The decoherence effect due to emission of gravitons is examined. It shows the same qualitative features of the QED effect which has already been investigated, it is obviously much weaker, wholly universal and shows a stronger energy dependence. The result can be extended to photons, they also may undergo decoherence due to graviton emission. For this limited aim the incomplete status of the quantum gravity, in comparison with QED, is not source of severe difficulties because all the effects are attributed to the infrared sector of the dynamics.
I. Introduction

The loss of quantum coherence due to bremsstrahlung process has been studied in detail [1,2] for the electromagnetic case. The electromagnetic radiation is certainly present whenever charged particles undergo scattering processes, so, without assuming that the radiation is the unique process giving rise to the loss of quantum coherence, we may be sure that electrodynamical effects are there. It is also evident that this kind of decoherence mechanism makes a sharp distinction between charged and uncharged particles, it becomes therefore natural to look for other kinds of radiation that may implement the same effect in a universal way. The obvious candidate is the gravitational radiation since the gravity is the most universal form of interaction known at present, so in this note the effects of gravitational bremsstrahlung are examined. Some possible source of difficulties, appears immediately: when we investigate the effects of photon emission we have at our disposal QED which probably the best settled (and the oldest) branch of quantum field theory, the quantum theory of gravitation still waits a final shape. The small sector of the general quantum mechanics of gravitation that concerns the infrared radiation can be treated in close analogy with QED as it can be seen from the general treatment of the infrared radiations given by Weinberg [3]. This treatment will be systematically followed in this note, some points will be recalled or re-elaborated when necessary. The difference that cannot be neglected is the fact that the emission of photon is due to the electric charge which is a Lorentz scalar and that it is quite possible to consider situations where every interacting particle keeps its charge, the emission of gravitons is due to the energy-momentum four vector and that this quantity is certainly not kept by the single interacting particle although it is conserved in the overall process. This fact is conceptually obvious but gives rise to some technical difficulties: for this reason in this paper the perturbative treatment is presented formerly, in sect. II because there the effects of the overall conservation are better shown, after an extension of the Bloch-Nordsieck model in proposed in sect.III where one sees that the effect of decoherence can be studied following each particle individually. The fact that also photons may undergo decoherence due to gravitational bremsstrahlung is noted.

II. Infrared gravitons

A. Infrared compensation for standard states

Some results given in ref.[3] will be explicitly needed, so they are now recalled and adapted to the particular problem under investigation. Since the source of the gravitons is the energy-momentum tensor one must consider the whole scattering process, where this source is conserved, it is not enough to follow a particular line as it is allowed in QED [5]. To avoid unessential complications the scattering process is an elastic two-body collision, the masses are taken to be equal.

The units $c = \hbar = 1$ and the coupling $\kappa = \sqrt{8\pi G}$ are used ($G$ is the Newtonian...
constant), the Minkowski metric is denoted by $\eta_{\mu\nu}$, the Mandelstam invariant $s, t, u$ are also employed.

If one calls $M_o$ the amplitude for the scattering without further radiation

Fig.1

then one obtains the amplitude for the scattering with the radiation of a soft graviton [3]:

Fig.2

$$M_r = M_o \cdot i\kappa \left[ \frac{q^\mu q'^\nu}{q \cdot k} + \frac{q'^\mu q^\nu}{q' \cdot k} - \frac{p^\mu p'^\nu}{p \cdot k} - \frac{p'^\mu p^\nu}{p' \cdot k} \right] (2\pi)^{-3/2} f_{\mu\nu}$$  \hspace{1cm} (1)

One can get this expression working out e.g. the soft limit of the emission by a spinorial particle, using the vertex[6]*:

$$\frac{i}{2} i\kappa \left\{ \frac{1}{2} [\gamma^\mu (p_1 + p_2)^\nu + \gamma'^\nu (p_1 + p_2)^\mu] + \eta^{\mu\nu} [\hat{p}_1 + \hat{p}_2 - 2m] \right\}$$

but it is quite general, it does not depend on the spin of the emitting particle. The polarization vector is $f_{\mu\nu}$ and since the source is conserved the sum over polarization is simply given by:

$$\sum_i f^i_{\mu\nu} f^i_{\rho\sigma} = \Pi_{\mu\nu\rho\sigma} = \frac{1}{2} (\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\nu\rho}\eta_{\mu\sigma} - \eta_{\mu\nu}\eta_{\rho\sigma})$$  \hspace{1cm} (2)

