Polarization effects in metallic films perforated with a bidimensional array of subwavelength rectangular holes

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For several years, periodical arrays of subwavelength cylindrical holes in thin metallic layers have taken a crucial importance in the context of the results reported by Ebbesen et al., on particularly attractive optical transmission experiments. It had been underlined that the zeroth order transmission pattern does not depend on the polarization of the incident light at normal incidence. In the present paper, we show that it is not the case for rectangular holes, by contrast to the case of circular holes. In this context, we suggest a new kind of polarizer that present the advantages brought by the original Ebbesen devices. Assuming the recent technological interest for these kinds of metallic gratings, such a kind of polarizer could lead to new technological applications.

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Introduction

For several years, the motivation for investigating surface plasmon properties takes a crucial importance in the context of plasmonics. In addition, the consideration of metallic gratings has been renewed in the context of the results report by Ebbesen et al. They reported on particularly attractive optical transmission experiments on periodical arrays of subwavelength cylindrical holes in a thin metallic layer deposited on glass, a dielectric substrate. Two specific characteristics of their results are the transmission which is higher than the simple sum of individual holes contributions and the interesting pattern of the zeroth order transmission versus wavelength [1].

During the last few years, these experimental results have received considerable attention and they have implied several theoretical and experimental works [1-17]. Many assumptions were suggested to explain the transmission properties of these devices. For instance, it has been suggested that these phenomena can be described in terms of short range diffraction of evanescent waves [11], or in terms of dynamical diffraction [13]. Another explanation suggest the role of cavity resonance into the holes to explain the transmission enhancement [12,15].

However, the main results tend to prove that surface plasmons (SPs) play a key role in the features observed in the transmission curves [1-7,9,10]. Nevertheless, it remains many questions which must be answered to clarify the processes involved in these experiments. For instance, the exact role of SPs was not clearly assessed. Indeed, the observed transmission presents a set of peaks and minima. Many authors suggested that transmission peaks correspond to SPs resonance [1-7,9,10] whereas others suggested that SPs resonance were related to minima [12,14].

In a recent work [14], we have presented many results which tend to qualify these two hypotheses. We have shown that the concept of resonant Wood anomalies [14,18,19] can be invoked to interpret the role of SPs in the Ebbesen experiments. We have shown that the transmission pattern is better described by Fano’s profiles [19] correlated with interferences between non resonant processes and the resonant response of SPs coupled with nonhomogeneous diffraction orders [14]. We have shown that each maximum of transmission (preceded by a minimum) corresponds to one maximum of a Fano’s profile (preceded by the related minimum) [14,19]. Moreover, whereas such Fano’s profile in transmission is related to a resonant process, the location of its maximum (or minimum) does not necessarily correspond to the location of the resonance [14,19].

In such devices made of subwavelength hole arrays, polarization properties start only to be underlined in some experimental works [16,17]. In the original experiments, for square gratings made of circular holes in chromium films for instance [4], it has been underlined that the zeroth order transmission pattern does not depend on the polarization of the incident light at normal incidence [4]. Obviously, for a square grating made of rectangular holes for instance, one can expect a polarization dependent transmission. Indeed, for a square grating with circular holes the main axies of the grating can be inverted whereas for a square grating with rectangular holes these axies cannot be inverted without break the grating symmetry.

In the present paper, we will show the polarization dependency of the transmission for rectangular holes arrays contrary to circular holes arrays. We will present a simple analytical interpretation of our numerical results. However, we will underline the limits of this simple interpretation as one must not forgot the real complexity of the problem which yet requires a numerical study. Despite a simple appearance, the present problem is not trivial. Our results will lead us to suggest a new kind of polarizer which presents the advantages brought by the original Ebbesen devices [1,4].
In the following, our simulations rest on a coupled modes method associated with the use of the scattering matrix (S matrix) formalism. Taking into account the periodicity of the device, the permittivity is first described by a Fourier series. Then, the electromagnetic field is described by Bloch’s waves which can also be described by a Fourier series. In this context, Maxwell’s equations take the form of a matricial first order differential equation along the z axis perpendicular to the x and y axes where the permittivity is periodic \([14,20,21]\). The heart of the method is to solve this equation. One approach deals with the propagation of the solution step by step by using the S matrix formalism. More explicitly, we numerically divide the grating along the z axis. Indeed, it is well known that S matrices and their combinations are much better conditioned than transfer matrices \([21]\). In this way, we can calculate the amplitudes of the reflected and transmitted fields, for each diffracted order (which corresponds to a vector g of the reciprocal lattice) according to their polarization \((s\ or\ p)\) \([14]\). Note that our algorithm has been accurately compared with other methods such as FDTD or KKR \([22]\). In the present work the convergence is obtained from two harmonics only, i.e. for 25 vectors of the reciprocal lattice. Furthermore, there is no convergence problem associated with discontinuities here, and we do not need to use Li’s method \([23,24]\).

