Mixing and Screening of Photons and Gluons in a Color Superconductor

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We calculate the Debye and Meissner masses of photons and gluons for a spin-one color superconductor in the polar and color-spin-locked phases as well as for a spin-zero color superconductor in the 2SC and color-flavor-locked phases. A general derivation for photon-gluon mixing is provided in terms of the QCD partition function. We show qualitatively and quantitatively which of the gauge bosons attain a mass via the Anderson-Higgs mechanism. Contrary to the spin-zero phases, both spin-one phases exhibit an electromagnetic Meissner effect.

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I. INTRODUCTION

Sufficiently cold and dense quark matter is a color superconductor. Due to asymptotic freedom, one-gluon exchange is the dominant interaction in a dense quark system. The one-gluon exchange interaction is attractive in the color-antitriplet channel. Similar to electron Cooper pairing in ordinary superconductors [1], this leads to the formation of Cooper pairs at the quark Fermi surface [2, 3] (for a review, see [4]). Cold and dense matter could occur in the interior of compact stellar objects, such as neutron stars [5].

In ordinary superconductivity, because of spontaneous symmetry breaking of the electromagnetic gauge group $U(1)_{em}$, the photon attains a mass via the Anderson-Higgs mechanism. Physically, this mass corresponds to screening of the magnetic field in superconducting matter. One of the consequences is the electromagnetic Meissner effect, i.e., the expulsion of magnetic fields. In order to investigate the Anderson-Higgs mechanism in a color superconductor, one has to take into account the color gauge group $SU(3)_c$ in addition to the electromagnetic $U(1)_{em}$. The symmetry breaking pattern of $SU(3)_c \times U(1)_{em}$ determines which gauge bosons (gluons and photons) attain a mass. The order parameter for quark Cooper pairing defines the specific symmetry breaking pattern. In this paper we provide a general treatment of the influence of color superconductivity on color and electromagnetic fields and apply it to four different phases of superconducting quark matter.

First, we consider a system of two quark flavors. Here, two quarks can form Cooper pairs in the color-antitriplet, flavor-singlet channel [2], commonly called the 2SC phase. The Cooper pairs in this phase carry total spin zero, $J = 0$. Moreover, in the 2SC phase the quarks of one color are supposed to remain unpaired.

Second, we study three quark flavors in the color-flavor-locked (CFL) phase [6]. In this phase, the condensate consists of pairs in the color-antitriplet, flavor-antitriplet channel. All colors and flavors participate in pairing and the condensate is invariant under joint color-flavor rotations. Also in this phase the Cooper pairs carry spin zero.

In the third and fourth phases considered here, quarks form spin-one Cooper pairs, $J = 1$ [2, 7, 8]. For symmetry reasons, a total spin $J = 1$ is required for Cooper pairs of quarks which carry the same flavor. The simplest case is a system of only one quark flavor. Another possibility is a many-flavor system where each flavor separately forms Cooper pairs. In the polar phase [9, 10], the spin of the pair points in a fixed spatial direction and, as in the 2SC phase, one color remains unpaired. In the color-spin-locked (CSL) phase [2], each spatial direction is associated with a direction in color space. The case of spin-one Cooper pairing of quarks is in some respect similar to atomic pairing in superfluid Helium 3 [11]. Therefore, similar phases occur in superconducting quark matter, e.g., the CSL phase corresponds to the “B phase” in Helium 3.

Since the order parameter has a particular color-flavor-spin structure in each phase, the pattern of symmetry breaking differs from phase to phase, which consequently have to be investigated separately in order to identify the gauge bosons which attain a mass through the Anderson-Higgs mechanism. In some phases, namely the 2SC, CFL, and polar phases, it turns out that the physical fields are not the original gluon and photon fields but linear combinations

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of these, i.e., there is mixing between photons and gluons [12, 13]. The mixing angle is the analogue of the Weinberg angle in the standard model of electroweak interactions.

Our paper is organized as follows. In Sec. II we discuss the QCD partition function for massless quarks in the presence of gluon and photon gauge fields. We include a diquark condensation term to account for color superconductivity. The goal of this section is to determine the gauge boson propagators. This fundamental study provides the basis for the following sections. It is general in the sense that we consider gauge fields for arbitrary four-momenta \( P \) while afterwards we focus on the limiting case \( P \to 0 \) in order to discuss screening of static and homogeneous electric and magnetic fields.

In Sec. III, we use a group-theoretical argument, namely the invariance of the gap matrix under some subgroup of \( SU(3)_c \times U(1)_{em} \), to show which of the gauge bosons attain a mass via the Anderson-Higgs mechanism. In this section we determine in a qualitative manner if electric and magnetic gauge fields are screened. Moreover, we derive the mixing angles describing the rotation of the gauge fields.

In the following two sections we extend the results of Sec. III by performing a quantitative calculation of the Debye and Meissner masses for all gauge bosons. These masses correspond to the inverse screening lengths for electric and magnetic fields, respectively. They are obtained from the longitudinal and transverse components of the polarization tensors in the zero-energy, low-momentum limit. The first of these two sections, Sec. IV, deals with the technical details of the calculation. In its first part, Sec. IV A, we perform all calculations that do not depend on the special color-superconducting phase. A part of these calculations, namely the integrals over the absolute value of the quark momentum, are deferred to App. A. In the second part, Secs. IV B - IV E, we specify the results for the above mentioned four color-superconducting phases, i.e., we derive the particular expressions for the relevant components of the polarization tensors. Readers who are not interested in the technical details may skip this section. All results and a discussion are given in Sec. V. We present all screening masses for photons and gluons which yield the mixing angles and the masses for the (physically relevant) mixed, or rotated, gauge bosons. One of the points we discuss in this section is in which cases these mixing angles differ from the ones obtained in Sec. III. We conclude and summarize our study in Sec. VI.

Our convention for the metric tensor is \( g^{\mu \nu} = \text{diag}\{1, -1, -1, -1\} \). Our units are \( \hbar = c = k_B = 1 \). Four-vectors are denoted by capital letters, \( K = K^\mu = (k_0, \mathbf{k}) \), and \( k \equiv |\mathbf{k}| \), while \( k \equiv k/k \). We work in the imaginary-time formalism, i.e., \( T/V \sum_k = T \sum_n \int d^3k/(2\pi)^3 \), where \( n \) labels the Matsubara frequencies \( \omega_n = i k_0 \). For bosons, \( \omega_n = 2n\pi T \), for fermions, \( \omega_n = (2n+1)\pi T \).

II. MIXING OF GLUONS AND PHOTONS

In this section we provide the theoretical basis for the calculation of the Debye and Meissner masses in a color superconductor. We clarify how “mixing” of gluon and photon fields has to be understood. The results of this section are the final form for gluon and photon propagators, Eq. (38), and the mixing angle between gluon and photon fields, Eq. (43).

A. Gluon and photon propagators

We start from the grand partition function of QCD for massless quarks in the presence of gluon fields \( A^a_\mu \) and a photon field \( A^\mu \),

\[
Z = \int \mathcal{D}A e^{S_A \mathcal{Z}_q[A]}.
\]

Here, \( S_A \) is the action for gluon and photon fields. It consists of three parts,

\[
S_A = S_{F^2} + S_{gf} + S_{FPG},
\]

where \( S_{gf} \) and \( S_{FPG} \) are the gauge fixing and ghost terms, respectively, and

\[
S_{F^2} \equiv -\frac{1}{4} \int_X (F^a_{\mu \nu} F^{a \mu \nu} + F^{\mu \nu} F_{\mu \nu})
\]

is the gauge field part. The space-time integration is defined as \( \int_X \equiv \int_0^{1/T} d\tau \int_V d^3x \), where \( T \) is the temperature and \( V \) the volume of the system. The field strength tensors \( F_{\mu \nu}^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc} A^b_\mu A^c_\nu \) correspond to the gluon
fields $A_\mu$, while $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ corresponds to the photon field $A_\mu$. The functional $Z_q[A]$ is the grand partition function of massless quarks in the presence of gluon and photon fields and a chemical potential $\mu$. It is given by

$$ Z_q[A] = \int D\hat{\psi}D\psi \exp \left[ \int_X \hat{\psi} \left( i \gamma^\mu \partial_\mu + \mu \gamma_0 + g \gamma^\mu A_\mu a + \epsilon \gamma^\mu A_\mu Q \right) \psi \right] , \quad (4) $$

where $T_a$ are the generators in the fundamental representation of the gauge group of strong interactions, $SU(3)_c$, and $Q$ is the generator of the electromagnetic $U(1)_em$. The coupling constants for the strong interaction and electromagnetism are denoted by $g$ and $\epsilon$, respectively. The quark fields $\psi$ are spinors in color, flavor, and Dirac space.

In order to take into account Cooper pairing of the quarks we include a diquark source term. This has been done for instance in Refs. [2, 14]. We generalize this treatment by also taking into account the photon field. Then, after integrating out the fermion fields, the partition function is given by [14]

$$ Z = \int DA \exp \left[ S_A + \frac{1}{2} \text{Tr} \ln(S^{-1} + A_\mu^a \hat{\Gamma}_a^\mu) \right] , \quad (5) $$

where the trace runs over space-time, Nambu-Gor’kov, color, flavor, and Dirac indices. The sum over $a$ now runs from 1 to 9, where $A_\mu^a \equiv A_\mu^a$ is the photon field. We also defined the $2 \times 2$ Nambu-Gor’kov matrices

$$ \hat{\Gamma}_a^\mu = \text{diag} \left( \Gamma_a^\mu, \Gamma_a^\mu \right) \equiv \begin{cases} \text{diag}(g \gamma^\mu T_a, -g \gamma^\mu T_a^T) & \text{for } a = 1, \ldots, 8 , \\ \text{diag}(\epsilon \gamma^\mu Q, -\epsilon \gamma^\mu Q) & \text{for } a = 9 . \end{cases} \quad (6) $$

$S \equiv S(X,Y)$ is the quasiparticle propagator in Nambu-Gor’kov space. In Eq. (5), we did not explicitly keep the fluctuations of the order parameter field, cf. Eq. (26) of Ref. [14]. This is possible, since we are only interested in the gluon and photon propagator. Nevertheless, it should be kept in mind that, like in any theory with spontaneously broken gauge symmetry, these fluctuations mix with the (4-)longitudinal (unphysical) components of the gauge field. As usual, using a suitable choice of ’t Hooft gauge, the fluctuations can be decoupled from the gauge field. Then, in unitary gauge, one finds that the gauge boson propagator is explicitly (4-)transverse [14], in accordance with general principles [15, 16].

Expanding the logarithm in Eq. (5) to second order in the gauge fields, we obtain

$$ \frac{1}{2} \text{Tr} \ln(S^{-1} + A_\mu^a \hat{\Gamma}_a^\mu) \simeq \frac{1}{2} \text{Tr} \ln S^{-1} + \frac{1}{2} \text{Tr}[A_\mu^a \hat{\Gamma}_a^\mu S] - \frac{1}{4} \text{Tr}[A_\mu^a \hat{\Gamma}_a^\mu S A_\nu^a \hat{\Gamma}_a^\nu S] . \quad (7) $$

Following Ref. [14], the sum of all terms which are quadratic in the gauge fields will be denoted by $S_2$. $S_2$ does not only contain the pure gluon and photon terms but also two terms which mix gluon and photon fields. In order to perform the trace over space-time, we introduce the Fourier transforms

$$ S(X,Y) = \frac{T}{V} \sum_K e^{-iK \cdot (X-Y)} S(K) , \quad (8a) $$

$$ A_\mu^a(X) = \sum_P e^{-iP \cdot X} A_\mu^a(P) , \quad (8b) $$

where we used translational invariance, $S(X,Y) = S(X-Y)$. Then we obtain

$$ S_2 = -\frac{1}{4} \int_{X,Y} \text{Tr}[A_\mu^a(X) \hat{\Gamma}_a^\mu S(X,Y) A_\mu^b(Y) \hat{\Gamma}_b^\nu S(Y,X)] $$

$$ = -\frac{1}{4} \sum_{K, P} \text{Tr}[A_\mu^a(-P) \hat{\Gamma}_a^\mu S(K) A_\mu^b(P) \hat{\Gamma}_b^\nu S(K-P)] $$

$$ = -\frac{V}{2T} \sum_P A_\mu^a(-P) \Pi_{\mu\nu}^{ab}(P) A_\mu^b(P) , \quad (9) $$

where the trace now runs over Nambu-Gor’kov, color, flavor, and Dirac indices and where we defined the polarization tensor

$$ \Pi_{\mu\nu}^{ab}(P) \equiv \frac{V}{2} \sum_K \text{Tr}[\hat{\Gamma}_a^\mu S(K) \hat{\Gamma}_b^\nu S(K-P)] . \quad (10) $$
The Nambu-Gor’kov quasiparticle propagator in momentum space is defined as
\[
S(K) = \begin{pmatrix}
G^+(K) & \Xi^-(K) \\
\Xi^+(K) & G^-(K)
\end{pmatrix},
\]
with the quasiparticle propagators \(G^\pm(K)\) and the “anomalous” propagators \(\Xi^\pm(K)\),
\[
G^\pm(K) = \left\{ [G_0^{\mp}]^{-1} - \Phi^\mp(K) G_0^\mp K \Phi^\pm(K) \right\}^{-1}, \quad \Xi^\pm(K) = -G_0^\mp K \Phi^\pm(K) G^\pm(K).
\]
Here, \(G_0^\pm(K) = (\gamma_\mu K^\mu \pm \mu_0^\gamma) \Phi^{-1}(K)\) is the free propagator. In these expressions, we have put the regular self-energies (cf. Ref. [4]) to zero since, to the order we are computing, they do not influence the results for the polarization tensors. Following Ref. [10], we write the gap matrix \(\Phi^+\) for color-superconducting quark matter, where ultrarelativistic quarks form Cooper pairs in the even-parity channel, as
\[
\Phi^+(K) = \sum_{n=\pm} \phi^e(K) M_k \Lambda_k^e.
\]
Here, \(\phi^e(K)\) is the gap function, \(\Lambda_k^e = (1 + e\gamma_0 \gamma \cdot \hat{k})/2\) are projectors onto states of positive or negative energy, and \(M_k\) is a matrix in color, flavor, and spin space that determines the color-superconducting phase. One can choose \(M_k\) such that \([M_k, \Lambda_k^e] = 0\). Furthermore, \(\Phi^{-}(K) \equiv \gamma_0 (\Phi^+(K))^\dagger \gamma_0\). For the color-superconducting phases considered here, \(\gamma_0 M_k^\dagger M_k \gamma_0 = M_{k_1} M_{k_1}^\dagger\). In this case,
\[
G^\pm(K) = [G_0^{\mp}(K)]^{-1} \sum_{e=\pm} \sum_{r=1,2} \mathcal{P}_k^{e,r} \Lambda_k^{e,r} \frac{1}{k_0^2 - \left[\epsilon_{k,r}(\phi^e)\right]^2},
\]
where \(\mathcal{P}_k^{e,r}\) are projectors onto the eigenspaces of the hermitian matrix \(\gamma_0 M_k^\dagger M_k \gamma_0 - \lambda_{1,2}\). In all phases considered here, there are two eigenvalues of this matrix, denoted by \(\lambda_1\) and \(\lambda_2\). They appear in the quasiparticle excitations energies
\[
\epsilon_{k,r}(\phi^e) \equiv [(\mu - ek)^2 + \lambda_r |\phi^e|^2]^{1/2}.
\]
The projectors are given by
\[
\mathcal{P}_k^{1,2} = \frac{\gamma_0 M_k^\dagger M_k \gamma_0 - \lambda_{1,2}}{\lambda_{1,2} - \lambda_{2,1}}.
\]
The anomalous propagators can be written as
\[
\Xi^+(K) = -\sum_{e,r} \gamma_0 M_k \mathcal{P}_k^{e,r} \Lambda_k^{e,r} \phi^e(K) \frac{1}{k_0^2 - \left[\epsilon_{k,r}(\phi^e)\right]^2}, \quad \Xi^-(K) = -\sum_{e,r} M_k^\dagger \mathcal{P}_k^{e,r} \Lambda_k^{e,r} \phi^{e*}(K) \frac{1}{k_0^2 - \left[\epsilon_{k,r}(\phi^e)\right]^2}.
\]
As in Ref. [14], we introduce a set of complete, orthogonal projectors for each 4-vector \(P^\mu = (p_0, \mathbf{p})\),
\[
Q_1^{\mu\nu} \equiv g^{\mu\nu} - Q_2^{\mu\nu} - Q_3^{\mu\nu}, \quad Q_2^{\mu\nu} \equiv \frac{N^\mu N^\nu}{N^2}, \quad Q_3^{\mu\nu} \equiv \frac{P^\mu P^\nu}{P^2}.
\]
With \(N^\mu \equiv (p_0 p^2 / P^2, p_0^0 p^2 / P^2)\), the projector \(Q_2^{\mu\nu}\) projects onto the one-dimensional subspace that is \((4-)\) orthogonal to \(P^\mu\) but \((3-)\) parallel to \(\mathbf{p}\). The operator \(Q_3^{\mu\nu}\) projects onto the one-dimensional subspace parallel to \(P^\mu\). Consequently, \(Q_2^{\mu\nu}\) projects onto a two-dimensional subspace that is \((4-)\) orthogonal to both \(P^\mu\) and \(N^\nu\). Furthermore, this subspace is \((3-)\) orthogonal to \(\mathbf{p}\). With the additional tensor
\[
Q_4^{\mu\nu} \equiv N^\mu P^\nu + P^\mu N^\nu
\]
we can decompose the polarization tensor [16],
\[
\Pi^{\mu\nu}_{ab}(P) = \sum_i \Pi_{ab}^i(P) Q_i^{\mu\nu}.
\]
(In the notation of Refs. [14, 16], \(Q_1 \equiv A, Q_2 \equiv B, Q_3 \equiv E, Q_4 \equiv C\)). Since for \(i = 1, 2, 3, Q_i^{\mu\nu} Q_{4\nu\mu} = 0\), the coefficients \(\Pi_{ab}^i(P)\) of the projection operators are given by
\[
\Pi_{ab}^i(P) = \frac{\Pi_{ab}^{\mu\nu}(P) Q^{\mu\nu}}{Q_i^{\lambda\lambda}}, \quad i = 1, 2, 3.
\]
The remaining coefficient corresponding to $Q_4$ is

$$\Pi^{4}_{ab}(P) = \frac{\Pi^{\mu
u}_{ab}(P)Q_{4\mu\nu}}{Q^{\alpha}_{4}\overline{Q}_{4\alpha}}.$$  \hspace{1cm} (22)

