Automatically R-Conserving Supersymmetric SO(10) Models and Mixed Light Higgs Doublets

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Abstract

In automatic R-parity conserving supersymmetric (SUSY) SO(10) models, the simplest way to accommodate realistic fermion masses is to demand that the light Higgs doublets are linear combinations of the $\{10\}$ and $\{126\}$ grand unified Higgs representations. We study the realization of this mixed light Higgs property (MLHP) consistent with doublet-triplet splitting in a minimal R-conserving SUSY SO(10) models. We then discuss predictions for neutrino masses and mixings in this model as well its implications for proton decay.
1 Introduction

Supersymmetric (SUSY) SO(10) model has a number of desirable features that make it an ideal candidate to describe physics beyond the standard model in a grand unified framework. They are:

(a) unification of all fermions of each generation into a single spinor representation\[?] restoring quark-lepton symmetry to weak interactions;

(b) a natural implementation of the see-saw mechanism\[?] for understanding small neutrino masses, which in the minimal version of the model are of the right order of magnitude to explain the solar neutrino puzzle via the MSW\[?] mechanism;

(c) a simple mechanism for explaining the origin of matter starting with a zero baryon and lepton asymmetry of the universe for temperature above the grand unification scale\[?].

In this paper, we wish to discuss a subclass of SUSY SO(10) models which have another highly desirable feature: automatic R-parity conservation which leads to natural conservation of baryon and lepton number symmetries prior to symmetry breaking. As is well-known, this property is not present in the SUSY standard model nor in the SUSY SU(5) model\[?], where extra symmetries have to be imposed by hand to ensure B and L conservation. On the other hand, in the SO(10) model if all Higgs representations are chosen to have congruence number zero (such as 45, 54, 210, etc.) and two (such as 10, 120, 126, 126, etc.), the R-parity symmetry is automatic. Two possible minimal models with this property are given below:

Model A: The Higgs particles belong to representations \{210\}, \{126\} + \{126\} and \{10\}\[?]. The role of \{210\} is to break SO(10) down to SU(2)_L × SU(2)_R × SU(4)_C; that of \{126\} (denoted by \(\Delta\)) is to break SU(2)_R × SU(4)_C down to U(1)_Y × SU(3)_C while at the same time giving heavy Majorana mass to the right-handed
neutrino ($\nu_H$) to implement the see-saw mechanism for neutrino masses; the role of $\{126\}$ (denoted by $\Delta$) is to cancel the $\bar{\Delta}$ contribution to the $D$-term so that supersymmetry is maintained down to the weak scale; the role of $\{10\}$ (denoted by $H$) is of course to break the $SU(2)_L \times U(1)_Y$ to $U(1)_{\text{em}}$ and generate fermion masses. The model is minimal in the sense that omitting any one of these multiplets will leave extra undesirable local symmetries at low energies or break SUSY at GUT scale.

**Model B:** The Higgs particles belong to the multiplets $\{45\} \oplus \{54\}$ (denoted by $A$ and $S$ respectively), and $\{126\} \oplus \{\mathbf{126}\} \oplus \{10\}$. Apart from being more economical compared to model A, this model has another difference from model A. i.e., here SO(10) is broken down to $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ before breaking to $U(1)_Y \times SU(3)_C$.

It is worth pointing out that in general superstring models, the $\{126\} \oplus \{\mathbf{126}\}$ (or higher) SO(10) representations do not emerge below the Planck scale after compactification. Therefore, several SO(10) models discussed in the literature have only $\{45\}$, $\{16_H + 10_H\}$, and $\{10\}$, where $\{45\} \oplus \{16_H\} \oplus \{10_H\}$ are used to break SO(10) down to the standard model. In these models, the vacuum expectation values (VEV’s) of $\{16_H\} \oplus \{10_H\}$ lead to R-parity breaking terms at low energies. For instance, a term of the form $\Psi \Gamma_a \Psi \Gamma_a \bar{\Psi} H / M_{pl}$ will lead to a B-violating term of the form $\langle \tilde{\nu}^c \rangle \left( u^c \bar{d}^c \bar{d}^c + Q L \bar{c} \right) / M_{pl}$, which can lead to catastrophic baryon number violation. One must invoke additional symmetries to prevent the R-non-invariant terms. It may very well be that such symmetries emerge from superstring compactification. But this remains to be demonstrated. Moreover, with the above minimal set, it is impossible to break the SO(10) symmetry down to the standard model without including large Planck scale induced non-renormalizable terms in the superpotential.