As a continuation of our previous works, we turn our attention to the case of chromium films \([14]\). The values of chromium and glass permittivities are those obtained from experiments \([25]\). For instance, glass permittivity is approximately equal to 2.10 for wavelengths ranging from 1000 to 1500 nm. Here, the thickness of the metallic film is taking equal to \(h = 100\ \text{nm}\). The film is perforated with rectangular holes of sides \(a = 350\ \text{nm}\) and \(b = 700\ \text{nm}\) respectively. The shorter side is along the x axis and the longer side is along the y axis. The holes shape a square grating of parameter \(c = 1000\ \text{nm}\) (fig.1a). Incident light is normal to the interface i.e. normal to the \(Oxy\) plane. The incident light is linearly polarized and the orientation of the incident electric field \(E_{\text{inc}}\) is given by the angle \(\theta\) between \(E_{\text{inc}}\) and the y axis (fig.1a) in the \(Oxy\) plane. Because the transmitted zeroth order is normal to the \(Oxy\) plane, its polarization is described in the \(Oxy\) plane too. The transmitted zeroth order electric field will be described by the complex electric field components \(E_x\) and \(E_y\) along the x and y axes respectively. The polarization of the transmitted zeroth order is then described by the modulus of the amplitudes \(|E_x|\) and \(|E_y|\) and the dephasing \(\delta = \text{arg} \{E_y/E_x\}\). The amplitude of the incident light is equal to 1 V.m\(^{-1}\).

First, we represent in fig.1b a schematic view of the reciprocal lattice related to the square grating of parameter \(c\) in reduced coordinates. Each point corresponds to a reciprocal lattice node, i.e. to one diffraction order \(g = \frac{2\pi}{\lambda} (i,j)\). Solid lines represent a contours representation of the modulus of the Fourier transform of the rectangular hole. For the most common diffraction orders we give the values of the modulus of the Fourier transform corresponding to the rectangular hole. Such values are representative of the typical order of magnitude for the relative amplitudes of the diffraction orders. For instance the orders \((\pm 1,0)\) are associated with a relative amplitude of 0.810 whereas it is of 0.318 for the orders \((0,\pm 1)\), i.e. 2.5 times less. Obviously, for a circular hole, the Fourier transform would be symmetric if we substitute \(i\) by \(j\) and vice versa. Then, the relative amplitude of the orders \((\pm 1,0)\) and \((0,\pm 1)\) would be equal. This difference between circular and rectangular holes is crucial. Indeed, in our recent work \([14]\) we have specified the role of SPs in the Ebbesen experiment through the concept of resonant Wood anomalies \([14,18,19]\). As explained above, the transmission pattern is described by Fano’s profiles correlated with interferences between nonhomogeneous diffraction orders and the resonant response of the surface plasmons coupled with nonhomogeneous diffraction orders. More precisely, the key process in those devices appears as follows. We suppose that the incident light is polarized along the \([1,0]\) axis of the grating. The incident light then diffracts against the grating and generates nonhomogeneous resonant diffraction orders e.g. \((1,0)\) which is \(p\) polarized. Such a resonant order is coupled with a surface plasmon (obviously \((0,1)\) is also generated but it is \(s\) polarized and then it is not a resonant order, i.e. it is not coupled with surface plasmons). It becomes possible to excite this surface plasmon which leads to a feedback reaction on the order \((1,0)\). Then this diffraction order can diffract too and generate a contribution to the homogenous zeroth diffraction order \((0,0)\). In fact \(s\) polarized diffraction orders give no contribution to the enhancement of the transmission, contrary to \(p\) polarized diffraction orders \([14]\). Obviously, the contribution of the order \((1,0)\) depends on the relative amplitude related to the Fourier transform of the hole. Now, if the incident light is polarized along the \([0,1]\) axis of the grating, the order \((0,1)\) becomes the resonant \(p\) polarized order whereas \((1,0)\) becomes the non-resonant \(s\) polarized order. Then two cases occur. If the Fourier transform is symmetric, the substitution of the \(p\) polarized order \((1,0)\) by the \(p\) polarized order \((0,1)\) let invariant the transmission. If the Fourier transform is not symmetric, then the contribution to zeroth order of both \(p\) polarized orders will be not the same according to the incident polarization. In our case, if the incident light is polarized along the \([1,0]\) axis of the grating, the \(p\) polarized order \((1,0)\) (which is associated with a relative amplitude of 0.810 of the Fourier transform) gives an important contribution.
to the zeroth order transmission. If the incident light is polarized along to the $[0, 1]$ axis of the grating, the $p$ polarized order $(0, 1)$ (which is associated with a relative amplitude of 0.318 of the Fourier transform) gives a weak contribution to the zeroth order transmission. Then the transmission $t_x$ for light polarized along the $x$ axis will be greater than the transmission $t_y$ for light polarized along the $y$ axis. This let to some issues. As explained above, the electric field of the incident light can be written as

