Higher Dimensional Recombination of Intersecting D-branes

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Abstract

We study recombination of D-branes system intersecting at more than one angle using super Yang-Mills theory. We find the condensation of an off-diagonal tachyon mode relates to the recombination, as was clarified in branes at one angle in hep-th/0303204. In branes at two angles, after the tachyon mode between two D2-branes condensed, D2-brane charge distributes in the bulk near an intersection point. We also find that when two intersection angles are equal, the off-diagonal lowest mode becomes massless, and a new stable non-abelian configuration, which is supersymmetric up to a quadratic order in the fluctuation, is found by the deformation by this mode.

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1 Introduction

Low energy dynamics of D-brane is well known by the analysis using super Yang-Mills theory [1]. One of the D-branes systems which can be discussed in Yang-Mills theory is intersecting D-branes. Fluctuation spectra of intersecting D-branes on a torus are discussed using Yang-Mills action in [2]. In intersecting branes system, recombination is important in a context of string phenomenology. Standard Model Higgs mechanism appears as the brane recombination [3]. On the other hand, it plays an important role to realize the inflation in brane world scenario [4]. In Yang-Mills theory, recombination is discussed in [5, 6] in branes at one angle*. Fundamental string stretching between intersecting D-branes is also discussed in Yang-Mills theory in [10]. We expand the discussions of recombination of intersecting D-branes to more than one intersection angle. This enables us to discuss more complicated intersecting branes systems which appear in the context of Standard Model on intersecting D-branes. To clarify the mechanism of recombination in higher dimensions is one of the aims of this paper.

When we consider two intersection angles, we find a supersymmetric configuration where two intersection angles are equal. It is known that supersymmetric intersecting branes are embedded into spacetime on a calibrated surface to minimize their worldvolume †. Calibration equations are realized as BPS conditions in abelian Dirac-Born-Infeld action [12]. An embedding followed by the calibration geometry is called in [13] as abelian embedding. The dynamics of multiple D-branes is described by the non-abelian Born-Infeld(NBI) action. In non-abelian case, there are another embedding which cannot be diagonalized. We call such embeddings as non-abelian embedding. There are various types of non-abelian embedding and they include many interesting insight but it is not yet considered except for a few cases [13]. It is difficult to describe the non-abelian embedding because we do not know the explicit form of the NBI action. It is difficult to know the full NBI action because in the non-abelian case, slowly varying field approximation becomes meaningless [14] and we must consider the derivative terms together with the field strength ‡. But there are some regions where even Yang-Mills analysis is appropriate to discuss the non-abelian embedding and we believe that the result of such Yang-Mills analysis can be lifted smoothly to a full NBI analysis.

In this paper, we discuss recombination of intersecting D2-branes at more than one angle. When two intersection angles are not equal, a lowest mode of Neveu-Schwarz sector becomes tachyonic and considering a condensation of this mode, we obtain a deformed intersecting

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*It is also discussed in tachyon field theory [7] and Matrix theory [8]. Intersecting D-branes with separation are discussed in [9].

†For a recent review, see [11].

‡We know up to and including $F^6$ terms of the NBI action now [15, 16].
branes. In branes at one angle, the recombination happens locally near the intersection point and we can describe this phenomenon by the condensation of the tachyon mode which is localized at the intersection point. The final state of decay process is two parallel D-strings. In branes at two angle, the final state after recombination happens is considered as a state which preserves 1/4 supersymmetry [17]. The tachyon mode we consider here is localized at the intersection point even in this case, therefore, the condensation of tachyon mode describes a first step of the recombination, behaves as a trigger. After tachyon mode condensed, we can not diagonalize two transverse scalar fields by a gauge transformation simultaneously, and there is D2-brane charge distributing in the bulk near the intersection point. In the case when two intersection angles are equal, we obtain a massless mode in the off-diagonal spectrum in Yang-Mills theory. By considering the deformation by this massless mode, we obtain a configuration which is supersymmetric up to a quadratic order in the fluctuation. In this configuration, D2-brane charge is also distributed in the bulk near the intersection point. The massless deformation considered here does not seem to correspond to the abelian calibration geometry§ and this is an interesting example of non-abelian embedding.

In section 2, we discuss the recombination of D2-branes intersecting at two angles. When two intersecting angles are equal, we can obtain the supersymmetric configuration and there is an off-diagonal massless mode, which is discussed in section 3. The effect of higher order corrections of field strength is discussed in section 4. Section 5 is devoted to conclusion and discussion.

2 Recombination of D-branes intersecting at two angles

Before considering Yang-Mills analysis, we see a mass spectrum of the string stretching between two intersecting D-branes at multiple angles $\theta_i (i = 1, \ldots, a)$. It is obtained by the worldsheet analysis as [19, 20] ¶

$$m_j^2 = \frac{1}{2\pi \alpha'} \sum_{i=1}^{a} (2n_i + 1) \theta_i \pm 2\theta_j ,$$

(2.1)

where $n_i \in \mathbb{N}$.

§Calibration of supersymmetric intersecting D-branes in terms of world-volume field theory is discussed in [18].

