Annihilation radiation from neutralino dark matter at the Galactic center (GC) would be greatly enhanced if the dark matter were strongly clustered around the supermassive black hole (SBH). The existence of a dark-matter “spike” is made plausible by the observed, steeply-rising stellar density near the GC SBH. Here the time-dependent equations describing gravitational interaction of the dark matter particles with the stars are solved. Scattering of dark matter particles by stars would substantially lower the dark matter density near the GC SBH over $10^{10}$ yr, due both to kinetic heating, and to capture of dark matter particles by the SBH. This result suggests that enhancements in the dark matter density around a SBH would be modest whether or not the host galaxy had experienced the scouring effects of a binary SBH.

1. Introduction

Neutralinos in supersymmetry are likely candidates for the non-baryonic dark matter (Pagels & Primack 1982; Goldberg 1983). If neutralinos make up a large fraction of the dark matter in the galactic halo, pair annihilations will produce an excess of photons which may be observed in gamma ray detectors (Bergstrom et al. 1999). The galactic center (GC) is a promising target for such searches since the dark matter density is predicted to rise as $\rho \sim r^{-1}$ at the centers of dark matter halos (Navarro et al. 2003). In addition, the GC contains a supermassive black hole (SBH) with mass $M_\bullet \sim 10^{6.5} M_\odot$ (Schoedel et al. 2003). “Adiabatic growth” models in which the SBH remains stationary as it accumulates matter predict the formation of a steep power-law density profile around the SBH, a “spike,” and an increase by many orders of magnitude in the amplitude of the neutralino annihilation signal (Gondolo & Silk 1999).

The bulges of galaxies like the Milky Way are believed to have formed via mergers of pre-existing stellar systems. If the latter contained SBHs, a merger would result in the formation of a binary SBH (Begelman, Blandford & Rees 1980). The density of stars and dark matter around a binary SBH drops rapidly as the binary ejects matter via the gravitational slingshot.
Evidence for the scouring effect of binary SBHs is seen at the centers of the brightest galaxies (Milosavljevic et al. 2002; Ravindranath, Ho, & Filippenko 2002) where the stellar density profiles are nearly flat and sometimes even exhibit a central minimum (Lauer et al. 2002). However in fainter elliptical galaxies and in the bulges of spiral galaxies like the Milky Way, steeply-rising stellar densities are observed: $\rho_* \sim r^{-\gamma}, 1.5 \lesssim \gamma \lesssim 2.5$. In these galaxies, the most recent mergers may have taken place before the era at which SBHs formed, allowing the stellar density near the SBH to remain high (Silk 2002). Since stars and dark matter respond similarly to the presence of a SBH, galaxies with steeply-rising stellar densities are the most plausible sites for high dark matter densities and hence for the detection of annihilation radiation.

Stars near the SBH would also interact with the dark matter via gravitational scattering (Ilyin, Zybin, & Gurevich 2003; Gnedin & Primack 2003). Here, the time-dependent equations describing the scattering of dark matter particles off of stars in the presence of a SBH are solved. Scattering decreases the density of a dark matter spike by kinetic heating, and by driving particles into the SBH. The result, after $10^{10}$ yr, is the virtual dissolution of the dark matter spike. This result suggests that enhancements in the dark matter density around a SBH are likely to be modest whether or not the host galaxy had experienced the scouring effects of a binary SBH.

2. Method and Dissolution Time Scales

Define $r_h$ to be the radius of gravitational influence of the SBH, with $M_*(r < r_h) = 2M_*$. For the GC SBH, $r_h \approx 1.67$ pc (Genzel et al. 2003). After growth of the SBH, the dark matter density is approximately

$$\rho(r) = \rho(r_b) \times \begin{cases} (r/r_b)^{-\gamma_{sp}}, & r \lesssim r_b \\ (r/r_b)^{-\gamma_c}, & r \gtrsim r_b \end{cases}$$

where $\gamma_{sp} = 2 + 1/(4 - \gamma_c)$, $\rho \propto r^{-\gamma_c}$ is the dark matter density before growth of the SBH, and $r_b \approx 0.2r_h$ (Merritt 2003).