The explicit forms for $\Pi^{i}_{ab}(P)$ are given in Ref. [14], Eqs. (42) and (43). Employing the decomposition of the polarization tensor in Eq. (9), we obtain

$$S_2 = -\frac{1}{2}V \left( \sum\sum A^{a}_{\mu}(-P)Q_{4\mu}^{\nu}\Pi^{\nu}_{ab}(P)A^{b}_{\nu}(P) \right).$$  \hspace{1cm} (23)

Now we add the free field term

$$S^{(0)}_{F2} = -\frac{1}{2}V \left( \sum\sum A^{a}_{\mu}(-P)(P^{2}g^{\mu\nu} - P^{\mu}P^{\nu})A^{b}_{\nu}(P) \right) = -\frac{1}{2}V \left( \sum\sum A^{a}_{\mu}(-P)P^{2}(Q_{1}^{\mu\nu} + Q_{2}^{\mu\nu})A^{b}_{\nu}(P) \right).$$  \hspace{1cm} (24)

We obtain

$$S_2 + S^{(0)}_{F2} = -\frac{1}{2}V \sum\sum A^{a}_{\mu}(-P)\left\{ \sum_{i=1}^{2}Q_{i}^{\mu\nu}\left[ \delta_{ab}P^{2} + \Pi^{i}_{ab}(P) \right] + \sum_{i=3}^{4}Q_{i}^{\mu\nu}\Pi^{i}_{ab}(P) \right\}A^{b}_{\nu}(P).$$  \hspace{1cm} (25)

Since we finally want to read off the gluon and photon propagators, we have to transform this expression in two ways. First, concerning the Dirac structure it is necessary to get rid of the term proportional to $Q_{4}$ which mixes the longitudinal mode (3-parallel to $p$) with the unphysical mode (4-parallel to $P^{\mu}$). Then, inverting the inverse propagator becomes trivial, because it is just a linear combination of the complete, orthogonal projectors $Q_{1}, Q_{2}, Q_{4}$. Second, in order to obtain the physical modes we have to diagonalize the resulting $9 \times 9$ matrices which, after eliminating $Q_{4}$, will replace $\delta_{ab}P^{2} + \Pi^{i}_{ab}(P), i = 1, 2, \text{and } \Pi^{i}_{ab}(P), i = 3, 4$ in Eq. (25).

We first write $S_2 + S^{(0)}_{F2}$ as

$$S_2 + S^{(0)}_{F2} = -\frac{1}{2}V \sum\sum \left\{ \sum_{i=1}^{2}A^{a}_{\mu}(-P)\left[ \delta_{ab}P^{2} + \Pi^{i}_{ab}(P) \right] A^{b}_{\nu}(P) + A^{a}_{\mu}(-P)\Pi^{3}_{ab}(P)A^{b}_{3\mu}(P) \right.$$

$$\left. + A^{a}_{3\mu}(P)N_{\mu}^{i}\Pi^{i}_{ab}(P)P_{\nu}A^{b}_{3\nu}(P) + A^{a}_{3\mu}(P)\Pi^{4}_{ab}(P)N_{\mu}A^{b}_{2\nu}(P) \right\}. \hspace{1cm} (26)$$

where $A^{a}_{1\mu}(P) \equiv Q_{i}^{\mu\nu}A^{a}_{\nu}(P)$ are the gauge fields projected on the subspace corresponding to $Q_{i}$. Now one can “unmix” the fields $A^{a}_{2\nu}(P)$ and $A^{a}_{3\nu}(P)$ by the following transformation of the unphysical field component $A^{a}_{3\mu}(P)$, which does not affect the final result since we integrate over all fields in the partition function,

$$A^{a}_{3\mu}(P) \rightarrow A^{a}_{3\mu}(P) - P_{\mu}^{-1}\Pi^{3}(P)_{ab}^{-1}A^{b}_{2\nu}(P). \hspace{1cm} (27a)$$

$$A^{a}_{3\mu}(P) \rightarrow A^{a}_{3\mu}(P) - Q^{ab}_{\nu}\Pi^{3}(P)_{ab}^{-1}A^{b}_{2\nu}(P). \hspace{1cm} (27b)$$

After this transformation, one is left with quadratic expressions in the projected fields. The transformation modifies the term corresponding to $i = 2$,

$$S_2 + S^{(0)}_{F2} = -\frac{1}{2}V \sum\sum A^{a}_{\mu}(-P)\left( Q_{i}^{\mu\nu}\left[ \delta_{ab}P^{2} + \Pi^{i}_{ab}(P) \right] \right.$$

$$\left. + Q_{i}^{\mu\nu}\left[ \delta_{ab}P^{2} + \Pi^{3}_{ab}(P) - P^{2}N^{2}\Pi^{i}_{ab}(P) \right] \right.\Pi^{3}(P)_{ab}^{-1}A^{b}_{2\nu}(P) \right) \left. + Q_{i}^{\mu\nu}\Pi^{i}_{ab}(P)A^{b}_{2\nu}(P) \right\}. \hspace{1cm} (28)$$

Before we do the diagonalization in the 9-dimensional gluon-photon space, we add the gauge fixing term $S_{gf}$. We choose the following gauge with gauge parameter $\lambda$,

$$S_{gf} = -\frac{1}{2\lambda}V \sum\sum A^{a}_{\mu}(-P)P^{\mu}P^{\nu}A^{b}_{\nu}(P) - \frac{1}{2\lambda}V \sum\sum A^{a}_{\mu}(-P)P^{2}Q^{\mu\nu}A^{b}_{\nu}(P). \hspace{1cm} (29)$$

This looks like a covariant gauge but, including the fluctuations of the order parameter, which we did not write explicitly, it is actually some kind of ’t Hooft gauge, cf. Eq. (50) of Ref. [14]. Moreover, had we fixed the gauge prior
to the shift (27) of the gauge fields, we would have to start with an expression which is non-local and also involves the (3-)longitudinal components $A_i^a \mu$ of the gauge field.

Adding the gauge fixing term (29) to Eq. (28) leads to

$$S_2 + S_f^{(0)} + S_{gf} = -\frac{1}{2} V \sum_P \sum_{i=1}^{3} A_i^a(-P) Q_i^{\mu\nu} \Theta_i^{ab}(P) A_b^b(P) ,$$

with

$$\Theta_i^{ab}(P) = \begin{cases} 
\delta_{ab} P^2 + \Pi_1^{ab}(P) & \text{for } i = 1 , \\
\delta_{ab} P^2 + \Pi_2^{ab}(P) - P^2 N^2 \Pi_4^{ab}(P) \left[ \Pi_3^{ab}(P) \right]^{-1} \Pi_4^{cd}(P) & \text{for } i = 2 , \\
\delta_{ab} P^2 + \Pi_3^{ab}(P) & \text{for } i = 3 . 
\end{cases}$$

In order to obtain the physical modes, we have to diagonalize the $9 \times 9$ matrices $\Theta_i^{ab}(P)$. Since $\Theta_i^{ab}(P)$ is real and symmetric, diagonalization is achieved via an orthogonal transformation with a $9 \times 9$ matrix $O_i(P)$,

$$S_2 + S_f^{(0)} + S_{gf} = -\frac{1}{2} V \sum_P \sum_{i=1}^{3} \tilde{A}_i^a(-P) Q_i^{\mu\nu} \tilde{\Theta}_i^{ab}(P) \tilde{A}_b^b(P) ,$$

where

$$\tilde{A}_i^a(P) = O_i^{ab}(P) A_b^b(P)$$

are the rotated gauge fields and

$$\tilde{\Theta}_i^{ab}(P) = O_i^{ab}(P) \Theta_i^{bc}(P) O_i^{ac}(P)$$

are diagonal matrices. The index $i$ in $\tilde{A}_i^a$ has a different origin than in $A_i^a$ introduced in Eq. (26). For $\tilde{A}_i^a$, it indicates that for each $i = 1, 2, 3$ one has to perform a separate diagonalization. For $A_i^a$ it denoted the projection corresponding to the projector $Q_i^{ab}$.

Note that the orthogonal matrix $O_i(P)$ depends on $P^\mu$. For energies and momenta much larger than the superconducting gap parameter the polarization tensor is explicitly $(4)$-transverse, $\Pi^3 = 0$, and diagonal in $a,b$. In this case, $O_i(P) \rightarrow 1$. Consequently, the gauge fields are not rotated. However, in the limit $p_0 = 0$, $p \rightarrow 0$ it is known that gluons and photons mix at least in the 2SC and CFL phases [12], $O_i(P) \neq 1$. Thus, the mixing angle between gluons and photons, which will be discussed in Sec. II B, is in general a function of $P^\mu$ and interpolates between a nonvanishing value at $p_0 = 0$, $p \rightarrow 0$ and zero when $p_0, p \rightarrow \infty$. Note also that the orthogonal matrix $O_i(P)$ depends on $i$, i.e., it may be different for longitudinal and transverse modes. We comment on this in more detail below.

From Eq. (32), we can immediately read off the inverse propagator for gluons and photons. It is

$$\Delta^{-1}_{\mu\nu}(P) = \sum_{i=1}^{3} Q_i^{\mu\nu} \tilde{\Theta}_i^{ab}(P) .$$

From the definition of $\tilde{\Theta}_i^{ab}(P)$ we conclude

$$\tilde{\Theta}_i^{ab}(P) = \begin{cases} 
P^2 + \tilde{\Pi}_i^{ab}(P) & \text{for } i = 1, 2 , \\
\frac{1}{4} P^2 + \tilde{\Pi}_3^{ab}(P) & \text{for } i = 3 , 
\end{cases}$$

where

$$\tilde{\Pi}_i^{ab}(P) = \begin{cases} 
O_i^{ab}(P) \Pi_i^{bc}(P) O_i^{ac}(P) & \text{for } i = 1, 3 , \\
O_2^{ab}(P) \left\{ \Pi_1^{bc}(P) - P^2 N^2 \Pi_2^{bd}(P) \left[ \Pi_3^{bc}(P) \right]^{-1} \Pi_4^{cd}(P) \right\} O_2^{bc}(P) & \text{for } i = 2 .
\end{cases}$$

In the case $p_0 = 0$, using Eqs. (42) and (43) of Ref. [14] one realizes that the extra term involving $\Pi_3$ and $\Pi_4$ for $i = 2$ vanishes. Thus, in this case one only has to diagonalize the original polarization tensors $\Pi_{ab}$.
Finally, we end up with the gauge boson propagator

$$\Delta_{aa}^{\mu\nu}(P) = \frac{1}{P^2 + \Pi_{aa}^{1}(P)} Q_{1}^{\mu\nu} + \frac{1}{P^2 + \Pi_{aa}^{2}(P)} Q_{2}^{\mu\nu} + \frac{\lambda}{P^2 + \lambda \Pi_{aa}^{3}(P)} Q_{3}^{\mu\nu} . \quad (38)$$

Setting the gauge parameter $\lambda = 0$, we are left with the transverse and (3-)longitudinal modes, in accordance with general principles [15, 16]. The static color and electromagnetic properties of the color superconductor are characterized by the Debye masses $\tilde{m}_{D,a}$ and the Meissner masses $\tilde{m}_{M,a}$ which are defined as

$$\tilde{m}_{D,a}^2 \equiv - \lim_{p \to 0} \Pi_{aa}^{2}(0, p) = - \lim_{p \to 0} \tilde{\Pi}_{aa}^{0}(0, p) , \quad (39a)$$

$$\tilde{m}_{M,a}^2 \equiv - \lim_{p \to 0} \Pi_{aa}^{1}(0, p) = \frac{1}{2} \lim_{p \to 0} (\delta^{ij} - \hat{p}^i \hat{p}^j) \tilde{\Pi}_{aa}^{ij}(0, p) . \quad (39b)$$

Since the orthogonal matrices $O_i(P)$ are regular in the limit $p_0 = 0$, $p \to 0$, the masses can also be obtained by first computing $\lim_{p \to 0} \Pi_{\mu\nu}^{ab}(0, p)$ and then diagonalizing the resulting $9 \times 9$ mass matrix. In Sec. IV we use this method to compute $\tilde{m}_{D,a}^2$ and $\tilde{m}_{M,a}^2$, since the diagonalization of the matrix $\Pi_{ab}^{\mu\nu}(P)$ for arbitrary $P^\mu$ is too difficult.

