Top Quark As A Topological Color Soliton

X. Zhang

Department of Physics and Astronomy
Iowa State University
Ames, Iowa 50011

Abstract

In this paper I propose an scenario for the top quark as a topological color soliton. I illustrate some features of the top quark as a soliton in a toy model based on nonlinear realization of the $SU_L(3) \times SU_R(3)/SU_{L+R}(3)$ with $SU_{L+R}(3)$ being the color gauge group.
Recently the CDF collaboration[1] presented evidence for the top quark with mass $m_t \sim 174$ GeV. Since $m_t$ is of order the Fermi scale, the top quark is expected to hold the clue to the physics of the electroweak symmetry breaking[2]. The scale associated with top quark mass generation can be estimated by imposing unitarity on the processes $t \bar{t} \rightarrow V_L V_L[3]$, where $V_L$ is a longitudinal $W^\pm$ or $Z$. In the absence of the Higgs boson, such an amplitude is proportional to $G_F m_t \sqrt{s}$, and will violate unitarity at high energy $\Lambda$. The largest zeroth-partial-wave amplitude is the color-singlet, spin-zero combination of $t \bar{t}$ and the zero-isospin combination of $W_L^+ W_L^-$ and $Z_L Z_L$, from which one has[4]

$$\Lambda \leq \frac{4\sqrt{2}\pi}{3G_F m_t} \sim 2.9 \text{ TeV}.$$  \hspace{1cm} (1)

Since this energy scale is around $4\pi v \sim 3.1$ TeV[5][6], the scale of dynamical electroweak symmetry breaking, it suggests a possible connection between the top quark mass and the Fermi scale.

One interesting observation, based on the fact that the top quark Yukawa coupling is of order one, is that the top quark may be compared to the constituent quarks in QCD, whose mass ratios to the pion decay constant is also a number of order unity. Since the constituent quarks get masses dynamically, one would expect logically that the top quark also gets its mass dynamically.

The above argument not only makes a heavy top quark natural, but also results in a lot of interesting phenomena associated with the top quark. As an example, one expects that the axial coupling constant of the top quark to
the neutral gauge boson, $g_A$, will deviate from the standard model because the similar quantity for the constituent quark, $G_A$, is renormalized[6]. These predictions can be tested in the future Colliders. And if found experimentally to be true, it will provide a real clue to the mechanism of the breakdown in the electroweak theory. Note that in QCD chiral symmetry is broken dynamically by quark condensation. We should mention that if the top quark has different properties from the standard model prediction, it will affect experimental quantities associated with the bottom quark. Indeed, an anomalous $\Delta g_A$ at the level of the order $\Delta G_A \simeq 0.25$ for the constituent quarks can make the discrepancy of the $Z \rightarrow b\bar{b}$ width measured at LEP with the standard model expectation disappear[7].

Furthermore, the constituent quarks are argued to have a non-trivial structure[8], i.e., consisting of a valence quark plus many quark-antiquark pairs. D.B. Kaplan has shown that the constituent quarks can be considered as color solitons (Kaplan names them “qualitons”) [9]. In this picture, the constituent quark Q would look like a current quark q carrying around with it a significant deformation of the $<\bar{q}q>$ background. The spin, color and baryon number of the constituent quark are topologically induced and not localized at a point.

In this brief report, we suggest that the top quark is a soliton. For simplicity, we consider here the basic properties of the soliton, such as mass, radius, baryon number and its color and spin representation[F.1]. The model

[F.1] The flavor quantum number of the soliton in the coset space $SU(2) \times U(1)/U(1)$ has been worked out by C. Arnade and J. Bagger[10].
we use for the top quark soliton\[F.2\] is based on a non-linear realization of the $SU_L(3) \times SU_R(3)/SU_{L+R}(3)$. The unbroken group $SU_{L+R}(3)$ is the color gauge group $SU_c(3)$, so the Goldstone bosons $\pi^a$ are in the adjoint representation of $SU_c(3)$. The effective action describing the soliton solution is given by,

$$S = \frac{F_\pi^2}{16} \int d^4x \ Tr(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32\epsilon^2} \int d^4x \ Tr([\partial_\mu UU^\dagger, \partial_\nu UU^\dagger])^2 + n \Gamma_{WZ} + \frac{m_\pi^2 F_\pi^2}{8} \int d^4x \ Tr(U - 3) \ ,$$

(2)

where $U = \exp(2i\lambda^a \frac{\pi^a}{F_\pi})$, and $m_\pi$ is the Goldstone boson mass. It should be pointed out that we have neglected in (2) the operators with more powers of derivatives suppressed by powers of the symmetry breaking scale $\sim 4\pi v \sim O(1 \text{ TeV})$. The colored Goldstone bosons $\pi^a$ can decay into two gluons and the top quark pairs if $m_\pi > 2m_t$. In this paper, we will not discuss the phenomena associated with the $\pi^a$ fields, instead we take the effective action in (2) as a toy model to illustrate some features of the top quark as a soliton.