An estimate of the local heating rate of dark matter particles due to gravitational encounters with stars is the change per unit time of $\epsilon = \frac{1}{2}mv^{2}_{\text{rms}}$, the mean kinetic energy of the dark matter particles. Assuming Maxwellian velocity distributions for the stars and
dark matter,

\[
\frac{d\epsilon}{dt} = \frac{8 (6\pi)^{1/2} G^2 \rho_* m \ln \Lambda}{(v_{r}\nolimits_{rms}^2 + v_{\epsilon}\nolimits_{rms}^2)^{3/2}} (\epsilon_* - \epsilon)
\]

where \( \epsilon_* = \frac{1}{2} m_* v_{\epsilon}\nolimits_{rms}^2 \), \( \rho_* \) is the stellar mass density and \( \ln \Lambda \approx 15 \) is the Coulomb logarithm (Spitzer 1987). Taking the limit \( m \ll m_* \) and assuming \( v_{\epsilon}\nolimits_{rms} \approx v_{\epsilon}\nolimits_{rms}, \) appropriate shortly after the dark matter spike forms, the local heating time becomes

\[
T_{\text{local}} \equiv \left| \frac{1 \epsilon}{d\epsilon/dt} \right| = \frac{0.0814 v_{\epsilon}\nolimits_{rms}^3}{G^2 m_* \rho_* \ln \Lambda} \approx 1.8 \times 10^{9} \text{yr} \left( \frac{r}{1 \text{pc}} \right)^{-0.1};
\]

the latter expression uses the observed stellar mass density near the GC SBH, \( \rho_* \approx 3.2 \times 10^5 \mathcal{M}_\odot \text{pc}^{-3}(r/1\text{pc})^{-\gamma}, \gamma = 1.4 \pm 0.1 \) (Genzel et al. 2003), and \( v_{\epsilon}\nolimits_{rms} \approx 1.12(GM_*/r)^{1/2} \) with \( M_* = 3 \times 10^6 \mathcal{M}_\odot \). The time to heat the dark matter is nearly independent of radius and shorter by a factor \( \sim 5 \) than the age of the stellar bulge.

The change with time of the dark matter density may be computed from the Fokker-Planck equation describing the evolution of \( f(r, v, t) \), the mass density of dark matter particles in phase space, due to gravitational interactions with stars (Merritt 1983). We assume that \( f \) is isotropic in velocity space, \( f = f(E, t) \), with \( E = -v^2/2 + \phi(r) \) the binding energy per unit mass of a dark matter particle, and \( \phi(r) = -\Phi(r) \), with \( \Phi(r) \) the gravitational potential due to the SBH and the stars. The kinetic equation describing the evolution of \( f(E, t) \) due to scattering off of stars with mass \( m_* \gg m \) and phase-space density \( f_*(E) \) is

\[
4\pi^2 p(E) \frac{\partial f(E, t)}{\partial t} = -\frac{\partial F_E}{\partial E} - F_{lc}(E, t),
\]

\[
F_E(E, t) = -D_{EE}(E) \frac{\partial f}{\partial E},
\]

\[
D_{EE}(E) = 64\pi^4 G^2 m_* \ln \Lambda \left[ q(E) \int_{-\infty}^{E} dE' f_*(E') + \int_{E}^{\infty} dE' q(E') f_*(E') \right].
\]