¶See also [21].
2.1 Fluctuation modes

We start from the two D2-branes worldvolume effective action which is obtained by the dimensional reduction of (9+1) dimensional SU(2) Yang-Mills theory as

\[ S = -T \text{Tr} \int d^2x dt \left[ (D_a Y^i)^2 + \frac{1}{2} F^2_{ab} - \frac{1}{2} [Y^i, Y^j]^2 \right] . \]  

(2.2)

We take \( a, b = 0, 1, 2 \) which denote directions along the worldvolume and \( i, j \) as \( i, j = 3, \ldots, 9 \) which denote directions of transverse collective coordinates. \( F_{ab} \) and \( D_a Y^i \) are described as

\[ F_{ab} = \partial_a A_b - \partial_b A_a - i[A_a, A_b] , \]
\[ D_a Y^i = \partial_a Y^i - i[A_a, Y^i] , \]

(2.3)

where \( A_a \) are worldvolume gauge fields and \( Y^i \) are transverse scalar fields. Let us consider intersecting D2-branes system. It is sufficient for us to consider the situation that two D2-branes are embedded in 4-dimension and do not spread out in other directions. We take the embedded 4 directions as 1, 2, 8 and 9 and others as 3, \( \cdots \), 7. We describe intersecting angles as \( \theta_1 \) in \( x_1-Y^9 \) plane and \( \theta_2 \) in \( x_2-Y^8 \) plane here. We consider a classical solution written as

\[ Y^9 = q_1 x_1 \sigma^3 , \quad Y^8 = q_2 x_2 \sigma^3 , \quad A_a = 0 , \]

(2.4)

where \( q_\alpha (\alpha = 1, 2) \) are related to \( \theta_\alpha \) as \( \theta_\alpha \equiv 2 \tan^{-1}(2\pi \alpha' q_\alpha) \) and we set \( q_\alpha > 0 \) here.

Let us consider off-diagonal parts of fluctuations which are written as

\[ Y^9 = q_1 x_1 \sigma^3 + f_1(x_a) \sigma^1 + \tilde{f}_1(x_a) \sigma^2 , \quad Y^8 = q_2 x_2 \sigma^3 + f_2(x_a) \sigma^1 + \tilde{f}_2(x_a) \sigma^2 , \]
\[ A_1 = g_1(x_a) \sigma^1 + \tilde{g}_1(x_a) \sigma^2 , \quad A_2 = g_2(x_a) \sigma^1 + \tilde{g}_2(x_a) \sigma^2 . \]

(2.5)

We set gauge condition as \( A_0 = 0 \). Diagonal parts of fluctuations decouple from off-diagonal fluctuations at the quadratic level, therefore we neglect those fluctuations.
The Lagrangian quadratic in the fluctuation is calculated as

\[
L = \sum_{i,j=1,2} \left( -(\partial_0 f_i)^2 - 4q_i f_i \tilde{g}_i + (\partial_i f_j + 2\tilde{g}_i q_j x_j)^2 - (\partial_0 \tilde{g}_i)^2 + (\partial_i \tilde{g}_j - \partial_j \tilde{g}_i)^2 
+ 4(q_i x_i f_j - q_j x_j f_i)^2 + (f_\alpha \to \tilde{f}_\alpha, \tilde{g}_\alpha \to g) \right).
\]

(2.6)

The combinations of \(f_\alpha, \tilde{g}_\alpha\) and \(\tilde{f}_\alpha, g_\alpha\) are decoupled in the quadratic fluctuations, therefore we can neglect the \(\tilde{f}_\alpha, g_\alpha\) pairs. From now on, we denote \(\tilde{g}\) as \(g\). The equations of motion for the fluctuations are written as

\[
4 \begin{pmatrix} O_{11} & O_{12} \\ O_{21} & O_{22} \end{pmatrix} \begin{pmatrix} \tilde{V}_1 \\ \tilde{V}_2 \end{pmatrix} = 0,
\]

(2.7)

where

\[
O_{11} = \begin{pmatrix} \partial_0^2 - \partial_1^2 - \partial_2^2 + 4q_1^2 x_1^2 & -4q_1 - 2q_1 x_1 \partial_1 \\ -2q_1 + 2q_1 x_1 \partial_1 & 4q_1^2 x_1^2 + 4q_2^2 x_2^2 + \partial_0^2 - \partial_2^2 \end{pmatrix},
\]

\[
O_{12} = O_{21} = \begin{pmatrix} -4q_1 q_2 x_1 x_2 & -2q_1 x_1 \partial_2 \\ 2q_2 x_2 \partial_1 & \partial_1 \partial_2 \end{pmatrix},
\]

\[
O_{22} = \begin{pmatrix} \partial_0^2 - \partial_1^2 - \partial_2^2 + 4q_1^2 x_1^2 & -4q_2 - 2q_2 x_2 \partial_2 \\ -2q_2 + 2q_2 x_2 \partial_2 & 4q_1^2 x_1^2 + 4q_2^2 x_2^2 + \partial_0^2 - \partial_1^2 \end{pmatrix},
\]

(2.8)

and

\[
\tilde{V}_1 = \begin{pmatrix} f_1(x_a) \\ g_1(x_a) \end{pmatrix}, \quad \tilde{V}_2 = \begin{pmatrix} f_2(x_a) \\ g_2(x_a) \end{pmatrix}.
\]