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1The existence of the $u^c \bar{d}^c \bar{d}^c$ terms was pointed out to R. N. M. by A. Farragí
Let us now discuss the question of fermion masses in the R-parity conserving SO(10) models. It is well known[?] that if only a complex 10 Higgs representation contributes to fermion masses, then one gets the undesirable mass relations between leptons and quarks e.g. \(m_d/m_s = m_e/m_{\mu}\), which is a factor of 10 off compared to observations. One way to correct it is to have the Higgs doublets contained in the 10 contribute to the fermion masses through their direct dimension 3 couplings. However, if we naively added a separate 10, for this purpose, there would be an extra pair of Higgs doublets at low energies which is unacceptable from the point of view of gauge coupling unification[?]. Moreover, the usual problem of doublet-triplet splitting will also get worse. It was pointed out in Ref.[?] that there is a simple solution to this problem: the same 10, which contributes to the breaking of \(B-L\) symmetry and to the see-saw mechanism, can have its weak doublets acquire an induced VEV without leaving any extra light doublet and without any fine tuning provided there is a coupling between the 10 and the 10 in the superpotential via other Higgs multiplets that only have VEV’s of order of the GUT scale. (Such couplings can arise, for instance, if there is a 210 multiplet in the theory.) The induced VEV then will correct the bad charged fermion mass relations.

This property of inducing VEV for \((2,2,15)\) of \(\Sigma\) can also be stated in another way. When SO(10) breaks down to the standard model at \(M_U\), there are two light doublets (say \(\phi_u\) and \(\phi_d\)). To obtain realistic quark lepton mass relations, they must be a linear combination of the doublets in \(\{10\}\) and \(\{\overline{10}\}\). In the rest of the paper, we will call this “mixed light Higgs property” (MLHP). It is simple to see that models with MLHP do not have the property of Yukawa unification widely discussed in recent literature[?] but strict Yukawa unification is anyway not realistic for the first and second generation. This property, MLHP does impose non-trivial constraints on model building. For instance, maintaining MLHP while implementing the doublet-triplet splitting (DTS) is rather nontrivial in general. In the Dimopoulos-Wilczek
scheme\cite{5b} for DTS, an example was constructed\cite{5b}, where this property emerges consistent with some discrete symmetries. In fact, it was suggested in Ref.\cite{5d} that any new DTS scheme must have this property. In this paper, we will demonstrate a simple SO(10) model which has this good light Higgs property and study the consequences of this general class of SO(10) models with the additional minimality criterion.

This paper has been organized as follows: in Sec. 2, we argue in favour of the necessity of the MLHP in R-conserving SO(10) models; in Sec. 3, we discuss the superpotential of the model B and introduce the doublet-triplet splitting mechanism with MLHP; in Sec. 4, we study the quark and lepton mass matrices and discuss its implications for neutrino masses and mixings; in Sec. 5, we discuss the implications for proton decay; Sec. 6 is devoted to an R-conserving SO(10) model where the Higgsino mediated contributions to proton decay is naturally suppressed while maintaining MLHP; in Sec. 7, we summarize our results and conclude; in an appendix, we discuss the minimum of the Higgs potential consistent with the desired symmetry breaking pattern.

2 Mixed versus Pure Light Higgs

In this section, we explore to what extent, it is an absolute necessity to have mixed light Higgs doublets in an automatically R-parity conserving SO(10) model. \textit{i.e.}, could the light Higgs below GUT scale be purely light Higgs (PLH) arising solely from the complex \{10\}? It is obvious that if non-renormalizable Planck induced terms are not included in the superpotential, then PLH scheme will not work since it will lead to the bad fermion mass relation $m_d/m_s = m_e/m_\mu$ already mentioned. However, once the non-renormalizable Planck induced terms are included, the result is not obvious. To see what happens, let us assume first that the SO(10) is broken
down to the standard model by \{210\} Higgs, via VEV’s for the components (1,1,1) ⊕ (1,1,15) ⊕ (1,3,15). The possible non-renormalizable terms involving the matter superfields are of the form: \(\Psi \Phi H\), \(\Psi \Phi^2 H\), etc. It is then easy to verify that fermion mass matrices will have the general form:

\[
M_u = (h + h' + f + f')\kappa_u, \\
M_d = (h + h' - f - f')\kappa_d, \\
M_l = (h + h' + 3f + 3f')\kappa_d.
\]

(1)