So the transition probability, with emission of an infrared graviton is given by

$$X = \frac{1}{(2\pi)^3 |M_o|^2} \int \frac{d^3k}{2\omega} \kappa^2 \left[ \frac{1}{2} m^4 \left( \frac{1}{(q \cdot k)^2} + \frac{1}{(q' \cdot k)^2} + \frac{1}{(p \cdot k)^2} + \frac{1}{(p' \cdot k)^2} \right) + \frac{2(q \cdot q')^2 - m^4}{q \cdot kq' \cdot k} + \frac{2(p \cdot p')^2 - m^4}{p \cdot kp' \cdot k} - \frac{2(q \cdot k)^2 - m^4}{q \cdot kp \cdot k} + \frac{2(q' \cdot q')^2 - m^4}{p' \cdot kq' \cdot k} - \frac{2(p' \cdot p')^2 - m^4}{p' \cdot kp' \cdot k} \right]$$  \hspace{1cm} (3)

The notation $X_1$ will be used to indicate the sum of the first four terms, while $X_2$ will be used to indicate the sum of the last six terms. The integration over the energy of the radiated graviton goes from a minimum $\lambda$, and a maximum $\Lambda$ that has no role in the discussion, but for allowing the low-energy approximation. Performing the integration the following result is obtained:

$$\frac{1}{2\pi^2} |M_o|^2 \kappa^2 m^2 [1 + D(p \cdot p'/m^2) - D(p \cdot q/m^2) - D(p' \cdot q/m^2)] \ln(\Lambda/\lambda)$$  \hspace{1cm} (4)

where

$$D(x) = \frac{2x^2 - 1}{\sqrt{x^2 - 1}} \arccosh x \hspace{1cm} D(1) = 1 \hspace{1cm} D(x) \to 2x \ln 2x \text{ for } x \to \infty.$$

* In that paper the $\kappa$ is twice the $\kappa$ used here.
Now one must look at the virtual corrections: there are six terms corresponding to the correction in the channels $s, t, u$ respectively, working always in the i.r. limit the corresponding amplitude may be expressed as:

$$\mathcal{M}_v = 2[\mathcal{M}_s + \mathcal{M}_t + \mathcal{M}_u]$$  \hfill (6)

and the correction, at the order $\kappa^2$ is given by $2\Re(\mathcal{M}_o \mathcal{M}_v^*)$

$$\mathcal{M}_s = \frac{-i}{(2\pi)^4} \kappa^2 \mathcal{M}_o \int \frac{d^4l}{l^2 + i\epsilon} \frac{(p \cdot p')^2 - \frac{1}{2}m^4}{(p \cdot l - i\epsilon)(p' \cdot l + i\epsilon)}$$  \hfill (7a)

$$\mathcal{M}_t = \frac{i}{(2\pi)^4} \kappa^2 \mathcal{M}_o \int \frac{d^4l}{l^2 + i\epsilon} \frac{(p \cdot q)^2 - \frac{1}{2}m^4}{(p \cdot l - i\epsilon)(q \cdot l - i\epsilon)}$$  \hfill (7b)

$$\mathcal{M}_u = \frac{i}{(2\pi)^4} \kappa^2 \mathcal{M}_o \int \frac{d^4l}{l^2 + i\epsilon} \frac{(p \cdot q')^2 - \frac{1}{2}m^4}{(p \cdot l - i\epsilon)(q' \cdot l - i\epsilon)}$$  \hfill (7c)

The integration over the virtual momenta which takes into account the contribution of the photon pole gives the result needed to cancel one part of the real i.r. divergence [4,5], the term $\mathcal{X}_2$. The contribution of the second pole of the s-channel term is easily calculated in the frame $\vec{p}' = 0$, where it corresponds to $l_o = -i\epsilon$ with the result

$$\mathcal{M}_s^{(2)} = \frac{i}{4\pi} \kappa^2 \mathcal{M}_o [(p \cdot p')^2 - \frac{1}{2}m^4] \frac{1}{m|\vec{p}|} \int \frac{d\ell}{\ell} \quad \ell = |\vec{l}|$$  \hfill (7d)

This term is imaginary, so it does not contribute to the correction to order $\kappa^2$, it corresponds (in QED) to the perturbative expansion of the Coulomb phase[4,7], here it correspond to the gravitational elastic rescattering of the incoming or outgoing particles.

