New Vacuum State of the Electromagnetic Field-Matter Coupling System and the Physical Interpretation of Casimir Effect

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A new concise method is presented for the calculation of the ground-state energy of the electromagnetic field and matter field interacting system. With the assumption of squeezed-like state, a new vacuum state is obtained for the interacting system. The energy of the new vacuum state is lower than that given by the second-order perturbation theory in existing theories. In our theory, the Casimir effect is attributed neither to the quantum fluctuation in the zero-point energy of the genuine electromagnetic field nor to that in the zero-point energy of the genuine matter field, but to that in the vacuum state of the interacting system. Both electromagnetic field and matter field are responsible for the Casimir effect.

The phenomenon that an attractive force exists between two uncharged conducting metal plates is called the Casimir effect because of the celebrated formula that Casimir derived for the force between the two plates. Casimir calculated the sum of the zero-point energies of the normal modes of the electromagnetic (EM) fields and showed that the sum and the Casimir force, as the spatial derivative of the total energy, depend on the distance between the two plates. Later, Lifshitz and van Kampen made the generalization of Casimir effect to the case of two dielectric plates.

The origin of the Casimir effect has usually been attributed to the quantum fluctuations in the EM zero-point energy due to the presence of the two plates. However, Schwinger and his cooperators proposed the Schwinger’s source theory of the Casimir effect and showed that the Casimir force can be derived with no explicit reference to the zero-point energy fluctuations of the...
EM field. Milonno and Shih\[7\] developed a new source theory within the framework of completely conventional quantum electrodynamics. In this theory, the Casimir force can be derived in terms of the quantum fluctuations of atomic dipoles in the dielectric where the EM field mediates the EM interactions between those dipoles. Recently, Koashi and Ueda\[7\] published the matter-field theory of Casimir force in which the matter and EM field can be treated on an equal footing. Their strategy is to explicitly diagonalize the matter-field Hamiltonian, which is quadratic in its dynamic variables. The result shows that both EM field and matter field contributes to the zero-point energy and therefore to the Casimir force.

So it is unclear up to now what the physical interpretation of the Casimir effect is. It seems like it is largely a matter of taste whether we attribute the Casimir force to the quantum nature of the EM field or to that of the matter field.\[7, 8\] However, according to the postulates of quantum mechanics, to any well-defined observable (or dynamical variable) in physics, there corresponds an operator such that measurement of the observable yields values which are eigenvalues of the corresponding operator. Therefore, any physical effect should have unambiguous interpretations. On the other hand, the interpretation of the physical effect should reflect the physical reality.

In this letter, we will treat the EM field and the matter collective modes (plasmons in dielectric plates) on the same basis as in Ref. \[7\]. A new concise method will be presented for the calculation of the ground-state energy of the EM field-matter interacting system. A qualitative interpretation of the Casimir effect will be given based on the new vacuum state obtained in this letter. In our theory, the Casimir effect is attributed neither to the quantum fluctuations in zero-point energy of the genuine EM field nor to that in zero-point energy of the genuine matter field, but to that in zero-point energy of the combined vacuum state of the EM field and the matter field.

We now consider the interacting system of EM field and the matter field. In quantum mechanics, the EM field can be described as a set of harmonic oscillators corresponding to the normal mode frequencies, which are determined from the Maxwell equations with proper boundary conditions. The so-called matter field represents the collective motions of various kinds of charged particles in the dielectric substance.\[7\] They are surface mode plasmons that can also be described as a set of harmonic oscillators. Considering the interactions between these two sets of harmonic oscillators, the Hamiltonian of the system is as follows:

\[ H = H_0 + H_I, \]  

(1)

where

\[ H_0 = \sum_k \hbar \omega_{a,k} \left( a^+_k a^+_k + \frac{1}{2} \right) + \sum_i \hbar \omega_{b,i} \left( b^+_i b^+_i + \frac{1}{2} \right) \]  