Here, \(f\), \(f'\), and \(h'\) are of order \(\sqrt{8\pi M_U/M_{pl}}\); \(f\) is symmetric coming from an effective \(126\) operator; \(f'\) and \(h'\) are antisymmetric coming from effective \(120\) operators. One then has the relation

\[
M_l = 2r M_u - M_d.
\]

(2)

Taking trace of both sides of this equation, we get \(r \simeq m_e/m_t\) or zero. The right hand side of Eq.2 is completely determined by the known values of the quark masses and CKM angles. These values have to be extrapolated to the GUT scale in order to test the sumrule in eq.2. The details of this extrapolation procedure is described in Sec.4 and is applied to the model B. Here we use those extrapolated values of the parameters to study the validity of Eq.2. We find that generically, the electron mass comes out a factor of five or so too large compared to the observed value with no free parameter left to adjust. Therefore, we feel that it is highly unlikely that a pure light Higgs possibility would be realistic.

Similar arguments apply if the GUT symmetry is broken by a combination of \{45\} ⊕ \{54\}. Thus, we feel that the mixed light Higgs doublet property provides a better chance to get a realistic fermion spectrum in a minimal R-conserving SO(10) model. In the next section, we give an example of an explicit model where MLHP is realized and proceed to discuss its implications in subsequent sections.
3 Mixed Light Higgs in Model B

In this section, we present a simple minimal SO(10) model, where the light Higgs doublets have the desirable property (MLHP) described in the introduction. As already mentioned, the key step is to have a term in the superpotential that couples the 10 Higgs with the 126 Higgs(s) via Higgs fields that have VEV’s of order of the GUT scale. If we do not allow for Planck scale induced non-renormalizable terms, then we need a 210 Higgs to achieve this goal; but as shown in[?], achieving this together with doublet-triplet splitting is not very simple, although we did manage to construct an example which is technically natural. In this paper, we will take the point of view that one should include non-renormalizable Planck scale induced terms. In the first example that we give in this section, we will keep only the lowest order Planck induced terms. In a subsequent section, we present a slight generalization of this model where undesirable higher order terms are forbidden by a symmetry. As we will see this allows for the construction of a rather simple model with MLHP.

As usual, we assign the fermions to the 16-dimensional spinor representation of SO(10). We denote them by $\Psi_a$ (where $a = 1, 2, 3$ stands for generations) and we use the following minimal set of Higgs bosons needed for complete symmetry breaking:

(i) $\{45\} \oplus \{3\}$ (denoted by $A$ and $S$ respectively) to break SO(10) down to the left-right symmetric group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$;

(ii) $\{126\} \oplus \{1\}$ to break the $SU(2)_L \times U(1)_{B-L}$ symmetry down to $U(1)_Y$ while keeping supersymmetry in fact down to the $M_W$ scale;

(iii) A single $\{10\}$ (denoted by $H$) to break the $SU(2)_L \times U(1)_Y$ down to $U(1)_{e.m.}$.

The superpotential of the model is chosen to consist of the following parts:

$$W = W_f + W_s + W_p,$$

(3)
where

\[ W_f = h_{ab} \Psi_a \Psi_b H + f_{ab} \Psi_a \Psi_b \Delta, \]

\[ W_s = (\mu_H + \lambda S) HH + \mu_s S^2 + \lambda_s S^3 + \mu_A A^2 + \mu_\Delta \Delta \bar{\Delta} \]
\[ + \lambda_\Delta \Delta A \bar{\Delta} + \lambda_S (S \Delta \Delta + S \bar{\Delta}\Delta), \]

\[ W_p = \frac{\sqrt{8\pi} \lambda_p}{M_{pl}} A^2 H. \]

As noted in Ref. [?], if we show that the light doublets in the model are linear combinations of the doublets in \{10\} and \{\overline{126}\} multiplets, then \( W_f \) can accommodate a realistic charged fermion spectrum for all generations. This is, of course, intimately connected with the question of the doublet-triplet splitting.

It is easy to see (see Appendix), using the above superpotential, that the vanishing of F-terms at the scale \( M_U \) and \( v_R \) (which are equal to fit low energy LEP data[?]), is guaranteed by the following choice of VEV’s for the Higgs fields \( S \), \( A \), \( \Delta \), and \( \bar{\Delta} \):

\[
\begin{align*}
<S> &= \text{diag}(1,1,1,1,1,-\frac{3}{2},-\frac{3}{2},-\frac{3}{2},-\frac{3}{2}) M_U, \\
<A> &= i\tau_2 \otimes \text{diag}(b,b,b,c,c), \\
<\Delta>_{(1,3,10)} &= <\bar{\Delta}>_{(1,3,10)} = v_R.
\end{align*}
\]

