Detectability of the Sgr Dwarf Leading Tidal Stream with Auger, EUSO or OWL

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Abstract

We point out that if heavy metastable particles composing the dark matter of our galaxy are responsible for the ultra-high energy cosmic rays (UHECR) then the leading tidal stream of the Sagittarius dwarf galaxy could be detected through UHECR. The signal would be an anisotropy in the UHECR flux smaller than the telltale anisotropy towards the galactic center that would first establish unstable dark matter as the origin of the UHECR.

Cosmic ray particles with energies above the Greisen-Zatsepin-Kuzmin cutoff \(^{[1]}\) of about \(5 \times 10^{19}\) eV have been detected by a number of independent experiments over the last decade \(^{[2]}\). The existence of these ultra-high energy cosmic rays (UHECR) presents us with a problem. Nucleons and photons with those energies have short attenuation lengths and could only come from distances of 100 Mpc or less, while plausible astrophysical sources for those energetic particles are much farther away. Recently, results from the HiRes Collaboration \(^{[3]}\) brought the violation of the GZK cutoff into question and this issue will only be resolved conclusively by the Pierre Auger Observatory.

Among the solutions proposed for the origin of the UHECR is the decay of supermassive relic particles which may constitute all or a fraction of the dark matter \(^{[4]}\). Such particles must be metastable, with lifetimes exceeding the age of the Universe. In this scenario the flux \(\phi\) of UHECR coming from a particular volume element in the Universe is proportional
to the density of dark matter in this volume, and inversely proportional to the square of the distance $r$ between us and this particular volume element. Thus the flux is the integral of the column density over the solid angle

$$\phi \sim \int \frac{d^3 r \rho(\vec{r})}{r^2} = \int d\Omega \int \rho(\vec{r}) dr.$$  

Consequently, in decaying dark matter models the UHECR are produced mostly in our galaxy and the key test is the expected anisotropy in the arrival directions caused by the offset of the Sun from the center of the galaxy $[5]$. In these models cosmic rays from dark matter dominate at energies above about $6 \times 10^{19}$ eV, and are mostly photons $[6]$. Even for protons the effects of magnetic fields on UHECR trajectories can be neglected at these energies $[7]$, thus the trajectories of these UHECR are straight lines and point towards the place of origin. Spherical halo models yield an anisotropy symmetric around the direction of the Galactic Center (GC) (while triaxial models, which will not be considered here for simplicity, give considerable angular deviations). The amplitude of this anisotropy is controlled by the halo core radius $R_c$, a parameter which appears in the halo density models, the simplest of which is the isothermal halo model

$$\rho_{\text{halo}} = \rho_o \frac{R_c^2}{r'^2 + R_c^2}$$  

where $r'$ is the radius from the GC.

The halo core radius $R_c$ is not well determined from astrophysical means and could in fact be measured through the anisotropy itself. In isothermal halo models, the GC asymmetry amplitude, $A_{\text{GC}}(\theta)$, defined as the ratio of the flux in the direction of the GC and the flux from the galactic anti-center (AC) within an integration cone of aperture $\theta$, depends strongly on $R_c$. For example in $[8]$, for $R_c = 5$ kpc, the amplitudes are found to be (see Fig. 5 of $[8]$) $A_{\text{GC}}(5^\circ) \simeq 5$ and $A_{\text{GC}}(80^\circ) \simeq 3$, while for $R_c = 10$ kpc, $A_{\text{GC}}(5^\circ) \simeq 3$ and $A_{\text{GC}}(80^\circ) \simeq 2$.

The observation of a GC-AC asymmetry would prove the decaying particle scenario for the origin of the UHECR. In this case, the leading tail of the Sagittarius Dwarf Galaxy would give rise to a different asymmetry in the UHECR fluxes. This is the new asymmetry we are considering here.