The action in (2) has the same form as that for a ordinary $SU(3)$ Skyrmion\[11\]. However, the parameters are quantitatively different. In the original $SU(3)$ Skyrmion model, $F_\pi \sim O(100 \text{ MeV})$ and $n = N_c = 3$, but here $F_\pi \sim v \sim O(100 \text{ GeV})$ and $n = 1$. Thus the soliton will be much heavier than the ordinary Skyrmion and its quantum numbers will also be

\[F.2\] This model is similar to that considered by Kaplan\[9\] for qualitons.
completely different from that of the $SU(3)$ Skyrmion. In the Skyrmion model, the baryon number of the soliton is one, and the lowest baryonic state is in the octet representation of $SU_{L+R}(3)$. Instead, we have here that the baryon number is $\frac{1}{3}$, and the lowest baryonic state is in the fundamental representation of $SU_{L+R}(3)$ (for a detailed discussion, see ref.[9]). Thus the soliton carries the quantum number of the top quark[F.3].

The technique of studying the properties of the soliton is well-known. First, take the “hedgehog” Ansatz for $U$:

$$U_0 = \exp\{iF(r)\vec{r} \cdot \vec{\tau}\}, \quad (3)$$

where the $\tau^i$ are the Pauli matrices embedden in the color $SU_{L+R}(3)$. In this Ansatz,

$$F(0) = \pi, \quad F(\infty) = 0. \quad (4)$$

The energy of the solution (3) is given by

$$M = M_{cl}[F] + m_{cl}[F], \quad (5)$$

where

$$M_{cl}[F] = 4\pi \frac{F_\pi}{e} \int_0^\infty dx \left[ \frac{x^2}{8} \left( \frac{dF}{dx} \right)^2 + \frac{\sin^2 F}{4} \right] \quad + \frac{\sin^4 F}{2x^2} + \left( \frac{dF}{dx} \right)^2, \quad (6.a)$$

[F.3] The soliton considered by D’Hoker and Farhi[12] in the effective action generated by integrating out the heavy quark of the standard model is a color neutral object with integer baryon number.
\[ m_{cl}[F] = \frac{m_\pi^2}{e^3F_\pi} \int_0^\infty x^2 \, dx \left[ 1 - \cos F \right] \] , \quad (6.b)

where \( x = eF_\pi r \).

The soliton state can be constructed by using the standard quantization method[13]. For our purpose, we just give the expression of the Hamiltonian for the soliton,

\[ H = M + \frac{1}{2} \left( \frac{1}{I_A} - \frac{1}{I_B} \right) j(j+1) + \frac{1}{2I_B}(C_2 - \frac{1}{12}) \] , \quad (7)

where \( j \) is the spin and \( C_2 \) is the color \( SU_c(3) \) Casimir. In (7),

\[ I_A = \frac{2\pi}{3e^3F_\pi} \int_0^\infty dx \, \sin^2 F \left[ x^2 + 4(\sin^2 F + x^2 \left( \frac{dF}{dx} \right)^2) \right] \] , \quad (8.a)

\[ I_B = \frac{\pi}{e^3F_\pi} \int_0^\infty dx \, \sin^2 F \left[ \frac{x^2 + 2 \sin^2 F}{2} \frac{x^2 + 2 \sin^2 F + x^2 \left( \frac{dF}{dx} \right)^2} \right] \] . \quad (8.b)

For a spin \( \frac{1}{2} \), color triplet soliton, \( j = \frac{1}{2} \) and \( C_2 = \frac{4}{3} \). Then the mass of the soliton is given by

\[ M_{(soliton)} = M + \frac{3}{8I_A} + \frac{1}{4I_B} \] . \quad (9)