(2.9)

Expanding the fluctuations by the mass eigen functions as

\[
\tilde{V}_1(x_1, x_2, t) = \sum_{n \geq 0} \begin{pmatrix} \tilde{f}_{1n}(x_1, x_2) \\ \tilde{g}_{1n}(x_1, x_2) \end{pmatrix} C_{1n}(t), \quad \tilde{V}_2(x_1, x_2, t) = \sum_{n \geq 0} \begin{pmatrix} \tilde{f}_{2n}(x_1, x_2) \\ \tilde{g}_{2n}(x_1, x_2) \end{pmatrix} C_{2n}(t),
\]

(2.10)

where \(C_{in}(i = 1, 2)\) satisfy equations as

\[
(\partial_0^2 + m_{in}^2)C_{in}(t) = 0,
\]

(2.11)

we can obtain the eigen equations written as

\[
4 \begin{pmatrix} O'_{11} & O'_{12} \\ O_{21} & O'_{22} \end{pmatrix} \begin{pmatrix} \tilde{V}'_{1n} \\ \tilde{V}'_{2n} \end{pmatrix} = \begin{pmatrix} m_{1n}^2 \tilde{V}'_{1n} \\ m_{2n}^2 \tilde{V}'_{2n} \end{pmatrix},
\]

(2.12)
where

\[
O'_{11} = \begin{pmatrix}
-\partial_1^2 - \partial_2^2 + 4q_2^2 x_2^2 & -4q_1 - 2q_1 x_1 \partial_1 \\
-2q_1 + 2q_1 x_1 \partial_1 & 4q_1^2 x_1^2 + 4q_2^2 x_2^2 - \partial_2^2
\end{pmatrix},
\]

\[
O'_{22} = \begin{pmatrix}
-\partial_1^2 - \partial_2^2 + 4q_1^2 x_1^2 & -4q_2 - 2q_2 x_2 \partial_2 \\
-2q_2 + 2q_2 x_2 \partial_2 & 4q_1^2 x_1^2 + 4q_2^2 x_2^2 - \partial_1^2
\end{pmatrix},
\]

and

\[
\tilde{V}'_{1n} = \left( \begin{array}{c}
\tilde{f}_{1n} \\
\tilde{g}_{1n}
\end{array} \right), \quad \tilde{V}'_{2n} = \left( \begin{array}{c}
\tilde{f}_{2n} \\
\tilde{g}_{2n}
\end{array} \right).
\]

By solving these differential equations, the eigen function localized at the intersection point is obtained as

\[
\tilde{V}'_{10} = e^{-q_1 x_2^2 - q_2 x_2^2} C_{10}(t) \left( \begin{array}{c} 1 \\ 1 \end{array} \right),
\]

\[
\tilde{V}'_{20} = e^{-q_1 x_2^2 - q_2 x_2^2} C_{20}(t) \left( \begin{array}{c} 1 \\ 1 \end{array} \right).
\]

The squared of the mass eigenvalue is obtained as

\[
m_{10}^2 = 2(q_2 - q_1) \approx \frac{1}{2\pi\alpha'}(\theta_2 - \theta_1),
\]

\[
m_{20}^2 = 2(q_1 - q_2) \approx \frac{1}{2\pi\alpha'}(\theta_1 - \theta_2).
\]

There are massive and tachyonic modes, which coincide with the mass seen in the lowest mode of (2.1) in the small \(q\) region. Thus, we obtain the correct lowest mode of the string perturbation mass spectrum in [19].

A brane system with tachyon mode in open string theory is unstable and it rolls down to the stable vacuum by the condensation of the tachyon mode \[22\]. When we consider the condensation of the tachyon mode, we obtain the configuration written as

\[
Y^9 = \left( \begin{array}{c}
q_1 x_1 \\
\tilde{f}_{10} \\
\tilde{f}_{10} & q_2 x_1
\end{array} \right), \quad Y^8 = \left( \begin{array}{cc}
q_2 x_2 & 0 \\
0 & -q_2 x_2
\end{array} \right),
\]

\[
A_1 = \left( \begin{array}{cc}
0 & -i\tilde{g}_{10} \\
i\tilde{g}_{10} & 0
\end{array} \right), \quad A_2 = 0.
\]

where we consider a case \(q_1 > q_2\). We cannot simultaneously diagonalize \(Y^8\) and \(Y^9\) in a gauge transformation, therefore we do not find the simple recombination effect in the

\[5\]

\[\text{\textbf{Stability of branes at angles is discussed by considering the potential between two branes in [23].}}\]
geometrical picture as was discussed in [5]. We will discuss more about this in the next subsection.

