In this paper we first derive solutions which can be interpreted as branes wrapping nontrivial curved manifolds, and then study their cosmological implications. We find that at early times the branes tend to shrink the internal manifold, while allowing the “unwrapped” dimensions to expand in congruence with what has already been observed in the case when the internal manifold is flat (tori). However, at late times the internal curvature terms become important leading to potentially interesting differences.


INTRODUCTION

Inspired by String/M-Theory brane-physics have blossomed in recent years. Branes are \( p \)-dimensional extended objects embedded in a higher \( \hat{D} \geq p+1 \) dimensional universe which arise naturally in String theory as hyperplanes where open strings can end, and in Supergravity/M-Theory as solitons [?]. Mostly, branes have found applications in two virtually opposite set ups: In the “brane-world” scenario [?] the brane dimensions coincide with the three observed dimensions of our universe, while the spatial dimensions perpendicular to the brane, which can be both compact or non-compact correspond to the “unseen internal dimensions”. Contrastingly in the “brane-gas” scenario [?,?] the branes wrap around compact internal dimensions, while the directions perpendicular to the branes become the observed dimensions of our universe. While the brane-world picture have several virtues including being able to address the hierarchy and the cosmological constant problem, the brane gas model has the virtue of explaining why the internal manifold remained small as compared to the observed dimensions, and a simple counting argument also yields the dimensionality of the observed universe to be three! Invoking T-duality brane gas cosmology (BGC) also seems to be able to avoid the big bang singularity. Thus, it may be an interesting venture to combine the two scenarios by considering branes some (three) of whose dimensions are noncompact (which becomes our observed universe) while the others wrap compact internal manifolds, which potentially can be non-trivially curved - the isometries of this curved internal manifold can then be associated with gauge fields living on the brane. In this paper however, we only restrict ourselves to the usual set up of BGC but we generalize the internal manifold from being a flat tori to a curved space.

In [?] Brandenberger and Wafa observed that string winding states would tend to prevent expansion since the energy of the states increases if the circumference increases. However, strings with opposite winding numbers can annihilate if their world volume intersects and this happens most efficiently in three dimensions. Thus in a three dimensional subspace the string winding states can annihilate letting these dimensions grow while strings winding other directions will not be able to annihilate each efficiently and ultimately fall out of thermal equilibrium thereby stopping the expansion. This idea of ‘string gas’ has been generalized to ‘brane gas’ scenarios [?], with essentially similar results, except that now we have a hierarchy in the sizes of the extra dimensions, coming from the contributions of the \( p \)-branes, with different \( p \), as in general \( p \)-branes annihilate most effectively in \( 2p + 1 \) dimensions. Although BGC has been successful in solving several problems associated with standard big bang cosmology, there are some unresolved issues as pointed out in [?]. For example, it is not clear exactly how the three dimensions that we observe today are chosen among all the other dimensions, perhaps through random quantum and thermal fluctuations. In that case it is possible that our universe may contain different patches where different directions have become large! Also, it is known that branes with different values of \( p \) can interact with each other, changing their winding numbers, thereby allowing thermal equilibrium to be maintained among the brane winding states. A simple way to eliminate these issues is to assume the universe to be a direct product of three non-compact dimensions and a compact internal manifold. The BGC in such a set up has been extensively studied in [?] when the internal manifold is a tori. Here, we adopt the same approach, but generalize the program to nontrivial curved internal manifolds to study how, if at all, the internal curvature may change the cosmological dynamics. Of course, in this set up the question of why the universe is a product space with specifically three non-compact dimensions is an open issue, and the nice BGC argument concerning the dimensionality of our observed universe becomes redundant. We will come back to this at the end.

For a flat \( p \)-brane the \( p \) longitudinal (along the brane) directions are flat, or in other words a flat \( p \)-brane solution preserves the isometries of the \( p \) dimensional flat space. These branes can then wrap around flat \( p \)-dimensional tori. If one now considers branes wrapping a curved internal manifold, say \( M_p \) then the brane solution should accordingly, preserve the isometries of \( M_p \). In section I, we derive such \( p \)-brane solutions which look like de Sitter black holes from the four dimensional point of view\(^1\). This is to be expected as after reduction ordinary \( p \)-branes resemble black hole solutions,

\(^1\) Previously branes wrapping curved dimensions have been studied using boundary state conformal field theory [?], but here we take a supergravity approach.