The isoscalar radius of the soliton is given by

\[ r_0 = \left< r^2 \right>^{1/2} = \frac{1}{eF_\pi} \left\{ -\frac{2}{\pi} \int_0^\infty dx \, x^2 \, \sin^2 F \left( \frac{dF}{dx} \right) \right\}^{1/2} \] . \quad (10)

Now let us calculate \( M_{(soliton)} \) and \( r_0 \) numerically. We take a variational approach in ref.[14] and assume that \( F(r) \) has the following form:
\[ F(x) = 2 \arctan\left(\frac{x}{x_0}\right)^2 , \tag{11} \]

where \( x_0 \), the soliton size, is the variational parameter. For simplicity, we consider first the case where \( m_\pi = 0 \), then

\[ M = \frac{F_\pi}{e} \pi \frac{3\sqrt{2}}{16} (4x_0 + \frac{15}{x_0}) ; \tag{12.a} \]

\[ I_A = \frac{1}{e^3 F_\pi} \frac{\pi^2 \sqrt{2}}{12} (6x_0^3 + 25x_0) ; \tag{12.b} \]

\[ I_B = \frac{1}{e^3 F_\pi} \frac{\pi^2 \sqrt{2}}{16} (4x_0^3 + 9x_0) . \tag{12.c} \]

The \( x_0 \) can be estimated by minimizing (12.a) with respect to \( x_0 \), which gives \( x_0 = \sqrt{15/4} \). Then we have,

\[ M_{(\text{soliton})} \simeq F_\pi \left( \frac{40.544}{e} + e^3 \times 0.007 \right) , \tag{13} \]

and

\[ r_0 \simeq \frac{2.19}{e F_\pi} . \tag{14} \]

Following Ref.[15], let us consider a quantity \( M_{(\text{soliton})} r_0 \),

\[ M_{(\text{soliton})} r_0 \simeq 2.19 \times \left( \frac{40.544}{e^2} + 0.007 \times e^2 \right) . \tag{15} \]

Since \( M_{(\text{soliton})} r_0 \) depends only on one parameter \( e \), we would be able to find out a possible minimum value of the \( M_{(\text{soliton})} r_0 \) by minimizing (15)
with respect to $e$. It gives that

$$M_{(soliton)} \ r_0 \geq 2.333 \ ,$$

which is comparable to $2.52^{[15]}$, Keaton’s numerical value. One can see that for a top quark $m_t \sim 174$ GeV, we have $r_0 \sim \frac{1}{v}$. It should be pointed out that the minimum value of eq.(16) corresponds to $e^2 = 76.11$. In such a situation the quantum and the classical mass term in eq.(13) are exactly equal and the collective quantization, as carried out for the soliton here, is not well justified. So a smaller $e^2$ should be choosen. Consequently, $M_{(soliton)}r_0$ is expected to be larger than 2.333. For example, If one takes $e^2 = 50$, the quantum mass is only about half of the classical mass and now $M_{(soliton)}r_0$ is about 2.542, which is slightly larger than the minimum value 2.333.

In summary, we have shown that top quark can be described by a soliton. The spin, color and the baryon number of the top quark are topologically induced and its radius is of order $\sim \frac{1}{v}$. The top quark field is not localized at a point and will have color form factors. These new physics effects may show up experimentally as excessive or anomalous top production rates and distributions at the Hadron Colliders[16].

Before concluding, we would like to point out that in order to construct the color topological solitons, we have made use of the chiral color symmetry $SU_L(3) \times SU_R(3)$. Whether it exists or not depends on the dynamics of the fundamental theory of the top quark. As an existence proof, let us consider the gauge version of the top quark condensation theory[17], where new strong physics of the top quark at the $\sim 1$ TeV scale is introduced[18]. In a spe-
cific model proposed by Lindner and Ross[19], the four fermion interaction responsible for the top quark condensation is,

\[ \mathcal{L}_{\text{eff}} \sim \bar{T}_L \gamma_\mu T_L \bar{t}_R \gamma^\mu t_R, \]  

(17)

where \( T_L \) stands for the left-handed top and bottom quark doublet, \( t_R \) the right-handed top quark. In (17) the chiral color symmetry does exist[20].
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References


[20] In the real world, the chiral color symmetry is explicitly broken, at least “weakly” (note that the QCD coupling constant at the electroweak symmetry breaking scale is small), by QCD gauge interaction. So non-vanishing Goldstone boson mass in (2) will cause the results of the variational approach in (11) less reliable[14]. However the lower limit in (16) is still valid.