There are still some other contributions, that are needed to cancel the divergent terms in $\mathcal{X}_1$. Here a difference with respect to QED is found, in fact in QED either one chooses the Yennie gauge [8], so that the virtual corrections till now considered will completely cancel the i.r. divergent part coming from real radiation, or, when using the Lorentz gauge, one look at the i.r. divergent part of the renormalization constant $Z_2$ for the wave function of the charged particle. Here there is no simple analogous of the Yennie gauge and a general renormalization procedure is not available. However when a self energy correction is inserted with the procedure:

$$\frac{1}{\hat{p} - m} \to \frac{1}{\hat{p} - m} \Sigma(p) \frac{1}{\hat{p} - m}$$

one must keep in the limit $p \to \hat{p} \quad \hat{p}^2 = m^2$, i.e. when the insertion is performed on one external leg, the correct position of the mass pole and the correct residuum at the pole,
this amount to two subtractions that, in the i.r. regime are well defined; in particular the second subtraction term contains the factor $\frac{\partial \Sigma}{\partial p}\bigg|_{p \rightarrow \bar{p}}$, when the derivative acts on the numerator of $\Sigma$ it does not yield i.r. divergences, because $\Sigma$ itself is not i.r. divergent, on the denominator the derivative acts in the same way as in QED, and yields therefore the divergent terms needed to cancel the term $X_1$. The important, and unfortunate, difference is that now these and similar subtractions does not avoid the ultraviolet divergences, but for our limited aim only the infrared sector matters.

In simple formulation the result is that the i.r. radiation has a matrix element

$$\mathcal{M}_r(k) = \mathcal{M}_o \cdot \kappa \beta(k)$$  \hspace{1cm} (8)

The virtual correction has an i.r. contribution

$$\mathcal{M}_v = \mathcal{M}_o \cdot \kappa^2 \rho_v$$  \hspace{1cm} (9)

and that

$$\int |\beta(k)|^2 \frac{d^3k}{2\omega} + 2\Re \rho_v = C$$  \hspace{1cm} (10)

The term $C$ is finite, in the true i.r. limit is zero.

B. Superposition of two states of motion

If the two-particle final state is the superposition of two states of motion, like $| \mathcal{F} >= [|q_1, q'_1 > + |q_2, q'_2 >]/\sqrt{2}$, then the transition probability without any soft particle correction is:

$$\frac{1}{2} |\mathcal{M}_o^{(1)} + \mathcal{M}_o^{(2)}|^2$$  \hspace{1cm} (11)

The transition probability with emission of one soft particle of momentum $k$ is:

$$\frac{1}{2} |\mathcal{M}_o^{(1)} \cdot \kappa \beta^{(1)}(k) + \mathcal{M}_o^{(2)} \cdot \kappa \beta^{(2)}(k)|^2$$  \hspace{1cm} (12a)

The transition probability including the correction for one virtual soft particle is:

$$\frac{1}{2} |\mathcal{M}_o^{(1)} \cdot (1 + \kappa^2 \rho_v^{(1)}) + \mathcal{M}_o^{(2)} \cdot (1 + \kappa^2 \rho_v^{(2)})|^2$$  \hspace{1cm} (13a)

so the virtual correction to the order $\kappa^2$ may be written as:

$$C_v = \kappa^2 \left[ |\mathcal{M}_o^{(1)}|^2 \rho_v^{(1)} + |\mathcal{M}_o^{(2)}|^2 \rho_v^{(2)} \right] + 2\Re \left( \mathcal{M}_o^{(1)} \mathcal{M}_o^{(2)*} \right) (\rho_v^{(1)} + \rho_v^{(2)})$$  \hspace{1cm} (13b)

while the total infrared real correction is:

$$C_r = \frac{1}{2} \int \left[ |\mathcal{M}_o^{(1)} \beta^{(1)}(k)|^2 + |\mathcal{M}_o^{(2)} \beta^{(2)}(k)|^2 + 2\Re \left( \mathcal{M}_o^{(1)} \beta^{(1)}(k) \mathcal{M}_o^{(2)*} \beta^{(2)}(k) \right) \right] \frac{d^3k}{2\omega}$$  \hspace{1cm} (12b)
By using the compensation discussed previously between real and virtual correction we find that the only a surviving addendum contains the interference term.

\[ C = C_r + C_v = -\kappa^2 \Re(\mathcal{M}_o^{(1)} \mathcal{M}_o^{*(2)}) \int [\beta^{(1)}(k) - \beta^{(2)}(k)]^2 \frac{d^3k}{2\omega} \]  