Using this, we find that the mass matrix for the fermionic doublets in \( H, \Delta, \) and \( \bar{\Delta} \) can be written as:

\[
\begin{pmatrix}
\bar{\Delta}_u & \Delta_u & H_u \\
0 & \mu_\Delta + \lambda_\Delta c & 0 \\
\mu_\Delta - \lambda_\Delta c & 0 & \mu_1 \\
0 & \mu_2 & 0
\end{pmatrix},
\]

where we have fine tuned \( \mu_H - (3/2)\lambda_H M_U \approx M_W \) (assumed to be zero in writing Eq. (8)). In Eq. (8), the rows and columns denote the down- and up-type doublets.
contained in $H$, $\Delta$, and $\overline{\Delta}$ respectively. The $\Delta A^2 H$ entries are induced by the Planck scale corrections to $W_p$ leading to $\mu_i \approx \lambda_p \sqrt{8\pi M_\ell^2 / M_p}$ ($i = 1, 2$) which is of order $10^{-1} M_U$ to $10^{-2.5} M_U$. Note that the $(1,3,1)$ VEV in $< A >$ makes the entry $\Delta_u H_d$ different from $\Delta_d H_u$ (i.e., $\mu_1 \neq \mu_2$).

It is easy to see from Eq. (8), that the light Higgs doublets (denoted by $\phi_u$ and $\phi_d$) are given by

$$\phi_u = \cos \alpha H_u + \sin \alpha \overline{\Delta}_u,$$
$$\phi_d = \cos \gamma H_d + \sin \gamma \overline{\Delta}_d,$$

where $\tan \alpha = \mu_1 / (\mu_\Delta - \lambda_\Delta c)$ and $\tan \gamma = \mu_2 / (\mu_\Delta + \lambda_\Delta M_2)$. It is important to note that in general $\alpha \neq \gamma$. We find this to be a rather simple way to get the light doublets with the correct group theoretical property at low energies, for fermion masses. Furthermore, we expect $\alpha$ and $\gamma$ to be much smaller than one so that the departure from strict Yukawa unification [?] is small.

The triplet mass matrix is a four-by-four matrix as follows:

$$
\begin{pmatrix}
H & \Delta & \overline{\Delta} & \overline{\Delta}_R \\
\Delta & 2\mu_H + \frac{2}{\sqrt{3}} \lambda M_U & q_1 (b^2 + \alpha' c^2) & 0 \\
\overline{\Delta} & 0 & \mu_\Delta - \lambda_\Delta \beta' b & 0 \\
\Delta & q_1 (-b^2 + \alpha' c^2) & 0 & \mu_\Delta + \lambda_\Delta \beta' b \\
\overline{\Delta}_R & q_2 \beta c & 0 & \mu_\Delta + \lambda_\Delta \gamma' \beta
\end{pmatrix}
$$

In Eq. (10), the first three rows denote the anti-quark-type Higgsinos contained in $(1,1,6)$ of $H$, $\overline{\Delta}$, $\Delta$, and the last row denotes the same in the $(1,3,10)$ of $\Delta$ respectively; similarly, the first three columns denote the quark-type Higgsinos contained in $(1,1,6)$ of $H$, $\Delta$, $\overline{\Delta}$, and the last column the same in $(1,3,10)$ of $\overline{\Delta}$. And the primed symbols represent non-zero group theoretical factors and $q_i$ are proportional to $\mu_i$. It is certain that all eigenvalues are of order $M_U$. This solves the doublet-triplet splitting problem.
We want to point out that the specific form of the superpotential \( W_S + W_p \) can be derived by requiring invariance under a \( Z_2 \) symmetry, under which \( S, H, \) and \( \Delta \) are even and \( A, \overline{\Delta} \) are odd. This symmetry for instance forbids the \( \overline{\Delta} A^2 H \) term. Coming to the matter part of the superpotential, \( W_f \), the \( \Psi \Psi \Delta \) term is forbidden but there is an allowed Planck induced term \( f' \Psi \Psi A \Delta \) which essentially plays the same role as the second term in \( W_f \). The effective coupling in the mass matrix is then given by \( f' v_R \sqrt{8 \pi} / M_{pl} \).