**B. The mixing angle**

In this section we investigate the structure of the orthogonal matrices $O_i(P)$ which diagonalize the gauge field part of the grand partition function. In general, the matrices $O_i(P)$ mix all gluon components among themselves and with the photon. However, in the limit $p_0 = 0$, $p \to 0$ it turns out that the only non-zero off-diagonal elements of the tensor $\Pi_{ab} \equiv \lim_{p \to 0} \Pi_{aa}(0, p)$ are $\Pi_{88} = \Pi_{8\gamma} = \Pi_{8\gamma}$. Physically speaking, gluons do not mix among themselves and only the eighth gluon mixes with the photon. In this case, Eq. (37) reduces to the diagonalization of a $2 \times 2$ matrix. Consequently, the diagonalization is determined by only one parameter $\theta_i$ and the (nontrivial part of the) transformation operator reads

$$O_i = \begin{pmatrix} \cos \theta_i & \sin \theta_i \\ -\sin \theta_i & \cos \theta_i \end{pmatrix} . \quad (40)$$

The new fields are

$$\hat{A}_{\mu,i}^8 = \cos \theta_i A_{\mu}^8 + \sin \theta_i A_{\mu}^\gamma , \quad (41a)$$

$$\hat{A}_{\mu,i} = -\sin \theta_i A_{\mu}^8 + \cos \theta_i A_{\mu}^\gamma . \quad (41b)$$

The new polarization functions (eigenvalues of $\Pi_{ab}^i$) are

$$\hat{\Pi}_{88}^i = \Pi_{88}^i \cos^2 \theta_i + 2 \Pi_{8\gamma}^i \sin \theta_i \cos \theta_i + \Pi_{\gamma\gamma}^i \sin^2 \theta_i , \quad (42a)$$

$$\hat{\Pi}_{\gamma\gamma}^i = \Pi_{88}^i \sin^2 \theta_i - 2 \Pi_{8\gamma}^i \sin \theta_i \cos \theta_i + \Pi_{\gamma\gamma}^i \cos^2 \theta_i . \quad (42b)$$

The mixing angle $\theta_i$ is given by

$$\tan 2\theta_i = \frac{2 \Pi_{8\gamma}^i}{\Pi_{88}^i - \Pi_{\gamma\gamma}^i} . \quad (43)$$

If $[\Pi_{8\gamma}^i]^2 = \Pi_{88}^i \Pi_{\gamma\gamma}^i$, the determinant of $\Pi_{ab}^i$ is zero, which means that there is a vanishing eigenvalue. In this case, we have

$$\cos^2 \theta_i = \frac{\Pi_{88}^i}{\Pi_{88}^i + \Pi_{\gamma\gamma}^i} , \quad (44)$$

and the new polarization tensors, Eqs. (42), have the simple form

$$\hat{\Pi}_{88}^i = \Pi_{88}^i + \Pi_{\gamma\gamma}^i , \quad \hat{\Pi}_{\gamma\gamma}^i = 0 . \quad (45)$$

Physically, the vanishing polarization tensor for the rotated photon corresponds to the absence of the Meissner effect for $i = 1$, or the absence of Debye screening for $i = 2$. 
III. SYMMETRIES AND MIXING

In Section II B we have discussed the mixing angle $\theta_i$ that combines the electromagnetic field and the eighth gluon. The index $i$ refers to the projectors $Q'^\mu$ and thus, in general, the mixing angle could be different for transverse modes, $i = 1$, and longitudinal modes, $i = 2$.

In this section, we present a method to determine the mixing angles via a simple group-theoretical consideration. As we know from ordinary superconductors, the non-zero electric charge of a Cooper pair leads to the Meissner effect and thus corresponds to a non-vanishing Meissner mass $m_{M,\gamma}$ for the photon. Besides this magnetic screening there is also electric screening of photons described by the photon Debye mass $m_{D,\gamma}$. Of course, in a color superconductor, in addition to the electric charge we also have to take into account the color charge. The group-theoretical method applied in the following allows us to investigate whether there exists a (new) charge which generates an unbroken symmetry. In this case, a Cooper pair is neutral with respect to this charge, and consequently one expects that there is neither a Meissner effect nor Debye screening. The new charge is a linear combination of electric and color charges. Correspondingly, the associated gauge field is a linear combination of the photon and the eighth gluon field, which, in turn, defines the mixing angle. The group-theoretical method only allows to identify a new charge, and thus does not distinguish between electric Debye or magnetic Meissner screening. Consequently, the mixing angles for longitudinal (electric) and transverse (magnetic) modes deduced by this method are identical, $\theta_1 = \theta_2 = 0$.

A Cooper pair is neutral with respect to a charge, if the gap matrix $\Phi$ is invariant under a transformation generated by an operator $\tilde{Q}$ corresponding to this charge. In general, the gap matrix is a matrix in color, flavor, and spin space,

$$
\Phi = \phi_{ijk} e_i^c \otimes e_j^f \otimes e_k^d,
$$

where $\phi_{ijk}$ is the order parameter and $e_i^c \otimes e_j^f \otimes e_k^d$ is a basis for the color×flavor×spin representation of the group $SU(3)_c \times SU(N_f)_f \times SU(2)_j$. Since $\tilde{Q}$ generates a subgroup of the product group of strong and electromagnetic interactions, $SU(3)_c \times U(1)_{em}$, it can be found by the following invariance condition,

$$
\Phi = (g_c \times g_{em}) \Phi (g_c^T \times g_{em}^T),
$$

where $g_c \in SU(3)_c$ and $g_{em} \in U(1)_{em}$ act on $e_i^c$ and $e_j^f$, respectively. With $g_c = \exp(i\kappa_a T_a)$, $g_{em} = \exp(i\kappa Q)$, for infinitesimal transformations this condition can be written as

$$
0 = \phi_{ijk} \left[ i \kappa_a (T_a e_i^c e_j^f T_a^T \otimes e_k^d) + i \kappa e_i^c \otimes (Q e_j^f + e_j^f Q) \otimes e_k^d \right],
$$

and finding the charge $\tilde{Q}$ is equivalent to determining the real coefficients $\kappa_a$, $\kappa$, where $a = 1, \ldots, 8$. Using the representations and the order parameters for each phase as given in Table I we obtain

<table>
<thead>
<tr>
<th>Phase</th>
<th>Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>2SC:</td>
<td>$\kappa_1, \kappa_2, \kappa_3$ arbitrary, $\kappa_4 = \ldots = \kappa_7 = 0$, $\kappa_8 = -\frac{1}{\sqrt{3}} \kappa$</td>
</tr>
<tr>
<td>CFL:</td>
<td>$\kappa_1 = \ldots = \kappa_7 = 0$, $\kappa_8 = \frac{2}{\sqrt{3}} \kappa$</td>
</tr>
<tr>
<td>CSL:</td>
<td>$\kappa_1 = \ldots = \kappa_8 = \kappa = 0$</td>
</tr>
<tr>
<td>polar:</td>
<td>$\kappa_1, \kappa_2, \kappa_3$ arbitrary, $\kappa_4 = \ldots = \kappa_7 = 0$, $\kappa_8 = -2\sqrt{3} \kappa$</td>
</tr>
</tbody>
</table>

The fact that $\kappa_1, \kappa_2, \kappa_3$ are arbitrary in the 2SC and polar phases means that there is an unbroken $SU(2)_c$ symmetry under which the order parameter is invariant. This is obvious because in these phases Cooper pairs carry anti-blue color charge which is not seen by the generators of this $SU(2)_c$, since they operate exclusively on the red-green subspace. The relation between $\kappa_8$ and $\kappa$ determines a new $\tilde{U}(1)$ symmetry, generated by

$$
\tilde{Q} = Q + \eta T_8,
$$

where $\eta$ is an arbitrary complex number.

The mixing angle $\theta$ is determined by

$$
\theta = \frac{1}{2} \tan^{-1} \left( \frac{\kappa_8}{\kappa} \right).
$$

The mixing angles for longitudinal and transverse modes are identical, $\theta_1 = \theta_2 = 0$. Therefore, $\tilde{Q}$ generates a subgroup of the product group of strong and electromagnetic interactions, $SU(3)_c \times U(1)_{em}$, and the associated gauge field is a linear combination of the photon and the eighth gluon field, which, in turn, defines the mixing angle. The group-theoretical method only allows to identify a new charge, and thus does not distinguish between electric Debye or magnetic Meissner screening. Consequently, the mixing angles for longitudinal (electric) and transverse (magnetic) modes deduced by this method are identical, $\theta_1 = \theta_2 = 0$. A Cooper pair is neutral with respect to a charge, if the gap matrix $\Phi$ is invariant under a transformation generated by an operator $\tilde{Q}$ corresponding to this charge. In general, the gap matrix is a matrix in color, flavor, and spin space,
In this section, we calculate the polarization tensors $\Pi_{\mu\nu}^\gamma(P)$ given in Eq. (10) in the limit $p_0 = 0, p \to 0$. In this case, the Debye and Meissner masses, cf. Eqs. (39), are obtained from the coefficients of the first two projectors in the decomposition (20). They will be calculated in the next section. Here, we first derive a general expression for the polarization tensor that holds for all different phases. Then we insert the order parameters of the 2SC, CFL, polar, and CSL phases, and show the results for each phase separately.
A. General structure of $\Pi_{ab}^{\mu\nu}(0)$

We start from Eq. (10) and first perform the trace over Nambu-Gor’kov space,

$$
\Pi_{ab}^{\mu\nu}(P) = \frac{1}{2V} \sum_K \left\{ \text{Tr}[\Gamma_0^\mu G^+(K) \Gamma_0^\nu G^+(K - P)] + \text{Tr}[\Gamma_0^\mu G^-(K) \Gamma_0^\nu G^-(K - P)] + \text{Tr}[\Gamma_0^\mu \Xi^-(K) \Gamma_0^\nu \Xi^+(K - P)] + \text{Tr}[\Gamma_0^\mu \Xi^+(K) \Gamma_0^\nu \Xi^-(K - P)] \right\},
$$

(57)

where the traces now run over color, flavor, and Dirac space. In the following we first consider the traces with the quark propagators $G^{\pm}$ and afterwards investigate the traces containing the anomalous propagators $\Xi^{\pm}$.

In order to find the results for the former, we first perform the Matsubara sum. This is completely analogous to the calculation of Ref. [17]. The only difference is that our more compact notation with the help of the projectors $P_k$, cf. Eq. (14). Thus, abbreviating $K_1 \equiv K$, $K_2 \equiv K - P$, and $k_i \equiv |k_i|$ for $i = 1, 2$, we conclude

$$
T \sum_{k_0} \text{Tr} \left[ \Gamma_0^\mu G^+(K_1) \Gamma_0^\nu G^+(K_2) \right] = \sum_{\epsilon_1, \epsilon_2, r, s} \text{Tr} \left[ \Gamma_0^\mu \gamma_0 P_{k_1} \Lambda_{k_1}^{\epsilon_1} \Gamma_0^\nu \gamma_0 P_{k_2} \Lambda_{k_2}^{\epsilon_2} \right] v^{r+s}_{\epsilon_1 \epsilon_2}(k_1, k_2, p_0),
$$

(58a)

$$
T \sum_{k_0} \text{Tr} \left[ \Gamma_0^\mu G^-(K_1) \Gamma_0^\nu G^-(K_2) \right] = \sum_{\epsilon_1, \epsilon_2, r, s} \text{Tr} \left[ \Gamma_0^\mu \gamma_0 P_{k_1} \Lambda_{k_1}^{\epsilon_1} \Gamma_0^\nu \gamma_0 P_{k_2} \Lambda_{k_2}^{\epsilon_2} \right] v^{-r-s}_{\epsilon_1 \epsilon_2}(k_1, k_2, p_0),
$$

(58b)

where (cf. Eq. (40) of Ref. [17])

$$
v^{r+s}_{\epsilon_1 \epsilon_2}(k_1, k_2, p_0) = - \frac{n_1, r(1 - n_2, s)}{p_0 + \epsilon_1, r + \epsilon_2, s} - \frac{(1 - n_1, r)n_2, s}{p_0 - \epsilon_1, r - \epsilon_2, s} (1 - N_{1, r} - N_{2, s})
$$

$$
- \frac{(1 - n_1, r)(1 - n_2, s)}{p_0 - \epsilon_1, r - \epsilon_2, s} \frac{n_1, r n_2, s}{p_0 + \epsilon_1, r + \epsilon_2, s} (N_{1, r} - N_{2, s}),
$$

(59a)

$$
v^{-r-s}_{\epsilon_1 \epsilon_2}(k_1, k_2, p_0) = - \frac{(1 - n_1, r)n_2, s}{p_0 + \epsilon_1, r + \epsilon_2, s} - \frac{n_1, r(1 - n_2, s)}{p_0 - \epsilon_1, r - \epsilon_2, s} (1 - N_{1, r} - N_{2, s})
$$

$$
- \frac{n_1, r n_2, s}{p_0 - \epsilon_1, r + \epsilon_2, s} - \frac{(1 - n_1, r)(1 - n_2, s)}{p_0 + \epsilon_1, r - \epsilon_2, s} (N_{1, r} - N_{2, s}).
$$

(59b)

Here, we abbreviated

$$
\epsilon_{i, r} \equiv \epsilon_{k_i, r}, \quad n_{i, r} \equiv \frac{\epsilon_{i, r} + \mu - \epsilon_{k_i}}{2\epsilon_{i, r}} , \quad N_{i, r} \equiv \frac{1}{\exp(\epsilon_{i, r}/T) + 1} \quad (i = 1, 2).
$$

(60)

Note that, for $p_0 = 0$, we have

$$
v^{r+s}_{\epsilon_1 \epsilon_2}(k_1, k_2, 0) = v^{-r-s}_{\epsilon_1 \epsilon_2}(k_1, k_2, 0) \equiv v^{r+s}_{\epsilon_1 \epsilon_2}(k_1, k_2, 0).
$$

(61)

Next we discuss the traces containing the anomalous propagators. Again the Matsubara sum is completely analogous to the calculation in Ref. [17]. Therefore, using Eq. (17), we obtain

$$
T \sum_{k_0} \text{Tr} \left[ \Gamma_0^\mu \Xi^-(K_1) \Gamma_0^\nu \Xi^+(K_2) \right] = \sum_{\epsilon_1, \epsilon_2, r, s} \text{Tr} \left[ \Gamma_0^\mu \gamma_0 M_{k_1}^\dagger P_{k_1} \Lambda_{k_1}^{\epsilon_1} \Gamma_0^\nu \gamma_0 P_{k_2} \Lambda_{k_2}^{\epsilon_2} \right] w^{r+s}_{\epsilon_1 \epsilon_2}(k_1, k_2, p_0),
$$

(62a)

$$
T \sum_{k_0} \text{Tr} \left[ \Gamma_0^\mu \Xi^+(K_1) \Gamma_0^\nu \Xi^- (K_2) \right] = \sum_{\epsilon_1, \epsilon_2, r, s} \text{Tr} \left[ \Gamma_0^\mu \gamma_0 M_{k_1}^\dagger P_{k_1} \Lambda_{k_1}^{\epsilon_1} \Gamma_0^\nu \gamma_0 P_{k_2} \Lambda_{k_2}^{\epsilon_2} \right] w^{-r-s}_{\epsilon_1 \epsilon_2}(k_1, k_2, p_0),
$$

(62b)

where (cf. Eq. (93) of Ref. [17])

$$
w^{r+s}_{\epsilon_1 \epsilon_2}(k_1, k_2, p_0) \equiv \frac{\phi_{1, r} \phi_{2, s}}{4\epsilon_{1, r} \epsilon_{2, s}} \left[ \frac{1}{p_0 + \epsilon_{1, r} + \epsilon_{2, s}} - \frac{1}{p_0 - \epsilon_{1, r} - \epsilon_{2, s}} \right] (1 - N_{1, r} - N_{2, s})
$$

$$
- \frac{1}{p_0 - \epsilon_{1, r} + \epsilon_{2, s}} - \frac{1}{p_0 + \epsilon_{1, r} - \epsilon_{2, s}} (N_{1, r} - N_{2, s}).
$$

(63)

Here,

$$
\phi_{i, r} \equiv \phi^{r_i}(\epsilon_{i, r}, k_i)
$$

(64)
is the gap function on the quasiparticle mass shell given by the excitation branch \( k_0 = \epsilon_{i,r} \). In the derivation of Eqs. (58) and (62), we assumed that, in the 2SC and CFL phases, the chemical potentials for all \( N_f \) quark flavors are the same, \( \mu_1 = \ldots = \mu_{N_f} = \mu \). In this case, the functions \( v \) and \( w \) only depend on the single chemical potential \( \mu \) and can thus be factored out of the flavor trace. In the cases where quarks of the same flavor form Cooper pairs, i.e., in the polar and CSL phases, our formalism also allows for the treatment of a system with \( N_f > 1 \) and different chemical potentials \( \mu_n, n = 1, \ldots, N_f \). Then, \( v \) and \( w \) depend on the quark flavor through \( \mu_n \) and have to be included into the trace over flavor space.