Here \( p(E) = 4\sqrt{2} \int_0^{r_{\text{max}}(E)} dr r^2 \sqrt{\phi(r) - E} \) is the phase space volume accessible per unit of energy; \( p(E) = -\partial q/\partial E \); and \( \phi(r_{\text{max}}) = E \). The mass density of dark matter particles in \( E \)-space is \( N(E) dE = 4\pi^2 p(E) f(E) dE \). \( F_{lc}(E) \) is the flux of stars that are scattered from low angular momentum orbits into the SBH and is discussed in more detail below. Eqs. (4) assume that small-angle scatterings dominate the evolution of \( f \) and that the gravitational potential changes on a time scale long compared with \( T_{\text{local}} \); the latter assumption is valid as long as the gravitational acceleration is not produced dominantly by the dark matter particles themselves.
Neglecting $F_{lc}$, the total energy $\mathcal{E} = \int_{E_1}^{E_2} N(E) dE$ of dark matter particles in the energy range $E_1 < E < E_2$ changes with time according to

$$\frac{d\mathcal{E}}{dt} = - \int_{E_1}^{E_2} E \frac{\partial F_E}{\partial E} dE$$

$$= - [E F_E]_{E_1}^{E_2} - [f D_{EE}]_{E_1}^{E_2} - \int_{E_1}^{E_2} dE N(E) Q(E), \quad (5a)$$

$$Q(E) = 16\pi^2 G^2 m_* \ln \Lambda \int_0^E dE' f_*(E'). \quad (5c)$$

The third term in eq. (5c), which is always negative, represents heating of the dark matter. We accordingly define the (non-local) time scale for heating of the dark matter particles to be

$$T_{\text{heat}}^{-1} = \frac{\int_{E_1}^{E_2} dE N(E) Q(E)}{\int_{E_1}^{E_2} dE EN(E)}. \quad (6)$$

This expression may be used to estimate the dissolution time of a dark matter spike. Assume that both stars and dark matter particles initially have power-law density profiles near the SBH: $\rho(r, t = 0) \propto r^{-\gamma_{sp}}$, $\rho_*(r) \propto r^{-\gamma}$, $r \lesssim r_h$, and that $\phi(r) = G M_*/r$. The isotropic distribution function corresponding to an $r^{-\gamma}$ density profile in an $r^{-1}$ potential is $f(E) \propto E^{\gamma-3/2}$. Setting $E_1 = \phi(r_h) = G M_*/r_h$ and $E_2 \rightarrow \infty$, $T_{\text{heat}}$ becomes

$$T_{\text{heat}} = A(\gamma, \gamma_{sp}) \frac{M_*}{m_*} \left( \frac{G M_*}{r_h^3} \right)^{-1/2} \frac{1}{\ln \Lambda}. \quad (7a)$$

$$A(\gamma, \gamma_{sp}) = \frac{1}{2} \sqrt{\frac{\pi}{3}} \frac{(\gamma - 1/2)(7/2 - \gamma - \gamma_{sp})}{(3 - \gamma)(2 - \gamma_{sp})} \frac{\Gamma(\gamma + 1/2)}{\Gamma(\gamma + 1)}. \quad (7b)$$

When $\gamma = 3/2$, equal within the uncertainties with the slope of the stellar cusp around the Milky Way SBH (Genzel et al. 2003), the coefficient in eq. (7) is independent of $\gamma_{sp}$ and

$$T_{\text{heat}} = \frac{4\sqrt{3} M_*}{27 m_*} \left( \frac{G M_*}{r_h^3} \right)^{-1/2} \frac{1}{\ln \Lambda}. \quad (8a)$$

$$= 1.25 \times 10^9 \text{yr} \times \left( \frac{M_*}{3 \times 10^6 M_\odot} \right)^{1/2} \left( \frac{r_h}{2 \text{ pc}} \right)^{3/2} \left( \frac{m_*}{M_\odot} \right)^{-1} \left( \frac{\ln \Lambda}{15} \right)^{-1}. \quad (8b)$$

We take $T_{\text{heat}}$ as defined in eq. (8) as our unit of time in what follows, with $\tau \equiv t/T_{\text{heat}}$.