$$E_{\text{inc}} = E_{\text{inc}} \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix}$$

Assuming the fact that both components of the incident electric field are not transmitted in the same way, the zeroth order transmitted field is written as

$$E = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} t_x E_{\text{inc}} \sin \theta \\ t_y E_{\text{inc}} \cos \theta \end{bmatrix}$$

where $t_x$ and $t_y$ respectively are the transmission coefficients along the $x$ and $y$ axis respectively. The modulus of the amplitudes are then

$$\begin{cases} |E_x| = |t_x E_{\text{inc}}| \sin \theta \\ |E_y| = |t_y E_{\text{inc}}| \cos \theta \end{cases}$$

As the zeroth order transmission can be written as

$$T = \frac{1}{\varepsilon_x} \frac{1}{|E_{\text{inc}}|^2} \left( |E_x|^2 + |E_y|^2 \right)$$

it is easier to show that one can write $T$ as

$$T = T_{90^\circ} + (T_{0^\circ} - T_{90^\circ}) \cos^2 \theta$$

where $T_{0^\circ} = \sqrt{\frac{\varepsilon_x}{\varepsilon_v}} |t_y|^2$ and $T_{90^\circ} = \sqrt{\frac{\varepsilon_x}{\varepsilon_v}} |t_x|^2$ ($\varepsilon_x$ and $\varepsilon_v$ are the permittivities of the substrate and the vacuum respectively). At last, note that the dephasing $\delta$ is such that

$$\delta = \arg \left\{ \frac{t_y}{t_x} \right\}$$

One shows that eq.5 is similar to the well known Malus Law for polarizer. To illustrate our assumptions, fig.2 shows the zeroth order transmission against wavelength for different $\theta$ values. We clearly show the dependence of the transmission pattern on the incident polarization. For $\theta = 90^\circ$, i.e. the incident electric field is along the $x$ axis, the transmission is maximal and identical to what one observes in the Ebbesen experiments [1,4]. It is shown that the transmission increases with the wavelength, and that it is characterized by sudden changes in the transmission marked 1 to 2 on the figure. These values are shifted toward larger wavelengths when the grating size increases [14]. Moreover, for $\theta = 0^\circ$, i.e. when the incident electric field is along the $y$ axis, the transmission is minimal. This result is, at least qualitatively, in agreement with our hypothesis. Note that wavelengths 1 and 2 are close to Rayleigh’s wavelengths. In diffraction gratings such wavelengths are those for which a difthred order becomes tangent to the plane of the grating. At normal incidence, and for a diffraction order $g = \frac{2\pi}{\lambda}$, Rayleigh’s wavelength is defined as

$$\lambda_R^{u,i,j} = c\sqrt{\varepsilon_u (i^2 + j^2)^{-\frac{1}{2}}}$$

where $\varepsilon_u$ represents either the permittivity of the vacuum ($\varepsilon_v$), or of the dielectric substrate ($\varepsilon_d$). For the diffraction orders such that $(\pm 1, 0)$ or $(0, \pm 1)$, the values of Rayleigh’s wavelengths in the present case are 1000 nm for the vacuum/metal interface, and 1445.29 nm for the substrate/metal interface.