Inserting Eqs. (58) and (62) into Eq. (57), we obtain for the general polarization tensor

\[
\Pi^{\mu\nu}_{ab}(P) = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \sum_{e_1, e_2} \sum_{r,s} \left\{ \text{Tr} \left[ \Gamma^{\mu}_a \gamma_0 P^{\mu}_k \Lambda_{k_1}^{\epsilon_1} \Gamma^{\nu}_b \gamma_0 P^{\nu}_k \Lambda_{k_2}^{\epsilon_2} \right] v_{e_1 e_2}^{rs}(k_1, k_2, p_0) + \text{Tr} \left[ \Gamma^{\mu}_a \gamma_0 P^{\mu}_k \Lambda_{k_1}^{\epsilon_1} \Gamma^{\nu}_b \gamma_0 P^{\nu}_k \Lambda_{k_2}^{\epsilon_2} \right] v_{e_2 e_1}^{rs}(k_1, k_2, p_0) \right. \\
+ \left. \text{Tr} \left[ \Gamma^{\mu}_a \gamma_0 P^{\mu}_k \Lambda_{k_1}^{\epsilon_1} \Gamma^{\nu}_b \gamma_0 P^{\nu}_k \Lambda_{k_2}^{\epsilon_2} \right] w_{e_1 e_2}^{rs}(k_1, k_2, p_0) \right. \\
+ \left. \text{Tr} \left[ \Gamma^{\mu}_a \gamma_0 P^{\mu}_k \Lambda_{k_1}^{\epsilon_1} \Gamma^{\nu}_b \gamma_0 P^{\nu}_k \Lambda_{k_2}^{\epsilon_2} \right] w_{e_2 e_1}^{rs}(k_1, k_2, p_0) \right\} .
\]

(65)

In the following we focus on the special case \( p_0 = 0 \) and \( p \to 0 \), i.e., \( k_2 \to k_1 \equiv k \). In this limit, the traces only depend on \( k \) and the functions \( v \) and \( w \) only on \( k \equiv |k| \). Thus, the \( d^3 k \) integral factorizes into an angular and a radial part. With the abbreviations

\[
\mathcal{V}_{ab, e_1 e_2}^{\mu\nu, rs} = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \left\{ \text{Tr} \left[ \Gamma^{\mu}_a \gamma_0 P^{\mu}_k \Lambda_{k_1}^{\epsilon_1} \Gamma^{\nu}_b \gamma_0 P^{\nu}_k \Lambda_{k_2}^{\epsilon_2} \right] + \text{Tr} \left[ \Gamma^{\mu}_a \gamma_0 P^{\mu}_k \Lambda_{k_1}^{\epsilon_1} \Gamma^{\nu}_b \gamma_0 P^{\nu}_k \Lambda_{k_2}^{\epsilon_2} \right] \right\} ,
\]

(66a)

\[
\mathcal{W}_{ab, e_1 e_2}^{\mu\nu, rs} = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \left\{ \text{Tr} \left[ \Gamma^{\mu}_a \Lambda_{k_1}^{\epsilon_1} \Gamma^{\nu}_b \Lambda_{k_2}^{\epsilon_2} \right] + \text{Tr} \left[ \Gamma^{\mu}_a \Lambda_{k_1}^{\epsilon_1} \Gamma^{\nu}_b \Lambda_{k_2}^{\epsilon_2} \right] \right\} ,
\]

(66b)

we can write the polarization tensor as

\[
\Pi^{\mu\nu}_{ab}(0) \equiv \lim_{p \to 0} \Pi^{\mu\nu}_{ab}(0, p) = \sum_{e_1, e_2} \sum_{r,s} \left[ \mathcal{V}_{ab, e_1 e_2}^{\mu\nu, rs} \lim_{p \to 0} \int dk^2 v_{e_1 e_2}^{rs}(k_1, k_2, 0) + \mathcal{W}_{ab, e_1 e_2}^{\mu\nu, rs} \lim_{p \to 0} \int dk^2 w_{e_1 e_2}^{rs}(k_1, k_2, 0) \right] .
\]

(67)

Note that only the angular integrals over the color, flavor, and Dirac traces, defined by \( \mathcal{V} \) and \( \mathcal{W} \), depend on the symmetries of the order parameter and thus have to be calculated separately for each phase. Therefore, we first consider the \( dk \) integrals which are the same for all cases.

In order to see how the two different quasiparticle excitations branches (labelled by \( r, s \), as well as normal and anomalous propagation of the respective excitations (represented by the functions \( v, w \)) contribute in the final expressions for the polarization tensors in the zero-energy, zero-momentum limit, it is convenient to define the quantities

\[
v^{rs} = \frac{1}{\mu^2} \lim_{p \to 0} \int dk^2 \left[ v^{rs+}(k_1, k_2, 0) + v^{rs-}(k_1, k_2, 0) \right] ,
\]

(68a)

\[
v^{rs} = \frac{1}{\mu^2} \lim_{p \to 0} \int dk^2 \left[ v^{rs+}(k_1, k_2, 0) + v^{rs-}(k_1, k_2, 0) \right] ,
\]

(68b)

\[
w^{rs} = \frac{1}{\mu^2} \lim_{p \to 0} \int dk^2 \left[ w^{rs+}(k_1, k_2, 0) + w^{rs-}(k_1, k_2, 0) \right] ,
\]

(68c)

\[
w^{rs} = \frac{1}{\mu^2} \lim_{p \to 0} \int dk^2 \left[ w^{rs+}(k_1, k_2, 0) + w^{rs-}(k_1, k_2, 0) \right] .
\]

(68d)

These quantities are dimensionless since \( v^{rs+}_{e_1 e_2}(k_1, k_2, 0) \) and \( w^{rs+}_{e_1 e_2}(k_1, k_2, 0) \) have the dimension \([1/\text{energy}]\) (cf. the definitions in Eqs. (59) and (63)). Combining particle-particle \( (e_1 = e_2 = +) \) and antiparticle-antiparticle \( (e_1 = e_2 = -) \), as well as particle-antiparticle \( (e_1 = - e_2 = \pm) \) contributions is possible since the corresponding integrals \( \mathcal{V}, \mathcal{W} \) multiplying these terms turn out to be the same. In the definitions of \( v^{rs}, \bar{v}^{rs} \) and \( w^{rs}, \bar{w}^{rs} \) we divided by the square of the quark chemical potential \( \mu^2 \) in order to make these quantities independent of the quark flavor. This will be convenient for the results of the spin-one phases, where the formation of Cooper pairs is possible for different chemical potentials for each quark flavor. In App. A we present the calculation of the relevant integrals defined in Eqs. (68).
and gluons in each phase. For the 2SC phase [17] and the CFL phase [18, 19], the results for the gluons are already explicit since the special form of the gap matrix, Eq. (13), is explicitly involved. Therefore, in the following sections we show the relevant color-flavor-Dirac matrices in Table III. The fields with no entry indicate that these values do not occur in the calculations.

<table>
<thead>
<tr>
<th>( T = 0 )</th>
<th>( \mathcal{M}_k )</th>
<th>( P^1_{\mu\nu} )</th>
<th>( P^2_{\mu\nu} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2SC</td>
<td>( J_\mu \gamma_\mu )</td>
<td>( J_\mu^2 )</td>
<td>( 1 - J_\mu^2 )</td>
</tr>
<tr>
<td>CFL polar</td>
<td>( J_\mu \gamma_\mu )</td>
<td>( \frac{1}{4}(</td>
<td>J_\mu</td>
</tr>
<tr>
<td>CSL</td>
<td>( J_\mu [k^a + \gamma_\perp(k)] )</td>
<td>( J_\mu^2 )</td>
<td>( 1 - J_\mu^2 )</td>
</tr>
</tbody>
</table>

TABLE III: Relevant color-flavor-Dirac matrices for the calculation of the Debye and Meissner masses in a given color-superconducting phase. The matrix \( \mathcal{M}_k \) reflects the symmetries of the various gap matrices. For the definition of the projectors \( P^r_{\mu\nu} \), see Eq. (16). In color space, we use the matrices \((J_j)_k = -i\epsilon_{ijk}, (i,j,k = 1,2,3); \) in flavor space, we use \((I_i)_j = -i\epsilon_{ijk}\) and the second Pauli matrix \( \tau_2 \). In Dirac space, we defined \( \gamma_\perp(k) \equiv \gamma - k\gamma \cdot k \).

As in Refs. [17, 18], we neglect the antiparticle gap, \( \phi^- \simeq 0 \), and compute the integrals up to leading order. In Table II we collect all results. Some integrals vanish since they are proportional to a vanishing gap. This is the case for \( \bar{w}r^s = 0 \) (for all \( r,s \)), since these integrals are proportional to at least one antiparticle gap. Clearly, for \( T \geq T_c \), the gap vanishes in all phases and thus \( w^{rs} = \bar{w}^{rs} = 0 \).

In order to discuss the traces and the angular integral in Eq. (67), we have to distinguish between the several phases since the special form of the gap matrix, Eq. (13), is explicitly involved. Therefore, in the following sections we discuss the 2SC, CFL, polar, and CSL phases separately and compute the Debye and Meissner masses for photons and gluons in each phase. For the 2SC phase [17] and the CFL phase [18, 19], the results for the gluons are already known. Also the masses of the rotated gauge bosons, where the rotation is given by the new generator \( Q \), cf. Eq. (51), were considered for these two phases [13]. Nevertheless, we will briefly discuss also these cases, since we first want to establish our notation and second, we will show that for the 2SC phase, there is actually no mixing between electric gluons and photons, i.e., the longitudinal mixing angle \( \theta_2 \) is zero. Consequently, there is no rotated electric photon.

The basic symmetry properties of all considered phases have been discussed in detail in Ref. [10]. We show the relevant color-flavor-Dirac matrices in Table III.

### B. The 2SC phase

1. **Gluon polarization tensor \((a,b \leq 8)\)**

Inserting the matrices given in the second line of Table III into Eqs. (66), we obtain

\[
\nu_{ab, e_1 e_2}^{\mu \nu, r s} = g^2 \left\{ \text{Tr} \left[ T_a \mathcal{P}^r T_b \mathcal{P}^s \right] T_{\mu \nu}^{e_1 - e_2} + \text{Tr} \left[ T_a^T \mathcal{P}^s T_b^T \mathcal{P}^r \right] T_{\mu \nu}^{e_1 - e_2} \right\},
\]

\[
\mathcal{W}_{ab, e_1 e_2}^{\mu \nu, r s} = g^2 \left\{ \text{Tr} \left[ T_a J_3 \mathcal{P}^r T_b J_3 \mathcal{P}^s \right] \mathcal{U}_{\mu \nu}^{e_1 - e_2} + \text{Tr} \left[ T_a^T J_3 \mathcal{P}^s T_b^T J_3 \mathcal{P}^r \right] \mathcal{U}_{\mu \nu}^{e_1 - e_2} \right\},
\]

where the traces only run over color space and where we defined

\[
T_{\mu \nu}^{e_1 e_2} = \int \frac{d\Omega_k}{(2\pi)^3} \text{Tr} \left[ \gamma^\mu \gamma_0 \Lambda^{e_1}_k \gamma^\nu \gamma_0 \Lambda^{e_2}_k \right],
\]

\[
\mathcal{U}_{\mu \nu}^{e_1 e_2} = \int \frac{d\Omega_k}{(2\pi)^3} \text{Tr} \left[ \gamma^\mu \gamma^5 \Lambda^{e_1}_k \gamma^\nu \gamma^5 \Lambda^{e_2}_k \right].
\]

Here, the traces only run over Dirac space. We used the fact that the projectors \( \mathcal{P}^{r s} \equiv P^{r s}_k \) do not depend on the quark momentum \( k \) and that the color and Dirac traces factorize. Furthermore, the trivial flavor trace was already
performed, yielding a factor 2. The angular integrals over Dirac traces, Eqs. (70), are easily evaluated,

\[
\begin{align*}
\mathcal{T}^{00}_{e_1,e_2} &= -\mathcal{U}^{00}_{e_1,-e_2} = \frac{1}{2\pi^2}(1 + e_1e_2), \\
\mathcal{T}^{0i}_{e_1,e_2} &= \mathcal{T}^{0i}_{e_1,-e_2} = \mathcal{U}^{0i}_{e_1,-e_2} = \mathcal{U}^{0i}_{e_1,e_2} = 0, \\
\mathcal{T}^{ij}_{e_1,e_2} &= \mathcal{U}^{ij}_{e_1,-e_2} = \frac{1}{2\pi^2}\delta_{ij}(1 - \frac{1}{3}e_1e_2),
\end{align*}
\]

where \(i, j = 1, 2, 3\). For the evaluation of the color traces note that \(J_3P^1 = J_3\) and \(J_3P^2 = 0\). We find that \(\mathcal{V}\) and \(\mathcal{W}\), given in Eqs. (69), are diagonal in the adjoint color indices \(a\) and \(b\).

(a) \(\mu = \nu = 0\). With Eq. (71a) we obtain after performing the color traces

\[
\Pi^{00}_{ab}(0) = \delta_{ab} \frac{2g^2\mu^2}{\pi^2} \begin{cases} \\
\frac{1}{2}v^{11} + \frac{1}{2}w^{11} & \text{for } a = 1, 2, 3, \\
\frac{1}{4}(v^{12} + v^{21}) & \text{for } a = 4 - 7, \\
\frac{1}{6}v^{11} + \frac{1}{3}v^{22} - \frac{1}{6}w^{11} & \text{for } a = 8.
\end{cases}
\]

(b) \(\mu = 0, \nu = i\) and \(\mu = i, \nu = 0\). Due to Eq. (71b),

\[
\Pi^{0i}_{ab}(0) = \Pi^{0i}_{ba}(0) = 0.
\]

(c) \(\mu = i, \nu = j\). Due to Eq. (71c), the polarization tensor is diagonal in spatial indices \(i, j\). We obtain

\[
\Pi^{ij}_{ab}(0) = \delta_{ab}\delta^{ij} \frac{2g^2\mu^2}{3\pi^2} \begin{cases} \\
\frac{1}{2}(v^{11} + 2\bar{v}^{11}) - \frac{1}{2}(w^{11} + 2\bar{w}^{11}) & \text{for } a = 1, 2, 3, \\
\frac{1}{4}[(v^{12} + v^{21}) + 2(\bar{v}^{12} + \bar{v}^{21})] & \text{for } a = 4 - 7, \\
\frac{1}{6}(v^{11} + 2\bar{v}^{11}) + \frac{1}{3}(v^{22} + 2\bar{v}^{22}) + \frac{1}{6}(w^{11} + 2\bar{w}^{11}) & \text{for } a = 8.
\end{cases}
\]

2. Mixed polarization tensor \((a \leq 8, b = 9\) and \(a = 9, b \leq 8\))

For a system in the color-superconducting 2SC phase where quarks with the electric charges \(q_1\) and \(q_2\) form Cooper pairs, the electric charge generator, introduced in Eq. (4), is given by \(Q = \text{diag}(q_1, q_2)\). Thus, we obtain

\[
\begin{align*}
\mathcal{V}^{\mu\nu,rs}_{a\gamma_1, e_1 e_2} &= \frac{1}{2}eg(q_1 + q_2) \left\{ \text{Tr}[T_a P^r P^s T^{\mu\nu}_{e_1,-e_2}] + \text{Tr}[T_a T^T J_3 P^r J_3 P^s T^{\mu\nu}_{e_1,e_2}] \right\}, \\
\mathcal{W}^{\mu\nu,rs}_{a\gamma_1, e_1 e_2} &= \frac{1}{2}eg(q_1 + q_2) \left\{ \text{Tr}[T_a J_3 P^r J_3 P^s] U^{\mu\nu}_{e_1,-e_2} + \text{Tr}[T_a T^T J_3 P^r J_3 P^s] U^{\mu\nu}_{e_1,e_2} \right\}.
\end{align*}
\]

It is not difficult to show that the polarization tensor is symmetric under the exchange of photon and gluon indices,

\[
\Pi^{\mu\nu}_{a\gamma}(0) = \Pi^{\mu\nu}_{a\gamma}(0).
\]

Using \(J_3^3 = J_3\), all color traces in Eq. (75) reduce to \(\text{Tr}[T_a J_3^2] = \delta_{a8}/\sqrt{3}\). Therefore, we obtain for the various Dirac components of the tensor:

(a) \(\mu = \nu = 0\).

\[
\Pi^{00}_{a\gamma}(0) = (q_1 + q_2) \frac{eg\mu^2}{\sqrt{3}\pi^2} \begin{cases} \\
0 & \text{for } a = 1 - 7, \\
v^{11} - v^{22} - w^{11} & \text{for } a = 8.
\end{cases}
\]

(b) \(\mu = 0, \nu = i\) and \(\mu = i, \nu = 0\).

\[
\Pi^{0i}_{a\gamma}(0) = \Pi^{0i}_{a\gamma}(0) = 0.
\]
(c) $\mu = i, \nu = j$.

\[
\Pi^{ij}_{a\gamma}(0) = \delta_{ij}(q_1 + q_2) \frac{eg\mu^2}{3\sqrt{3}\pi^2} \begin{cases} 
0 & \text{for } a = 1 - 7, \\
(v^{11} + 2v^{11}) - (v^{22} + 2v^{22}) + (w^{11} + 2w^{11}) & \text{for } a = 8.
\end{cases}
\]

(79)

3. Photon polarization tensor ($a = b = 9$)

In this case, the tensors $\mathcal{V}, \mathcal{W}$ are

\[
\mathcal{V}^{\mu\nu,rs}_{\gamma,\eta,\epsilon_1\epsilon_2} = \frac{1}{2} e^2 (q_1^2 + q_2^2) \text{Tr}[P^r P^s] \left( T^{\mu\nu}_{\epsilon_1\epsilon_2} + T^{\epsilon_1\epsilon_2}_{\mu\nu} \right),
\]

\[
\mathcal{W}^{\mu\nu,rs}_{\gamma,\eta,\epsilon_1\epsilon_2} = e^2 q_1 q_2 \text{Tr}[J_3 P^r J_3 P^s] \left( \mathcal{U}^{\mu\nu}_{\epsilon_1\epsilon_2} + \mathcal{U}^{\epsilon_1\epsilon_2}_{\mu\nu} \right).
\]

(80a)

(80b)

Here, we performed the flavor traces $\text{Tr}[Q^2] = q_1^2 + q_2^2$ and $\text{Tr}[Q^2 Q^2] = 2q_1 q_2$. After performing the color traces and the sums over $\epsilon_1, \epsilon_2$ and $r, s$, the results for the different components are as follows.

