The 27-plet Baryons from Chiral Soliton Models

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The purpose of the present note is to give a clear picture of all the 27-plet baryons from chiral soliton models and check the validity of this picture by symmetry. Though there are criticisms of the validity of chiral soliton models to study pentaquark states \[19\], we find that we can identify candidates for all non-exotic members in the 27-plet with spin 3/2, consistent with the experimental results in \[16\]. We also make predictions about the masses and widths for all exotic members in the 27-plet.

The action of Skyrme model is of the form \[20, 21\]

\[
I = \frac{1}{2} \int d^4x (\partial_\mu U \partial_\mu U^\dagger) + \frac{1}{32\mu^2} \int d^4x \left[ \partial_\mu UU^\dagger, \partial_\nu UU^\dagger \right]^2 + N_\tau \Gamma,
\]

and the SU(3) chiral field is expressed as

\[
U(x) = \exp \left[ \frac{\lambda_8 \phi(x)}{f_\pi} \right] = A(t)U_1(x)A(t)^{-1}, \quad A \in SU(3),
\]

where \(f_\pi \approx 93\) MeV is the observed pion decay constant, the dimensionless parameter \(\epsilon\) is introduced to stabilize the solitons by Skyrme, \(\Gamma\) is the Wess-Zumino term, \(\lambda_8\) are the eight Gell-Mann SU(3) matrices, \(\phi(x)\) are the eight pseudoscalar meson fields, and \(U_1(x)\) is a solitonic solution (with unit baryonic charge) of the equation of motion

\[
U_1(x) = \begin{pmatrix}
\exp \left[ i(\mathbf{\hat{r}} \cdot \mathbf{\tau})F(r) \right] & 0 & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

where \(F(r)\) is the spherical-symmetric profile of the soliton, \(\mathbf{\tau}\) are the three Pauli matrices, and \(\mathbf{\hat{r}}\) is the unit vector in space. The eigenvalues of the collective Hamiltonian are

\[
E_J^{(p,q)} = M_{cl} + \frac{1}{27I_1} \left[ p^2 + q^2 + pq + 3(p+q) - \frac{1}{4}(N_cB)^2 \right] + \left( \frac{27}{2I^2_1} - \frac{1}{27I_2} \right) J(J+1),
\]

where \((p,q)\) denotes an irreducible representation of the SU(3) group, \(M_{cl}\), \(I_1\) and \(I_2\) are given by the 3-dimensional space coordinate integrals of even functions of \(F(r)\) and \(\epsilon\), treated model-independently and fixed by experimental data, \(M_{cl}\) is the classical soliton mass, \(I_1\) and \(I_2\) are moments of inertia. From the energy eigenvalues above, it can be seen that \(\{27\}\) multiplet with spin 3/2 is next to the anti-decuplet, whose quark content is suggested in Fig. \[1\]
Using the wave function $\Psi^{(\mu)}_{\nu\nu'}(A)$ of baryon $B$ in the collective coordinates

$$\Psi^{(\mu)}_{\nu\nu'}(A) = \sqrt{\text{dim}(\mu)}D^{(\mu)}_{\nu\nu'}(A), \quad (5)$$

where $(\mu)$ denotes an irreducible representation of the SU(3) group; $\nu$ and $\nu'$ denote $(Y, I, J)$ and $(1, J_3)$ quantum numbers collectively; $Y$ is the hypercharge of $B$; $I$ and $I_3$ are the isospin and its third component of $B$ respectively; $J_3$ is the third component of spin $J$; and $D^{(\mu)}_{\nu\nu'}(A)$ are representation matrices, we can deal with the symmetry breaking Hamiltonian perturbatively

$$H' = \alpha D^{(8)}_{88} + \beta Y + \gamma \sum_{i=1}^{3} D^{(8)}_{i8} J_i, \quad (6)$$

where the coefficients $\alpha$, $\beta$, $\gamma$ are proportional to the strange quark mass and model dependent. In this note they are treated model-independently and fixed by experiments. $D^{(8)}_{mm}(A) = \frac{1}{2}\text{Tr}(A^T \lambda^m A \lambda^n)$ is the adjoint representation of the SU(3) group.

