1 Introduction

The present design of the beam screen of the LHC calls for a stainless steel pipe with a thin copper layer (about 50 μm thick) coated on its inner surface. The purpose of the copper coating is to reduce the surface resistance, thus suppressing the possible beam instability caused by the resistive wall and reducing the beam-induced wall heating. The resistive wall instability is a relatively low frequency phenomenon, ranging from a few kHz to a few MHz in the LHC. The surface resistance in this frequency range has been carefully studied both at room temperature and at liquid helium temperature, with and without a strong magnetic field [1, 2]. The estimate of the surface resistance for the beam heating, however, is a different issue. This is due to the so-called anomalous skin effect, which plays an important role at high frequencies (a fraction of GHz and above). Because of the short bunch length in the LHC (7.5 cm, r.m.s., at top energy), the bunch spectrum will cover a wide high frequency region (637 MHz, r.m.s.).

This situation is further complicated by the fact that the anomalous skin effect is accompanied by a strong magnetic field and by phenomena associated with the surface roughness. To the best of our knowledge, there is no valid theory for estimating the surface resistance in this complex environment. There were several early efforts of measurements [3, 4]. Due to the importance of this problem, the SSC Laboratory established a measurement program with the help of the Los Alamos National Laboratory. After the demise of the SSC, this program was stopped. Partial results are presented in this note that may be useful to the heating analysis for the LHC.

One thing missing in this analysis is the change of the anomalous surface resistance in the presence of a strong magnetic field. The measurement at the SSC

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was not completed. It is recommended to start a surface resistance measurement program at CERN for the LHC beam screen.

2 SSC measurements vs theoretical predictions

The measurements used the SSC beam tubes, which are copper-coated stainless steel pipes similar to the LHC beam screen. Table 1 lists the measured surface resistance at several frequencies. These data are compared to those calculated using the normal skin effect theory as well as the anomalous skin effect theory.

Table 1: Surface resistance of copper coated stainless steel pipe with measured dc† resistivity $\rho = 1.7 \times 10^{-8}$ $\Omega$m at room temperature and $\rho = 2.8 \times 10^{-10}$ $\Omega$m at liquid helium temperature (RRR = 61).

<table>
<thead>
<tr>
<th>Frequency $\omega/2\pi$ [GHz]</th>
<th>Room Temperature</th>
<th>Liquid Helium Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal</td>
<td>Measured</td>
</tr>
<tr>
<td>0.96</td>
<td>8</td>
<td>n/a††</td>
</tr>
<tr>
<td>1.87</td>
<td>11</td>
<td>n/a††</td>
</tr>
<tr>
<td>7</td>
<td>22</td>
<td>24</td>
</tr>
</tbody>
</table>

†† The actual measurement was done at 43 Hz. See the Appendix.

In the case of the normal skin effect, the skin depth is expressed by

$$\delta = \sqrt{\frac{2\rho}{\omega\mu_0}},$$

(1)

where $\mu_0$ denotes the vacuum permeability and the resistivity $\rho$ of the material is frequency independent. The surface resistance is

$$R_s = \frac{\rho}{\delta} = \sqrt{\frac{\omega\mu_0\rho}{2}},$$

(2)

and increases as the square root of the (angular) frequency $\omega$.

The anomalous skin effect theory, which was initially developed by London[5], Pippard[6], Reuter and Sondheimer[7], Chambers[8] and others, attributes the anomalous increase of the surface resistance of metals at high frequencies and low temperatures to the long mean free path $\lambda$ of the electrons. When the skin
depth $\delta$ becomes much smaller than $\lambda$, only a fraction of the conduction electrons moving almost parallel to the metal surface is effective in carrying current and the classical theory breaks down.