Fig.3 shows two zoomed plots around wavelengths 1 (fig.3a) and 2 (fig.3b). In fig.3a, one shows that the transmission minima are localized at the same wavelength whatever $\theta$. Surprisingly, in this narrow wavelength domain, transmission for $\theta = 0^\circ$ is greater than for $\theta = 90^\circ$ contrary to previously. The minimum which corresponds to the wavelength 1, is located at 1001.44 nm just after the related Rayleigh’s wavelength. Fig.3b shows that transmission minima are weakly shifted toward large wavelengths as $\theta$ increases (minima locations for each $\theta$ value, are marked by vertical black dashs). For $\theta = 0^\circ$, the minimum is located at 1445.29 nm, i.e. the wavelength 2 which corresponds to the related Rayleigh’s wavelength. For $\theta$ greater than zero, minimum is shifted. Moreover, one notes a sudden change in the transmission at the Rayleigh’s wavelength whatever $\theta$. The minima observed here correspond to minima of Fano’s profiles [14]. In fig.3a, the minima are not of the same kind of the minima in Fig.3b. This is not true minima of the Fano’s profile. All occur as if the minimum of the Fano’s profile disappears behind the Rayleigh’s wavelength towards low wavelength. In other words, the minima in Fig.3a come from the cutoff and the discontinuity introduced in Fano’s profiles at the Rayleigh’s wavelength. This had been widely explained in ref. [14].

For more details, in fig.4, we have reported the wavelengths $\lambda_m$ related to the transmission minima (related to fig.3b) against $\theta$ (black dots). For comparison, and by analogy with eq.5, the solid line is related to the following equation

$$\lambda_{m,\theta} = \lambda_{m,0^\circ} + (\lambda_{m,0^\circ} - \lambda_{m,90^\circ}) \cos^2 \theta$$

Quantitatively, fig.5 shows the variation of the transmission against the angle of polarization $\theta$ for both wavelengths 1300 nm and 1900 nm (fig.5a) and both wavelengths 1147.64 nm and 1001.44 nm (fig.5b) in the neighborhood of the above mentioned minima. The numerical computations give results indicated by the dots on the figure. Using the numerical values of the transmission for $\theta = 90^\circ$ and $\theta = 0^\circ$ respectively, i.e. $T_{90^\circ}$ and $T_{0^\circ}$, one plots the transmission against $\theta$ for both wavelength by using eq.5 (solid lines). One observes then a quantitative agreement between the pattern from eq.5 and the results obtained numerically. Note the weakly decreasing transmission at 1001.44 nm as already mentioned, which is also in full agreement with eq.5.
In order to more detail our approach, we present on fig.6a the amplitudes $|E_x|$ and $|E_y|$ of the zeroth transmitted order against wavelength for two different values of $\theta$. One shows that the $|E_y|$ amplitude does not exhibit high convex regions in transmission contrary to the $|E_x|$ amplitude. As explained, the convex regions in the zeroth order transmission (i.e. the high transmission domains between minima 1 and 2, and just after minimum 2) are induced by the scattering of non-homogenous resonant diffraction orders such as $(\pm 1, 0)$ or $(0, \pm 1)$. For the $|E_x|$ amplitude, the orders $(\pm 1, 0)$ are associated with a relative amplitude of 0.810 of the Fourier transform of the hole profile. This large contribution of these orders enables an enhanced transmission via multi-scattering. This effect appears as the reason of the existence of the convex region of high transmission as shown in previous work [14]. On the contrary, for the $|E_y|$ amplitude the resonant orders $(0, \pm 1)$ are associated with a relative amplitude of 0.318 of the Fourier transform of the hole profile. This contribution is not large enough to obtain a significant enhanced transmission. For $\theta = 30^\circ$, the $|E_y|$ amplitude contribution to the whole transmission is greater than the $|E_x|$ contribution. Although the transmission along the $x$ axis is greater than along the $y$ axis, as $E_{inc}$ components depend on $\theta$ (see eq.1) the $x$ axis component of the incident electric field is small enough to lead $|E_x|$ to be lower than $|E_y|$ (see eq.2). On the contrary for $\theta = 60^\circ$, the $|E_x|$ amplitude contribution to the whole transmission is greater than the $|E_y|$ contribution. Indeed, the transmission along the $x$ axis is obviously greater than along the $y$ axis, but now the $x$ axis component of the incident electric field is also greater than the one along the $y$ axis. Then, the amplitude $|E|$ pattern for $\theta = 60^\circ$ is essentially defined by the pattern of the amplitude $|E_x|$ whereas the amplitude $|E|$ pattern for $\theta = 30^\circ$ is essentially defined by the pattern of the amplitude $|E_y|$. In addition, the whole zeroth order transmission would be greater for $\theta = 60^\circ$ than for $\theta = 30^\circ$. Again, these results are in qualitative agreement with our assumptions. Quantitatively, fig.6b shows the variation of the amplitudes $|E_x|$ and $|E_y|$ against the angle of polarization $\theta$ for both wavelength 1300 nm and 1900 nm. The numerical computations give the results indicated by the dots on the figure. Using the numerical values of the amplitudes for $\theta = 90^\circ$ and $\theta = 0^\circ$, one plot the transmission against $\theta$ for both wavelength by using eq.3 (solid lines). As previously, one observes a quantitative agreement between the pattern from eq.3 and the results obtained numerically.