(2)

and \( H_I \) is the interaction energy of linearly interacting subsystems, \( \hbar \) the Planck constant. \( a^+_k (a_k) \) represents the creation (annihilation) operator of EM field
with the wavevector $k$ and frequency $\omega_{a,k}$, and $b_k^\dagger$ ($b_k$) the creation (annihilation) operator of the collective modes in the dielectric substance characterized by frequency $\omega_{b,k}$. $a_k$, $a_k^\dagger$, $b_k$ and $b_k^\dagger$ are Bosonic operators and satisfy the relations $[a_k,a_k^\dagger]=\delta_{k,k'}$, $[b_k,b_k^\dagger]=\delta_{i,j}$ and $[a_k,b_j]=[a_k,b_j^\dagger]=[a_k^\dagger,b_j]=[a_k^\dagger,b_j^\dagger]=0$, where $\delta_{i,j}$ and $\delta_{k,k'}$ are the Kronecker $\delta$-functions.

In general, the ground-state energy of the system can be obtained by directly diagonalizing the Hamiltonian. However, in order to obtain analytic results so as to have an explicit interpretation of the Casimir effect, without losing generality, we consider the interacting system with single mode of EM field and single mode matter field. In this case, we have the following simple form of Hamiltonian:

$$ H_0 = \hbar \omega_a (a^\dagger a + 1/2) + \hbar \omega_b (b^\dagger b + 1/2) $$

and

$$ H_I = v_+ a^\dagger a + v_- a^\dagger b + v_+^* a b + v_-^* a b^\dagger. $$

where $a^\dagger$ ($a$) and $b^\dagger$ ($b$) are the creation (annihilation) operators of the single mode EM field and matter field respectively, while $v_+$, $v_-$, $v_+^*$ and $v_-^*$ the coupling coefficients between the two fields.

From the Hamiltonian, Eq. (4), we know that the two subsystems are coupled linearly with each other and the numbers of photons (of the EM field) and plasmons (of the matter field) are not conservative respectively. Therefore it is appropriate to write the ground-state wave function for the interacting system in the form of squeezed-like state as follows:

$$ |g\rangle = A \exp \left( \alpha a^\dagger a + \beta b^\dagger b + \gamma a^\dagger b^\dagger \right) |0,0\rangle, $$

where $|0,0\rangle$ is the unperturbed vacuum state of the EM and matter fields with the property $a |0,0\rangle = b |0,0\rangle = 0$ and $\alpha$, $\beta$ and $\gamma$ are the parameters to be determined in the following calculation. $A$ is a constant for the normalization of the state $|g\rangle$ according to the relation $\langle g | g \rangle = 1$. With the aid of closure relations of coherent states of $a$ and $b$, we have

$$ A = \left\{ \int d(Re\phi) \pi \int d(Im\phi) \pi \right\} \exp \left[ 2\text{Re}(\alpha \phi \phi^* + \beta \phi^* \phi \varphi) \right] $$

where $\phi$ and $\varphi$ are respectively eigenvalues of $a$ and $b$, corresponding to their coherent states $|\phi\rangle$ and $|\varphi\rangle$, $i.e.$, $a |\phi\rangle = \phi |\phi\rangle$ and $b |\varphi\rangle = \varphi |\varphi\rangle$. The state $|g\rangle$ satisfies the following relations

$$ a |g\rangle = (2\alpha a^\dagger + \gamma b^\dagger) |g\rangle $$

and

$$ b |g\rangle = (2\beta b^\dagger + \gamma a^\dagger) |g\rangle. $$
The assumed vacuum state \( |g \rangle \) should be the solution of the Schrödinger wave equation and satisfy
\[
H |g \rangle = E |g \rangle ,
\] (8)
from which the parameters \( \alpha, \beta \) and \( \gamma \) and therefore the vacuum state \( |g \rangle \) and the corresponding energy \( E \) can be determined.

