B\textsubscript{s}\textsuperscript{0} mixing measurement in ATLAS

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Abstract

This note presents a study of the measurement of the B\textsubscript{s}\textsuperscript{0} mixing parameter $x_s$ in ATLAS. Full GEANT simulation of the ATLAS Inner Detector including the recently proposed ‘B-physics layer’ at $R = 6$ cm was followed by charged particle reconstruction using the new XRECON program. The impact parameter resolution for reconstructed tracks shows good agreement with previous calculations. Using precise secondary vertex reconstruction a resolution of $0.07\,\text{ps}$ in proper B-decay time can be reached, enabling $x_s$ measurement up to $\sim 35$.

1 Introduction

The B\textsubscript{s}\textsuperscript{0} meson mixing parameter $x_s = (\Delta M)_{B_s}/\Gamma_{B_s}$, where $\Delta M$ is the mass difference between the $B_s$ mass eigenstates and $\Gamma$ is the average width, is proportional to the CKM matrix element $|V_{ts}|^2$. Compared to the mixing in the $B_d$ meson system, the ratio $x_s/x_d$ is proportional to $|V_{ts}/V_{td}|^2$ which is much larger than one. The mixing parameter $x_s$ is therefore expected to be large, and the mixing in the $B_s$ meson system proceeds faster than in the $B_d$ system in which the mixing parameter $x_d$ has been measured to be about 0.7. Using the measured values of the B-meson masses and lifetimes, and the recently measured value of the top quark mass, the allowed range of $x_s$ is estimated to be between 5.4 and 60.2 at 90\% C.L. in \cite{1}.

Since the value of $x_d$ is already known to good precision, the measurement of $x_s$ gives a measurement of the ratio $|V_{ts}/V_{td}|$. It is expected that $|V_{td}|$ equals to $|V_{ts}|$, and therefore the ratio provides an independent measurement of one of the sides of the unitarity triangle, $|V_{td}/AV_{td}|$.

The mixing parameter $x_s$ has not yet been measured. A precise measurement of $x_s$ requires sufficient statistics of fully reconstructed $B_s$ mesons and an excellent decay time resolution. This note describes how these requirements are met with the ATLAS detector at LHC.

2 Impact parameter resolution

2.1 Motivation for the Si-strip ‘B-physics’ detector layer

The impact parameter resolution $\sigma_{IP}$ in the transverse plane with respect to the beam axis for a solenoidal magnetic field with the field parallel to the beam can be parame-
terized as:
\[ \sigma_{ip} = A + B \frac{p_T}{p_T} \]
where \( A \) is the asymptotic resolution for high transverse momentum charged particles, and \( B \) is a multiple scattering term. Both terms depend on the layer positions. \( A \) also depends on the layer resolutions while \( B \) depends on the amount of material in the tracking detector layers. By using a two-layer model together with a parameterized tracker momentum resolution it is possible to derive approximate analytical formulae for \( A \) and \( B \):

\[
A = \frac{R_3 - R_1}{R_3 - R_2} \sigma_1 + \frac{R_1}{R_3 - R_2} \sigma_2 + 0.3 R_1 R_2 (\frac{\Delta p_T}{p_T^2})
\]

\[
B = 13.6 \sqrt{x/X_0 (1 + 0.038 \ln(x/X_0)) R_1 + 0.3 R_1 R_2 (\frac{\Delta p_T}{p_T})}
\]

where \( \sigma_1 \) and \( \sigma_2 \) are detector layer resolutions, \( \frac{\Delta p_T}{p_T^2} \) represents an asymptotic term of momentum resolution and \( \frac{\Delta p_T}{p_T} \) is a constant term. It is assumed here that the momentum resolution terms of the ATLAS ID are \( 5 \times 10^{-4} \) and \( 1.2\% \), respectively. In formulae 2 and 3, the layer radii \( R_{1,2} \) are defined in mm, resolutions \( \sigma_{1,2} \) in \( \mu m \), \( A \) in \( \mu m \) and \( B \) in \( \mu m \times GeV/c \). \( x/X_0 \) is the fractional radiation length of the first detector layer. Dependence of \( A \) and \( B \) on the radius of the first tracking layer is shown in Fig. 1 for some values of \( \sigma_1 \) and \( x/X_0 \).