The anomalous skin effect theory predicts the following surface resistance (based on an interpolation formula for the so-called diffusion model [6, 8]):

$$R_s = R_\infty \left(1 + 1.157 \alpha^{-0.276}\right), \quad \text{for } \alpha \geq 3,$$

(3)

where the dimensionless parameter $\alpha$ is given by

$$\alpha = \frac{3}{2} \left(\frac{\lambda}{\delta}\right)^2 = \frac{3}{4} \omega \mu_0 (\rho \lambda)^2 \rho^{-3},$$

(4)

and $R_\infty$ is a quantity independent of temperature and impurity, having a $\omega^{2/3}$ dependence on frequency:

$$R_\infty = \left(\frac{\sqrt{3}}{16\pi} \rho \lambda (\omega \mu_0)^2\right)^{\frac{1}{3}}.$$  

(5)

The product $\rho \lambda$ is a characteristic of the metal and for copper one has

$$\rho \lambda = 6.6 \times 10^{-16} \, \Omega \, \text{m}^2,$$

which gives

$$R_\infty = 1.123 \times 10^{-3} \, \Omega \times \left(\frac{\omega}{2\pi \, \text{GHz}}\right)^{\frac{2}{3}}.$$  

Table 2: Skin depth $\delta$ and dimensionless parameter $\alpha$ for copper, assuming $\rho = 1.7 \times 10^{-8} \, \Omega \text{m}$ at room temperature and $\rho = 2.8 \times 10^{-10} \, \Omega \text{m}$ at $T = 4$° K (RRR = 61).

<table>
<thead>
<tr>
<th>Frequency $\omega/2\pi$ [GHz]</th>
<th>Room Temperature $\delta$ [\mu m]</th>
<th>$\alpha$</th>
<th>Liquid Helium Temperature $\delta$ [\mu m]</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.96</td>
<td>2.1</td>
<td>$5.0 \times 10^{-4}$</td>
<td>0.27</td>
<td>110</td>
</tr>
<tr>
<td>1.87</td>
<td>1.5</td>
<td>$9.8 \times 10^{-4}$</td>
<td>0.19</td>
<td>220</td>
</tr>
<tr>
<td>7</td>
<td>0.8</td>
<td>$3.7 \times 10^{-3}$</td>
<td>0.10</td>
<td>820</td>
</tr>
</tbody>
</table>

Table 2 shows the values of the skin depth $\delta$ and of the parameter $\alpha$, proportional to $(\lambda/\delta)^2$, corresponding to the frequencies of Table 1. The normal skin effect theory is valid at room temperature when $\alpha$ is small, whereas the anomalous skin effect theory applies at liquid helium temperature when $\alpha$ is large.
In Figure 1(a) it is plotted the surface resistance $R_s$ versus frequency, for copper at $4^\circ$ K, predicted by the normal skin effect theory (dashed line), by the anomalous skin effect theory (solid line) and measured at Los Alamos (dots). It is seen that, although the theory-predicted anomalous surface resistance is larger than that of the normal skin effect, it is still an underestimate (by a factor of 40-70%) when compared to the measured data. This discrepancy could be attributed to some factors that are not taken into account by the theory, such as the roughness and strain of the surface. (Note: The theoretical values are in agreement with the measured data using lab-prepared metal samples, which are polished and strain-free. The SSC tubes did not meet these requirements.)

Figure 1(b) shows the inverse of the surface resistance for copper at 1 GHz, versus the square root of the resistivity. The dashed straight line corresponds to the normal skin effect theory, while the solid line gives the anomalous skin effect saturation towards the limit $1/R_{\infty}$.

![Figure 1](image)

Figure 1: (a) Surface resistance in m$\Omega$: measured (dots) anomalous (solid line) and classical (dashed line) for copper at $4^\circ$ K (RRR = 61) versus the frequency $f = \omega/2\pi$ in GHz. (b) Inverse of the surface resistance in m$\Omega$: anomalous (solid line) and classical (dashed line) for copper at 1 GHz versus the inverse of the square root of the resistivity $\rho$ in $\Omega m$.