We can use the relations between the masses of the octet and decuplet baryons, then we need two additional equations to fix the parameters. Up to know, we have two methods to fix all the parameters in [14] and [15] model-independently: (I) take $\Theta^+$ as the member of the anti-decuplet, and use relations about the mass $(1.54 \text{ GeV})$ and width $(<25 \text{ MeV})$ of $\Theta^+$; (II) take both $\Theta^+$ and the candidate for $\Xi_{3/2}$ as members of the anti-decuplet and only use the mass relations of the anti-decuplet baryons [14] [15]. For method I, we have the results

$$\alpha = -766 \text{ MeV}, \quad \beta = 22 \text{ MeV}, \quad \gamma = 254 \text{ MeV},$$

$$1/I_1 = 154 \text{ MeV}, \quad 1/I_2 = 376 \text{ MeV};$$

and for method II we have the results

$$\alpha = -663 \text{ MeV}, \quad \beta = -12 \text{ MeV}, \quad \gamma = 185 \text{ MeV},$$

$$1/I_1 = 154 \text{ MeV}, \quad 1/I_2 = 399 \text{ MeV}.$$

The predicted mass of $\Xi_{3/2}$ from method I is 1.81, which is compatible with the experimental observation [17] and can be further adjusted to meet the data within uncertainties.

We find that the masses of the 27-plet calculated by the two methods are nearly equal, shown in Table 1. These results are close to those calculated by Walliser and Kopeliovich [14], with differences in the mass of $\Delta_{27}$. This shows the validity of the use of the perturbation method in chiral soliton models. In Table 1, we also list the candidates for the 27-plet baryons. We can find all the candidates for the non-exotic members by considering their masses and $I(J^P)$ in the baryon listing [23]. To verify this identification, we calculate the widths of the 27-plet baryons.

**Table 1. The masses (GeV) of baryons in the {27} multiplet**

| B | $\langle B|H'|B\rangle$ | method I | method II | $I(J^P)$ | PDG |
|---|------------------|---------|---------|--------|-----|
| $\Delta^*$ | $\frac{1}{11}\alpha - \frac{\beta - 95}{141}\gamma$ | 1.62 | 1.64 | $\frac{1}{2}(3^+)$ | 1.55 to 1.70 |
| $N_{27}$ | $\frac{1}{2}\alpha + \beta - \frac{90}{112}\gamma$ | 1.73 | 1.73 | N(1720) $\frac{1}{2}(1^+)$ | 1.65 to 1.75 |
| $\Sigma(27)$ | $-\frac{1}{2}\alpha + \frac{\beta}{112}\gamma$ | 1.79 | 1.80 | $\Sigma(1840)$ $\frac{1}{2}(3^+)$ | 1.72 to 1.93 |
| $\Xi_{27}$ | $-\frac{1}{27}\alpha + \beta + \frac{85}{221}\gamma$ | 1.95 | 1.96 | $\Xi(1950)$ $\frac{1}{2}(3^+)$ | 1.95 ± 0.15 |
| $\Lambda_{27}$ | $-\frac{1}{2}\alpha + \frac{\beta}{14}\gamma$ | 1.86 | 1.86 | $\Lambda(1890)$ $0(\frac{1}{2}^+)$ | 1.85 to 1.91 |
| $\Theta^*$ | $\frac{1}{2}\alpha + 2\beta - \frac{96}{111}\gamma$ | 1.61 | 1.60 | ? | ? |
| $X_{13}$ | $\frac{1}{2}\alpha - \frac{\beta - 25}{112}\gamma$ | 1.64 | 1.68 | ? | ? |
| $X_{23}$ | $-\frac{1}{2}\alpha - \beta + \frac{25}{112}\gamma$ | 1.84 | 1.87 | ? | ? |
| $\Omega^*$ | $-\frac{1}{2}\alpha - 2\beta + \frac{95}{141}\gamma$ | 2.06 | 2.07 | ? | ? |