Similar formulae can be calculated for the longitudinal impact parameter resolution along the beam direction, \( \sigma_Z \), where \( Z \) refers to the z-coordinate of the particle in the point of closest approach to the primary vertex. In this case there is no contribution from track curvature uncertainty and terms depending on momentum resolution are not present.

Although a two-layer model is not always sufficiently accurate is enables one to identify the main issues concerning the secondary vertex resolution. It is clear that in order to optimize the tracker detector for heavy quark studies at the initial low luminosity operation period, it is important to reduce the radius of the innermost detector layer.

In \( b \bar{b} \) events produced at LHC with \( \sqrt{s} = 14 \) TeV and selected with the ATLAS level-1 single muon trigger with \( p_T > 6 \) GeV/c and \( |y| < 1.6 \), typical charged particle momenta are a few GeV/c, and therefore, the multiple scattering dominates the impact parameter resolution. In order to improve the resolution, it is important to minimize the radiation length of the first layer. Its’ mechanical construction could be similar to that of the DELPHI vertex detector [2], which has no electronics and almost no support material in the active detector area. The radiation length of such a layer is dominated by the silicon detectors themselves and can be below 0.5\% including the detector overlaps. A layer of this type was recently proposed for ATLAS [3].

Recent radiation-damage studies show, that a radius of 6 cm is still safe for low-luminosity operation. A silicon strip detector should survive 5 to 6 years at \( L = 10^{20} cm^{-2} s^{-1} \) or 1 year at \( L = 5 \times 10^{20} cm^{-2} s^{-1} \) [4]. Because of the small area of this layer it should also be possible to replace it at affordable cost. It seems advisable to further reduce its radius to a limit created by the minimal possible beam-pipe diameter. For this study it was assumed that \( R_1 = 6 \) cm.
2.2 Impact parameter resolution of the present vertex detector layout

It is necessary to use a Monte Carlo program to calculate the impact parameter resolutions of different layouts of the vertex detector, because the two-layer approximation used for analytical calculations, discussed above, it not always sufficient. A simplified Monte Carlo program was used for this purpose. Layer positions, resolutions and radiation lengths were taken into account in the simulation. For ATLAS Inner Detector equipped with the ‘B-physics layer’ the following resolutions are reached:

\[
\sigma_{IP} = 14 + 72/(p_T \sqrt{\sin \theta})
\]  
\[
\sigma_Z = 20 + 83/(p_T \sqrt{\sin \theta})^3
\]

where \( \theta \) is the angle with respect to the beam line. The parameterization of \( \theta \) dependence assumes cylindrical detector layers. Half length of a layer at 6 cm should be around 20 cm. The parameterization is therefore valid for \(|\eta| < 2\). In the absence of the first layer, for higher luminosity operation, the resolution is

\[
\sigma_{IP} = 30 + 220/(p_T \sqrt{\sin \theta})
\]  
\[
\sigma_Z = 39 + 240/(p_T \sqrt{\sin \theta})^3
\]

assuming that the layer closest to the beam pipe is a Si pixel layer at \( R = 11.5 \text{ cm} \). By installing another pixel layer we improve the asymptotic term of the impact parameter resolution at an expense of further worsening the multiple scattering term.

Presence of the ‘B-physics’ layer was assumed in this study. The ‘Panel’ or the ‘simplified’ layout of Inner Detector [5] was used. It has to be pointed out, however, that as long as momentum resolution and track finding abilities are not changed, secondary vertex measurement should not depend on choices made for all tracking layers above \( R = 20 \text{ cm} \).

3 Reconstruction of events simulated with the GEANT full simulation

3.1 XRECON program

A full simulation of \( b\bar{b} \)-events was done using the SLUG/DICE program. For charged particle reconstruction, a development version of the XRECON program (charged particle reconstruction package in ATRECON), written by Igor Gavrilenko, was used. The program uses the Kalman-Filter formalism which allows one to include measurements one after another into the fit.