(14)

This expression can be recast in a form similar to eq.(4)

\[ C = -2\pi\kappa^2 m^2 \Re(\mathcal{M}_o^{(1)} \mathcal{M}_o^{*(2)}) \left[ 1 + D(q_1 \cdot q'_1/m^2) - D(q_1 \cdot q_2/m^2) - D(q_1 \cdot q'_2/m^2) \right] \ln(\Lambda/\lambda). \]  

(15)

If the two final states \(|q_1, q'_1\rangle\) and \(|q_2, q'_2\rangle\) which build up the superposition are not very different, it is convenient to use the expansion:

\[ q_2 = q_1 + \delta q \quad q'_2 = q'_1 - \delta q \]

In the centre-of-mass frame the common three momentum is \(Q^2 = \frac{1}{2}s - m^2\) and \(\delta q \cdot q_1 = -\delta q \cdot q'_1 = 2Q^2 \sin^2 \phi/2\), being \(\phi\) the angle, that is now assumed small, between the two final directions \(q_2\) and \(q_1\)

\[ C = -4\pi\kappa^2 \Re(\mathcal{M}_o^{(1)} \mathcal{M}_o^{*(2)}) Q^2 \sin^2 \phi/2 \left[ \hat{D}(p \cdot p'/m^2) - \hat{D}(1) \right] \ln(\Lambda/\lambda). \]  

(16a)

From previous expressions

\[ \hat{D}(1) = \frac{3}{2} \quad \hat{D}(x) \to 2(1 + \ln 2x) \quad \text{for} \quad x \to \infty \]  

(16b)

When the radiating particle is massless some minor modification are required: if \(m \to 0\) then \(x \to \infty\) in eq. (5) so the asymptotic form of \(D(x)\) is certainly correct

\[ C = -2\pi\kappa^2 \Re(\mathcal{M}_o^{(1)} \mathcal{M}_o^{*(2)}) \left[ m^2 + 2q_1 \cdot q'_1 \ln(2q_1 \cdot q'_1/m^2) - 2q_1 \cdot q_2 \ln(2q_1 \cdot q_2/m^2) - 2q_1 \cdot q'_2 \ln(2q_1 \cdot q'_2/m^2) \right] \ln(\Lambda/\lambda) \]

or also

\[ C = -4\pi\kappa^2 \Re(\mathcal{M}_o^{(1)} \mathcal{M}_o^{*(2)}) \left( [q_1 \cdot q'_1 \ln(2q_1 \cdot q'_1/s) - q_1 \cdot q_2 \ln(2q_1 \cdot q_2/s) - q_1 \cdot q'_2 \ln(2q_1 \cdot q'_2/s)] + \left[ (q_1 \cdot q'_1 - q_1 \cdot q_2 - q_1 \cdot q'_2) \ln(s/m^2) \right] \right) \ln(\Lambda/\lambda) \]  

(17)

Energy-momentum conservation gives \(q_1 + q'_1 = q_2 - q'_2 = 0\) so that in the limit \(q_1^2 = m^2 \to 0\) the second parenthesis goes to zero.

In analogy with eq. (16) it is possible to give the expression for small \(\phi\)

\[ C = 4\pi\kappa^2 \Re(\mathcal{M}_o^{(1)} \mathcal{M}_o^{*(2)}) 2Q^2 \sin^2 \phi/2 [2 \ln \sin \phi/2 - 1] \]  

(18)

Comparing this expression with eq.(16a) it appears that the dependence on the presence of a logarithmic term is the only remnant of the massless condition, at least
for small angles. The fact the it is possible to deal with the i.r. divergence coming from massless particles emitted by a massless source in a simple way is, as shown in [3], a peculiarity of the graviton emission, with photon emission the situation would be much worse; experimentally we know that there are massless particle having gravitational interactions, the photons, but there are not massless charged particles and if we look for charged massless particles in quantum chromodynamics we find indeed a very complicated infrared behaviour.