Diffusion in energy will cause a modest loss of stars to the SBH, $\dot{M} = - F_E(E_2)$, $E_2 \approx c^2$. A much greater capture rate is implied by scattering of dark matter particles on low angular momentum (eccentric) orbits into the SBH (Frank & Rees 1976). The loss rate is given
approximately by (Lightman & Shapiro 1977)

\[ F_{lc}(E) \approx \frac{4\pi^2 P(E) J_c^2(E) \overline{\mu}(E) R}{\partial R} \frac{\partial f}{\partial R}, \quad (9a) \]

\[ \approx S(E) f(E), \quad (9b) \]

\[ S(E) = 4\pi^2 P(E) J_c^2(E) \overline{\mu}(E) \left[ \ln R_0(E)^{-1} \right]^{-1}. \quad (9c) \]

Here \( R \equiv J^2/J_c^2(E) \) is a scaled angular momentum variable with \( J_c(E) \) the angular momentum of a circular orbit of energy \( E \); \( P \) is the period of a radial orbit; \( \overline{\mu} \) is the orbit-averaged angular momentum diffusion coefficient \( \langle (\Delta R)^2 \rangle/2R \); and \( R_0 \) is the value of the angular momentum variable at which \( f \) drops to zero due to the competing effects of capture and diffusion. The final line of eq. (9) assumes \( f \approx \ln[1/(R/R_0(E))] \) near the loss cone (Lightman & Shapiro 1977; Cohn & Kulsrud 1978). Cohn & Kulsrud (1978) give expressions for \( R_0 \) as a function of \( E \) based on solutions to the \( R \)-dependent Fokker-Planck equation; we adopt their expressions here. The angular momentum diffusion coefficients used by these authors, for modelling systems containing a single stellar mass, may be shown to remain unchanged when the scattered objects (here dark matter) have masses much less than those of the scatterers (stars). The rate of loss of stars predicted by eq. (9) depends only weakly on the radius of the capturing sphere, which we set to \( 2GM_*/c^2 \). The diffusion coefficient \( \overline{\mu} \) is of order \( T_{\text{heat}}^{-1} \), hence \( F_{lc}(E) \approx N(E)/T_{\text{heat}}(E) \ln R_0^{-1} \).

### 3. Results

The detailed evolution of the dark matter density around the GC SBH was computed by integrating eq. (4a) forward in time. The stellar density was modelled via Dehnen’s (1993) density law, \( \rho_\star(r) \propto (r/r_0)^{-\gamma}(1 + r/r_0)^{-\gamma-4} \) with \( \gamma = 1.4 \) and \( r_0 \) chosen to match the observed stellar density at \( r \lesssim r_h \) (Genzel et al. 2003). Figure 1 shows the evolution of the dark matter density assuming \( \gamma_{sp} = 7/3 \) and \( \gamma_c = 1 \), the values corresponding to a spike that developed in response to adiabatic growth of a SBH in a dark matter halo with \( \gamma_c = 1.0 \). In terms of the dimensionless variable \( \tau = t/T_{\text{heat}} \), the age of the Galactic bulge is \( \tau \sim 10 \). Fig. 1 shows that scattering of dark matter particles by stars causes the dark matter density to drop and the spike to flatten, within a radius \( \sim r_h \) where the heating time is shorter than the age of the bulge. Fig. 2 shows the dark matter density at \( \tau = 10 \) for a range of initial spike profiles, \( 1.5 \leq \gamma_{sp} \leq 2.75 \). The mean density within \( r_h \) drops by several orders of magnitude when \( \gamma_{sp} \approx 2 \).
Fig. 1.— Evolution of the dark matter phase space density $f(E)$ and mass density $\rho(r)$ due to gravitational interactions with stars around the GC SBH. The initial dark matter density satisfied eq. 1 with $\gamma_{sp} = 7/3$ and $\gamma_c = 1$. Times shown are $\tau = 0$ (heavy curves) and $\tau = 2, 4, ..., 20$ where $\tau$ is the time in units of $T_{\text{heat}}$ (eq. 8); $\tau \sim 10$ corresponds roughly to the age of the galactic bulge.
In the absence of scattering into the SBH, eq. (4) admits of the time-independent solution $f(E) = \text{constant}, \rho \sim r^{-3/2}$ (Gnedin & Primack 2003). As Figs. 1 and 2 show, there is indeed a tendency to evolve toward this characteristic profile, although a number of factors keep it from being precisely reached, including the finite evolution time; the presence of the loss term $F_{lc}$; and the fact that $f = \text{constant}$ can only hold true over a finite range of energies given the boundary conditions on $f$. Nevertheless, at late times ($\tau \gtrsim 20$), the solutions found here are generally well described by $\rho \sim r^{-3/2}$ at radii $10^{-5} \lesssim r/r_h \lesssim 10^{-2}$.