Putting Eq. (5) into Schrödinger equation (8) and using Eqs. (6) and (7), we get
\[
0 = +v_+^*\gamma |g \rangle + \frac{1}{2} (\hbar \omega_a + \hbar \omega_b) |g \rangle - E |g \rangle \\
+2\hbar \omega_a a^+ a^+ |g \rangle + v_- \alpha a^+ a^+ |g \rangle + 2v_+^* \alpha \gamma a^+ a^+ |g \rangle \\
+2\hbar \omega_b b^+ b^+ |g \rangle + v_+^* \gamma b^+ b^+ |g \rangle + 2v_+^* \beta \gamma b^+ b^+ |g \rangle \\
+(\hbar \omega_a + \hbar \omega_b) \gamma a^+ b^+ |g \rangle + v_+ a^+ b^+ |g \rangle + v_+^* \gamma a^+ b^+ |g \rangle \\
+2v_+^* \alpha a^+ b^+ |g \rangle + 2v_- \beta a^+ b^+ |g \rangle + 4v_+^* \alpha \beta a^+ b^+ |g \rangle .
\] (9)

In the above equation, we have put those terms, such as, \( |g \rangle, a^+ a^+ |g \rangle, b^+ b^+ |g \rangle \) and \( a^+ b^+ |g \rangle \) together. In order to get a solution of the Schrödinger equation, we may let the coefficients of those terms equal zero respectively. Then we have
\[
E = v_+^*\gamma + \frac{1}{2} (\hbar \omega_a + \hbar \omega_b) ,
\] (10)
\[
2\hbar \omega_a \alpha + v_- \gamma + 2v_+^* \gamma \alpha = 0, \tag{11}
\]
\[
2\hbar \omega_b \beta + v_+^* \gamma + 2v_+^* \gamma \beta = 0, \tag{12}
\]
\[
(\hbar \omega_a + \hbar \omega_b) \gamma + v_+ + v_+^* \gamma^2 + 2v_+^* \alpha + 2v_- \beta + 4v_+^* \alpha \beta = 0. \tag{13}
\]

In this case we can get a solution with above conditions although we could not assert that it is a unique one. We know that \( |g \rangle, a^+ a^+ |g \rangle, b^+ b^+ |g \rangle \) and \( a^+ b^+ |g \rangle \) are not orthogonal to each other but they are linearly independent. All the states with \( (a^+)^m (b^+)^n |g \rangle \) build a complete base of Hilbert space. Therefore the solution of Eqs. (10)-(13) should be a state of the system. In addition, we will see below that the state energy is lower than that of the ordinary vacuum state. This fact means that the new state could be regarded as the physical vacuum state of the given system.

From Eqs. (10)-(12) we can get the relations among the parameters \( \alpha, \beta \) and \( \gamma \) and the state energy \( E \). Considering the following confining conditions derived from the normalization of the state \( |g \rangle \)
\[
|\alpha| < 1/2, \quad |\beta| < 1/2, \quad |\gamma| < 1,
\] (14)
we obtain the following unique possible energy value for the ground state from equation (13):
\[
E = \frac{1}{2} \left[ \left( \frac{(\hbar \omega_a)^2 + (\hbar \omega_b)^2}{2} \right) - 2 \left( |v_+|^2 - |v_-|^2 \right) \right]
+2 \left[ \frac{(\hbar \omega_a \hbar \omega_b - |v_+|^2 + |v_-|^2)^2 - 4 |v_-|^2 (\hbar \omega_a \hbar \omega_b)^{1/2}}{2} \right] \frac{1}{2}.
\] (15)
which is obviously lower than the unperturbed vacuum state, i.e., $E < E_0 = (\hbar \omega_a + \hbar \omega_b)/2$.

For the weak interaction, i.e., $|v_+|, |v_-| << \hbar \omega_a$, and $\hbar \omega_b$, we have

$$E = \frac{1}{2} \left( \frac{|v_+|^2}{(\hbar \omega_a + \hbar \omega_b)} - \frac{2 |v_+|^2}{(\hbar \omega_a + \hbar \omega_b)^2} + \frac{|v_-|^2}{(\hbar \omega_a \hbar \omega_b)} \right)$$

(16)

to the fourth order of $|v_\pm|/\hbar \omega_a, b$. This result agrees exactly with that obtained by Koashi and Ueda\cite{Koashi1993} to the second order of $|v_\pm|/\hbar \omega_a, b$ and it is lower than that in Ref.[\?] to the fourth order. At the same time, our theory applies not only for the weak interacting system but also for the relatively strong interacting system. We can get the analytic result for strong interacting system.