Next, we see the excited mode. We assume the form of the eigen function of the fluctuation as the product of exponential function $e^{-q_1 x_1^2 - q_2 x_2^2}$ and linear function of $x_a$. There are the following three eigen functions. The first eigen function is written as

$$
\begin{pmatrix}
\tilde{f}_{11}(x_1, x_2, t) \\
\tilde{g}_{11}(x_1, x_2, t)
\end{pmatrix}
= x_2 e^{-q_1 x_1^2 - q_2 x_2^2} C_{11}(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix},
$$

$$
\begin{pmatrix}
\tilde{f}_{21}(x_1, x_2, t) \\
\tilde{g}_{21}(x_1, x_2, t)
\end{pmatrix}
= x_1 e^{-q_1 x_1^2 - q_2 x_2^2} C_{21}(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix}.
$$

(2.18)

The corresponding squared of the mass eigenvalue is obtained as

$$m_{11}^2 = 2(3q_2 - q_1) \sim \frac{1}{2\pi\alpha'} (3\theta_2 - \theta_1),$$

$$m_{21}^2 = 2(3q_1 - q_2) \sim \frac{1}{2\pi\alpha'} (3\theta_1 - \theta_2).$$

(2.19)

The mass eigenvalue obtained here is found in the string mass spectrum in (2.1). The second eigen function is written as

$$
\begin{pmatrix}
\tilde{f}_{12}(x_1, x_2, t) \\
\tilde{g}_{12}(x_1, x_2, t)
\end{pmatrix}
= x_1 e^{-q_1 x_1^2 - q_2 x_2^2} C_{12}(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix},
$$

$$
\begin{pmatrix}
\tilde{f}_{22}(x_1, x_2, t) \\
\tilde{g}_{22}(x_1, x_2, t)
\end{pmatrix}
= x_2 e^{-q_1 x_1^2 - q_2 x_2^2} C_{22}(t) \begin{pmatrix} -1 \\ -1 \end{pmatrix},
$$

(2.20)

where the corresponding mass eigenvalue is common in both eigen function, therefore corresponding coefficients are $C_{22}(t) = C_{22}(t)$ and mass eigenvalue is written as

$$m_{12}^2 = 2(q_1 + q_2) \sim \frac{1}{2\pi\alpha'} (\theta_1 + \theta_2).$$

(2.21)

The mass eigenvalue obtained here is found in (2.1).

The third eigen function written by the product of exponential function and linear function of $x_i$ is written as

$$
\begin{pmatrix}
\tilde{f}_{13}(x_1, x_2, t) \\
\tilde{g}_{13}(x_1, x_2, t)
\end{pmatrix}
= q_1 x_1 e^{-q_1 x_1^2 - q_2 x_2^2} C_{13}(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix},
$$

$$
\begin{pmatrix}
\tilde{f}_{23}(x_1, x_2, t) \\
\tilde{g}_{23}(x_1, x_2, t)
\end{pmatrix}
= q_2 x_2 e^{-q_1 x_1^2 - q_2 x_2^2} C_{23}(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix},
$$

(2.22)

where the squared of the mass eigenvalue is obtained as

$$m_1^2 = 0.$$

(2.23)
There is no massless mode in the string perturbation mass spectrum between intersecting branes at two different angles, except for some fixed angles. This mode is considered as Nambu-Goldstone mode of broken U(2) symmetry. To see this explicitly, let us consider a gauge transformation for the intersecting brane solution written as

\begin{align*}
Y^9 &= q_1 x_1 \sigma^3, \quad Y^8 = q_2 x_2 \sigma^3, \quad A_1 = A_2 = 0 .
\end{align*}

(2.24)

When we consider the gauge transformation written as

\begin{align*}
Y^8 &\rightarrow \tilde{Y}^8 = U Y^8 U^{-1}, \quad Y^9 \rightarrow \tilde{Y}^9 = U Y^9 U^{-1}, \\
A_1 \rightarrow \tilde{A}_1 = U A_1 U^{-1} - i(\partial_1 U) U^{-1}, \quad A_2 \rightarrow \tilde{A}_2 = U A_2 U^{-1} - i(\partial_2 U) U^{-1} ,
\end{align*}

(2.25)

where

\begin{align*}
U(x_1, x_2) &= e^{i \Lambda(x_1, x_2)} , \\
\Lambda(x_1, x_2) &= -\frac{c_3}{2} e^{-q_1 x_1^2 - q_2 x_2^2} \sigma_2 ,
\end{align*}

(2.26)

we obtain the configuration as follows:

\begin{align*}
\tilde{Y}^9 &= q_1 x_1 (\sigma_3 + C_3 e^{-q_1 x_1^2 - q_2 x_2^2}) + O(C_3^2) , \\
\tilde{Y}^8 &= q_2 x_2 (\sigma_3 + C_3 e^{-q_1 x_1^2 - q_2 x_2^2}) + O(C_3^2) , \\
\tilde{A}_1 &= q_1 x_1 C_3 e^{-q_1 x_1^2 - q_2 x_2^2} \sigma_2 + O(C_3^2) , \\
\tilde{A}_2 &= q_2 x_2 C_3 e^{-q_1 x_1^2 - q_2 x_2^2} \sigma_2 + O(C_3^2) .
\end{align*}

(2.27)

We find that the off-diagonal part in (2.27) is equivalent with that in (2.22). An off-diagonal massless mode which is not the Nambu-Goldstone mode is considered in the next section.