(a) $\mu = \nu = 0$.

\[
\Pi^{00}_{\gamma\gamma}(0) = \frac{e^2 \mu^2}{\pi^2} \left[ (q_1^2 + q_2^2) (2v^{11} + v^{22}) - 4q_1 q_2 w^{11} \right].
\]

(81)

(b) $\mu = 0, \nu = i$ and $\mu = i, \nu = 0$.

\[
\Pi^{0i}_{\gamma\gamma}(0) = \Pi^{ii}_{\gamma\gamma}(0) = 0.
\]

(82)

(c) $\mu = i, \nu = j$.

\[
\Pi^{ij}_{\gamma\gamma}(0) = \delta_{ij} \frac{e^2 \mu^2}{3\pi^2} \left\{ (q_1^2 + q_2^2) \left[ 2(v^{11} + 2v^{11}) + (v^{22} + 2v^{22}) \right] + 4q_1 q_2 (w^{11} + 2w^{11}) \right\}.
\]

(83)

C. The CFL phase

1. Gluon polarization tensor ($a, b \leq 8$)

With the matrix $\mathcal{M}_k$ and the projectors $P^1_{k,2}$ for the CFL phase, given in Table III, Eqs. (66) become

\[
\mathcal{V}^{\mu\nu,rs}_{ab,\epsilon_1\epsilon_2} = \frac{1}{2} g^2 \left\{ \text{Tr}[T_a^T P^r T_b^T P^s] T^{\mu\nu}_{\epsilon_1\epsilon_2} + \text{Tr}[T_a^T P^r T_b^T P^s] T^{\epsilon_1\epsilon_2}_{\mu\nu} \right\},
\]

\[
\mathcal{W}^{\mu\nu,rs}_{ab,\epsilon_1\epsilon_2} = \frac{1}{2} g^2 \left\{ \text{Tr}[T_a J \cdot I P^r T_b^T J \cdot I P^s] \mathcal{U}^{\mu\nu}_{\epsilon_1\epsilon_2} + \text{Tr}[T_a^T J \cdot I P^r T_b J \cdot I P^s] \mathcal{U}^{\epsilon_1\epsilon_2}_{\mu\nu} \right\}.
\]

(84a)

(84b)

As in the 2SC phase, the projectors $P^r, P^s$ do not depend on momentum. Consequently, the angular integrals defined in Eq. (70) also appear in the CFL phase. Therefore, also in the CFL phase the $(0i)$ and $(i0)$ components of the polarization tensor vanish, and the $(ij)$ components are proportional to $\delta^{ij}$. Contrary to the 2SC phase, color and flavor traces cannot be performed separately, since the projectors $P^r, P^s$ are nontrivial matrices both in color and in flavor space. In order to perform the color-flavor trace, one uses the relations $\text{Tr}[T_a P^1 T_b P^1] = 0$ and $J \cdot I P^1 = -2 P^1$. The polarization tensor is not only diagonal in color, but also has the same value for all eight gluons. One finally obtains the following expressions for the gluon polarization tensors.

(a) $\mu = \nu = 0$.

\[
\Pi^{00}_{ab}(0) = \delta_{ab} \frac{g^2 \mu^2}{6\pi^2} \left[ (v^{12} + v^{21}) + 7v^{22} + 2(w^{12} + w^{21}) + 2w^{22} \right].
\]

(85)
(b) $\mu = 0, \nu = i$ and $\mu = i, \nu = 0$.

$$\Pi_{\alpha \gamma}^{0 \mu}(0) = \Pi_{\alpha \gamma}^{0 \mu}(0) = 0 . \tag{86}$$

(c) $\mu = i, \nu = j$.

$$\Pi_{\alpha \gamma}^{ij}(0) = \delta_{ij} \frac{eg \mu^2}{18\pi^2} \left[ (v_{12} + v_{21}) + 2(\bar{v}_{12} + \bar{v}_{21}) + 7(\nu_{22} + 2\bar{v}_{22}) 
- 2(w_{12} + w_{21}) - 4(w_{12} + \bar{w}_{21}) - 2(2w_{22} + 2\bar{w}_{22}) \right] . \tag{87}$$

2. Mixed polarization tensor ($a \leq 8, b = 9$ and $a = 9, b \leq 8$)

To compute the mixed polarization tensors, we need the electric charge generator $Q$. Since we consider a system with three quark flavors of electric charges $q_1, q_2, q_3$, we have $Q = \text{diag}(q_1, q_2, q_3)$. In the final result we will insert the charges for $u, d,$ and $s$ quarks. We obtain

$$\begin{align*}
V_{\alpha \gamma, rs}^{\mu \nu, c_1, c_2} &= \frac{1}{2} eg \left\{ \text{Tr}[T_a \mathcal{P}^r Q \mathcal{P}^s] T_{c_1 c_2}^{\mu \nu} + \text{Tr}[T_a^T \mathcal{P}^r Q \mathcal{P}^s] T_{c_1 c_2}^{\mu \nu} \right\} , \\
W_{\alpha \gamma, rs}^{\mu \nu, c_1, c_2} &= \frac{1}{2} eg \left\{ \text{Tr}[T_a \mathcal{J} \cdot \mathbf{1} \mathcal{P}^r Q \mathcal{J} \cdot \mathbf{1} \mathcal{P}^s] T_{c_1 c_2}^{\mu \nu} + \text{Tr}[T_a^T \mathcal{J} \cdot \mathbf{1} \mathcal{P}^r Q \mathcal{J} \cdot \mathbf{1} \mathcal{P}^s] T_{c_1 c_2}^{\mu \nu} \right\} .
\end{align*} \tag{88a}$$

First we note that Eq. (76) also holds for the CFL phase. With the help of the relations

$$\text{Tr}[T_a \mathcal{P}^1 Q \mathcal{P}^1] = 0 , \quad \text{Tr}[T_a Q] = \text{Tr}[T_a] \text{Tr}[Q] = 0 , \tag{89}$$

and

$$\text{Tr}[T_a \mathcal{J} \cdot \mathbf{1} Q \mathcal{J} \cdot \mathbf{1}] = 3 \text{Tr}[T_a \mathcal{P}^1 Q] = 3 \text{Tr}[T_a \mathcal{P}^1 Q \mathcal{J} \cdot \mathbf{1}] = \delta_{a3} \frac{1}{2}(q_1 - q_2) + \delta_{a8} \frac{1}{2\sqrt{3}}(q_1 + q_2 - 2q_3) \tag{90}$$

we obtain the following results.

(a) $\mu = \nu = 0$.

$$\Pi_{\alpha \gamma}^{00}(0) = \frac{eg \mu^2}{6\pi^2} \begin{cases} 0 & \text{for } a = 1, 2, 4 - 7 , \\
(q_1 - q_2) \left[ (v_{12} + v_{21}) - 2v_{22} + 2(w_{12} + w_{21}) - 7w_{22} \right] & \text{for } a = 3 , \\
\frac{1}{\sqrt{3}}(q_1 + q_2 - 2q_3) \left[ (v_{12} + v_{21}) - 2v_{22} + 2(w_{12} + w_{21}) - 7w_{22} \right] & \text{for } a = 8 . \end{cases} \tag{91}$$

(b) $\mu = 0, \nu = i$ and $\mu = i, \nu = 0$.

$$\Pi_{\alpha \gamma}^{0 i}(0) = \Pi_{\alpha \gamma}^{i 0}(0) = 0 . \tag{92}$$

(c) $\mu = i, \nu = j$.

$$\Pi_{\alpha \gamma}^{ij}(0) = \delta_{ij} \frac{eg \mu^2}{18\pi^2} \begin{cases} 0 & \text{for } a = 1, 2, 4 - 7 , \\
(q_1 - q_2) \left[ (v_{12}^2 + v_{21}^2) + 2(\bar{v}_{12}^2 + \bar{v}_{21}^2) - 2v_{22} - 4\bar{v}_{22} \\
- 2(w_{12}^2 + w_{21}^2) - 4(\bar{w}_{12}^2 + \bar{w}_{21}^2) + 7w_{22}^2 + 14\bar{w}_{22}^2 \right] & \text{for } a = 3 , \\
\frac{1}{\sqrt{3}}(q_1 + q_2 - 2q_3) \left[ (v_{12}^2 + v_{21}^2) + 2(\bar{v}_{12}^2 + \bar{v}_{21}^2) - 2v_{22} - 4\bar{v}_{22} \\
- 2(w_{12}^2 + w_{21}^2) - 4(\bar{w}_{12}^2 + \bar{w}_{21}^2) + 7w_{22}^2 + 14\bar{w}_{22}^2 \right] & \text{for } a = 8 . \end{cases} \tag{93}$$
3. Photon polarization tensor \((a = b = 9)\)

In this case, the tensors \(\mathcal{V}, \mathcal{W}\) read

\[
\mathcal{V}^\mu\nu_{\gamma\gamma,\epsilon_1\epsilon_2} = \frac{1}{2} e^2 \text{Tr}[Q^\dagger \mathcal{P}\gamma Q^\dagger \mathcal{P}\gamma] \left( T_{\epsilon_1\epsilon_2}^{\mu\nu} + T_{\epsilon_1\epsilon_2}^{\nu\mu} \right),
\]

\[
\mathcal{W}^{\mu\nu}_{\gamma\gamma,\epsilon_1\epsilon_2} = \frac{1}{2} e^2 \text{Tr}[Q^\dagger \mathbf{J} \cdot \mathbf{I}^\dagger \mathcal{P}\gamma Q^\dagger \mathbf{J} \cdot \mathbf{I}^\dagger \mathcal{P}\gamma] \left( U_{\epsilon_1\epsilon_2}^{\mu\nu} + U_{\epsilon_1\epsilon_2}^{\nu\mu} \right).
\]

(94a)

(94b)

In order to perform the color-flavor traces, we abbreviate the following sums over the three quark charges,

\[
x \equiv \sum_{n,m=1}^{3} q_n q_m, \quad y \equiv \sum_{n=1}^{3} q_n^2.
\]

(95)

Then, with \(\text{Tr}[Q^\dagger \mathcal{P}\gamma Q^\dagger] = x/9\), \(\text{Tr}[Q^\dagger \mathbf{1}_c] = 9 \text{Tr}[Q^\dagger \mathbf{1}] = 3y\) (where \(\mathbf{1}_c\) is the unit matrix in color space), and \(\text{Tr}[\mathbf{J} \cdot \mathbf{I}^\dagger \mathcal{P}\gamma \mathbf{J} \cdot \mathbf{I}] = 3 \text{Tr}[\mathbf{J} \cdot \mathbf{I}^\dagger \mathcal{P}\gamma \mathbf{J} \cdot \mathbf{I}] = 2(x - y)\), we obtain the following results.

(a) \(\mu = \nu = 0\).

\[
\Pi_{\gamma\gamma}(0) = \frac{\varepsilon^2 \mu^2}{9\pi^2} \left[ x(v^{11} + 3y - x)(v^{12} + v^{21}) + (21y + x)v^{22} - 4xw^{11} + (6y - 2x)(w^{12} + w^{21}) + (6y - 10x)w^{22} \right].
\]

(96)

In a system of \(d\), \(s\), and \(u\) quarks we have the electric charges \(q_1 = q_2 = -1/3\) and \(q_3 = 2/3\). In this case,

\[
x = 0, \quad y = \frac{2}{3}.
\]

(97)

Inserting these values into Eq. (96), the result becomes proportional to the gluon polarization tensor, Eq. (85),

\[
\Pi_{\gamma\gamma}(0) = \frac{4\varepsilon^2}{3g^2} \Pi_{\alpha\alpha}(0).
\]

(98)

(b) \(\mu = 0, \nu = i\) and \(\mu = i, \nu = 0\).

\[
\Pi_{\gamma\gamma}(0) = \Pi_{\alpha\alpha}(0) = 0.
\]

(99)

(c) \(\mu = i, \nu = j\).

\[
\Pi_{\gamma\gamma}^{ij}(0) = \delta_{ij} \frac{\varepsilon^2 \mu^2}{2\pi^2} \left\{ x(v^{11} + 2\bar{v}^{11}) + (3y - x)[v^{12} + \bar{v}^{21} + 2(\bar{v}^{12} + \bar{v}^{22})] + (21y + x)(v^{22} + 2\bar{v}^{22}) + 4x(w^{11} + 2\bar{w}^{11}) - (6y - 2x)[w^{12} + w^{21} + 2(\bar{w}^{12} + \bar{w}^{21})] - (6y - 10x)(w^{22} + 2\bar{w}^{22}) \right\}.
\]

(100)

Again, using the quark charges of \(d\), \(s\), and \(u\) quarks that lead to Eq. (97), we obtain a result proportional to that given in Eq. (87),

\[
\Pi_{\gamma\gamma}^{ij}(0) = \frac{4\varepsilon^2}{3g^2} \Pi_{\alpha\alpha}(0).
\]

(101)

D. The polar phase

In the polar phase, Cooper pairs are formed by quarks of a single flavor and carry total spin one, \(J = 1\). This phase is defined by the matrix \(\mathcal{M}_k = J_k [\hat{k}^z + \gamma^z \vec{k}]\), cf. Table III. The spin of the Cooper pair is aligned with the spatial \(z\) direction which means that rotational \(SO(3)\) is broken to \(SO(2)_J\), cf. Table I. As in the 2SC phase, the condensate also points in a fixed color direction. Physically, this means that quarks of one color, say blue, remain unpaired. This similarity to the 2SC phase can also be seen in the projectors \(\mathcal{P}^{1,2}\) which are identical in both phases. Note that \(\mathcal{P}^{1,2}\) in the polar phase do not depend on \(\vec{k}\) although \(\mathcal{M}_k\) does. In the following we consider a system of quarks with \(N_f\) different flavors where each quark flavor forms Cooper pairs separately. Each quark flavor has a separate electric charge, \(q_1, \ldots, q_{N_f}\), and chemical potential, \(\mu_1, \ldots, \mu_{N_f}\).
1. Gluon polarization tensor \((a, b \leq 8)\)

In this case, we have

\[
\mathcal{V}^{\mu\nu,rs}_{ab,1_e} = \frac{1}{2} g^2 \left\{ \text{Tr} [ T_a \mathcal{P}^r \mathcal{T}^s_b ] T^{\mu\nu}_{-c_1,-c_2} + \text{Tr} [ T_a^{T} \mathcal{P}^r \mathcal{T}^s_b ] T^{\mu\nu}_{c_1,c_2} \right\},
\]

(102a)

\[
\mathcal{W}^{\mu\nu,rs}_{ab,1_e} = \frac{1}{2} g^2 \left\{ \text{Tr} [ T_a J_3 \mathcal{P}^r \mathcal{T}^s_b ] \hat{U}^{\mu\nu}_{c_1,c_2} + \text{Tr} [ T_a^{T} J_3 \mathcal{P}^r \mathcal{T}^s_b ] \hat{U}^{\mu\nu}_{c_1,c_2} \right\},
\]

(102b)

where

\[
\hat{U}^{\mu\nu}_{c_1,c_2} \equiv - \frac{1}{(2\pi)^3} \int \frac{d\Omega_k}{(2\pi)^3} \text{Tr} [ \gamma^\mu (\hat{k}z - \gamma^z (\hat{k})) \Lambda^{c_1}_{k} \gamma^\nu (\hat{k}z - \gamma^z (\hat{k})) \Lambda^{c_2}_{k} ].
\]

(103)

Since

\[
\text{Tr} [ \gamma^\mu (\hat{k}z - \gamma^z (\hat{k})) \Lambda^{c_1}_{k} \gamma^\nu (\hat{k}z - \gamma^z (\hat{k})) \Lambda^{c_2}_{k} ] = \text{Tr} [ \gamma^\mu \Lambda^{c_1}_{k} \gamma^\nu \Lambda^{c_2}_{k} ] = \text{Tr} [ \gamma^\mu \Lambda^{c_1}_{k} \gamma^\nu \Lambda^{c_2}_{k} ],
\]

(104)

we find

\[
\hat{U}^{\mu\nu}_{c_1,c_2} = \mathcal{U}^{\mu\nu}_{c_1,c_2}.
\]

(105)

Consequently, the gluon polarization tensor in the polar phase is almost identical to that in the 2SC phase. The only difference is the flavor trace which here yields a factor \(\sum_{n=1}^{N_f} \mu^2_n\). Therefore, Eqs. (72) – (74) hold also for the polar phase after replacing \(2\mu^2\) with \(\sum_{n=1}^{N_f} \mu^2_n\).