The Two Micron All Sky Survey (2MASS) and the Sloan Digital Sky Survey (SDSS) $[9, 10]$ have traced the tidal stream of the Sagittarius (Sgr) dwarf galaxy more than $360^\circ$ around the sky. This galaxy is a dwarf spheroidal of roughly $10^9 M_\odot$. It is a satellite of the Milky Way Galaxy, located inside the Milky Way, $\sim 12$ kpc behind the Galactic Center (GC) and $\sim 12$ kpc below the Galactic Plane (GP) $[11]$. There are two streams of matter that extend outwards from the main body of the Sgr galaxy and wrap around the GC. These
streams, known as the leading and trailing tidal tails, are made of matter tidally pulled away from the Sgr galaxy. They are on the orbital plane of the Sgr galaxy, which crosses the Galactic plane near the position of the Solar system. The leading tidal tail results from stars (and dark matter particles [see [12]]) that were originally between the Sgr dwarf center and the center of the Milky Way. These stars that are closer to the GC orbit faster than the Sgr dwarf center, they move at lower gravitational potential and have shorter orbital periods around the GC. Thus, these faster orbiting stars (and dark matter particles) stream out ahead of the main body of the Sgr dwarf and create the leading tail. The stars that give rise to the leading tail are stripped at roughly a distance of 10 kpc from the GC, on the opposite side of the GC from the Sun. This distance of 10 kpc is determined [12] by subtracting the current distance between the GC and the center of the Sgr main body, \( \sim 16 \) kpc, minus the Sgr tidal radius, \( \sim 6 \) kpc. This distance of 10 kpc determines the perigalacticon of the leading tidal tail, both where it was removed from the Sgr main body and again on this side (the solar side) of the GC. Similarly, the trailing tail is formed from stars that were on the opposite side of the dwarf when they were stripped, they are further out and thus move slower than the Sgr galaxy main body.

The trailing stream roughly extends in a doughnut shape in the Southern galactic hemisphere around the position of the Sun, at a distance of about 20 kpc from us. The leading tail goes around the GC in the Northern galactic hemisphere, turns around at a radius of about 35 kpc, then approaches the galactic plane from the Northern galactic hemisphere and crosses this plane passing through the Solar system (see Fig. 11 of [9]) in the general direction orthogonal to the Galactic plane. The streams carry dark matter as well as visible stars. Thus the leading stream brings additional dark matter into the vicinity of the Solar system. The effect of the leading Sgr stream for direct detection experiments, if the DM consists of weakly interacting massive particles, has recently been studied in [12] where the estimated range of DM density in the stream was found to be

\[
\rho_{\text{stream}} = [2, 200] \times 10^4 M_{\odot}/\text{kpc}^3 = [0.001, 0.08] \text{ GeV/cm}^3. \tag{3}
\]

This density corresponds to a fraction \( \chi=(0.3-25)\% \) of the local density of the isothermal Galactic halo for the usual estimate \( \rho_h = 0.3 \text{ GeV/cm}^3 \), and to a fraction \( \chi=(0.45-37.5)\% \) of the local halo density for a lower estimate \( \rho_h = 0.2 \text{ GeV/cm}^3 \) (e.g. see the compilation of possible local halo density values for spherically symmetric halo models in [13]). This evaluation assumes a constant density along the stream [12].

The trailing stream may lead to some small increase of UHECR events coming from the Sgr dwarf plane, but this increase will be very small and would be more difficult to detect than the leading trail. Recall that the flux depends on the integral of the density over the line of sight (see Eq. (1)), and we can see along the axis of the leading trail but only transversally through the trailing tail.
We concentrate here on the leading tidal stream, which passes through the solar system. The width of the tidal stream in 2MASS M stars is estimated to be 4-8 kpc and the best-fit projection brings the center of the leading tail within 2 kpc of the Sun [9]. We can see in Fig. 11 of [9] that the stream starts turning around at about (25-35) kpc from the Sun, but for smaller distances can be approximated by a cylinder. For simplicity here we model the leading stream as a cylinder of radius \( R \) and height \( L \) "above" the plane of the galaxy, i.e. on the side of the North Galactic Pole (NGP). We assume that the central axis of the cylinder passes through the position of the Sun. The NGP is located at \( \delta = +27^\circ 24' \) of declination and \( \alpha =12^h 49^m \) of right ascension.