Thus, we obtain the correct string mass of the lowest and first excited part of the spectrum in the approximation of small angles \(O(\theta_i)\). We generalize these results to D3-branes intersecting at three angles, which are written in appendix A.

### 2.2 Tachyon Condensation and Recombination

We consider only the tachyon mode (2.15) here. By the condensation of the tachyon mode, we obtain the configuration written as

\begin{align*}
Y^9 &= q_1 x_1 \sigma_3 + \tilde{f}_{10} \sigma_1, \quad Y^8 = q_2 x_2 \sigma_3 , \\
A_1 &= \tilde{g}_{10} \sigma_2, \quad A_2 = 0 .
\end{align*}

(2.28)

**In one angle system, it is discussed in [24].**
We can not simultaneously diagonalize $Y^8$ and $Y^9$ in a gauge transformation, but we can diagonalize $Y^8$ and $Y^9$ in some region simultaneously. Let us look for the region where we can diagonalize $Y^8$ and $Y^9$ simultaneously. In the region,

$$|x_1| \gg \frac{C_1}{q_1},$$  \hfill (2.29)

we can diagonalize $Y^8$ and $Y^9$ as

$$Y^9 \sim q_1 x_1 \sigma_3, \quad Y^8 \sim q_2 x_2 \sigma_3.$$  \hfill (2.30)

Far away from the intersection point along $x_1$ direction, the brane configuration remains unchanged, because the tachyon mode, which is due to the brane deformation, is localized near the intersection point. Thus, this result is easily explained. In the region $x_2 \sim 0$, we can also diagonalize $Y^8$ and $Y^9$ simultaneously as

$$Y^9 \sim \sqrt{(q_1 x_1)^2 + \tilde{f}_{10}^2 \sigma_3}, \quad Y^8 \sim 0.$$  \hfill (2.31)

We find that the intersection point is resolved, and ‘recombination’ happens on the $x_1 Y^9$ plane near the intersection point. The other region can not be diagonalized and the D2-branes diffuse in the $x_1 x_2 Y^8 Y^9$ region near the intersection point. We will see the distribution of the D2-brane charge in the next subsection. Finally, let us remark a comment. If $q_1 \gg q_2$, there are many tachyon modes with masses $2(2nq_2 + q_2 - q_1)$ ($n = 0, 1, 2, \cdots$). In the limit $q_2 \to 0$, these infinite number of modes will be summed up by the form written as $f \sim C e^{-q_1 x_1^2} \sigma_3$, which is equivalent to the fluctuation mode found in branes at one angle in [5].

### 2.3 D2-brane charge

D2-brane charge density in terms of the D2-brane worldvolume effective action is obtained by [25, 26] as

$$J_{0kl} = \frac{1}{12} \text{Tr}(-iF_{ij}[Y^k, Y^l] - D_i Y^k D_j Y^l + D_i Y^l D_j Y^k),$$  \hfill (2.32)

where $i$ and $j$ are parametrizations of worldvolume directions, and D2-brane charge is already integrated along the direction $Y^k$ and $Y^l$.

We consider the configuration written as

$$Y^9 = q_1 x_1 \sigma_3 + \tilde{f}_{10} \sigma_1, \quad Y^8 = q_2 x_2 \sigma_3 + \tilde{f}_{20} \sigma_1,$$

$$A_1 = \tilde{g}_{10} \sigma_2, \quad A_2 = \tilde{g}_{20} \sigma_2.$$  \hfill (2.33)
If \( q_1 > q_2 \), the combinations of \( \tilde{f}_{10} \) and \( \tilde{g}_{10} \) correspond to tachyon modes and \( \tilde{f}_{20} \) and \( \tilde{g}_{20} \) correspond to massive modes. We calculate the charge density of this configuration. By the result,

\[
-i \text{Tr} F_{12}[Y^8, Y^9] = 8(C_2 q_1 x_1 - C_1 q_2 x_2)^2 e^{-2q_1 x_1^2 - 2q_2 x_2^2},
\]

\[
-\text{Tr} D_1 Y^8 D_2 Y^9 = 8(C_2 q_1 x_1 - C_1 q_2 x_2)^2 e^{-2q_1 x_1^2 - 2q_2 x_2^2},
\]

\[
\text{Tr} D_1 Y^9 D_2 Y^8 = 2q_1 q_2 - 4(C_2 q_1 + C_1 q_2) e^{-2q_1 x_1^2 - 2q_2 x_2^2},
\]

we obtain the D2-brane charge density as

\[
J_{089} = \frac{1}{6} q_1 q_2 + \left( \frac{2}{3} (C_2 q_1 x_1 - C_1 q_2 x_2)^2 - \frac{1}{3} C_2^2 q_1 - \frac{1}{3} C_1^2 q_2 \right) e^{-2q_1 x_1^2 - 2q_2 x_2^2}.
\]

(2.34)

\( C_1 \) is exponentially growing function of \( t \). We consider the situation where the tachyon mode condenses here, therefore we take \( C_2 = 0 \) because \( C_2 \) is the coefficient of a massive mode. D2-brane charge is rewritten as

\[
\frac{1}{6} q_1 q_2 + \left( \frac{2}{3} (C_1 q_2 x_2)^2 - \frac{1}{3} C_2^2 q_2 \right) e^{-2q_1 x_1^2 - 2q_2 x_2^2}.
\]

(2.35)

The first term is background D2 charge. The second term is induced by the tachyon mode and localized near the intersection point. Total D2-brane charge is obtained by the integration of \( x \) as

\[
\int dx_1 dx_2 e^{-2q_1 x_1^2 - 2q_2 x_2^2} \left( \frac{2}{3} (C_2 q_1 x_1 - C_1 q_2 x_2)^2 - \frac{1}{3} C_2^2 q_1 - \frac{1}{3} C_1^2 q_2 \right) = 0.
\]

(2.36)

Thus, we confirm that D2-brane charge is conserved after tachyon mode has condensed.