Before turning to a discussion of the fermion masses in the model, we wish to emphasize two points: first, in discussing the fermion masses in any R-conserving SO(10) model, one must first ensure that the light Higgs doublets have the correct MLHP property consistent with the doublet-triplet splitting. The absence of color singlet Higgs doublets in 45 representation makes our mechanism a good starting point for model building; secondly, this property of the mixed light Higgs doublets is not trivial to ensure while keeping all color triplet Higgsinos heavy. For instance, if instead of 45, we used a 210 to break SO(10), the doublet mass matrix becomes a four-by-four matrix since it includes the (2,2,10) submultiplet of 210 and several elements in this matrix must be engineered to zero value to attain the MLHP goal[?]. Of course, one could double the number of 126 \( \oplus \) \( \overline{126} \) multiplets such that one set contributed to the light Higgs doublets while the other breaks B-L symmetry and the two remain totally separate. However, in this case, one must worry about the possibility of unwanted pseudo-Goldstone supermultiplets which spoil gauge unification.
4 The fermion sector and predictions for the neutrinos:

Let us now turn to the fermion mass matrices in this model. It was shown that in Ref. [?], the fermion mass matrices in this model are characterized by 12 parameters, which can all evaluated given the six quark masses, three charged lepton masses and three CKM angles. The neutrino masses and mixings are then completely predicted. We repeat this discussion with two differences from Ref. [?]. First, we take the effect of the superpartners on the running of the gauge and Yukawa couplings. Second, we take the effect of the top quark Yukawa couplings on the running of the masses[?]. We will follow Naculich[?] below. The low energy superpotential for the model is given by:

\[ W_0 = h_u Q \phi_u u^c + h_d Q \phi_d d^c + h_e L \phi_e e^c + \mu \phi_u \phi_d, \]  

(11)

where \( h_u, h_d, \) and \( h_e \) are three-by-three matrices expressible in terms of the SO(10) coupling matrices \( h \) and \( f \) in Eq. (4) as follows:

\[ h_u = h \cos \alpha + f \sin \alpha, \]  

(12)

\[ h_d = h \cos \gamma + f \sin \gamma, \]  

(13)

\[ h_e = h \cos \gamma - 3 f \sin \gamma. \]  

(14)

As emphasized earlier \( \alpha \neq \gamma \) is required. Otherwise all CKM angles vanish. Moreover, in a strict supergravity framework, where supersymmetry breaking is implemented in the hidden sector, at \( \mu = M_{pl} \), we have \( m_{\phi_u}^2 = m_{\phi_d}^2 = m_\phi^2 \). Their extrapolation down to the electroweak scale is governed predominantly[?] by \( h_u,33 \) and \( h_d,33 \). Correct symmetry breaking pattern (i.e., \( \tan \beta > 1 \)) also requires that \( \alpha \) and \( \gamma \) be different from each other. The electroweak symmetry is then broken
radiatively so that,

\[ < \phi_u^0 > = v \sin \beta, \]

\[ < \phi_d^0 > = v \cos \beta. \quad (15) \]

The mass matrices at GUT scale can then be written as: \( \mu = M_U \)

\[ M_u = (\overline{h} + \overline{f})v, \]

\[ M_d = (\overline{h} r_1 + \overline{f} r_2)v, \]

\[ M_l = (\overline{h} r_1 - 3 \overline{f} r_2)v, \quad (16) \]

where

\[ \overline{h} = h \cos \beta; \overline{f} = f \sin \alpha \sin \beta; r_1 = \frac{\cos \gamma}{\cos \alpha} \cot \beta; r_2 = \frac{\sin \gamma}{\sin \alpha} \cot \beta. \quad (17) \]

This is now in the same notation as in Ref. \[ ? \]. In order to evaluate the matrices \( \overline{h} \) and \( \overline{f} \), \( r_1 \) and \( r_2 \), we use the following sum rule derived in Ref. \[ ? \], \( i.e., \)

\[ M_l = \frac{4 r_2 r_1}{r_2 - r_1} M_u - \frac{3 r_2 + r_1}{r_2 - r_1} M_d. \quad (18) \]

We then take \( \text{Tr} M_l \), \( \text{Tr} M^2_l \), and \( \text{Tr} M^3_l \) to obtain \( r_2 \) and \( r_1 \[ ? \]. The light neutrino mass matrix is given by the see-saw formula[?] to be[?]

\[ M_\nu = - M_{\nu D} M^{-1}_{\nu M} M^T_{\nu D}, \quad (19) \]

where

\[ M_{\nu D} = \frac{3 r_1 + r_2}{r_2 - r_1} M_u - \frac{4}{r_2 - r_1} M_d. \quad (20) \]

\[ M_{\nu M} = - \frac{1}{R} \left[ \frac{r_1}{r_2 - r_1} M_u - \frac{1}{r_2 - r_1} M_d \right], \quad (21) \]

with

\[ R = \frac{v \sin \alpha \sin \beta}{v_R}. \]
Note that this light neutrino mass matrix defined at $\nu_R$ needs to be extrapolated to the weak scale; but since $\nu_R$ decouples below $\nu_R$, there are only some over all anomalous dimensions of the effective light-Majorana neutrino mass. This effect is small and we will ignore it.