The program starts from the TRT. The number of charged particle tracks is defined at this stage. The program proceeds inwards linking hits from Silicon layers to the track. At each step track parameters and their error matrix are propagated to the next shell. In order to take multiple scattering into account, errors are enlarged according to the radiation length of the detector layer and the extrapolation distance. The local measurement is added into a fit. If there are many possible hit combinations the
program follows all of them. A candidate trajectory is abandoned if there are no hits found in at least two consecutive layers. In the end, the best of the remaining hit combinations is selected based on the $\chi^2$-value of the fit.

### 3.2 Reconstruction of single charged particles

Using the full simulation and reconstruction software, we can cross-check the validity of the impact parameter resolution given by the ‘toy’ Monte Carlo calculations described in section 2. Figure 2 shows a comparison between the results obtained by using the full simulation and the XRECON program, and the parameterized resolution. A good agreement can be observed, which is an important justification for all track-level simulations utilizing a parameterized description of the impact parameter resolution.

### 3.3 Performance in $b\bar{b}$ events

The following $b$-decays were generated with the PYTHIA Monte Carlo program, and then passed through a GEANT detector simulation:

\[
\begin{align*}
    b\bar{b} & \rightarrow \mu \ (p_T^\mu > 6 \text{ GeV}/c, |\eta| < 1.6) \\
    \rightarrow B_s^0 \rightarrow D^- \pi^+ \ \\
    & \rightarrow \phi \pi^- \\
    & \rightarrow K^+ K^-
\end{align*}
\]

These events pass the first level muon trigger. The chosen $B$-decay is expected to provide a clean signature of $B_s^0$ mixing. A few other channels can be considered as well.

The low luminosity scenario ($L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$) was assumed, and therefore no pile-up of minimum bias events was added on top of the signal events. Counter inefficiencies and noise were not yet included in the simulation.

The reconstruction program was at an early development stage and was still being debugged and improved. In order to proceed a version had to be frozen by the middle of April. The program was limited to the barrel part of the TRT, and only tracks within $|\eta| < 0.7$ could be reconstructed. In the studied events, there were on the average 25 tracks in this $\eta$ region, 15 of which had $p_T > 1 \text{ GeV}/c$. No attempt was made to reconstruct tracks with lower $p_T$. The track finding efficiencies of the reconstruction program are summarized in table 1.

<table>
<thead>
<tr>
<th>particles</th>
<th>reconstruction efficiency [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>all with $p_T &gt; 1 \text{ GeV}$</td>
<td>86</td>
</tr>
<tr>
<td>$\mu^\pm$ (mostly trigger $\mu$)</td>
<td>98</td>
</tr>
<tr>
<td>$\pi^\pm$</td>
<td>86</td>
</tr>
<tr>
<td>$K^\pm$ (mostly from $\phi$)</td>
<td>81</td>
</tr>
</tbody>
</table>

The high efficiency for muons can be explained by their high $p_T$, since the majority of muons in these events are the ones that are needed for triggering. The low efficiency
for K⁺, on the other hand, is probably due to topological reasons. Two kaons from a φ⁰ decay are close in space, which increases the probability for the reconstruction program to be confused. Almost no fake tracks were observed.

Even at this early stage of the reconstruction software there seems to be no fake impact parameters which could confuse B-tagging. Fake impact parameters which appear due to misassignment of hits in the first few layers tend to be very large and can be easily identified. Such tracks are included in the loss quoted above.

Using reconstructed tracks one can obtain the invariant mass peaks corresponding to D⁺⁻⁻ and B⁺⁻⁻⁻⁻⁰⁻⁻⁻ decays (Fig.3). The mass resolutions are 10 MeV/c² and 37 MeV/c², respectively.
4 Vertex fit and proper time resolution

The B-decay vertex position was fitted using reconstructed tracks. The topology of a considered B$_s$ decay is shown in a schematic way in a figure:

![Diagram showing B$_s$ decay vertex](image)

Three out of four of the final decay products come from the D$_s^-$ decay vertex, separated from the B$_s$ decay point by the flight path of D$_s^-$ before its decay. Therefore, the vertex reconstruction has to be done in two steps:

1. Fit of three tracks to reconstruct the D$_s^-$
2. Fit of the D$_s^-$π$^+$ vertex

PXFVTX - a vertex fitting routine written by Pierre Billoir and used in the DELPHI experiment since 1989 was applied at both stages. The following resolution in the B-decay proper time was reached:

$$\sigma_t = 0.069 \pm 0.008 \text{ ps}$$

Corresponding resolution in the decay length of the B-meson in the transverse plane is $\approx 50 \mu$m. Distribution of the two residues are shown in Fig.4.
5 Signal and background for $B^0_s \rightarrow D^-_s \pi^+$

In order to evaluate the signal acceptance and the background rejection with necessary statistics, track level simulations were performed in parallel with the reconstruction exercises. Table 2 summarizes the analysis of the signal. The first level trigger is provided by the muon. The second level trigger was not yet studied for these events. Since the results on the track reconstruction efficiencies with the full simulation and reconstruction programs are still preliminary and subject to optimization, we assumed here a track reconstruction efficiency of 95% and a lepton efficiency of 80%.

The signal statistics could be increased by using other decay channels of the $B^0_s$ like $B^0_s \rightarrow D^-_s a^+_1$, $B^0_s \rightarrow J/\psi K^{*0}$, and including other decay modes of $D^-_s$ like $D^-_s \rightarrow K^- K^{*0}$.

Events selected by the first level muon trigger are predominantly $b\bar{b}$ events. Background can come from two sources:

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Table 2. Analysis cuts and acceptance for $B^0_s \rightarrow D^-_s \pi^+$ events. The statistics corresponds to one year of running of LHC at the initial luminosity.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{L}$ [cm$^{-2}$s$^{-1}$]</td>
<td>$10^{39}$</td>
<td></td>
</tr>
<tr>
<td>$t$ [s]</td>
<td>$10^5$</td>
<td></td>
</tr>
<tr>
<td>$\sigma(b\bar{b} \rightarrow \mu X)$ [$\mu$b]</td>
<td>1.7</td>
<td>$p_T^\mu &gt; 6$ GeV/c (</td>
</tr>
<tr>
<td>$N(b\bar{b} \rightarrow \mu X)$</td>
<td>$1.7 \times 10^{10}$</td>
<td></td>
</tr>
<tr>
<td>$Br(b \rightarrow B^0_s)$</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>$Br(B^0_s \rightarrow D^-_s \pi^+)$</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td>$Br(D^-_s \rightarrow \phi\pi^-)$</td>
<td>0.028</td>
<td></td>
</tr>
<tr>
<td>$Br(\phi \rightarrow K^+K^-)$</td>
<td>0.491</td>
<td></td>
</tr>
<tr>
<td>$N(K^+K^-\pi^+\pi^-)$</td>
<td>70,100</td>
<td></td>
</tr>
</tbody>
</table>

Acceptance of the cuts:

- $p_T > 1$ GeV/c
- $|\eta| < 2.0$
- $\Delta\varphi_{K\bar{K}} < 10^\circ$
- $\Delta\theta_{K\bar{K}} < 10^\circ$
- $|M_{K\bar{K}} - M_{\pi^0}| < 10$ MeV/c$^2$
- $|M_{K\bar{K}^*} - M_{D^-_s}| < 15$ MeV/c$^2$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^-_s$ vertex fit $\chi^2 &lt; 10.0$</td>
<td>Preliminary</td>
<td></td>
</tr>
<tr>
<td>$B^0_s$ vertex fit $\chi^2 &lt; 5.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B^0_s$ proper decay time $&gt; +0.4$ ps</td>
<td>4%</td>
<td></td>
</tr>
<tr>
<td>$B^0_s$ impact parameter $&lt; 55 \mu$m</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B^0_s$ $p_T &gt; 10.0$ GeV/c</td>
<td>4%</td>
<td></td>
</tr>
<tr>
<td>$N(K^+K^-\pi^+\pi^-)$ after cuts</td>
<td>2800</td>
<td></td>
</tr>
<tr>
<td>Lepton identification</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Track efficiency</td>
<td>$(0.95)^4$</td>
<td></td>
</tr>
<tr>
<td>$N(K^+K^-\pi^+\pi^-)$ reconstructed</td>
<td>1825</td>
<td></td>
</tr>
</tbody>
</table>
1. All charged particles come from a B-decay of the same or higher multiplicity. The following channels were studied:

\[ B_0^+ \rightarrow D^- \pi^+ \]. The branching ratio of this channel is smaller than for the signal, and the mass is shifted by \((M_{B_0} - M_{B_s}) \approx 100 \text{ MeV}/c^2\). It was checked with a Monte Carlo simulation that the background from this channel is negligible.