Fig. 2.— Dark matter density at $\tau = 0$ (dotted lines) and $\tau = 10$ (solid lines) given an initial density that satisfies eq. 1, with $\gamma_c = 1$ and $\gamma_{sp} = (1.50, 1.75, 2.00, 2.25, 2.50, 2.75)$.

The flux of dark matter annihilation photons along a direction that makes an angle $\psi$ with respect to the GC is proportional to the line-of-sight integral $\int_{\psi} \rho^2 dl$. Following earlier authors (Bergstrom et al. 1999), we define the dimensionless form factor $J(\psi) \equiv K \int_{\psi} \rho^2(l)dl$, $K^{-1} = (8.5 \text{ kpc})(0.3 \text{ GeV}/\text{cm}^3)$. Given a photon detector with angular accep-
tance $\Delta \Omega$ directed toward the GC, the signal is proportional to

$$\langle J \rangle \equiv \frac{1}{\Delta \Omega} \int_{\Delta \Omega} J(\psi) \, d\Omega.$$  

Figure 3 shows the evolution of $\langle J \rangle$ for $\Delta \Omega = 10^{-5}(10^{-3})$ sr; the first value is the approximate solid angle of the detectors in GLAST and in atmospheric Cerenkov telescopes like VERITAS, while the larger angle corresponds approximately to EGRET. The dark matter density was normalized to a fiducial value of $\rho = 100 M_\odot pc^{-3}$ at $r = r_h$; $J$ and $\langle J \rangle$ scale as $\rho^2(r_h)$. We note that $\rho(r_h)$ is very uncertain and could be much lower (Ullio, Zhao, & Kamionkowski 2001; Merritt et al. 2002; Evans, Ferrer, & Sarkar 2003). Figure 3 shows that the very large initial values of $\langle J \rangle$ are rapidly diminished as the spike is dissolved; by $\tau = 10$, $\langle J \rangle$ has dropped below $\sim (10^4, 10^3)$, $\Delta \Omega = (10^{-5}, 10^{-3})$ for all $\gamma_{sp} \lesssim 2.5$. These values are similar to what would be predicted for the central regions of a dark matter halo in the absence of a SBH (Bergstrom et al. 1999).

The dark mass captured by the SBH after $10^{10}$ yr is less than $10^4 M_\odot$ ($\rho(r_h) = 100 M_\odot pc^{-3}$) for all the integrations presented here. Various schemes have been discussed for increasing the captured mass in stars or dark matter to much greater values, perhaps of order $M_\bullet$. These include making the dark matter collisional (Ostriker 2000); assuming instantaneous replenishment of the loss cone (MacMillan & Henriksen 2002; Zhao, Haehnelt & Rees 2002) or allowing the stellar potential to be non-axisymmetric (Merritt & Poon 2003). The first two mechanisms are ad hoc; the third, if it applies to the GC, might allow the persistence of a dark matter spike in the face of scattering and capture, by increasing the mass in dark matter particles that can interact with the SBH.

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Fig. 3.— Evolution of \( \langle J \rangle \), a dimensionless measure of the cuspiness of the dark matter spike (eq. 10), for two values of the solid angle \( \Delta \Omega \) of a detector centered on the SBH. Curves are shown for \( \gamma_{sp} = 1.50 \) (lower), 1.75, 2.00, 2.25, 2.50, and 2.75 (upper). The dark matter density was normalized to an initial value of 100\( M_\odot \)pc\(^{-3} \) at \( r = r_h \); \( \langle J \rangle \) scales as \( \rho^2(r_h) \).
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