We now discuss the physical interpretation of the Casimir effect. Up to now, there are two quite different kinds of physical interpretations of the Casimir force, i.e., the field-theory\cite{Casimir1948, Schwinger1951} and the source theory\cite{Koashi1993, Schwinger1951, Schwinger1960}. In the field-theory, the Casimir force is attributable to the zero-point energy fluctuations of the genuine EM field, where the existence of the conducting plates can only change the boundary conditions of the EM field. However, Schwinger et al.\cite{Schwinger1951, Schwinger1960} got the same formula of the Casimir force based on the source theory, and they pointed out that the Casimir effect is attributable to the quantum fluctuations of the genuine matter field, where the EM field is only the medium to transport the electromagnetic interactions between atomic dipoles in the dielectric substances.\cite{Koashi1993} However, if we choose appropriate ordering of operators in the Hamiltonian of linearly coupled harmonic oscillators, the second-order correction to the ground-state energy can therefore be attributed solely to quantum fluctuations of either one of the oscillators, the EM field operator or the matter field one.\cite{Koashi1993, Schwinger1951} Then the Casimir force can be attributable either to the genuine EM field or to the genuine matter field.\cite{Koashi1993} According to the theories of Casimir,\cite{Casimir1948} Schwinger et al.\cite{Schwinger1951, Schwinger1960} and Koashi and Ueda,\cite{Koashi1993} it seems as if the physical interpretation of the Casimir force depends on what physical models or mathematical methods we used, but not on the expression of physical reality.

In this letter, we used a new concise method to calculate the ground-state energy of the EM field and matter interacting system. The obtained vacuum energy is obviously lower than that of the unperturbed vacuum state and that given by the second-order perturbation theory.\cite{Koashi1993} So the new state can be regarded as the real vacuum state of the interacting system. We may use this state to interpret the source of the Casimir force. In the discussion, we have dealt with the EM field and the matter field on the same basis. This is very important to the interpretation of the Casimir force because relevant physical observables
are expressed in terms of eigenvalues and eigenstates of the full Hamiltonian of the entire system \( H \) according to postulates in quantum mechanics.

In the Hamiltonian of the interacting system, i.e., \( H_0 + H_I \) (Eqs. (3) and (4)), there are two points in connection with the distance \( d \) between the two dielectric plates. On the one hand, the surface plasmon (such as its frequency \( \omega_b \)) does depend both on the parameters of the dielectric surface and on that of the space between the two plates, and therefore it must be related to the distance \( d \). On the other hand, the EM mode (such as the frequency \( \omega_a \)) is affected by the boundary condition and dielectric constant and therefore is related to the distance \( d \) too. The EM field can alter the properties of surface plasmon (such as the frequency \( \omega_b \)) by the interaction between the two fields. For example, this interaction exists only in a thin film beneath the surface of a perfectly conducting metal. Therefore we know that all the physical quantities, such as \( \omega_a \), \( \omega_b \), and all the coupling coefficients \( v_+ \), \( v_- \), \( v^*_+ \) and \( v^*_− \) in the Hamiltonian (3) and (4) depend on the distance \( d \) between the two dielectric plates. In summary, the vacuum state (Eq. (5)) and the eigenenergy \( E \) (Eq. (15)) of the interacting system depend on the distance \( d \).

Because the Casimir force is derived from the change in the ground-state energy \( E \) with respect to a virtual infinitesimal displacement of the dielectric plates, i.e., \( f_{\text{Casimir}} = -\frac{\partial E}{\partial d} \), we have the conclusion that Casimir effect is attributable to the quantum fluctuations of the combined vacuum state of the EM field and the matter field. It is the interaction between the two fields that produces the Casimir effect.

In summary, we have presented a new concise method for the calculation of the ground-state energy of the interacting system of the EM field and matter field. With the assumption of squeezed-like state, a new vacuum state is obtained for the interacting system. The energy of the vacuum state is lower than that given by the second-order perturbation theory. Based on the new vacuum state, the Casimir effect is attributed to the fluctuations in zero-point energy of the vacuum state of the interacting system. Both EM field and matter field are responsible for the Casimir effect.

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