We then compare the expected UHECR flux \( \phi_\parallel \) in a solid angle of aperture \( \theta \) in the direction of the NGP (i.e. along the axis of the cylinder), with the flux \( \phi_\perp \) coming from an identical cone taken in a direction perpendicular to both the directions to the NGP and the GC (a cone with axis in the galactic plane). There are two directions with this property, one pointing to \( \delta = -48^\circ \), \( \alpha =9^h \) and the other pointing to \( \delta = +48^\circ \), \( \alpha =21^h \). We define the asymmetry amplitude

\[
A_{\text{stream}}(\theta) = \frac{\phi_\parallel}{\phi_\perp}.
\]

(4)

In a spherically symmetric halo without the Sgr stream there should be no difference in the fluxes from both directions, thus \( A_{\text{stream}} =1 \). The presence of the stream leads to an asymmetry \( A_{\text{stream}}>1 \).

Taking the evaluation of the density of the leading stream from [12] to be \( \rho_{\text{stream}} = \chi \rho_h \), where \( \rho_h \) is the halo density at the location of the Sun, and assuming that the density is constant along the stream, we have computed \( A_{\text{stream}}(\theta) \), using the halo model in Eq. (2). Notice that \( A_{\text{stream}}(\theta) \) depends on the local overdensity \( \chi \) due to the stream and not on the local halo density itself. In Fig. 1 we plot the column density as seen from the location of the Sun for the halo model in Eq. (2) with \( R_c = 5 \) kpc and for several of our leading stream models. The increase in column density in the directions close to the axis of the stream is clearly visible, except for the lowest values of \( \chi \).

In Figs. 2 and 3 we plot the asymmetry \( A_{\text{stream}}(\theta) \) for different values of \( \chi \) and two values of \( R_c = 5 \) kpc and 10 kpc (Fig. 2 and 3, respectively). With \( R \) and \( L \) as the dimensions chosen for the cylinder representing the stream, \( A_{\text{stream}}(\theta) \) is constant for \( \theta < \arctan(R/L) \), and decreases rapidly with increasing values of \( \theta \), as can be seen in Figs. 2 and 3. The maximum asymmetry, \( A_{\text{stream}}(\theta \leq 6.5^\circ) = 1.7 \), is obtained for the largest cylinder, lowest halo density (i.e. highest \( \chi \)) and small core radius, namely for \( R=4 \) kpc, \( L=35 \) kpc, \( \chi = 0.375 \) and \( R_c=5 \) kpc. A larger core radius decreases the asymmetry to \( A_{\text{stream}}(\theta \leq 6.5^\circ) = 1.57 \). Reducing \( L \) to 25 kpc one obtains \( A_{\text{stream}}(\theta \leq 9^\circ) = 1.5 \) for \( R_c = 5 \) kpc (and 1.38 for \( R_c = 10 \) kpc). Reducing the radius of the cylinder to \( R = 3 \) kpc and reducing \( \chi \) to \( \chi = 0.25 \), with \( L=25 \) kpc the asymmetry amplitude becomes \( A_{\text{stream}}(\theta \leq 7^\circ) = 1.36 \) (1.28) for
\( R = 5 \) (10) kpc. These are some of the largest expected asymmetries, while the asymmetries for the smallest values of \( \chi \) are negligible. For example, \( A_{\text{stream}} < 1.005 \) for \( R=3 \) kpc, \( L=25 \) kpc and \( \chi = 0.003 \).

In order to detect the asymmetry \( A_{\text{stream}}(\theta) \) above the three-sigma level we need a minimum number of events \( N \) within the two cones of aperture \( \theta \) that we are comparing,

\[
\frac{N_{\parallel} - N_{\perp}}{\sqrt{N_{\parallel} + N_{\perp}}} = \sqrt{\frac{(A_{\text{stream}} - 1)}{A_{\text{stream}} + 1}} \geq 3,
\]

which means

\[
N \geq 9 \frac{(A_{\text{stream}} + 1)}{(A_{\text{stream}} - 1)^2}.
\]

For the maximum value of the asymmetry, \( A_{\text{stream}} = 1.7 \), we need at least \( N = 50 \) events; for \( A_{\text{stream}} = 1.4 \), we need \( N = 135 \) events; for \( A_{\text{stream}} = 1.2 \), we need \( N = 500 \) events; for \( A_{\text{stream}} = 1.1 \), we need at least \( N \) few \( 10^3 \) events; and for \( A_{\text{stream}} < 1.01 \), we would need \( N \) few \( 10^5 \) events. These are the number of events within the cones considered. While the number of events increases as the solid angle of the cones increases, the stream asymmetry decreases for cones of aperture larger than \( \arctan(R/L) \). Thus there is an optimum cone aperture which maximizes the reach of each observatory.