### 3 Supersymmetry

In this section, we consider the equal intersection angle case \( q_1 = q_2 \equiv q \) here. Now the tachyon mode becomes massless and the corresponding eigen functions are written as

\[
\tilde{V}_{10}' = \begin{pmatrix} \tilde{f}_{10}(x_1, x_2, t) \\ \tilde{g}_{10}(x_1, x_2, t) \end{pmatrix} = e^{-qr^2} C_{10} \begin{pmatrix} 1 \\ 1 \end{pmatrix},
\]

\[
\tilde{V}_{20}' = \begin{pmatrix} \tilde{f}_{20}(x_1, x_2, t) \\ \tilde{g}_{20}(x_1, x_2, t) \end{pmatrix} = e^{-qr^2} C_{20} \begin{pmatrix} 1 \\ 1 \end{pmatrix},
\]

(3.1)

where \( r^2 = x_1^2 + x_2^2 \). \( C_{10} \) and \( C_{20} \) are some numerical constant. It is known that two intersecting D-branes at equal two angles preserve 1/4 supersymmetry \( ^{\dagger\dagger} \). Therefore, let us

\( ^{\dagger\dagger} \)Black hole entropy in this system is discussed in [27].
check the supersymmetry of the configuration written as

\[ Y^9 = q_1 x_1 \sigma_3 + \tilde{f}_{10} \sigma_1, \quad Y^8 = q_2 x_2 \sigma_3 + \tilde{f}_{20} \sigma_1, \]
\[ A_1 = \tilde{g}_{10} \sigma_2, \quad A_2 = \tilde{g}_{20} \sigma_2. \] (3.2)

The supersymmetric variation of gaugino is written as

\[ \delta \psi = F_{\mu \nu} \Gamma^{\mu \nu} \epsilon. \] (3.3)

(3.2) satisfies the following BPS conditions up to the quadratic order in the fluctuations as

\[ F_{12} + i[Y^8, Y^9] = 0, \]
\[ D_1 Y^8 + D_2 Y^9 = \sigma^3 \mathcal{O}(C^2), \]
\[ D_1 Y^9 - D_2 Y^8 = \sigma^3 \mathcal{O}(C^2), \] (3.4)

and we obtain

\[ \delta \psi = \mathcal{O}(C^2) \epsilon. \] (3.5)

Thus, this configuration is supersymmetric up to the quadratic order in the fluctuation. The configuration with the supersymmetry up to quadratic order and where all supersymmetries are broken beyond this order is discussed in [28].

We can diagonalize \( Y^8 \) and \( Y^9 \) simultaneously in the region \( x_1 \sim x_2 \sim 0 \) as

\[ Y^9 \sim \tilde{f}_{10} \sigma_3, \quad Y^8 \sim \tilde{f}_{20} \sigma_3. \] (3.6)

Thus, the intersection point is resolved, which was also seen in the previous section. Note that in the previous section, only one mode, which is tachyonic, condenses. On the other hand, we consider the condensation of two massless modes here. Finally, we calculate the distribution of the D2-brane charge. D2-brane charge is written in (2.35) as

\[ J_{089} = \frac{1}{6} q^2 + \left( \frac{2}{3} (C_2 x_1 - C_1 x_2)^2 q^2 - \frac{1}{3} C_2^2 q - \frac{1}{3} C_1^2 q \right) e^{-2q^2}. \] (3.7)

Total charge is obtained as

\[
\int dx_1 dx_2 \left( \frac{1}{6} q^2 + \left( \frac{2}{3} (C_2 x_1 - C_1 x_2)^2 q^2 - \frac{1}{3} C_2^2 q - \frac{1}{3} C_1^2 q \right) e^{-2q^2} \right)
= \frac{q^2}{6} \cdot \text{(area of } x_1-x_2 \text{ plane)}. \] (3.8)

Thus, total D2-brane charge is conserved.
4 Higher order corrections of $F$

We consider the effect of the higher order corrections of $F$. The $F^4$ terms are the first nontrivial contribution to the fluctuation analysis, therefore we consider the mass spectrum and eigen functions including $F^4$ terms. The symmetrized traced Lagrangian in [29] is written as

$$L = \text{Str} \sqrt{-\det(\eta_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})},$$

and $F^4$ terms are obtained by the expansion of this action as

$$L = \text{Str}(2\pi\alpha')^2 \left( \frac{1}{8} F_{\mu\nu} F_{\nu\lambda} F_{\lambda\sigma} F_{\sigma\mu} - \frac{1}{32} F_{\mu\nu} F_{\nu\lambda} F_{\lambda\sigma} F_{\sigma\mu} \right).$$