2. Mixed polarization tensor \((a \leq 8, b = 9\) and \(a = 9, b \leq 8\))

Obviously, also for the mixed polarization tensor, the Dirac and color part is identical to the 2SC case. Consequently, in Eqs. (77) – (79) one has to replace the factor \((q_1 + q_2)^2\mu^2\) by \(\sum_{n=1}^{N_f} q_n^2 \mu^2_n\) in order to obtain the results for the polar phase.

3. Photon polarization tensor \((a = b = 9)\)

Analogously, for the photon polarization tensor, the 2SC results given in Eqs. (81) – (83) are valid for the polar phase with the following modifications. Since in the 2SC phase there is a nontrivial flavor structure in the matrix \(\mathcal{M}_k\), there are two different flavor factors in Eqs. (81) – (83), namely \((q_1^2 + q_2^2)^2\mu^2\) and \(2q_1 q_2 \mu^2\). Replacing each of these factors by the common factor \(\sum_{n=1}^{N_f} q_n^2 \mu^2_n\) yields the corresponding results for the polar phase.

E. The CSL phase

As in Sec. IV D, we consider a system of \(N_f\) quark flavors, each flavor forming Cooper pairs separately. In the CSL phase, all quark colors acquire two gapped excitation energies. This is similar to the CFL phase, where there is also a quasiparticle excitation branch with an energy gap \(\phi\) and a second one with an energy gap \(2\phi\). Formally, this structure has its origin in the two nonzero eigenvalues of the operator \(\gamma_0 \mathcal{M}_k^{1,0} \mathcal{M}_k^{0,0}\), cf. Eqs. (15) and (16). In the case of the CSL phase, the matrix \(\mathcal{M}_k\) as well as the projectors \(\mathcal{P}_k^{1,2}\) depend on the direction of the quark momentum \(\hat{k}\). Due to color-spin locking, the spatial components of the vector \(\hat{k}\) are aligned with the three color directions red, green, and blue.
1. **Gluon polarization tensor** \((a, b \leq 8)\)

Inserting the matrices \(\mathcal{M}_k\) and \(\mathcal{P}_k^{1,2}\) from Table III into Eq. (66) yields

\[
\mathcal{V}_{ab,e_1 e_2}^{\mu \nu, rs} = \frac{g^2}{2} \int \frac{d\Omega_k}{(2\pi)^3} \left\{ \text{Tr} \left[ \gamma^\mu T_a \mathcal{P}_k^{\mu \nu} \Lambda_k^{-e_1} \gamma^\nu \Lambda_k^{-e_2} + (e_{1,2} \rightarrow -e_{1,2}, T_{a,b} \rightarrow T_{a,b}^T) \right] \right\}, \tag{106a}
\]

\[
\mathcal{W}_{ab,e_1 e_2}^{\mu \nu, rs} = -\frac{g^2}{2} \int \frac{d\Omega_k}{(2\pi)^3} \left\{ \text{Tr} \left[ \gamma^\mu T_a \mathcal{J} \cdot (\mathbf{k} - \gamma_\perp(\mathbf{k})) \mathcal{P}_k^{\mu \nu} \Lambda_k^{-e_1} T_b^T \mathcal{J} \cdot (\mathbf{k} - \gamma_\perp(\mathbf{k})) \mathcal{P}_k^{\mu \nu} \Lambda_k^{-e_2} \right] \\
+ (e_{1,2} \rightarrow -e_{1,2}, T_{a,b} \rightarrow T_{a,b}^T) \right\}. \tag{106b}
\]

The trace over the 12 x 12 color-Dirac matrices is more complicated than in all previously discussed cases. For the \((00)\) components, both \(\mathcal{V}\) and \(\mathcal{W}\) are diagonal in color space. The diagonal elements are collected in Table IV.

They are divided into two parts, one corresponding to the symmetric Gell-Mann matrices, \(a = 1, 3, 4, 6, 8,\) the other corresponding to the antisymmetric Gell-Mann matrices, \(a = 2, 5, 7.\) For the \((ii)\) and \((0i)\) components, we find (omitting the indices \(r, s\) and \(e_1, e_2\))

\[
\mathcal{V}_{ab}^{00} = \mathcal{V}_{ba}^{00} = \mathcal{W}_{ab}^{00} = \mathcal{W}_{ba}^{00} = 0 . \tag{107}
\]

Therefore, also for the CSL phase, the \((0i)\) and \((i0)\) components of the gluon polarization tensor vanish.

For the \((ij)\) components, we find that \(\mathcal{V}\) and \(\mathcal{W}\) are neither diagonal with respect to color \(a, b\) nor with respect to spatial indices. Nevertheless, after inserting the \(k\) integrals from Table II, the gluon polarization tensor becomes diagonal, i.e., \(\Pi_{ab}^{ij}(0) \sim \delta_{ab} \delta_{ij}.\) The reason for the cancellation of all non-diagonal elements are the following properties of \(\mathcal{V}\) and \(\mathcal{W},\)

\[
\sum_{rs} \mathcal{V}_{ab,e_1 e_2}^{ij, rs} \sim \delta_{ab} \delta_{ij} \frac{1}{6\pi^2}, \tag{108a}
\]

\[
\mathcal{W}_{ab,e_1 e_2}^{ij, rs} \sim 1 - e_1 e_2 \quad \text{for } (i \neq j; a, b \text{ arbitrary}), \text{ and for } (i = j; a \neq b) . \tag{108b}
\]

Taking into account the trace over flavor space, which is the same as in the polar phase, we obtain

\(a) \mu = \nu = 0.\)

\[
\Pi_{ab}^{00}(0) = \delta_{ab} \frac{g^2}{18\pi^2} \sum_{n=1}^{N_f} \mu_n^2 \left\{ \begin{array}{ll} 3(v^{12} + v^{21}) + 3w^{22} & \text{for } a = 1, 3, 4, 6, 8, \\
+6(w^{12} + w^{21}) - 3w^{22} & \\
2v^{11} + (v^{12} + v^{21}) + 5w^{22} & \text{for } a = 2, 5, 7 . \end{array} \right. \tag{109}
\]

\(b) \mu = 0, \nu = i \text{ and } \mu = i, \nu = 0.\)

\[
\Pi_{ab}^{ii}(0) = \Pi_{ab}^{ii}(0) = 0 . \tag{110}
\]

\(c) \mu = i, \nu = j.\) The general result (keeping all functions \(v, w\)) is a complicated \(24 \times 24\) matrix and therefore not shown here. However, as stated above, the polarization tensor is diagonal after inserting the values for the functions \(v\) and \(w,\) and the result for the Debye and Meissner masses will be given in the next section.
2. Mixed polarization tensor ($a \leq 8, b = 9$ and $a = 9, b \leq 8$)

Here we have

\[
\mathcal{V}^{\mu \nu, rs}_{\alpha \gamma, e_1 e_2} = \frac{e^2}{2} \int \frac{dQ_{k}}{(2\pi)^3} \left\{ \text{Tr} \left[ \gamma^\mu \gamma_0 Q \mathcal{P}^r_{k} \Lambda_{k}^{-e_1} \gamma^\nu \gamma_0 Q \mathcal{P}^s_{k} \Lambda_{k}^{-e_2} \right] + (e_{1,2} \to -e_{1,2}, T_a \to T_a^T) \right\},
\]

(111a)

\[
\mathcal{W}^{\mu \nu, rs}_{\alpha \gamma, e_1 e_2} = -\frac{e^2}{2} \int \frac{dQ_{k}}{(2\pi)^3} \left\{ \text{Tr} \left[ \gamma^\mu T_a J \cdot (\mathbf{k} - \gamma_{\perp}(\mathbf{k})) \mathcal{P}^r_{k} \Lambda_{k}^{e_1} \gamma_0 Q J \cdot (\mathbf{k} - \gamma_{\perp}(\mathbf{k})) \mathcal{P}^s_{k} \Lambda_{k}^{-e_2} \right] + (e_{1,2} \to -e_{1,2}, T_a \to T_a^T) \right\},
\]

(111b)

where $Q = \text{diag}(q_1, \ldots, q_{N_f})$. All (00), (i0), and (0i) components of these integrals vanish,

\[
\mathcal{V}_{a\gamma}^{00} = \mathcal{V}_{a\gamma}^{0i} = \mathcal{V}_{a\gamma}^{i0} = \mathcal{W}_{a\gamma}^{00} = \mathcal{W}_{a\gamma}^{0i} = \mathcal{W}_{a\gamma}^{i0} = 0.
\]

(112)

(Here, we again omitted the indices $r$, $s$ and $e_1$, $e_2$.) The $(ij)$ components of $\mathcal{V}$ and $\mathcal{W}$ are nonvanishing and, as for the gluonic CSL case, we do not show them explicitly. However, the final result, i.e. the polarization tensor in the considered limit, vanishes because of the following relations,

\[
\sum_{rs} \mathcal{V}^{ij,rs}_{a\gamma, e_1 e_2} = 0,
\]

(113a)

\[
\mathcal{W}^{ij,rs}_{a\gamma, e_1 e_2} \sim 1 - e_1 e_2.
\]

(113b)

(a) $\mu = \nu = 0$. According to Eq. (112), we have

\[
\Pi^{00}_{a\gamma}(0) = 0.
\]

(114)

(b) $\mu = 0, \nu = i$ and $\mu = i, \nu = 0$.

\[
\Pi^{0i}_{a\gamma}(0) = \Pi^{i0}_{a\gamma}(0) = 0.
\]

(115)

(c) $\mu = i, \nu = j$. This polarization tensor has a complicated structure in terms of $v, w$ and is thus not given here. However, as stated above, the final result is zero.

3. Photon polarization tensor ($a = b = 9$)

For the photon polarization tensor in the CSL phase we need

\[
\mathcal{V}^{\mu \nu, rs}_{\gamma \gamma, e_1 e_2} = \frac{e^2}{2} \int \frac{dQ_{k}}{(2\pi)^3} \left\{ \text{Tr} \left[ \gamma^\mu \gamma_0 Q \mathcal{P}^r_{k} \Lambda_{k}^{-e_1} \gamma^\nu \gamma_0 Q \mathcal{P}^s_{k} \Lambda_{k}^{-e_2} \right] + (e_{1,2} \to -e_{1,2}) \right\},
\]

(116a)

\[
\mathcal{W}^{\mu \nu, rs}_{\gamma \gamma, e_1 e_2} = -\frac{e^2}{2} \int \frac{dQ_{k}}{(2\pi)^3} \left\{ \text{Tr} \left[ \gamma^\mu Q J \cdot (\mathbf{k} - \gamma_{\perp}(\mathbf{k})) \mathcal{P}^r_{k} \Lambda_{k}^{e_1} \gamma_0 Q J \cdot (\mathbf{k} - \gamma_{\perp}(\mathbf{k})) \mathcal{P}^s_{k} \Lambda_{k}^{-e_2} \right] + (e_{1,2} \to -e_{1,2}) \right\}.
\]

(116b)

The results for the (00) and (ij) components are given in Tables V and VI. The (0i) and (i0) components vanish (for all $r$, $s$, $e_1$, $e_2$),

\[
\mathcal{V}_{\gamma \gamma}^{i0} = \mathcal{V}_{\gamma \gamma}^{0i} = \mathcal{W}_{\gamma \gamma}^{i0} = \mathcal{W}_{\gamma \gamma}^{0i} = 0.
\]

(117)

One obtains the following photon polarization tensor.

(a) $\mu = \nu = 0$.

\[
\Pi^{00}_{\gamma \gamma}(0) = \sum_{n=1}^{N_f} q_n^2 \mu_n^2 \frac{e^2}{\pi^2} (v^{11} + 2v^{22} - 4w^{11} - 2w^{22})
\]

(118)
We distinguish between the normal-conducting and the superconducting phase. The screening properties of the normal-conducting phase are obtained with the numbers given in Table II for temperatures larger than the critical temperature for the superconducting phase transition, \( T \geq T_c \). For all \( a, b \leq 9 \), they lead to a vanishing Meissner mass, i.e., as expected, there is no Meissner effect in the normal-conducting state. However, there is electric screening for temperatures larger than \( T_c \). Here, the Debye mass solely depends on the number of quark flavors and their electric charge. We find (with \( a \leq 8 \))

\[
T \geq T_c : \quad m_{D,aa}^2 = 3 N_f \frac{g_a^2 \mu_\pi^2}{6\pi^2}, \quad m_{D,ag}^2 = 0, \quad m_{D,gg}^2 = 18 \sum_n q_n^2 \frac{e^2 \mu_n^2}{6\pi^2}. \tag{121}
\]

Consequently, the 9 × 9 Debye mass matrix is already diagonal. Electric gluons and electric photons are screened.

The masses in the superconducting phases are more interesting. The results for all phases are collected in Table VII (Debye masses) and Table VIII (Meissner masses). The physically relevant, or “rotated”, masses are obtained after diagonalization of the 9 × 9 mass matrices. We see from Tables VII and VIII that the special situation discussed in Sec. II B applies to all considered phases; namely, all off-diagonal gluon masses, \( a, b \leq 8 \), as well as all mixed masses for \( a \leq 7 \) vanish. Furthermore, in all cases where the mass matrix is not diagonal, we find \( m_{a}^2 = m_{88} m_{a\gamma} \). Therefore, the rotated masses \( \tilde{m}_{88} \) and \( \tilde{m}_{\gamma\gamma} \) (which are the eigenvalues of the mass matrix) are determined by Eqs. (45). The electric and magnetic mixing angles \( \theta_D \equiv \theta_2 \) and \( \theta_M \equiv \theta_1 \) are obtained with the help of Eqs. (44). Remember that the indices 1 and 2 originate from the spatially transverse and longitudinal projectors defined in Eqs. (18). They were associated with the Meissner and Debye masses in Eqs. (39). We collect the rotated masses and mixing angles for all phases in Table IX.

Let us first discuss the spin-zero cases, 2SC and CFL. In the 2SC phase, the gluon masses are obtained from Eqs. (72) and (74). Due to a cancellation of the normal and anomalous parts, represented by \( v \) and \( w \), the Debye and Meissner masses are equal:

\[
\tilde{m}_{88}^{2SC} = m_{D,aa}^{2SC} = m_{D,ag}^{2SC} = m_{D,gg}^{2SC} = 0, \quad \tilde{m}_{\gamma\gamma}^{2SC} = 0. \tag{75}
\]

For the CFL phase, we have

\[
\tilde{m}_{88}^{CFL} = m_{D,aa}^{CFL}, \quad \tilde{m}_{\gamma\gamma}^{CFL} = 0, \quad \tilde{m}_{\gamma8}^{CFL} = m_{D,ag}^{CFL}, \quad \tilde{m}_{\gamma\gamma}^{CFL} = m_{D,gg}^{CFL}. \tag{76}
\]

TABLE V: (00) components of the tensors \( V, W \) for the photon polarization tensor in the CSL phase, defined in Eqs. (116).