III. Explicit description of the time evolution

A. The Bloch and Nordsieck model reviewed

The usual tools of scattering theory with emission of soft massless bosons going back to Bloch and Nordsieck model will be used[4], in that form which has been reviewed in the previous treatment of the QED effects[2], so only the points that show significant differences will be presented. The main difference is that, as discussed in section II, it is not obvious that it is enough to follow only one particle in its fly, if we look to the final state, one should take into account both outcoming particles and together with their radiation and see how the compensation between real and virtual corrections is realized in this case. The following analysis shows, however, that the compensation happens in transparent way so that the consideration of only one particle, together with its radiation is, at the end, justified. The scattering process (for particles of equal masses) is described by the Hamiltonian [3]:

\[
H = H_o + V = H_o + \sum_{J=1}^{2} V_J + \Delta \tag{19a}
\]

\[
H_o = -i\vec{v}_1 \cdot \vec{\partial}_1 + \frac{m}{\gamma_1} - i\vec{v}_2 \cdot \vec{\partial}_2 + \frac{m}{\gamma_2} + \sum_{\nu} \int d^3k \omega a^{\dagger}_{k,\nu} a_{k,\nu} \tag{19b}
\]

\[
V_J = -\frac{\kappa m}{(2\pi)^{3/2}} \sum_{\nu} \int \frac{d^3k}{\sqrt{2\omega}} \gamma_J \left[ a_{k,\nu} \vec{v}_J \cdot f_{k,\nu} \cdot \vec{v}_J e^{i\vec{k} \cdot \vec{r}_J} + a^{\dagger}_{k,\nu} \vec{v}_J \cdot f_{k,\nu} \cdot \vec{v}_J e^{-i\vec{k} \cdot \vec{r}_J} \right]. \tag{19c}
\]

The polarization tensors for the gravitons are expressed here in non-covariant Coulomb gauge and satisfy the well-known relations

\[
\sum_{\nu} f^{cd}_{k,\nu} f_{k,\nu} = \frac{1}{2} \left[ u_{ac} u_{bd} + u_{ad} u_{bc} - u_{ab} u_{cd} \right] \quad u_{ij} = \delta_{ij} - \hat{k}_i \hat{k}_j.
\]

The Latin indices are purely spatial and \( \hat{k} \) is the unit vector \( \vec{k}/\omega \). The constant \( \Delta \) is an energy renormalization term, having the role that in a covariant treatment is played by mass counter-term.

It has been already shown[2] that the evolution operator takes the form

\[
\mathcal{U}(t) = \exp \left[ -\frac{1}{2} \int_0^t \int_0^{t'} D(\tau, \tau') d\tau, d\tau' \right] \mathcal{N} \exp \int_0^t [-i\tilde{V}(\tau) d\tau], \tag{20}
\]
with $\tilde{V}(t) = e^{iH_0 t} V e^{-iH_0 t}$.

Due to the actual form of $\tilde{V}(t)$, in the present case we have

$$D(\tau, \tau') = \sum_{I,J} D_{I,J}(\tau, \tau') \quad (21a)$$

and

$$D_{I,J}(\tau, \tau') = < |\mathcal{P}(\tilde{V}_I(\tau)\tilde{V}_J(\tau'))| > = \frac{\kappa^2 m^2}{(2\pi)^3} \sum_\nu \int \frac{d^3k}{2\omega} \gamma_{I,J} \bar{v}_I \cdot \bar{f}_{k,\nu} \cdot \bar{v}_J \cdot \bar{f}_{k,\nu} \cdot \bar{v}_J$$

$$[e^{i(\bar{k}(\bar{r}_I - \bar{r}_J)} e^{i(\omega - \bar{k} \cdot \bar{v}_J)\tau'} e^{-i(\omega - \bar{k} \cdot \bar{v}_J)\tau} \bar{v}(\tau - \tau') + (\tau, I) \leftrightarrow (\tau', J)] \quad (21b)$$

From the form of $\mathcal{U}$ one sees that the radiation of one soft particle involves the absolute square of the matrix element $< |\tilde{V}_I(\tau)| \bar{k}, \nu >$: which is made up by addenda of the form:

$$A_{I,J}(\tau, \tau') = < |\tilde{V}_I(\tau)| \bar{k}, \nu < | \tilde{V}_J(\tau')| >$$

$$= \frac{\kappa^2 m^2}{(2\pi)^3} \sum_\nu \int \frac{d^3k}{2\omega} \gamma_{I,J} \bar{v}_I \cdot \bar{f}_{k,\nu} \cdot \bar{v}_J \cdot \bar{f}_{k,\nu} \cdot \bar{v}_J [e^{i(\bar{k}(\bar{r}_I - \bar{r}_J)} e^{i(\omega - \bar{k} \cdot \bar{v}_J)\tau'} e^{-i(\omega - \bar{k} \cdot \bar{v}_J)\tau}$$

It is only a matter of calculation to verify that $\Re \int_o^t \int_o^t d\tau d\tau' D = \int_o^t \int_o^t d\tau d\tau' A$.