\[ B_0^+ \rightarrow D^- \pi^+ \]. Similarly to the previous channel, the mass is shifted by about 100 MeV/c^2.

\[ \Lambda_0 \rightarrow \Lambda_c^+ \pi^-, \Lambda_c^+ \rightarrow pK^-\pi^+ \]. It was checked with simulation that these events do not pass the analysis cuts.

2. Combinatorial background, in which some (or all) of the particles do not originate from a B-decay. A large Monte-Carlo statistics is needed to test the very high rejection factor required for this background. No candidate event with \(5 \text{ GeV}/c < M_{K\pi\pi} < 6 \text{ GeV}/c\) was found within 370,000 inclusive \(\mu X\) events. This brings an upper limit on the background under the signal peak to 1/500. It is expected that this combinatorial background should be reducible but still more Monte-Carlo statistics would be needed to improve the present upper limit. Experience from previously analyzed channels suggests that combinatorial background should be less dangerous than other four-body B-decays.

6 Sensitivity for \(x_s\)

Assuming one year of operation at \(L = 10^{35}\text{cm}^{-2}\text{s}^{-1}\) followed by one year at \(5 \cdot 10^{35}\text{cm}^{-2}\text{s}^{-1}\) (integrated luminosity corresponding to one lifetime of the layer at \(R = 6\text{ cm}\)) the number of collected \(B_0^+\) decays would reach \(\sim 11,000\) events from the analyzed channel alone. Estimates of \(x_s\) sensitivity were done using this number and assuming 1:1 signal/background ratio.

If the \(B_0^+\) meson does not oscillate before it decays, the \(D_s\) meson and the tagging muon have equal signs. If the \(B_0^+\) meson oscillates, then the sign of the \(D_s\) meson is opposite to that of the tagging muon. The decay time distributions for these two classes are:

\[
\frac{dn(++)}{d\tau} = \frac{N}{2T_{B_s}} e^{-\tau/T_{B_s}} (1 + D \cos(x_s \tau/T_{B_s}))
\]

\[
\frac{dn(+-)}{d\tau} = \frac{N}{2T_{B_s}} e^{-\tau/T_{B_s}} (1 - D \cos(x_s \tau/T_{B_s}))
\]

where \((++)\) refers to a like-sign combination of the \(D_s\) and \(\mu\), \((+-)\) refers to an unlike-sign combination, \(\tau\) is the proper time, \(D\) is the product of all dilution factors and \(N\) is the total sample size.

The distribution of \(B_0^+\) decay times for one sign combination of the tagging muon and \(D_s\) is shown in Fig.5 for \(x_s = 20\) and 35. Dilution from tagging \(D_{\text{tag}} = 0.52\), background \(D_{\text{back}} = 0.71\) and resolution in the proper time \(D_{\text{time}}\) dependent on \(x_s\) were taken into account as well as the analysis cut of 0.4 ps.

For \(x_s = 20\) oscillations are clearly visible on this plot. For \(x_s = 35\), however, the finite resolution in proper time introduces an extra dilution \(D_{\text{time}}\) which makes them almost disappear. On the corresponding asymmetry plot it is possible to see the first
few cycles even for this high value of $x_s$. Asymmetry $A(\tau)$, defined as

$$A(\tau) = \frac{dn(++)/d\tau - dn(+-)/d\tau}{dn(++)/d\tau + dn(+-)/d\tau} = D \cos(x_s \tau / \tau_{\text{bs}})$$

(10)

is shown in Fig. 6 again for $x_s = 20$ and 35. Fitting of the oscillation parameter was done for these two asymmetry distributions using the form of Eq. 10, in which the free parameters were $D$ and $x_s$, and assuming $\tau_{\text{bs}} = 1.29$ ps. Results of the fits were $x_s = 19.96 \pm 0.05$ and $x_s = 35.04 \pm 0.13$, respectively. Although $x_s = 35$ is on the limit of our proper time resolution, it still seems possible to determine its value with a good accuracy.