In order to carry out the numerical analysis, we choose the following values for the running masses:

\[
\begin{align*}
  m_u &= 5.1\text{MeV}; & m_c &= 1.27\text{GeV}; & m_t(m_t) &= 166\text{GeV}; \\
  m_d &= (8.9 + ds)\text{MeV}; & m_s &= (0.175 + ss)\text{GeV}; & m_b &= 4.25\text{GeV}; \\
  m_{\tau} &= .51\text{MeV}; & m_\mu &= 105.6\text{MeV}; & m_\tau &= 1.784\text{GeV}. \quad (22)
\end{align*}
\]

The $m_t(m_t)$ is obtained by taking the CDF mean value of 174GeV for the $m_t$ pole. The symbols $ds$ and $ss$ are left free to be fixed by the sum rule in Eq. (18) along with $r_1$ and $r_2$. The CKM angles are parameterized in terms of $s_{12}$, $s_{23}$, and $s_{13}$, with $s_{12} = -0.221$, $s_{23} = 0.043$, and $s_{13} = 0.0045$ as our choice corresponding to the mean values from experiments.

We then extrapolate all masses to the SUSY breaking scale. The extrapolation factors are defined as $\eta_i = m_i/m_i(\mu_{\text{SUSY}})$ or $1/\text{GeV}/m_i(\mu_{\text{SUSY}})$. They are

\[
\begin{align*}
  \eta_u &= 2.17; & \eta_c &= 1.89; & \eta_t &= 1; & \eta_d &= 2.16; \\
  \eta_s &= 2.16; & \eta_b &= 1.47; & \eta_{e,\mu,\tau} &= 1.02.
\end{align*}
\]

In order to extrapolate from $\mu_{\text{SUSY}}$ to $M_U$, we need to know $\tan\beta$. We follow Naculich and assume $\tan\beta < 40$ so that effects of all Yukawa couplings except that of the top quark can be ignored. In this limit, the top Yukawa coupling effect is accounted for by the factor $B_t = 0.88647$. The GUT scale values of the various masses (denoted with bars) are given by

\[
\begin{align*}
  m_u &= \overline{m}_u \eta_u A_u B_t^3; & m_c &= \overline{m}_c \eta_c A_u B_t^3; & m_t &= \overline{m}_t \eta_t A_u B_t^6; \\
  m_d &= \overline{m}_d \eta_d A_d B_t^0; & m_s &= \overline{m}_s \eta_s A_d B_t^0; & m_b &= \overline{m}_b \eta_b A_d B_t^1.
\end{align*}
\]
\[ m_e = \overline{m}_e \eta_e A_e B_1^0; \quad m_\mu = \overline{m}_\mu \eta_\mu A_\mu B_1^0; \quad m_\tau = \overline{m}_\tau \eta_\tau A_\tau B_1^0, \]  
\[ \tag{23} \]

where \( A \)-factors are the contributions of the gauge groups to the extrapolation and are numerically given by (choosing \( \mu_{SUSY} = 170 \) GeV)

\[ A_u = 3.21; \quad A_d = 3.13; \quad A_\tau = 1.48. \]  
\[ \tag{24} \]

Some of the mixing angles are also extrapolated and one has (for \( ij = ub, cb, td, and ts \))

\[ s_{ij} = \zeta_{ij} B_1^{-1}. \]  
\[ \tag{25} \]

In order to predict neutrino masses, we need the value of the parameter \( R = \frac{v \sin \alpha \sin \beta}{v_R} \). The value of \( \sin \alpha \) and \( \sin \beta \) are arbitrary, whereas \( v = 246 \) GeV and \( v_R \) is fixed by unification of the gauge couplings. In the absence of the heavy particle threshold corrections, one has \( v_R \approx M_U \approx 2 \times 10^{16} \) GeV\(^{-1}\); but as has been noted for the case of non-SUSY SO(10) models\(^{[?]\)}, the threshold corrections can easily introduce uncertainty of a factor of \( 10^{\pm 1} \) in \( M_U \) and \( v_R \). We will therefore assume that \( v_R \approx 10^{15} \) to \( 10^{16} \) GeV in what follows.