For the terms where $I = J$ this is precisely the result already used in [2], the exponential of $-\Re \int_o^t \int_o^t d\tau d\tau' D$ coming from eq. (20) is precisely compensated by the exponentiation of $\int_o^t \int_o^t d\tau d\tau' A$, which comes from the sum over the radiated bosons, when they build up a coherent state, the only difference comes from the polarization factors. The case $A_{1,2} + A_{2,1}$ involves explicitly the difference $\bar{r}_2 - \bar{r}_1$ through the term $R = \bar{k} \cdot (\bar{r}_2 - \bar{r}_1) = \omega(r_2 z_2 - r_1 z_1)$, having introduced the cosines, $z_J = \cos \theta_J$, of the angles between $\bar{k}$ and $\bar{v}_J$.

The actual expression we get for $A = \int_o^t \int_o^t d\tau d\tau' [A_{1,2} + A_{2,1}]$ is:

$$A = \frac{\kappa^2 m^2}{(2\pi)^3} \int d\Omega \frac{d\omega}{2} \frac{v_1 v_2^2 \gamma_1 \gamma_2 (1 - z_1^2)(1 - z_2^2)}{(1 - v_1 z_1)(1 - v_2 z_2)} \times$$

$$\left[ \cos \omega R - \cos \omega[R - (1 - v_1 z_1)\tau] - \cos \omega[R + (1 - v_2 z_2)\tau] + \cos \omega[R + v_1 z_1 \tau - v_2 z_2 \tau] \right] \quad (23)$$

and coincides with $\Re \int_o^t \int_o^t d\tau d\tau'[D_{1,2} + D_{2,1}]$. So here also there is a compensation between virtual and real contributions.

The term coming from $3D$, which remains there and cannot be eliminated by some renormalization procedure just because it depends on $\bar{r}_2 - \bar{r}_1$, is a part of the rescattering phase: the phase that in QED, we would call Coulomb phase *.

* It is not, however the complete rescattering phase: since the treatment is not covariant there is a contribution coming from the static gravitational interaction; in the covariant perturbative treatment, shown in sect. II, these two terms are never separated.
We are in any case interested in the behaviour for large $t$, so there is a question on how we deal with $R$. The simpler attitude, followed in [2], was to think of $\vec{r}_J$ as independent of $t$, plane wave for the outgoing particles, if however we think, more realistically, that we have wave packets, although very broad, we would take, say, $\vec{r}_J = \vec{s}_J + \vec{v}_J t$, with fixed $\vec{s}_J$ in the wave functions of the scattered particles, and so $R = S + (v_2 z_2 - v_1 z_1) t$, we see, however, that this substitution would amount in the previous expression eq.(23) to a substitution of $\vec{v}_J$ with $-\vec{v}_J$ and of $R$ with $S$, this will not affect the $t \to \infty$ limit.

These calculations show what really one expected i.e. that the Bloch-Nordsieck cancellation works also for two interacting outgoing particles; more precisely, that the diagonal terms compensate among themselves and the same do the cross terms. So now the study of the evolution of the superposition of two states of motion can be done, as in QED, looking only at one outgoing particle.

### B. Superposition of two states of motion

There are two differences with the QED: one is that instead of $e$ we find $\kappa m \gamma$, the second lies in the polarization term, but once we look at the evolution of one final particle with its radiation all the qualitative features remain the same as in QED, so only the initial definitions and the differences with respect to the previous case will be discussed. If the state after the scattering is a superposition of two states characterized by velocities $v_+$ and $v_-$ there are two transition amplitudes generated by the evolution operator $U(t)$, say $T_+$ and $T_-$ and then an interference term is produced in the transition probability at finite time:

$$\mathcal{I}(t) = \Re [T_+^* T_-]$$

Each of the two factors carries its virtual correction term as discussed before and in [2], but the real emission of soft bosons gives, for each boson which is emitted, a factor: *:

$$q(\omega_R) = \frac{(\kappa m \gamma)^2}{(2\pi)^3} \int d\Omega \sum_{pol} \frac{(\vec{v}_+ \cdot \vec{f} \cdot \vec{v}_+)(\vec{v}_- \cdot \vec{f} \cdot \vec{v}_-)}{(k \cdot \vec{v}_+ - \omega)(k \cdot \vec{v}_- - \omega)} \omega d\omega$$

$$[1 - \exp[-i(k \cdot \vec{v}_+ - \omega)t] - \exp[i(k \cdot \vec{v}_- - \omega)t] + \exp[i(k \cdot \vec{v}_- - k \cdot \vec{v}_+ t)]].$$

Summing over all the emitted bosons this term is exponentiated, whereas the virtual correction terms are noting but the negative exponentials of $\frac{1}{2} g$, once with all the velocities equal to $\vec{v}_+$ and once with all the velocities equal to $\vec{v}_-$.

