \approx 1$.

All these results suggest that near-maximal rotation of black holes is neither necessary nor likely. Black hole spins $j \sim 0.7 - 0.95$ are produced in a variety of scenarios. This corresponds to thin disk radiative efficiencies of 10%–19%, which is broadly consistent with the radiative efficiencies required by Soltan-type arguments (Yu & Tremaine 2002; Elvis, Risaliti, & Zamorani 2002). Such modest spins are also not in conflict with the idea that radio galaxies are powered by black hole spindown. The Blandford & Znajek (1977) luminosity of a black hole scales as $j^2 (B^r)^2$ where $B^r$ is the mean radial magnetic field on the event horizon. Unless $B^r$ is a sharply increasing function of $j$, the Blandford-Znajek luminosity of a black hole with $j \sim 0.9$ is not very different from that of a nearly maximally rotating black hole. Only if black holes were built up mainly through thin disk (sub-Eddington) accretion of material in a fixed orbital plane would near-maximal rotation be the norm.

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REFERENCES

Abramowicz, M., Jaroszinski, M., & Sikora, M. 1978, AA, 63, 221
Dreyer, O., Kelly, B., Krishnan, B., Finn, L.S., Garrison, D., Lopez-Aleman, R. 2003, gr-qc/0309007
Gnedin, O. Y. 2001, Class. & Quan. Grav., 18, 3983
Hughes, S. A. 2001, Phys. Rev. D, 64, 64004
Tanaka, Y. et al. 1995, Nature, 375, 659
Thorne, K. S. 1995, Seventeenth Texas Symposium on Relativistic Astrophysics and Cosmology, 759, 127
Fig. 1.— A snapshot from an evolution of a weakly magnetized Fishbone-Moncrief torus around a $j = 0.75$ black hole. Color corresponds to $\log(\rho_0)$. Red is high rest-mass density, and black is low.
Fig. 2.— Evolution of the mass, energy, and angular momentum accretion rate for a weakly magnetized Fishbone-Moncrief tori around a black hole with $j = 0.9375$.

Fig. 3.— Evolution of $s = (dj/dt)(M/\dot{M}_0)$ for a series of four Fishbone-Moncrief tori.
Fig. 4.— Time-averaged value of $s$ for a sequence of Fishbone-Moncrief tori. The squares indicate data points from simulations, while the thin line indicates values expected for a thin disk.
Table 1: Representative values for nonspinning binary black holes

<table>
<thead>
<tr>
<th>Reference</th>
<th>$E_b^a$</th>
<th>$J^b$</th>
<th>$\Omega^c$</th>
<th>$J/M^2d$</th>
<th>$(J/M^2)_{\text{max}}^e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schwarzschild</td>
<td>-0.0572</td>
<td>3.464</td>
<td>0.068</td>
<td>0.8913</td>
<td>0.9897</td>
</tr>
<tr>
<td>Cook (1994)</td>
<td>-0.09030</td>
<td>2.976</td>
<td>0.172</td>
<td>0.7788</td>
<td>0.9578</td>
</tr>
<tr>
<td>Baumgarte (2000)</td>
<td>-0.092</td>
<td>2.95</td>
<td>0.18</td>
<td>0.773</td>
<td>0.955</td>
</tr>
<tr>
<td>Grandclement et al. (2002)</td>
<td>-0.068</td>
<td>3.36</td>
<td>0.103</td>
<td>0.869</td>
<td>0.985</td>
</tr>
<tr>
<td>Damour et al. (2000)</td>
<td>-0.0668</td>
<td>3.27</td>
<td>0.0883</td>
<td>0.846</td>
<td>0.980</td>
</tr>
</tbody>
</table>

$^a$ Binding energy per unit reduced mass at the ISCO.
$^b$ Angular momentum per unit reduced mass at the ISCO.
$^c$ Orbital angular velocity at the ISCO.
$^d$ Estimated spin parameter of final black hole.
$^e$ Maximum spin parameter of final black hole (see text).