In the possibility of later experimental results, we wish to add some details for comparison. Fig.7a gives the dephasing $\delta$ against wavelength for both angle $\theta = 60^\circ$ and $\theta = 30^\circ$. One shows the peaks close to the minima of transmission, i.e. close to the surface plasmon resonance according to our previous work [14]. Despite the absence of an explicit dependence against $\theta$ for $\delta$ according to eq.6, one notes that such a dependence exist after all. So, fig.7b shows the dephasing against $\theta$ for both wave-length 1300 nm and 1900 nm. One shows for $\theta = 0$ that the dephasing is equal to zero, and the polarization of the zeroth transmitted order is thus the same as the incident field. Though the dephasing tends to be different from zero when $\theta = 90^\circ$, as $|E_y|$ tends to zero (see fig.6) the polarization is the same as the incident field. By contrast, the transmission polarization will be elliptical for other values of $\theta$. Such a dependence of $\delta$ against $\theta$ and wavelength probably express a dependence of $t_x$ and $t_y$ against $\theta$. This maybe corresponds to a propagating constant against z-axis that is different according to the $x$-axis or $y$-axis. In addition, as shown in ref.[14], $t_x$ and $t_y$ result from many contributions such as SPs via multi-scattering. Because SPs exhibit a polarization dependency, this could explain the $t_x$ and $t_y$ behaviour. Nevertheless, as one shows, the dependence of $\delta$ against $\theta$ and wavelength is not well understood and we hope to clarify this point in a later work. Moreover, it will be necessary to search for the effects induced by a parameters change.

At last, it is striking that our numerical results coincide with our simple and intuitive explanation which leads to predict that a square grating with rectangular holes can behave like a polarizer. Indeed, the whole multisattering processes are complex and there are numerous contributions to the zeroth order transmission as explained above. It was then not necessarily obvious, that our intuitive model match up with our numerical results, all the more that the results shown in fig.7 underline the limits of our simple interpretation.

**Conclusion**

In the present paper, we have shown that contrary to the original experiments (which used a grating made of circular holes) [4], the zeroth order transmission pattern depends on the polarization of the incident light when using rectangular holes. The results presently obtained match up with our previous work [14]. In this context, we obtain a device with properties globally similar to those of the original Ebbesen experiments, but which exhibits in addition, some properties of a polarizer. Assuming the recent technological interest for this kind of metallic gratings, we think that the kind of device introduced in the present paper could lead to new technological applications. At last, note that, during the submission process, Gordon et al [17] have published experimental results concerning transmission through a bidimensional array of elliptical holes in metallic film. Their results clearly establish the polarization dependence of the transmission with asymmetric holes. These experimental results corroborate the present theoretical approach and in this way, the experimental and theoretical works supply many complementary results.
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References


Captions

Figure 1 : (a) Diagrammatic view of the system unders study, (b) Simplified view of the Fourier transform of the holes grating.

Figure 2 : zeroth order transmission against wavelength for various polarization, i.e. for various values of θ.

Figure 3 : close view of zeroth order transmission against wavelength for various polarization, i.e. for various values of θ. (a) in the vicinity of wavelength (1). (b) in the vicinity of wavelength (2).

Figure 4 : Wavelengths related to the transmission minima against θ in the vicinity of wavelength (2) (see fig.3b).

Figure 5 : (a), zeroth order transmission against θ for both wavelength 1300 nm and 1900 nm. (b), zeroth order transmission against θ for both wavelength 1001.44 nm and 1447.64 nm.

Figure 6 : (a) $|E_x|$ and $|E_y|$ amplitudes against wavelength for θ = 30° and θ = 60°. (b) $|E_x|$ and $|E_y|$ amplitudes against θ for both wavelength 1300 nm and 1900 nm.

Figure 7 : (a) dephasing δ against wavelength for θ = 30° and θ = 60°. (b) dephasing δ against θ for both wavelength 1300 nm and 1900 nm.
(a) Transmission (%) vs. θ (Degrees)
- 1900 nm
- 1300 nm

(b) Transmission (%) vs. θ (Degrees)
- 1447.64 nm
- 1001.44 nm
![Graphs showing the relationship between wavelength (nm) and amplitude (V.m⁻¹) for different angles (Degrees) and wavelengths (1300 nm and 1900 nm).](image)