The long time evolution produces also here a dominant term in $\ln(\omega_R t)$ whose coefficient $X = \int d\Omega \xi$ can be calculated starting from eq.(24); assuming here also

* the real part of eq.(24) has the same form as the integrand of eq. (23), the quantities entering in them have however different meaning: here $v_+$ and $v_-$ refer to two different states of motion of the same particle, in eq.(23) $v_1$ and $v_2$ refer to the two final particles.
that the two speeds are equal and calling $\delta$ the angle between the two directions it results:

$$\xi = \frac{G m^2 \gamma^2}{\pi^2} v^4 \frac{(\cos \delta - \cos \theta_+ \cos \theta_-)^2 - \frac{1}{4} (\sin \theta_+ \sin \theta_-)^2}{(1 - v \cos \theta_+)(1 - v \cos \theta_-)}$$  \hspace{1cm} (25)

This expression could have been directly obtained from the corresponding one for the QED case with the appropriate substitutions of the coupling constant and of the spin projector, all the previous part of Sect. III may be interpreted as a justification of eq.(25).

At this point we must extract some simpler information out of the general form.

In the limit $\delta \to 0$ one gets

$$\xi_0 = \frac{G m^2 \gamma^2}{2 \pi^2} v^4 \frac{(\sin \theta)^4}{(1 - v \cos \theta)^2}$$

so that

$$X_0 = \frac{G m^2 \gamma^2}{\pi^2} v^2 \left[ 2v - \frac{4}{3} v^3 - (1 - v^2) \ln \frac{1 + v}{1 - v} \right].$$  \hspace{1cm} (26)

This expression has, in turn, the two limits

$$v \to 1 \quad X_0 \to \frac{8G m^2 \gamma^2}{3 \pi}$$

$$v \to 0 \quad X_0 \to \frac{G m^2}{\pi} v^4 \frac{16}{15} \left[ 1 + \frac{v^2}{7} \right].$$

For the real case $\delta \neq 0$ we consider in any case the configurations at small $\delta$, but with the condition $\delta \gamma$ not small, as in [2]. After some lengthy but straightforward calculations we obtain

$$v \to 1 \quad X_\delta \to \frac{G m^2 \gamma^2}{\pi} \left[ \frac{8}{3} - \delta^2 \left( \frac{7}{3} - \ln \frac{1}{\frac{4}{3} (\delta^2 + 1/\gamma^2)} \right) \right].$$

The difference between $X$ and $X_0$ expresses, as discussed in [2], the lack of compensation between the real corrections and the virtual corrections and gives the rate of decay of the interference term $I$ with time *:

$$\frac{I(t_1)}{I(t_2)} = \left[ \frac{t_1}{t_2} \right]^{-\nu} \quad \text{with} \quad \nu = \frac{G m^2 \gamma^2 \delta^2}{\pi} \left( \frac{7}{3} - \ln \frac{1}{\frac{4}{3} (\delta^2 + 1/\gamma^2)} \right).$$  \hspace{1cm} (27)

Since we are dealing with gravitational effects we investigate the possible effect for large (macroscopic) bodies, in this case the speeds are certainly small, we keep simply the term proportional to $v^4$. The resulting expression is:

$$v \to 0 \quad X_\delta \to \frac{G m^2}{\pi} v^4 \frac{2}{15} (8 - 7 \sin^2 \delta)$$

* The conditions that have been assumed for $\delta$ and $\gamma$ make the logarithm negative and so $\nu > 0$ holds.
And the comparison with the same expression at $\delta = 0$ gives for the suppression of the interference term the behaviour:

$$\frac{\mathcal{I}(t_1)}{\mathcal{I}(t_2)} = \left[\frac{t_1}{t_2}\right]^{-\nu} \quad \text{with} \quad \nu = \frac{Gm^2}{\pi} \frac{14}{15} \sin^2 \delta. \tag{28}$$