By solving the equations of motions of the fluctuations, we obtain the eigen functions of the tachyon mode and nontrivial contribution to the fluctuation analysis, therefore we consider the mass spectrum. The eigen functions remain the product of gaussian functions of $x$ and fluctuations (2.5), we obtain quadratic part of the fluctuations as

$$L = -Q_1 ((\partial_0 f_1)^2 + (\partial_0 g_1)^2) - Q_2 ((\partial_0 f_2)^2 + (\partial_0 g_2)^2) + Q_3 ((\partial_1 f_1 + 2g_1 q_1 x_1)^2 - 4q_1 f_1 g_1) + Q_4 ((\partial_2 f_2 + 2g_2 q_2 x_2)^2 - 4q_2 f_2 g_2) + Q_5 ((\partial_1 f_2 + 2g_1 q_2 x_2)^2 + (\partial_2 f_1 + 2g_2 q_1 x_1)^2 + 4(q_1 f_1 - q_2 f_2)^2 + (\partial_1 g_2 - p_2 g_1)^2),$$

$$Q_1 \equiv (1 - \frac{1}{6} \frac{q_1^2}{(2\pi\alpha')^2})(1 + \frac{1}{6} \frac{q_2^2}{(2\pi\alpha')^2}), \quad Q_2 \equiv (1 + \frac{1}{6} \frac{q_1^2}{(2\pi\alpha')^2})(1 - \frac{1}{6} \frac{q_2^2}{(2\pi\alpha')^2}),$$

$$Q_3 \equiv (1 - \frac{1}{2} \frac{q_1^2}{(2\pi\alpha')^2})(1 + \frac{1}{6} \frac{q_2^2}{(2\pi\alpha')^2}), \quad Q_4 \equiv (1 + \frac{1}{6} \frac{q_1^2}{(2\pi\alpha')^2})(1 - \frac{1}{2} \frac{q_2^2}{(2\pi\alpha')^2}),$$

$$Q_5 \equiv (1 - \frac{1}{6} \frac{q_1^2}{(2\pi\alpha')^2})(1 - \frac{1}{6} \frac{q_2^2}{(2\pi\alpha')^2}).$$

By solving the equations of motions of the fluctuations, we obtain the eigen functions of the lowest mode as

$$\vec{V}'_{10} = \begin{pmatrix} \tilde{f}_{10}(x_1, x_2, t) \\ \tilde{g}_{10}(x_1, x_2, t) \end{pmatrix} = n_{10} e^{-q_1 x_1^2 - q_2 x_2^2} C_{10}(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

$$\vec{V}'_{20} = \begin{pmatrix} \tilde{f}_{20}(x_1, x_2, t) \\ \tilde{g}_{20}(x_1, x_2, t) \end{pmatrix} = n_{20} e^{-q_1 x_1^2 - q_2 x_2^2} C_{20}(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix}. (4.4)$$

The squared of the mass eigenvalue is obtained as

$$m^2_{10} = \pm (2(q_1 - q_2) - \frac{2}{3(2\pi\alpha')^2(q_1^3 - q_2^3)}) = \pm \frac{(\theta_1 - \theta_2)}{2\pi\alpha'} + O(\theta^3)$$

We obtain the correct mass eigenvalue of tachyon mode at order $\theta^3$. The eigen function of tachyon mode is the product of gaussian functions of $x_1$ and $x_2$. In the analysis [16], the eigen functions of tachyon mode remain gaussian at order $F^6$, therefore we expect that the eigen functions remain the product of gaussian functions of $x_1$ and $x_2$ even in two angle's case in the action including the $F^6$ terms, or more higher order $F$ terms.
5 Conclusion and discussion

We considered the recombination of intersecting D-branes at more than one angle using super Yang-Mills theory. There are off-diagonal tachyon modes in the fluctuation and the condensation of this mode triggers the recombination. On the other hand, in supersymmetric intersecting branes system, there are two kinds of nontrivial massless deformations in the intersecting D-brane system, the deformation into calibrated geometry and the recombination by the condensation of the off-diagonal mode. The former is obtained by a minimal surface problem of membranes and described perfectly by using U(1) DBI action. On the other hand, to describe the latter phenomenon, we need to know the full knowledge of the non-abelian Born-Infeld action. In some regions, we can analyze this phenomenon even in Yang-Mills theory. We discussed the condensation of the tachyon mode and we found that there are D2-branes distributing in the bulk near the intersection point after tachyon mode condensed. Tachyon condensation diffuses D2-brane charge at the intersection point at first stage, and after that, the localization might be relaxed and recombined D2-branes which preserve 1/4 supersymmetry would emerge as a final state. In the case that two intersection angles are equal, the lowest off-diagonal mode becomes massless. The configuration deformed by this mode becomes supersymmetric up to the quadratic fluctuation, and the intersection point is resolved. This is an interesting example of non-abelian embedding between two intersecting D-branes. In this case, abelian mode which governs the calibration geometry and non-abelian mode which govrens the non-abelian embedding. The region that Yang-Mills analysis is appropriate is also up to the quadratic order in the fluctuation. In string theory, or NBI action, this supersymmetric nonabelian embedding might be valid beyond the quadratic order in the fluctuation. To discuss this point more deeply, it might be interesting to consider the higher order fluctuations. $F^4$ corrections are also discussed and the higher order corrections of $\theta$ are obtained. It is straightforward to discuss the recombination of $D_p$-branes intersecting at more angles. The expansion to three intersection angles is considered in Appendix A. In future direction, it might be interesting to discuss the Higgs mechanism of the Standard model in this direction.