Let us see how large an overdensity \( \chi \) the Pierre Auger Observatory, EUSO and OWL would be able to measure. The reach of each observatory is shown in Fig. 2 and Fig. 3 with thicker solid (red) lines. As can be seen in the figures, the integration cones for which the stream is best detectable through the asymmetry have an aperture \( \theta \) of about 15° to 20°. For larger cones, although the minimum asymmetries that can be detected decrease with increasing number of events \( N \) within each cone, the predicted stream asymmetry decreases faster, so that the reach in \( \chi \) does not improve.

To estimate the number of expected events, we need a value for the UHECR flux. At energies of \( 6 \times 10^{19} \) eV the AGASA and HiRes cosmic ray fluxes differ by about a factor of 3 (see e.g. Fig. 1 in [18]). Since it is the AGASA data that do not show a GZK cutoff, we adopt the AGASA value for the flux, with an energy dependence of \( E^{-2} \), which is typical of decaying dark matter models [6]. This gives us an isotropic UHECR flux integrated above \( 6 \times 10^{19} \) eV of about 0.07 (km\(^2\) sr yr\(^{-1}\). At higher energies the data points of AGASA, which are about one order of magnitude higher than those of HiRes, give an integrated isotropic flux of about 0.04 (km\(^2\) sr yr\(^{-1}\) for \( E > 1 \times 10^{20} \) eV.

The southern station of the Pierre Auger Observatory (see for example [14]) is already under construction since the year 2000 in the Province of Mendoza, Argentina, at a latitude of about 35.2° South. It will be completed by 2004. The northern station is proposed for Utah and would then enable continuous full-sky coverage. The southern array consists of 1600
particle detectors spread over 3000 km² with fluorescence telescopes placed on the boundaries of the surface array. The expected angular resolution is less than 1° for all energies (0.5° at 10²⁰ eV). This angular resolution is very good to localize the flux from the stream. The limiting effective aperture for the full southern array is 7350 km²sr, including only zenith angles less than 60°, which is the region of the sky where the reconstruction algorithm works best. Given that the latitude of the southern array is 35.2° South, this means that only objects at declination smaller than +24.8° can be reconstructed well. However, the stream is centered at the NGP, which has a declination of +27.4°, and so it is outside the best detection region of Auger South.

Thus, the stream asymmetry could not be studied by the Southern Auger site unless events below a 60° zenith angle can be analyzed effectively. In this case the aperture would increase by about 50%, becoming 11250 km²sr, and there would be a total of 790 events per year with $E \geq 6 \times 10^{19}$ eV. In the extreme case in which zenith angles as large as 85° could be reached, and we are not sure that this can be done, the number of events within the cones of solid angle $\Omega$ we are studying would be a fraction $f = \frac{\Omega}{(3.64\pi)} = \frac{(1 - \cos \theta)}{(1 + \cos 40°)}$ of the total number of events observed, and only cones with aperture $\theta < 22.4°$ would be completely included in the observable portion of the sky. In this case the minimum $A_{\text{stream}}(\theta)$ observable by Auger South in 10 years as function of $\theta$ for $\theta < 22.4°$ can be seen in Figs. 2 and 3 (thicker solid (red) line with the label Auger S). In particular, for $\theta = 20°$, we have $f = 0.034$, and after 10 years we would get $N = 268$ events with $E \geq 6 \times 10^{19}$ eV. This number of events would allow us to detect an asymmetry $A_{\text{stream}}(20°) \geq 1.28$. From Figs. 4 and 5, which plot the overdensity $\chi$ due to the stream as a function of the asymmetry within a cone of 20° aperture, we see that if Auger South could observe at zenith angles of up to 85°, it could detect the stream if $\chi$ is at least $\chi = 0.3$ (this value occurs for $R = 4$ kpc, $L = 35$ kpc and $R_c = 5$ kpc).