### 3 Resistive wall heating

The parasitic heating power attributed to the surface resistance can be calculated by:

$$P = \frac{I_{\text{sw}}^2}{M_f} \cdot \frac{e^2}{\pi} \int_0^\infty \tilde{X}^2(\omega) R_{\text{wall}}(\omega) \, d\omega$$  \hspace{2cm} (6)
in which $I_{av}$ is the average beam current, $M$ the number of bunches, $f_0$ the revolution frequency, $c$ the velocity of light, $\tilde{\lambda}(\omega)$ the bunch spectrum, which is

$$\tilde{\lambda} = \frac{1}{c} \exp \left\{ -\frac{\sigma_z^2 \omega^2}{2c^2} \right\} \tag{7}$$

where $\sigma_z$ denotes the r.m.s. bunch length. $R_{\text{wall}}$ is the wall resistance, which is related to the surface resistance by:

$$R_{\text{wall}} = \frac{L}{\ell} R_s \tag{8}$$

in which $L$ is the circumference of the machine and $\ell$ the periphery of the beam screen. By combining these expressions, one gets

$$P = \frac{L}{\pi \ell} \frac{I_{av}^2}{M f_0} \int_0^\infty \exp\left\{ -\frac{(\sigma_z/c)^2 \omega^2}{2} \right\} \cdot R_s(\omega) \, d\omega \tag{9}$$

Therefore, if the frequency dependence of the surface resistance $R_s(\omega)$ is known, the heating power can be calculated by performing the integral in Eq. (9). A useful formula in this integration is [9]:

$$\int_0^\infty \exp\{-\mu x^2\} x^{\nu-1} \, dx = \frac{1}{\nu^{\frac{3}{2}}} \Gamma\left(\frac{\nu}{2}\right) \tag{10}$$

For example, assuming that Eq. (2) could be applied to the whole bunch spectrum (i.e., ignoring the anomalous skin effect), then one obtains the familiar expression:

$$P = \Gamma\left(\frac{3}{4}\right) \frac{L}{2\pi \ell \sqrt{\frac{M f_0}{2}}} \left(\frac{c}{\sigma_z}\right)^{3/2} \frac{I_{av}^2}{M f_0} \tag{11}$$

As discussed above, the frequency dependence of $R_s(\omega)$ is different at low frequencies (the normal skin effect) from that at high frequencies (the anomalous skin effect). The difficulty, however, comes from the fact that the frequency boundary where the transition from the normal to the anomalous occurs is unknown. No such kind of data is available for the SSC tubes. (Note: The theory provides an estimate for this boundary. But it appears to be very low and needs to be checked by measurements.)

Assuming the interpolation formula Eq. (3) applies over the whole bunch spectrum, the relative increase of the heating power according to the theory is

$$\frac{P_{\text{anom}}}{P_{\text{norm}}} = \frac{\int_0^\infty \exp\left\{ -\frac{(\omega \sigma_z/c)^2}{2} \right\} P_s(\omega) \, d\omega}{\int_0^\infty \exp\left\{ -\frac{(\omega \sigma_z/c)^2}{2} \right\} P_s(\omega) \, d\omega} \tag{12}$$

For copper at 4° K, assuming $\text{RRR} = 61$ as in Table 1, and $\sigma_z = 7.5$ cm, this gives an increase of 28%.
In the case of LHC at top energy, owing to magneto-resistance the copper resistivity at 4° K is $\rho = 5.5 \times 10^{-10} \Omega m$, corresponding to an effective $RRR = 30$. The increase estimated by using Eq. (12) is 11%.

If, on the other hand, the measured data of $R_s$ at Los Alamos are used for estimating the heating power, the increase would be about 70%.

Due to this wide range of uncertainty and the importance of reliable data for establishing a realistic heating budget for the cryogenic system, it is recommended to start a surface resistance measurement program on samples of the LHC beam screen, taking all the three extreme conditions into account, i.e., low temperature, high magnetic field and high frequency. The appendix lists the methods that were used at Los Alamos and can be considered for part of the LHC measurement program.

4 Acknowledgements

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References