<table>
<thead>
<tr>
<th>( V_{\gamma\gamma}^{00} )</th>
<th>( V_{\gamma\gamma}^{01} )</th>
<th>( V_{\gamma\gamma}^{02} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{\gamma\gamma}^{00} )</td>
<td>( V_{\gamma\gamma}^{01} )</td>
<td>( V_{\gamma\gamma}^{02} )</td>
</tr>
</tbody>
</table>

\[
\delta^{ij}(11 + 7e_1e_2)/(54\pi^2) \quad 8\delta^{ij}(1 - e_1e_2)/(27\pi^2) \quad \delta^{ij}(19 - e_1e_2)/(27\pi^2)
\]

\[
W_{\gamma\gamma}^{00} = W_{\gamma\gamma}^{01} = W_{\gamma\gamma}^{02} = 0
\]

TABLE VI: (ij) components of the tensors \( V, W \) for the photon polarization tensor in the CSL phase, defined in Eqs. (116).

<table>
<thead>
<tr>
<th>( V_{\gamma\gamma}^{ij} )</th>
<th>( V_{\gamma\gamma}^{ij} )</th>
<th>( V_{\gamma\gamma}^{ij} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{\gamma\gamma}^{ij} )</td>
<td>( V_{\gamma\gamma}^{ij} )</td>
<td>( V_{\gamma\gamma}^{ij} )</td>
</tr>
</tbody>
</table>

\[
\delta^{ij}(11 + 7e_1e_2)/(54\pi^2) \quad 8\delta^{ij}(1 - e_1e_2)/(27\pi^2) \quad \delta^{ij}(19 - e_1e_2)/(27\pi^2)
\]

\[
W_{\gamma\gamma}^{ij} = W_{\gamma\gamma}^{ij} = W_{\gamma\gamma}^{ij} = 0
\]

\[\Pi_{\gamma\gamma}^{00}(0) = 0 \quad \Pi_{\gamma\gamma}^{01}(0) = 0. \tag{119}\]

\[
\Pi_{\gamma\gamma}^{ij}(0) = \delta^{ij} \sum_{n=1}^{N_f} q_n^2 \mu_n^2 \frac{e^2}{27\pi^2} \left[ 9v^{11} + 2v^{11} + 16(w^{12} + w^{21}) + 18v^{22} + 20v^{22} \right.
\]

\[
+ 36w^{11} - 8w^{11} + 32(w^{12} + w^{21}) + 18w^{22} - 20w^{22} \right]. \tag{120}\]

V. MIXING AND SCREENING
TABLE VII: Zero-temperature Debye masses. All masses are given in units of $N_f \mu^2/(6\pi^2)$, where $N_f = 2$ in the 2SC phase, $N_f = 3$ in the CFL phase, and $N_f = 1$ in the polar and CSL phases. We use the abbreviations $\zeta \equiv (21 - 8 \ln 2)/54$, $\alpha \equiv (3 + 4 \ln 2)/27$, and $\beta \equiv (6 - 4 \ln 2)/9$.

<table>
<thead>
<tr>
<th></th>
<th>$m^2_{D,aa}$</th>
<th>$m^2_D = m^2_{D,aa}$</th>
<th>$m^2_D,\gamma\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1 2 3 4 5 6 7 8</td>
<td>1 7 8 9</td>
<td></td>
</tr>
<tr>
<td>2SC</td>
<td>3$g^2$</td>
<td>7$g^2$</td>
<td>2$e^2$</td>
</tr>
<tr>
<td>CFL</td>
<td>3$\zeta g^2$</td>
<td>3$g^2$</td>
<td>0 $-2\sqrt{3}$ $\zeta g e$</td>
</tr>
<tr>
<td>polar</td>
<td>$\frac{3g^2}{4}$</td>
<td>$\frac{3g^2}{4}$</td>
<td>$\frac{3g^2}{4}$</td>
</tr>
<tr>
<td>CSL</td>
<td>$3\beta g^2$</td>
<td>$3\alpha g^2$</td>
<td>$3\beta g^2$</td>
</tr>
</tbody>
</table>

$$m^2_{D,aa}$$ in Table VII are used.

Meissner masses for the gluons 1, 2, and 3 vanish. Physically, this is easy to understand. Since the condensate picks one color direction, all quarks of the third color, say blue, remain unpaired. The first three gluons only interact with red and green quarks and thus acquire neither a Debye nor a Meissner mass. We recover the results of Ref. [17]. For the mixed and photon masses we inserted the electric charges for $u$ and $d$ quarks, i.e., in Eqs. (77), (79), (81), and (83) we set $q_1 = 2/3$ and $q_2 = -1/3$. Here we find the remarkable result that the mixing angle for the Debye masses is different from that for the Meissner masses, $\theta_D \neq \theta_M$. The Meissner mass matrix is not diagonal. By a rotation with the angle $\theta_M$, given in Table IX, we diagonalize this matrix and find a vanishing mass for the new photon. Consequently, there is no electromagnetic Meissner effect in this case. This fact is well-known [12]. The Debye mass matrix, however, is diagonal. The off-diagonal elements $m^2_{D,\alpha\gamma}$ vanish, since the contribution of the ungapped modes, corresponding to the blue quarks, $v^{22}$, cancels the one of the gapped modes, $v^{11} - w^{11}$, cf. Eq. (77). Consequently, the mixing angle is zero, $\theta_D = 0$. Physically, this means that not only the color-electric eighth gluon but also the electric photon is screened. Had we considered only the gapped quarks, i.e., $v^{22} = 0$ in Eqs. (72), (77), and (81), we would have found the same mixing angle as for the Meissner masses and a vanishing Debye mass for the new photon. This mixing angle is the same as predicted from simple group-theoretical arguments, Eq. (55). The photon Debye mass in the superconducting 2SC phase differs from that of the normal phase, Eq. (121), which, for $q_1 = 2/3$ and $q_2 = -1/3$ is $m^2_D,\gamma\gamma = 5 N_f \mu^2/(6\pi^2)$.

In the CFL phase, all eight gluon Debye and Meissner masses are equal. This reflects the symmetry of the condensate where there is no preferred color direction. For the mixed and photon masses, we used Eq. (97), i.e., we inserted the electric charges for $u$, $d$, and $s$ quarks into the more general expressions given in Sec. IV C. The results in Tables VII and VIII show that both Debye and Meissner mass matrices have nonzero off-diagonal elements, namely $m^2_{D,\alpha\gamma} = m^2_{M,\alpha\gamma}$. Diagonalization yields a zero eigenvalue in both cases. This means that neither electric nor magnetic (rotated) photons are screened. Or, in other words, there is a charge with respect to which the Cooper pairs are neutral. Especially, there is no electromagnetic Meissner effect in the CFL phase, either. Note that the CFL phase is

<table>
<thead>
<tr>
<th></th>
<th>$m^2_{D,aa}$</th>
<th>$m^2_D,\alpha\gamma$</th>
<th>$m^2_D,\alpha\gamma$</th>
<th>$m^2_D,\gamma\gamma$</th>
<th>$m^2_D,\gamma\gamma$</th>
<th>$m^2_D,\gamma\gamma$</th>
<th>$m^2_D,\gamma\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1 2 3 4 5 6 7 8</td>
<td>1 7 8 9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2SC</td>
<td>3$g^2$</td>
<td>2$e^2$</td>
<td>1 $\frac{3g^2}{4} + \frac{3g^2}{4} e^2$</td>
<td>0 $3g^2/(3g^2 + e^2)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CFL</td>
<td>$(4e^2 + 3g^2)\zeta$</td>
<td>0 $3g^2/(3g^2 + 4e^2)$</td>
<td>$(\frac{3g^2}{4} + g^2)\zeta$</td>
<td>0 $3g^2/(3g^2 + 4e^2)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>polar</td>
<td>$\frac{3g^2}{4}$</td>
<td>$\frac{3g^2}{4}$</td>
<td>$\frac{3g^2}{4}$</td>
<td>$\frac{3g^2}{4}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CSL</td>
<td>$3\beta g^2$</td>
<td>$18g^2 e^2$</td>
<td>1 $\frac{3g^2}{4} + 4g^2 e^2$</td>
<td>0 $g^2/(g^2 + 12g^2 e^2)$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE IX: Zero-temperature rotated Debye and Meissner masses in units of $N_f \mu^2/(6\pi^2)$ and mixing angles for electric and magnetic gauge bosons. The constants $\zeta$, $\alpha$, and $\beta$ are defined as in Tables VII and VIII.
the only one considered in this paper in which electric photons are not screened. Unlike the 2SC phase, both electric and magnetic gauge fields are rotated with the same mixing angle $\theta_D = \theta_M$. This angle is well-known [12, 13]. Note that, due to the spectrum of the matrix $\gamma_0 M_D^a \gamma_0 = (J \cdot \mathbf{1})^2$, there are two gapped branches. Unlike the 2SC phase, there is no ungapped quasiparticle excitation branch. This is the reason why both angles $\theta_D, \theta_M$ coincide with the one predicted in Eq. (55).

Let us now discuss the spin-one phases, i.e., the polar and CSL phases. For the sake of simplicity, all results in Tables VII, VIII, and IX refer to a single quark system, $N_f = 1$, where the quarks carry the electric charge $q$. After discussing this most simple case, we will comment on the situation where $N_f > 1$ quark flavors separately form Cooper pairs. The results for the gluon masses show that, up to a factor $N_f$, there is no difference between the polar phase and the 2SC phase regarding screening of color fields. This was expected since also in the polar phase the blue quarks remain unpaired. Consequently, the gluons with adjoint color index and the 2SC phase regarding screening of color fields. This was expected since also in the polar phase the blue quarks remain unpaired. Consequently, the gluons with adjoint color index $a = 1, 2, 3$ are not screened. Note that, due to Eq. (105), the spatial z-direction picked by the spin of the Cooper pairs has no effect on the screening masses. As in the 2SC phase, electric gluons do not mix with the photon. There is electromagnetic Debye screening, which, in this case, yields the same photon Debye mass as in the normal phase, cf. Eq. (121). The Meissner mass matrix is diagonalized by an orthogonal transformation defined by the mixing angle which equals the one in Eq. (55).

In the CSL phase, we find a special pattern of the gluon Debye and Meissner masses. In both cases, there is a difference between the gluons corresponding to the symmetric Gell-Mann matrices with $a = 1, 3, 4, 6, 8$ and the ones corresponding to the antisymmetric matrices, $a = 2, 5, 7$. The reason for this is, of course, the residual symmetry group $SO(3)_{c+J}$ that describes joint rotations in color and real space and which is generated by a combination of the generators of the spin group $SO(3)_J$ and the antisymmetric Gell-Mann matrices, $T_2, T_3,$ and $T_7$. The remarkable property of the CSL phase is that both Debye and Meissner mass matrices are diagonal. In the case of the Debye masses, the mixed entries of the matrix, $m_{D, \gamma a}$, are zero because of the vanishing traces, Eq. (112), indicating that pure symmetry reasons are responsible for this fact (remember that, in the 2SC phase, the reason for the same fact was a cancellation of the terms originating from the gapped and ungapped excitation branches). There is a nonzero photon Debye mass which is identical to that of the polar and the normal phase which shows that electric photons are screened in the CSL phase. Moreover, and only in this phase, also magnetic photons are screened. This means that there is an electromagnetic Meissner effect. Consequently, there is no charge, neither electric charge, nor color charge, nor any combination of these charges, with respect to which the Cooper pairs are neutral. This was also shown in Sec. III where we argued that in the CSL phase there is no nontrivial residual $\hat{U}(1)_{cm}$, cf. Table I. This feature of the CSL phase was already discussed in Ref. [20].

Finally, let us discuss the more complicated situation of a many-flavor system, $N_f > 1$, which is in a superconducting state with spin-one Cooper pairs. In both polar and CSL phases, this extension of the system modifies the results in Tables VII, VIII, and IX. We have to include several different electric quark charges, $q_1, \ldots, q_{N_f}$, and chemical potentials, $\mu_1, \ldots, \mu_{N_f}$, in a way explained below Eq. (105) and shown in the explicit results of the CSL phase, Sec. IVE. In the CSL phase, these modifications will change the numerical values of all masses, but the qualitative conclusions, namely that there is no mixing and electric as well as magnetic screening, remain unchanged. In the case of the polar phase, however, a many-flavor system might change the conclusions concerning the Meissner masses. While in the one-flavor case, diagonalization of the Meissner mass matrix leads to a vanishing photon Meissner mass, this is no longer true in the general case with arbitrary $N_f$. There is only a zero eigenvalue if the determinant of the matrix vanishes, i.e., if $m_{M, \gamma a} = m_{M, 88} m_{M, \gamma \gamma}$. Generalizing the results from Table VIII, this condition can be written as

$$\sum_{n,m} q_n(q_n - q_m) \mu_n^2 \mu_m^2 = 0 \ .$$

(122)

Consequently, in general and for fixed charges $q_n$, there is a hypersurface in the $N_f$-dimensional space spanned by the quark chemical potentials on which there is a vanishing eigenvalue of the Meissner mass matrix and thus no electromagnetic Meissner effect. All remaining points in this space correspond to a situation where the (new) photon Meissner mass is nonzero (although, of course, there might be a mixing of the eighth gluon and the photon). Eq. (122) is trivially fulfilled when the electric charges of all quarks are equal. Then we have no electromagnetic Meissner effect in the polar phase which is plausible since, regarding electromagnetism, this situation is similar to the one-flavor case. For specific values of the electric charges we find very simple conditions for the chemical potentials. In a two flavor system with $q_1 = 2/3, q_2 = -1/3$, Eq. (122) reads

$$\mu_1^2 \mu_2^2 = 0 \ .$$

(123)

while, in a three-flavor system with $q_1 = -1/3, q_2 = -1/3,$ and $q_3 = 2/3$, we have

$$(\mu_1^2 + \mu_2^2) \mu_3^2 = 0 \ .$$

(124)
Consequently, these systems always, i.e., for all combinations of the chemical potentials $\mu_n$, exhibit the electromagnetic Meissner effect in the polar phase except when they reduce to the above discussed simpler cases (same electric charge of all quarks or a one-flavor system).

VI. SUMMARY AND CONCLUSIONS

We have presented a calculation of the gluon and photon polarization tensors $\Pi^{\mu\nu}_{ab}(P)$, $a, b = 1, \ldots, 8, \gamma$, in a color superconductor which yield, in the zero-energy, low-momentum limit, the Debye and Meissner masses for the gauge bosons. These masses indicate in which cases electric and magnetic fields are screened. Here, magnetic screening is equivalent to the Meissner effect. We have derived the gauge field propagators in a general treatment starting from the QCD partition function. It has been shown that, in general, longitudinal and transverse modes of the gauge fields are mixed in a nontrivial way. An “unmixing” transformation leads to a modified longitudinal polarization tensor. However, in the static limit, this mixing is absent and longitudinal and transverse modes are well separated.

We have shown that a nondiagonal polarization tensor causes a mixing of the gauge fields. In general, the tensor has to be diagonalized via a transformation generated by an orthogonal operator $O(P)$. Consequently, also the gluon and photon fields, $A^a_{\mu}$, are transformed. The operator $O(P)$ defines new gauge fields, $\tilde{A}^a_{\mu}$, which are linear combinations of the original ones. This is in complete analogy to the standard model of electroweak interactions, where, due to spontaneous breaking of the electroweak symmetry $SU(2) \times U(1)$, the new fields are the gauge bosons of the weak and electromagnetic interaction. In this case, the three fields corresponding to the weak interaction attain a mass via the Higgs mechanism. Similarly, also in the case of the color-superconducting phase transition, some of the originally nine gauge fields can become massive (besides a possible mixing). In the framework of the formalism presented here, the masses of the new fields are the eigenvalues of the polarization tensor $\Pi^{i}_{ab}(0)$, where the index $i = 1, 2$ corresponds to the transverse and longitudinal modes (Meissner and Debye masses, respectively). Thus, the number of vanishing eigenvalues of $\Pi^{i}_{ab}(0)$ indicate the dimension of the unbroken subgroup of the color-electromagnetic product group $SU(3)_c \times U(1)_{em}$.