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A  Recombination of D-branes intersecting at three angles

D3-branes effective action is written as

$$S = -T \text{Tr} \int d^2 x dt \left[ (D_a Y^i)^2 + \frac{1}{2} F^2_{ab} - [Y^i, Y^j]^2 \right]. \quad (A.1)$$

where $a, b$ as $a, b = 0, 1, 2, 3$ and $i, j$ as $i, j = 4, \cdots, 9$. Two D3-branes are embedded in 6-dimension and do not spread out in other directions. The embedded directions are chosen as $1, 2, 3, 7, 8$ and $9$ and others as $4, 5$ and $6$. A classical solution we consider here is written as

$$Y^9 = q_1 x_1 \sigma^3, \quad Y^8 = q_2 x_2 \sigma^3, \quad Y^7 = q_3 x_3 \sigma^3, \quad A_a = 0, \quad (A.2)$$

which describes the two intersecting D3-branes.

Let us turn on the off-diagonal fluctuations as

$$Y^9 = q_1 x_1 \sigma^3 + f_1(x_a)\sigma^1 + \bar{f}_1(x_a)\sigma^2, \quad Y^8 = q_2 x_2 \sigma^3 + f_2(x_a)\sigma^1 + \bar{f}_2(x_a)\sigma^2, \quad A_1 = g_1(x_a)\sigma^1 + \bar{g}_1(x_a)\sigma^2, \quad A_2 = g_2(x_a)\sigma^1 + \bar{g}_2(x_a)\sigma^2. \quad (A.3)$$

The Lagrangian quadratic in the fluctuation is calculated as

$$L = \sum_{i,j=1,2,3} \left( - (\partial_0 f_i)^2 - 4q_i f_i \bar{g}_i + (\partial_i f_j + 2\bar{g}_i q_j x_j)^2 - (\partial_0 \bar{g}_i)^2 + (\partial_i \bar{g}_j - \partial_j \bar{g}_i)^2 + 4(q_i x_i f_j - q_j x_j f_i)^2 + (f_a \rightarrow \bar{f}, \bar{g}_a \rightarrow g) \right). \quad (A.4)$$

The combinations of $f_a, \bar{g}_a$ and $\bar{f}_a, g_a$ are decoupled in the quadratic fluctuations, therefore we can neglect the $\bar{f}, g$ pairs. From now on, we denote $\bar{g}$ as $g$.

By solving the equations of motion for the fluctuations, we obtain the eigen function of
the lowest mode of the fluctuation as

$$
\begin{align*}
\begin{pmatrix}
\tilde{f}_{10}(x_1, x_2, t) \\
\tilde{g}_{10}(x_1, x_2, t)
\end{pmatrix} &= \sum_{i=1,2,3} n_{1} e^{-q_{i} x_{i}^2} C_{10}(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \\
\begin{pmatrix}
\tilde{f}_{20}(x_1, x_2, t) \\
\tilde{g}_{20}(x_1, x_2, t)
\end{pmatrix} &= \sum_{i=1,2,3} n_{2} e^{-q_{i} x_{i}^2} C_{20}(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \\
\begin{pmatrix}
\tilde{f}_{30}(x_1, x_2, t) \\
\tilde{g}_{30}(x_1, x_2, t)
\end{pmatrix} &= \sum_{i=1,2,3} n_{3} e^{-q_{i} x_{i}^2} C_{30}(t) \begin{pmatrix} 1 \\ 1 \end{pmatrix}.
\end{align*}
$$

(A.5)

The lowest mode remains the product of gaussian of $x_i$. The squared of the mass eigenvalue is obtained as†

$$
\begin{align*}
m_{10}^2 &= \pm 2(q_1 + q_2 + q_3) \sim \frac{1}{2\pi\alpha'} (-\theta_1 + \theta_2 + \theta_3) \\
m_{20}^2 &= \pm 2(q_1 - q_2 + q_3) \sim \frac{1}{2\pi\alpha'} (\theta_1 - \theta_2 + \theta_3) \\
m_{30}^2 &= \pm 2(q_1 + q_2 - q_3) \sim \frac{1}{2\pi\alpha'} (\theta_1 + \theta_2 - \theta_3)
\end{align*}
$$

(A.6)

This result is consistent with the worldsheet analysis in [19]. The parameter region of the tachyonic configuration is considered in [32].

References


† The mass spectra of constant field strength configuration in Yang-Mills theory on $T^4$ are discussed in [30] and the generalizations on $T^{2n}(n = 1, 2, \cdots)$ are discussed in [31].