For Figs. 4 and 5 we have chosen a cone aperture of 20° because this aperture is close to that one that maximizes the reach, although the latter depends somewhat on the observatory (see Figs. 2 and 3).

If the Northern Auger site would become a reality, the NGP and thus the stream would be in full view of this new observatory. The projected total effective area of 44000 km²sr [15] of the combined Southern and Northern Auger sites, with total $4\pi$ sr sky coverage, would yield 3080 events with $E \geq 6 \times 10^{19}$ eV. The number of events within one of the cones of solid angle $\Omega$ we are studying is a fraction $f = \frac{\Omega}{(4\pi)} = \frac{(1 - \cos \theta)}{2}$. For $\theta = 20°$, this gives $f = 0.030$. Thus in 10 years of combined Southern and Northern Auger data we would obtain $N = 924$ events with $E \geq 6 \times 10^{19}$ eV. This would allow the detection of asymmetries above the thicker solid (red) line labeled Auger S+N shown in Figs. 2 and 3. In particular we could detect $A_{\text{stream}}(20°) \geq 1.14$, which would translate into the detection of a stream with $\chi \geq 0.14$ (see Figs. 4 and 5).
The Extreme Universe Space Observatory (EUSO) [16], a mission on-board the International Space Station, will image the air fluorescence of the extensive air showers produced by UHECR in the atmosphere. Its angular resolution is expected to be less than 2° and a conservative estimate of the duty cycle is 10%. It will operate for three years, with an effective trigger aperture which increases very fast with energy from $1.0 \times 10^4 \text{ km}^2 \text{ sr}$ at $E = 4 \times 10^{19} \text{ eV}$ to $4.5 \times 10^4 \text{ km}^2 \text{ sr}$ at $E \geq 1 \times 10^{20} \text{ eV}$ (these effective apertures include the 10% duty cycle). The sky coverage is expected to be very close to complete, thanks to a suitable inclination of the EUSO detector axis with respect to the vertical and to the characteristics of the International Space Station orbit (the ISS orbit has a typical inclination of about 52° with respect to the ecliptic, a period of about a 16th of a day, and a precession of the line of nodes of about 5° per day [17]). Thus, within $4\pi$ sr of sky coverage, we expect about 1800 events per year with $E \geq 1 \times 10^{20} \text{ eV}$. The number of events within the cones of solid angle $\Omega$ we are studying is a fraction $f = \Omega/(4\pi) = (1 - \cos \theta)/2$ ($f = 0.030$ for $\theta = 20°$). This means that in three years EUSO could detect asymmetries above the thicker solid (red) line labeled EUSO shown in Figs. 2 and 3. In particular, in three years of operation $N = 162$ events would be collected with $E \geq 1 \times 10^{20} \text{ eV}$ for $\theta = 20°$. Thus, EUSO could measure a minimum asymmetry $A_{\text{stream}}(20°) = 1.36$, which is larger than the largest we obtained in our models at $\theta = 20°$ (see Figs. 2 and 3).

The Orbiting Wide-angle Light-collectors (OWL) [18] mission would consist of a pair of satellites observing extended air showers in the atmosphere produced by UHECR with stereoscopic view. The angular resolution would be less than 1°. It would have an instantaneous aperture rapidly increasing with energy, from $4 \times 10^4 \text{ km}^2 \text{ sr}$ at $E = 3 \times 10^{19} \text{ eV}$ to about $2 \times 10^6 \text{ km}^2 \text{ sr}$ at $E \geq 1 \times 10^{20} \text{ eV}$. Taking into account that OWL can only view the dark side of the Earth, and accounting for the effect of the Moon, clouds and man-made light, the duty cycle is about 10%. In fact the continuous effective aperture would be $2.3 \times 10^5 \text{ km}^2 \text{ sr}$ for $E \geq 1 \times 10^{20} \text{ eV}$ [18] and about two orders of magnitude smaller at $E = 3 \times 10^{19} \text{ eV}$. Because of the steep rise of the aperture with energy, OWL would see more events at $E \geq 1 \times 10^{20} \text{ eV}$ than at $E \geq 3 \times 10^{19} \text{ eV}$. With a flux of 0.04 (km$^2$ sr s)$^{-1}$, OWL would collect 9200 events per year. The number of events within the cones of solid angle $\Omega$ we are studying is a fraction $f = \Omega/(4\pi) = (1 - \cos \theta)/2$ of these 9200 events per year. In five years of operation the minimum asymmetry OWL could detect is shown as a function of $\theta$ in Figs. 2 and 3 with the thicker solid (red) line labeled OWL. In particular, OWL would detect 276 events per year in a cone of aperture $\theta = 20°$. This corresponds to a detection of $N = 1380$ events within the same cone in 5 years. Thus OWL could detect stream asymmetries above 1.12, i.e. could reach local overdensities due to the stream $\chi \geq 0.12$. In terms of overdensity due to the stream, OWL’s five-year reach is better than the reach of the combined Auger South and North in 10 years.