We have presented a general formalism to calculate the polarization tensor for different color-superconducting phases. We have explicitly computed its zero-energy, low-momentum limit for the 2SC, CFL, polar, and CSL phases. Parts of the results were already known in the literature, namely the gluon Debye and Meissner masses for the spin-0 phases 2SC and CFL [17–19]. Our result for the photon Debye mass in the 2SC phase differs from that of Ref. [13] since our calculation shows that the photon-gluon mass matrix is already diagonal and thus electric gluons do not mix with the photon (while in Ref. [13] the same diagonal matrix was rotated and a “mixed mass” was obtained). The masses for the spin-1 phases have never been computed before. For the polar phase, we have shown that there is mixing between the magnetic gauge bosons but, as in the 2SC phase, no mixing of the electric gauge bosons. In a system of one quark flavor, this mixing leads to a vanishing Meissner mass. However, for more than one quark flavor, we have shown that, if the electric charges of the quark flavors are not identical, there is an electromagnetic Meissner effect in the polar phase, contrary to both considered spin-0 phases. For the CSL phase, we have found the remarkable result that, for any number of flavors, neither electric nor magnetic gauge fields are mixed. Since there is no vanishing eigenvalue of $\Pi^{i}_{ab}(0)$, all eight gluons and the photon (electric as well as magnetic modes) become massive and there is an electromagnetic Meissner effect. This might affect the electromagnetic properties of a neutron star that has a core of color-superconducting quark matter in the CSL phase. We have already discussed these possible astrophysical implications in Ref. [20]. There we also argued that, in spite of a suppression of the gap of three orders of magnitude compared to the spin-0 gaps [9, 10], spin-1 gaps might be preferred in a charge-neutral system. The reason for that is that a mismatch of the Fermi surfaces of different quark flavors has no effect on the spin-1 phases, where quarks of the same flavor form Cooper pairs.

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APPENDIX A: INTEGRALS OVER QUARK MOMENTUM $k$

In this appendix, we prove the results shown in Table II, i.e., we calculate the integrals over quark momentum defined in Eqs. (68). In this calculation we will neglect the antiparticle gap, $\phi^- \simeq 0$. 
1. Contributions from normal propagators at $T = 0$

We start from Eqs. (59) which, for $p_0 = 0$ reduce to (cf. Eq. (61))

$$v_{rs}^{s} = - \frac{\epsilon_1 \epsilon_2 s - \xi_1 \xi_2}{2 \epsilon_1 \epsilon_2 s (\epsilon_1, \epsilon_2, s)} (1 - N_{1,r} - N_{2,s}) + \frac{\epsilon_1 \epsilon_2 s + \xi_1 \xi_2}{2 \epsilon_1, \epsilon_2 s (\epsilon_1, \epsilon_2, s)} (N_{1,r} - N_{2,s}).$$

(A1)

Here, we inserted the definition of $n_i$ given in Eq. (60) and defined

$$\xi_i \equiv \epsilon_i k_i - \mu.$$  

(A2)

In the limit $p \to 0$ or, equivalently, $k_1 \to k_2$, we obtain for $r = 1, 2$

$$v^{rr} \simeq - \frac{1}{\mu^2} \int_0^\infty dk k^2 \left[ \frac{\lambda_r \phi^2}{4 e_r^2} (1 - 2 N_r) - \frac{\epsilon_r^2 + \xi_r^2}{2 e_r^2} dN_r \right],$$

(A3)

where we used $(N_{1,r} - N_{2,r})/(\epsilon_{1,r} - \epsilon_{2,r}) \simeq dN_r/d\epsilon_r$ and $\xi \equiv k - \mu$. All quantities now depend on $k_1 = k$ and correspond to positive energies, $\epsilon_1 = \epsilon_2 = 1$, since, due to $\phi \simeq 0$, the terms corresponding to negative energies vanish. Therefore, we omit the index $i = 1, 2$. Note that, up to subleading order, the gap function does not depend on the index $r, \phi_1 \simeq \phi_2 \equiv \phi$ [10]. We have to distinguish between the cases where $\lambda_r \neq 0$ and where $\lambda_r = 0$. When $\lambda_r = 0$, we obtain for zero temperature $T = 0$, where $N_r = 0$,

$$v^{rr} = - \frac{1}{\mu^2} \int_0^\infty dk k^2 \frac{\lambda_r \phi^2}{4 e_r^2} \simeq - \frac{1}{2} \int_0^\mu d\xi \lambda_r \phi^2 (\epsilon_r^2 + \lambda_r \phi^2)^{-3/2} \simeq - \frac{1}{2}.$$

(A4)

Here, we restricted the $k$-integration to the region $0 \leq k \leq 2\mu$ since the gap function is strongly peaked around the Fermi surface, $k = \mu$. In this region, we assume that the gap function does not depend on the momentum $k$. Since the eigenvalue $\lambda_r$ cancels out, this result does not depend on $r$. Therefore it gives rise to the value of $v^{11}$ for all four considered phases and for $v^{22}$ in the cases of the CFL and CSL phases. When $\lambda_r = 0$, we obtain (for arbitrary temperature $T$)

$$v^{rr} = \frac{1}{\mu^2} \int_0^\infty dk k^2 dN_F d\mu,$$

(A5)

where $N_F \equiv 1/[\exp(\xi/T) + 1]$ is the Fermi distribution. Using $\mu \gg T$, and the substitution $\zeta = \xi/2T$, this can be transformed to

$$v^{rr} \simeq - \int_0^\infty d\zeta \frac{1}{\cosh^2 \zeta} = -1.$$  

(A6)

Let us now compute $v^{rs}$ for $r \neq s$. Using Eq. (A1), we obtain for two nonvanishing eigenvalues $\lambda_1, \lambda_2$ and at zero temperature

$$v^{12} = v^{21} = - \frac{1}{\mu^2} \int_0^\infty dk k^2 \frac{\epsilon_1 \epsilon_2 - \xi^2}{2 \epsilon_1 \epsilon_2 (\epsilon_1 + \epsilon_2)} \simeq - \int_0^\mu d\xi \frac{1}{(\lambda_2 - \lambda_1) \phi^2} \left( \frac{\epsilon_2^2 + \xi_2^2}{\epsilon_2} - \frac{\epsilon_1^2 + \xi_1^2}{\epsilon_1} \right) \simeq - \frac{1}{2}.$$  

(A7)

Again, we neglected the integral over the region $k > 2\mu$. In the last integral, the leading order contributions cancel. The subleading terms give rise to $(\lambda_2 - \lambda_1) \phi^2/2$. Therefore, the eigenvalues cancel and the result does not depend on $\lambda_1, \lambda_2 \neq 0$. Eq. (A7) holds for the CFL and CSL phases.

When one of the excitation branches is ungapped (2SC and polar phase), for instance, $\lambda_2 = 0$, we have

$$v^{12} = v^{21} = - \frac{1}{\mu^2} \int_0^\infty dk k^2 \left[ \frac{n_1}{\epsilon_1 + \xi} (1 - N_1 - N_F) - \frac{1 - n_1}{\epsilon_1 - \xi} (N_1 - N_F) \right].$$  

(A8)

For $T = 0$, this becomes with the standard approximations

$$v^{12} \simeq - \frac{1}{\mu^2} \int_{-\mu}^\mu d\xi (\xi + \mu)^2 \left[ \frac{\epsilon_1 - \xi}{\epsilon_1 + \xi} \Theta(\xi) + \frac{\epsilon_1 + \xi}{\epsilon_1 - \xi} \Theta(-\xi) \right]$$

$$= - \frac{1}{\mu^2} \int_0^\mu d\xi \left( 1 + \frac{\xi^2}{\mu^2} \right) \frac{\epsilon_1 - \xi}{\epsilon_1 + \xi} \simeq - \frac{1}{2},$$

(A9)
where the last integral has already been computed in Ref. [17].

Next we compute $\bar{v}^{rs}$ for the various cases. Setting the thermal distribution function for antiparticles to zero, we obtain with the definition in Eq. (68)

$$\bar{v}^{rs} = -\frac{1}{\mu^2} \int_0^{\infty} dk k^2 \left[ \frac{(1 - n_r)(1 - N_r)}{\epsilon_r^+ + \epsilon_s^-} - \frac{N_r n_r}{\epsilon_r^+ + \epsilon_s^-} + \frac{(1 - n_s)(1 - N_s)}{\epsilon_r^- + \epsilon_s^+} - \frac{N_s n_s}{\epsilon_r^- + \epsilon_s^+} \right]$$  

(A10)

with $\epsilon_r^+ \equiv \epsilon_{k,r}^+$. Again, we first consider the situation where $r = s$. In this case, for $T = 0$ and $\lambda_r \neq 0$, (defining $\epsilon \equiv \epsilon^+$)

$$\bar{v}^{rr} = -\frac{2}{\mu^2} \int_0^{\infty} dk k^2 \left[ \frac{1}{\epsilon_r^+ + k + \mu} \right]$$

$$= \frac{1}{\mu^2} \int_0^{\infty} dk k \frac{\mu(\epsilon_r^+ + k + \mu) + \lambda_r \phi^2}{\epsilon_r^+(\epsilon_r^+ + k + \mu)} \approx \frac{1}{2}. \quad \text{(A11)}$$

Here, $-1/(2k)$ is a vacuum subtraction. The last integral has been computed in Ref. [17]. Note that the result does not depend on the value of $\lambda_r \neq 0$. Thus, it is valid for $\bar{v}^{11}$ in the 2SC and polar phases as well as for both $\bar{v}^{11}$ and $\bar{v}^{22}$ in the CFL and CSL phases.

For $\lambda_r = 0$, we obtain from Eq. (A10) at zero temperature and with the vacuum subtraction $1/(2k)$

$$\bar{v}^{rr} = -\frac{2}{\mu^2} \int_0^{\infty} dk k^2 \left[ \frac{\Theta(k - \mu)}{2k} \right] = \frac{1}{2}. \quad \text{(A12)}$$

This result holds for $\bar{v}^{22}$ in the 2SC and polar phases.

Next, we discuss the case $r \neq s$. From Eqs. (A10) and (A11) it is obvious, since the result for $\bar{v}^{rr}$ did not depend on $\lambda_r$, that we get the same result for $\bar{v}^{rs}$ with two nonvanishing eigenvalues $\lambda_1, \lambda_2 \neq 0$. Thus, in this case,

$$\bar{v}^{12} = \bar{v}^{21} = \frac{1}{2}. \quad \text{(A13)}$$

For $\lambda_2 = 0$, we obtain at zero temperature

$$\bar{v}^{12} = \bar{v}^{21} = -\frac{1}{\mu^2} \int_0^{\infty} dk k^2 \left[ \frac{1 - n_1}{\epsilon_1^+ + k + \mu} \right]$$

$$= \frac{1}{\mu^2} \int_0^{\infty} dk k \frac{\Theta(k - \mu) - \frac{3}{2k}}{2k} \approx \frac{1}{2}. \quad \text{(A14)}$$

where identical integrals as in Eqs. (A11) and (A12) were performed. Consequently, also for the 2SC and polar phases, Eq. (A13) holds.

### 2. Contributions from anomalous propagators at $T = 0$

We start from Eq. (63) which, for $p_0 = 0$, is

$$w^{rs}_{\epsilon_1 \epsilon_2} = \frac{\phi^{\epsilon_1 \phi^{\epsilon_2}}(1 - N_{1,r} - N_{2,s}) + \phi^{\epsilon_1 \phi^{\epsilon_2}}(N_{1,r} - N_{2,s})}{2\epsilon_1 \epsilon_2 (\epsilon_1 + \epsilon_2)} \left( 1 - \frac{1}{2N_r} \right) + \frac{\phi^{\epsilon_1 \phi^{\epsilon_2}}}{2\epsilon_1 \epsilon_2 (\epsilon_1 - \epsilon_2)} \left( N_{1,r} - N_{2,s} \right). \quad \text{(A15)}$$

Obviously, since $\phi^{-} \approx 0$, we have for all $r, s$ and all phases $\bar{w}^{rs} = 0$ (for the definition of $\bar{w}^{rs}$ cf. Eq. (68)). First, we calculate $w^{rs}$ for $r = s$. Neglecting the antiparticle gap and taking the limit $k_1 \to k_2$, we obtain, using the same notation as above, $\epsilon_r \equiv \epsilon_{k,r}^+$,

$$w^{rr} = \frac{1}{\mu^2} \int_0^{\infty} dk k^2 \left[ \frac{\phi^2}{4\epsilon_r^2} (1 - 2N_r) + \frac{\phi^2}{2\epsilon_r^2} dN_r \right]. \quad \text{(A16)}$$

For $\lambda_r \neq 0$ and at zero temperature, where $N_r = 0$, this expression reduces to

$$w^{rr} = \frac{1}{\mu^2} \int_0^{\infty} dk k^2 \frac{\phi^2}{4\epsilon_r^2} \approx \frac{1}{2} \int_0^{\mu} d\xi \phi^2 (\xi^2 + \lambda_r \phi^2)^{-3/2} \approx \frac{1}{2\lambda_r}. \quad \text{(A17)}$$

Unlike in $v^{rr}$, the eigenvalue $\lambda_r$ does not cancel out and we obtain different results for $\lambda_r = 4$ and $\lambda_r = 1$. In the cases of the 2SC and polar phases we obtain $w^{11} = 1/2$ whereas for the CFL and CSL phases, $w^{11} = 1/8$ and $w^{22} = 1/2$. In
the former two cases, the quantity \( w^{22} \) does not occur in our calculation. Thus we can turn to the case where \( r \neq s \).

Here, we conclude from Eq. (A15)

\[
w^{12} = w^{21} = \frac{1}{\mu^2} \int_0^\infty dk k^2 \left[ \frac{\phi^2}{2\epsilon_1 \epsilon_2} \left( 1 - N_1 - N_2 \right) + \frac{\phi^2}{2\epsilon_1 \epsilon_2} (N_1 - N_2) \right].
\]  

(A18)

For two nonvanishing eigenvalues \( \lambda_1, \lambda_2 \), this reads at \( T = 0 \)

\[
w^{12} = \frac{1}{\mu^2} \int_0^\infty dk k^2 \frac{\phi^2}{2\epsilon_1 \epsilon_2} \left( 1 - \frac{\lambda_1}{\lambda_2} \right) \simeq \frac{1}{\lambda_1 - \lambda_2} \int_0^\mu d\xi \left( \frac{1}{\epsilon_2} - \frac{1}{\epsilon_1} \right)
\]

\[
\simeq \frac{1}{2} \ln \frac{\lambda_1}{\lambda_2}.
\]  

(A19)

For \( \lambda_1 = 4 \) and \( \lambda_2 = 1 \) (CFL and CSL phases) we obtain

\[
w^{12} = w^{21} = \frac{1}{3} \ln 2.
\]  

(A20)

The corresponding expression for the case where the second eigenvalue vanishes, \( \lambda_2 = 0 \), does not occur in our calculation.

### 3. Integrals in the normal phase, \( T \geq T_c \)

For temperatures larger than the transition temperature \( T_c \) but still much smaller than the chemical potential, \( T \ll \mu \), all integrals defined in Eqs. (68) are easily computed. Since there is no gap in this case, \( \phi = 0 \), all contributions from the anomalous propagators vanish trivially, \( w^{rs} = \bar{w}^{rs} = 0 \). From Eq. (A1), which hold for all temperatures, we find with \( \phi = 0 \)

\[
v^{11} = v^{22} = v^{12} = \frac{1}{\mu^2} \int_0^\infty dk k^2 dN_F \simeq -1
\]

(A21)

and (with the vacuum subtraction \( 1/k \))

\[
\bar{v}^{11} = \bar{v}^{22} = \bar{v}^{12} = -\frac{1}{\mu^2} \int_0^\infty dk k^2 \left[ \frac{1 - N_F}{k} - \frac{1}{k} \right] \simeq \frac{1}{2}.
\]  

(A22)