An independent method of determining $x_s$ is based on Fourier transformation of the asymmetry curve. A discrete Fourier transform, defined by the formula

$$y_j = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} \exp \left( \frac{2\pi i j k}{N} \right) x_k, \quad (j = 0, 1, \ldots, N)$$

(11)

was used. In Eq. 11, $N$ is the total number of time bins and $x_k$ is the number of entries in time bin $k$. Fourier transform is usually complex. In order to get real numbers one can use so called 'power spectrum', which is defined as $|y_j|^2$. Power spectra of transforms of asymmetry distributions are shown in Fig. 7 for the same two values of $x_s$. The frequency axis is scaled in units of $x_s$. As the frequency increases and oscillations are smeared by the inaccuracy of the time reconstruction, the peak corresponding to the right frequency becomes lower. It is, however, clearly visible for $x_s = 35$.

$x_s$ sensitivity after one year of running at $L = 10^{33}$ cm$^{-2}$s$^{-1}$ was calculated as well, using 1800 signal events. The same signal/background ratio of $1:1$ was assumed. For this small statistics, the fit of the asymmetry curve was not successful for $x_s = 35$. The Fourier transform method turned out not to work either. The spectrum was dominated by noise and did not show a clear peak in the right place. Both methods were successful for $x_s \leq 30$. For example, for $x_s$ values of 20 and 30, fit results were $20.10 \pm 0.13$ and $29.94 \pm 0.16$, respectively. It can be concluded that after one year of low luminosity running, $x_s = 30$ should be within reach.

The main uncertainty of the estimation of ATLAS $x_s$ measurement range comes from the large error on $B_s^0$ lifetime. $B_s^0$ lifetime of 1.29 ps, which is the average value measured at LEP [6], was used for this study. The average lifetime measured by CDF is $1.42 \pm 0.27$ ps [7]. If the actual lifetime will turn out to be higher than the LEP value, the oscillation period corresponding to a given value of $x_s$ would increase as well and so would the $x_s$ sensitivity. For 1.5 ps lifetime, 16% higher than the value used here, it should be possible to reach $x_s = 40$.

### 7 Conclusions

The limit on the $x_s$ measurement sensitivity of ATLAS originates from the finite accuracy of proper time measurement rather than from background or available statistics. Profiting from the good secondary vertex resolution provided by the 'B-physics' layer, ATLAS can measure much higher values of $x_s$ than the LEP experiments, which are limited by statistics. It should be possible to measure the value of $x_s$ up to around 35, which is sufficient to cover the preferred range of Standard Model predictions for $x_s$.9
An outcome of the track finding exercise done for this study is also encouraging. It is demonstrated that tracks can be reconstructed in the ATLAS Inner Detector in a way which enables clean and precise impact parameter measurement in a realistic b\bar{b} event.

References


Figure 1: Asymptotic and multiple scattering terms of the impact parameter resolution versus the innermost layer radius. The lines represent the analytical calculations done using two-layer approximation. The two points represent the ‘toy Monte-Carlo’ calculations done for the ATLAS Inner Detector with and without the ‘B-physics layer’.
Figure 2: Impact parameter resolution for tracks reconstructed after GEANT simulation with the track-finding program vs. momentum for $\theta = 90^\circ$. Continuous line represents parameterized resolution.
Figure 3: Three and four particle invariant mass distributions showing $D^+_\pi$ and $B^0_n$ mass peaks in reconstructed events
Figure 4: Resolution in the proper time (top) and in the radius (bottom) for the reconstructed $B^0_s$ decay vertex.
Figure 5: Distribution of B^0_{s} proper decay times (for one sign combination) for \( x_s = 20 \) and \( x_s = 35 \).
Figure 6: Time-dependent asymmetry distributions for $x_s = 20$ and 35. Also shown are fit results, parameter number two of a fit corresponds to $x_s$. 
Figure 7: Fourier transforms of an asymmetry distribution for $x_N = 20$ and 35