SIMULTANEOUS NON-DISTURBING DETECTION OF INCOMPATIBLE PROPERTIES IN DOUBLE-SLIT EXPERIMENT

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We exploit the notion of which-slit detector introduced by Englert, Scully and Walther (ESW), to show that two incompatible properties can be detected together for each particle hitting the screen, without disturbing the center-of-mass motion of the particle.

1. Let us consider a typical two-slit experiment for a particle which possesses, besides the position observable described in the Hilbert space \( \mathcal{H} \), a further degree of freedom \( s \) corresponding to a dichotomic observable \( S \) with spectrum \( \sigma(S) = \{1, 0\} \), such that \( \partial H / \partial S = 0 \) (\( H \) is the Hamiltonian operator). Such a system can be described in the Hilbert space \( \mathcal{H} = K \otimes C^2 \). The projection operator representing the event ‘the particle crosses slit 1 (resp., 2)’ has the form \( E = L \otimes 1 \) (resp., \( (1 - L) \otimes 1 \)). Given any interval \( \Delta \) on the final screen, by \( F \) we denote the projection operator which represents the event ‘the particle hits the final screen in a point within \( \Delta \).’ Let \( \psi_1, \psi_0 \in K \) and \( \psi_0 \in K \) be state vectors respectively localized in slit 1 and 2 when the particle is in the region of the two-slit, i.e. such that \( L \psi_1 = \psi_1 \), \( L \psi_0 = 0 \).

If the complete state vector of the particle is \( \Psi = (1/\sqrt{2})[\psi_1 \otimes |1\rangle + \psi_0 \otimes |0\rangle] \), then the projection \( T = 1 \otimes |1\rangle \langle 1| \) represents a which-slit detector which does not disturb the center-of-mass motion of the particle, because

\[
[T, F] = 0 \quad (1)
\]

\[
[T, E] = 0 \quad \text{and} \quad T \Psi = E \Psi. \quad (2)
\]

Indeed, (1) implies that the measurement of \( T \) can be performed without the distribution of the particles on the final screen; (2) means that \( T \) and \( E \) represent directly correlated properties, so that the outcome 1 (resp., 0) for \( T \) detects the passage of the particle through slit 1 (resp.2). Since \( T \) is completely non-disturbing, it is an idealized version of the which-slit detector introduced by ESW [1][2].

The following definition generalizes this notion.

**DEFINITION.** Let \( L_+ \) be a projection of \( K \). A projection \( T_+ = 1 \otimes R \) is called a non disturbing ESW detector for \( E_+ = L_+ \otimes 1 \) if

\[
[T_+, F] = 0 \quad (3)
\]

\[
[T_+, E_+] = 0 \quad \text{and} \quad T_+ \Psi = E_+ \Psi. \quad (4)
\]

Hence, projection \( T \) above is a non-disturbing (which-slit) ESW detector for projection \( E \).

2. Now we face the following question: There are situations in which the simultaneous knowledge of which slit passage and an incompatible property is possible for each particle hitting the final screen, without affecting the distribution of particles? As we show by means of the following example, the answer is yes, and in such a case the two properties are directly correlated.

Let \( M_1, M_2, M_3, M_4 \) be four mutually orthogonal subspaces of \( K \), with respective projections \( P_1, P_2, P_3, P_4 \), such that \( L = P_1 + P_2, \quad 1 - L = P_3 + P_4 \).

Let the state of the particle be described by

\[
\Psi = (1/2)[(\psi_1 + \psi_2)|1\rangle + (\psi_3 + \psi_4)|0\rangle],
\]

where \( \psi_k \in M_k \) and \( \| \psi_k \| = 1 \), \( k = 1, 2, 3, 4 \). Since (1) and (2) continue to hold, projection \( T \) above is a non-disturbing ESW which-slit detector.

Now we consider the projection \( E_+ = L_+ \otimes 1 \), where \( L_+ = A + B + C + D \), with

\[
A = |\psi_1\rangle\langle 3/4|\psi_1| + (1/4)|\psi_2| - (1/4)|\psi_3| + (1/4)|\psi_4|,
\]

\[
B = |\psi_2\rangle\langle 1/4|\psi_1| + (3/4)|\psi_2| + (1/4)|\psi_3| - (1/4)|\psi_4|,
\]

\[
C = |\psi_3\rangle - (1/4)|\psi_1| + (1/4)|\psi_2| + (1/4)|\psi_3| - (1/4)|\psi_4|,
\]

\[
D = |\psi_4\rangle\langle 1/4|\psi_1| - (1/4)|\psi_2| - (1/4)|\psi_3| + (1/4)|\psi_4|.
\]

To grasp the physical meaning of \( E_+ \) the choice of \( \psi_1, \psi_2, \psi_3, \psi_4 \) must be specified, of course. However, rule \( [E_+, E] \neq 0 \) holds for any choice; therefore projection \( E_+ \) represents a property incompatible with \( E \). Now, equality \( T \Psi = E_+ \Psi \) holds; hence \( T \) is a non-disturbing ESW detector also for \( E_+ \), which is incompatible with \( E \).

Furthermore, since \( E \Psi = T \Psi = E_+ \Psi \), when the particles are prepared in the state \( \Psi \), there is an entanglement between the two incompatible properties \( E \) and \( E_+ \).

Outcome 1 (0) for which slit detector \( T \) implies that both \( E \) and \( E_+ \) have outcome 1 (0), \( E \) and \( E_+ \) being incompatible with each other. The measurement of \( T \) does not affect the final position.