In our input, the signs of the fermion masses can be arbitrary. There are many possibilities. Below we give the results for the choice of signs for masses that satisfy all the constraints of the model:

**Case I:** All masses chosen positive. In this case, we find \( ds = -1.581, \) \( ss = -0.03533, \) \( r_1 = 0.00952, \) and \( r_2 = 0.00479. \) The Eq. (18) for this choice of \( \{r_1, r_2\} \) leads to the values of \( \{\overline{m}_e, \overline{m}_\mu, \overline{m}_\tau\} = \{0.0003377, 0.0695543, 1.18177\} \) to be compared with extrapolated values: \( \{0.0003378, 0.0695548, 0.1.18177\} \). The predictions for neutrino masses and mixing for this case are given by

\[ M_\nu = R\{-0.679348, 43.2421, 704.332\}, \]
\[ V_l = \begin{pmatrix}
0.999238 & -0.0381721 & 0.00819604 \\
0.0379246 & 0.998875 & 0.0284804 \\
-0.00927398 & -0.0281479 & 0.999561
\end{pmatrix}. \] (26)

A natural value for \( R \approx 10^{-14} \) to \( 10^{-13} \) (since \( v_s \sin \beta \approx 10^2 \text{GeV} \)) depending on whether \( v_R \) is \( M_U \) or \( M_U/10 \), we get \( m_{\nu_e} \approx 7 \times 10^{-3} \text{eV} \) to \( 7 \times 10^{-2} \), and \( m_{\nu\mu} \approx 4 \times 10^{-4} \text{eV} \) to \( 4 \times 10^{-3} \text{eV} \). The \( \theta_{\nu e} \) mixing angle in this case is rather small; but \( \sin^2 \theta_{\nu e} \approx 5 \times 10^{-3} \), which is of right order of magnitude to resolve the solar neutrino puzzle via the MSW mechanism[?].

**Case II:** For the choice of all signs for masses to be negative, we get a consistent fit to all charged fermion masses for \( r_1 = 0.00637635 \), and \( r_2 = 0.0187798 \). The predictions for the neutrino sector in this case are

\[
M_\nu = R\{-0.910023, -21.4474, -547.607\},
\]

\[
V_l = \begin{pmatrix}
0.995493 & 0.0921015 & -0.0225942 \\
-0.0939442 & 0.990303 & -0.102345 \\
0.012949 & 0.104006 & 0.994492
\end{pmatrix}. \] (27)

In this case, both \( \theta_{\nu e} \) and \( \theta_{\nu\mu} \) mixing angles are outside the MSW two neutrino solution given by Hata and Langacker[?]. Therefore, if the solar neutrino deficit situation continues to remain as it is now, this solution will be ruled out.

**Case III:** We have found another fit to the sum rule in Eq. (18), for the choice of masses, where \( m_e, m_2 \), and \( m_1 < 0 \) whereas all the remaining masses are chosen positive. The values of \( r_1 \) and \( r_2 \) are: \( r_1 = 0.0100082 \) and \( r_2 = 0.072028 \). The predictions for neutrino masses and lepton mixing for this case are given by

\[
M_\nu = R\{0.0662987, 4.60097, -2041.97\},
\]
Again, here all \((mass \text{ difference})^2\) are outside the range of the small angle as well as the large angle MSW solutions to the solar neutrino puzzle. Again, this solution can be tested by the solar neutrino data.

**Case IV:** This case is obtained by changing the signs of masses in case III. This corresponds to the choices \(r_1 = 0.0175744\) and \(r_2 = 0.00699614\). The predictions for neutrino masses and lepton mixing in this case are

\[
M_\nu = R\{-0.460917, -30.2621, -663.422\},
\]

\[
V_l = \begin{pmatrix}
0.99973 & 0.0226327 & 0.00523126 \\
-0.0227722 & 0.999339 & 0.0283497 \\
-0.00458617 & -0.0284611 & 0.999584
\end{pmatrix}.
\]

(29)

Here, again the mixing angles are outside the range required by the MSW analysis of the present solar neutrino data.