In conclusion, EUSO would not be able to see the stream. The Southern Auger site alone
cannot study the stream asymmetry unless events below a 60° zenith angle can be analyzed effectively. In the hypothetical case in which zenith angles as large as 85° could be reached, in 10 years Auger South alone could test local overdensities in the stream of $\chi \geq 0.3$ (if $R_c = 5$ kpc). This is possible only if $\rho_h \leq 0.3$ GeV/cm$^3$. For example, if $\rho_h = 0.3$ GeV/cm$^3$, the minimum stream density testable by Auger South alone in 10 years would be $\rho_{\text{stream}} = 0.09$ GeV/cm$^3$, while if $\rho_h = 0.2$ GeV/cm$^3$, it would be $\rho_{\text{stream}} = 0.06$ GeV/cm$^3$.

The best chance of detecting the Sgr leading tidal stream in UHECR would come either with 5 years of observation with OWL or 10 years of observation with the combined Northern and Southern Auger sites. In fact, the combined Northern and Southern Auger data, integrated for about 10 years, would allow us to detect local stream overdensities of $\chi \geq 0.14$ (if $R_c = 5$ kpc). This means $\rho_{\text{stream}} \geq 0.042$ GeV/cm$^3$ if $\rho_h = 0.3$ GeV/cm$^3$, or $\rho_{\text{stream}} > 0.028$ GeV/cm$^3$ if $\rho_h = 0.2$ GeV/cm$^3$.

The OWL data, integrated for about 5 years, would allow us to detect local stream overdensities of $\chi \geq 0.12$ (if $R_c = 5$ kpc). This means $\rho_{\text{stream}} \geq 0.036$ GeV/cm$^3$ if $\rho_h = 0.3$ GeV/cm$^3$, or $\rho_{\text{stream}} > 0.024$ GeV/cm$^3$ if $\rho_h = 0.2$ GeV/cm$^3$.

The numbers given here are only indicative and we do not attempt to provide an estimate of their error. Our conclusions depend on the assumed UHECR fluxes which we took here to be at the level measured by AGASA. Clearly, the searches would become less sensitive for smaller fluxes. Our conclusions also depend on the halo model, in that the asymmetry due to the stream would decrease if the halo is flattened in the direction of the stream.

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References


Figure 1: Halo and stream column densities for several stream models, for halo core radius $R_c = 5$ kpc, as a function of the angle $\psi$ measured from the Galactic Center in the direction of the North Galactic Pole (the Galactic Center is at $\psi = 0$, the North Galactic Pole is at $\psi = 90^\circ$, and the Galactic Anticenter is at $\psi = 180^\circ$).
Figure 2: Asymmetry amplitudes $A_{\text{stream}}(\theta)$ as a function of the cone aperture $\theta$, for $R_c=5\text{ kpc}$ and several values of the overdensity $\chi$ due to the Sgr stream and the radius $R$ and length $L$ of the cylinder that models the Sgr stream. The thicker solid (red) superimposed lines indicate the reach of each of the observatories.
Figure 3: Same as Fig. 2 but for $R_c=10$ kpc.
Figure 4: Local overdensity due to the stream $\chi$ as a function of the asymmetry amplitude for a cone of $20^\circ$ aperture, $A_{\text{stream}}(20^\circ)$, for $R_e=5$ kpc.
Figure 5: Same as Fig. 4 but for $R_c=10$ kpc.

Figure 5: Same as Fig. 4 but for $R_c=10$ kpc.