The decay of a 27-plet baryon $B$ to an octet baryon $B'$ and a pseudoscalar meson $m$ is controlled by a pseu-
doscalar Yucawa coupling $\hat{g}_A \propto G_0 D^{(8)}_{m_3} - G_1 d_{3ab} D^{(8)}_{ma} j_b - \frac{G_2}{\sqrt{3}} D^{(8)}_{m_8} j_3$, \( (7) \)

where \( d_{3ab} \) is the SU(3) symmetric tensor, \( a, b = 4, 5, 6, 7, \)

\[
\Gamma(B \rightarrow B'm) = \frac{G_2^2}{4\pi} \frac{|p|}{m_B} \left[ (m_{B'}^2 + p^2)^{\frac{3}{2}} - m_{B'} \right] \left\{ \frac{\text{dim}(\mu')}{\text{dim}(\rho)} \sum_\gamma \left( \begin{array}{c} \rho, I_m, I_{\rho}, I_{\gamma} \end{array} \right) \left( \begin{array}{c} \mu' \mu' \end{array} \right) \right\}^2, \quad (8)
\]

where we postulate \( B \) with \( (Y, I, I_3; J^P, -J_3) = (Y, I, I_3; J^P, -J_3) = (Y, I, I_3; J^P, -J_3) = (Y, I, I_3; J^P, -J_3) = (Y, I, I_3; J^P, -J_3) = (Y, I, I_3; J^P, -J_3) = (Y, I, I_3; J^P, -J_3) \) and \( m \) with \( (Y, I, I_3; J^P, -J_3) = (Y, I, I_3; J^P, -J_3) = (Y, I, I_3; J^P, -J_3) = (Y, I, I_3; J^P, -J_3) = (Y, I, I_3; J^P, -J_3) = (Y, I, I_3; J^P, -J_3) = (Y, I, I_3; J^P, -J_3) \) and \( G_2^2 = 3.84(G_0 - \frac{1}{2} G_1)^2 \). If we postulate the width of \( \Theta^+ I_{0+} \prec 25 \text{ MeV} \), we can calculate the upper bounds of widths for all the 27-plet baryons, listed in Table 2. We can see that the candidates for non-exotic baryons manifest the approximate symmetry of the 27 representation of the SU(3) group. In the results above, we only consider the flavor SU(3) as an exact symmetry. If we take into account the effects of flavor asymmetry, the width of \( \Theta^+ \) will fall by about 30% \( [27] \).

<table>
<thead>
<tr>
<th>PDG estimation</th>
<th>modes</th>
<th>branching ratios</th>
<th>$\Gamma$ from data</th>
<th>width $\leq$ calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta(1600)$</td>
<td>250 to 450</td>
<td>$N\pi$</td>
<td>10 to 25%</td>
<td>10 to 40</td>
</tr>
<tr>
<td>$N(1720)$</td>
<td>100 to 200</td>
<td>$N\pi$</td>
<td>10 to 25%</td>
<td>10 to 40</td>
</tr>
<tr>
<td>$\Lambda(1890)$</td>
<td>60 to 200</td>
<td>$N\pi$</td>
<td>10 to 25%</td>
<td>10 to 40</td>
</tr>
<tr>
<td>$\Xi(1950)$</td>
<td>60 to 200</td>
<td>$N\pi$</td>
<td>10 to 25%</td>
<td>10 to 40</td>
</tr>
<tr>
<td>$\Theta^*$</td>
<td>?</td>
<td>$\Lambda K$</td>
<td>seen</td>
<td>90</td>
</tr>
<tr>
<td>$\bar{X}_{1s}$</td>
<td>?</td>
<td>$\Sigma\pi$</td>
<td>possibly seen</td>
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</tr>
<tr>
<td>$\bar{X}_{2s}$</td>
<td>?</td>
<td>$\Xi\pi$</td>
<td>seen</td>
<td>36</td>
</tr>
<tr>
<td>$\Omega^*$</td>
<td>?</td>
<td>$\Xi K$</td>
<td>seen</td>
<td>107</td>
</tr>
</tbody>
</table>

In summary, we use the perturbation method to deal with the 27-plet baryons with spin \( \frac{3}{2} \) from chiral soliton models. Calculations of the widths of the candidates for the non-exotic members manifest an approximate symmetry of the 27 representation of the SU(3) group. Thus, it seems that chiral soliton models are able to give us a clear picture of the 27-plet with spin \( \frac{3}{2} \), as well as the anti-decuplet \( [14, 15] \), beyond their validity of describing the octet and decuplet baryons. We also predict the masses and widths of the exotic members in 27-plet. The exotic members seem to be more difficult to be found experimentally for their larger widths compared with those of the anti-decuplet members. If this picture is right, the non-exotic member $\Xi(1950)$ should be with $J^P = \frac{3^+}{2}$.

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[16] NA49 Collaboration, C. Alt et al., hep-ex/0310014.