**Case V:** In this case, all masses are chosen positive except the electron mass and we find \(ds = -1.581, ss = -0.03833, r_1 = 0.00955,\) and \(r_2 = 0.00500\). The predictions for neutrino masses and mixing are given by

\[
M_\nu = R\{-0.945754, 42.8837, 714.726\},
\]

\[
V_l = \begin{pmatrix}
0.999542 & -0.029543 & 0.00655202 \\
0.0294036 & 0.999359 & 0.0204401 \\
-0.00715168 & -0.020238 & 0.99977
\end{pmatrix}.
\]

(30)
A natural value for $R \approx 10^{-14}$ to $10^{-13}$ (since $v \sin \alpha \sin \beta \approx 10^5$ GeV) depending on whether $v_R$ is $M_U$ or $M_U/10$, we get $m_{\nu_e} \approx 7 \times 10^{-3}$ eV to $7 \times 10^{-2}$, and $m_{\nu_\mu} \approx 4 \times 10^{-4}$ eV to $4 \times 10^{-3}$ eV. The $\theta_{e\mu}$ and $\theta_{e\tau}$ mixing angles in this case are rather small for the MSW mechanism to work.

**Case VI:** If we choose all masses to be negative except the electron mass, we get a fit consistent with all charged fermion masses for $r_1 = 0.00656483$, and $r_2 = 0.0187798$. The predictions for the neutrino sector in this case are

$$M_\nu = R\{-1.02584, -23.7002, -616.621\},$$

$$V_\nu = \begin{pmatrix}
0.993278 & 0.112199 & -0.028463 \\
-0.114706 & 0.98706 & -0.112052 \\
0.015506 & 0.114562 & 0.993295
\end{pmatrix}. \quad (31)$$

In this case, both $\theta_{e\tau}$ and $\theta_{e\mu}$ mixing angles are outside the MSW two neutrino solution [?]. Therefore, if the solar neutrino deficit situation continues to remain as it is now, this solution will also be ruled out.

If we set aside prejudices towards mixing angles coming from solar neutrinos, then our $\nu_\mu$-$\nu_\tau$ mixing angles are in the interesting ranges to be testable in the next generation of proposed acceleration $\nu_\mu$-$\nu_\tau$ oscillation experiments. In Fig. 1, we compare our predictions with the domains of $\Delta m^2$ and $\sin^22\theta_{\mu\mu}$ angles to be explored in the proposed CERN and Fermilab experiments.

## 5 Proton Decay

One of the key predictions of grand unified theories is the life-time of the proton and its decay mode. In non-SUSY GUT models, the dominant decay of the proton arises from the exchange of superheavy gauge bosons and the operators responsible
for this have dimension six. The primary decay mode is $p \to e^+\pi^0$. On the other hand, in SUSY GUT models, in addition to the above dim.-six operators, there also exist dim.-five operators, and in simple SUSY SU(5) or SUSY SO(10) models, the latter graphs dominate. The resulting dominant decay mode is $p \to \pi^+K^+$, which can be used to distinguish between the SUSY GUT theories from non-SUSY GUT ones.

Proton decay in SUSY SU(5) model has been extensively studied [8, 9] and it has been established that in this case, one requires the superheavy color-triplet Higgsino ($\tilde{H}_3$) mass $M_{\tilde{H}_3} \gg M_V$ in order to be consistent with the existing lower bounds on the $\tau_p-\tau_e K^+$[3]. We will show below that in the SUSY SO(10) model dim.-five proton decay operators receive contributions from two diagrams: one involving $\{\text{10}\}$ Higgs and the other involving $\{\text{126}\}$ Higgs and these graphs could interfere destructively, thereby reducing the effective $p$-decay amplitude. This in turn can relax the constraints on color-triplet Higgsino masses.

The color-triplet Higgsinos that mediate proton decay are part of a four-by-four matrix, given in Eq. (10). One can always find two four-by-four unitary matrices $V$ and $U$, for the triplet mass matrix Eq. (10), such that

$$(V\dagger)_{ik}(M_T)_{kl}U_{lj} = M_{ij}\delta_{ij}.$$ 

Then, the dim.-five operators at the GUT scale, which are to be turned into baryon-number violating four-fermion interactions by gaugino- or Higgsino-dressing at the electroweak scale, have the following common factor:

$$C_{abcd} = C_0 \sum_{i=1}^{4} \frac{(h_{ab}U_{1i} + f_{ab}f_0U_{2i})(h_{cd}V_{1i} + f_{cd}f_0V_{2i})}{M_i},$$

(32)

where the early Latin indices are family ones and run over 1,2,3; $C_0$ and $f_0$ represent some over all factors (Clebsch-Gordan coefficients).

From the definition Eq. (16), we obtain

$$\mathcal{F} = \frac{1}{v} \frac{r_2 M_{\nu} - M_d}{r_1 + r_2},$$

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