**IV. Some conclusions**

The results presented here are intended to be an extension of the analogous investigation concerning the photon emission [2]. The low-energy gravitational effects are quite negligible * for elementary particles, anyhow the explicit computation shows a time decay of the interference term which is exponential in $t$, the factor that multiplies the time is different in the two cases: its energy dependence is much stronger, the angular dependence shows some minor differences, if fact both in gravity and in QED the angular dependence is logarithmic for small angles. It is however quite plausible that the same mechanism is at work for macroscopic bodies where it could become comparable and ever larger than the electromagnetic effects. The actual expressions intended to refer to macroscopic bodies, eq.(28) may be questioned on the basis that the starting point was a Bloch-Nordsieck model designed for high energy collision, looking in detail to the procedure that has been used one sees that the essential point of the approximation was to neglect the change of velocity of the emitting body in the process of radiation, now it is clear that for a long wave-length emission having a massive body is enough to make the variation of the velocity negligible also for small velocities. In practice interactions with the rest of the world will be, for a macroscopic body, certainly more relevant than gravitational radiation, but this kind of radiative process last should be quite universal and for this fact it has some interest of principle. Another point to be stressed is the possibility that this kind of dynamics acts also on photons.

As it has discussed in [2] this particular decoherence effect is evidently due to the presence of massless particles, and so to the possibility of producing in the collision process of an indefinite number of them, so every exclusive channel gets, at the end, probability zero; in an S-matrix treatment the sum over different final states is not simply a practical procedure, but also a necessity in principle. As the radiated particles become softer and softer the time needed to radiate them becomes longer and longer, for every finite time no true i.r. divergence is present [9], the zero value for of the probability for every exclusive channel in anyhow attained with continuity in time.

There is a more complicated effect of decoherence, which affects in case of superposition of two (or more) states of motion the rescattering phase, see eq.(7d), but the possibility of observing these effects appear even more remote and also less relevant in principle, so it has not been computed.

It can also be noticed that the result does not comes purely from kinematics, the particular form of the coupling is relevant, if e.g. one would consider a chiral

* needless to say: the high-energy gravitational effects are unknown
The insertion of a massless pseudoscalar particle on an external line would give
\[
\frac{1}{\hat{p} - \hat{k} - m} \gamma_5 u(p) = \frac{\hat{k}}{2p \cdot \hat{k} - k^2} \gamma_5 u(p)
\]
and there would be no divergence in the limit \( k \to 0 \).

Tentatively one could argue that the emission both of photons and of gravitons arises from a dynamics giving a privileged role to the momentum variables and therefore the persistence of states which are superposition of two or more sharply different momenta is not tolerated. We know also that this kind of emission is related to the presence of long-range forces [3], it is not obvious whether this has to do with the decoherence process, but it is certainly the reason why both the electromagnetic and the gravitational interaction sum up starting from microscopic size to macroscopic size and thus making a qualitative continuity in the dynamics of small and large objects.

The effects of the infrared radiations are clearly due to a dynamics which is quite standard; this can be said beyond any doubt for the electromagnetic infrared emission, which is well confirmed experimentally, but also for the gravitational radiation, for which a direct experimental verification is not available, the common lore is to take for granted its existence. So these effects should be there even if the quantum mechanics should undergo some important modifications which, in very different forms have been postulated also in the aim of introducing a decoherence while keeping the relevant features of quantum physics. This is not the place to discuss the very large literature on this field, as an example one of the most carefully elaborated proposal is mentioned [10]. There a modification in the in the evolution of every quantum system is postulated, by introducing a non-Hamiltonian term, while the Hamiltonian part is not modified; in the present treatment the evolution remains Hamiltonian, but in the limit \( t \to +\infty \) the evolution operator is no longer unitary, while remaining isometric.

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References

5. J.M. Jauch and F. Röhrlich - The theory of photons and electrons, Ch.16 Springer Verlag 1980

Figure captions

1.) The basic $2 \rightarrow 2$ scattering graph, for particles $p, p'$ going into particles $q, q'$.
2.) The four graphs for $2 \rightarrow 2$ scattering with emission of a soft particle with very low four-momentum $k$.
3.) Three of the six graphs for $2 \rightarrow 2$ scattering with virtual correction in $s, t, u$ channels, the virtual particle bears the four-momentum $l$.
4.) One of the four $2 \rightarrow 2$ scattering graph with self-energy correction.