Three-body spin-orbit forces from chiral two-pion exchange

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Abstract

Using chiral perturbation theory, we calculate the density-dependent spin-orbit coupling generated by the two-pion exchange three-nucleon interaction involving virtual Δ-isobar excitation. From the corresponding three-loop Hartree and Fock diagrams we obtain an isoscalar spin-orbit strength $F_{so}(k_f)$ which amounts at nuclear matter saturation density to about half of the empirical value of $90 \text{MeVfm}^5$. The associated isovector spin-orbit strength $G_{so}(k_f)$ comes out about a factor of 20 smaller. Interestingly, this three-body spin-orbit coupling is not a relativistic effect but independent of the nucleon mass $M$. Furthermore, we calculate the three-body spin-orbit coupling generated by two-pion exchange on the basis of the most general chiral $\pi\pi NN$-contact interaction. We find similar (numerical) results for the isoscalar and isovector spin-orbit strengths $F_{so}(k_f)$ and $G_{so}(k_f)$ with a strong dominance of the p-wave part of the $\pi\pi NN$-contact interaction and the Hartree contribution.


The microscopic understanding the dynamical origin of the strong nuclear spin-orbit force is still one of the key problems in nuclear physics. The analogy with the spin-orbit interaction in atomic physics gave the hint that it could be a relativistic effect. This idea has lead to the construction of the (scalar-vector) mean-field models for nuclear structure calculations [1, 2]. In these models the nucleus is described as a collection of independent Dirac quasi-particles moving in self-consistently generated scalar and vector mean-fields. The footprints of relativity become visible through the large nuclear spin-orbit coupling which emerges in that framework naturally from the interplay of the two strong and counteracting (scalar and vector) mean-fields. The corresponding many-body calculations are usually carried out in the Hartree approximation, ignoring the negative-energy Dirac-sea. The NN-interaction underlying these models is to be considered as an effective one that is tailored to properties of finite nuclei but not constrained (completely) by the observables of free NN-scattering.

On the other hand it has long been known that calculations based on Hamiltonians which contain only realistic two-nucleon potentials (thus fitting accurately all NN-phase shifts and mixing angles below the NN$\pi$-threshold) often cannot predict the observed spin-orbit splittings of nuclear levels. In fact one of the original motivations for the Fujita-Miyazawa three-nucleon potential [3] was just the study of such spin-orbit splittings. In ref.[4] it has then been shown that one out of the Urbana family of three-nucleon forces makes a substantial contribution to the spin-orbit splitting in the nucleus $^{15}$N. Moreover, three-nucleon forces are actually needed in addition to realistic two-nucleon potentials in order to reproduce the correct saturation point of (isospin-symmetric) nuclear matter [5]. The long-range part of the three-nucleon interaction is generated in a natural way by two-pion exchange [6] and it can in fact be predicted by using chiral symmetry [7].

The purpose of this paper is present analytical results for the nuclear spin-orbit coupling generated by the (chiral) two-pion exchange three-nucleon interaction. In order to arrive at such results we will make use of the density-matrix expansion of Negele and Vautherin [8]. This
technique allows one to compute diagrammatically the nuclear energy density functional which includes the wanted spin-orbit coupling term proportional to the density gradient. We will first consider the three-nucleon interaction proposed originally by Fujita and Miyazawa [3] where two pions are exchanged between nucleons while the third nucleon is excited to a (p-wave) Δ-resonance. We are able to evaluate the corresponding three-loop Hartree and Fock diagrams in closed analytical form. Then, we will turn to the two-pion exchange three-nucleon interaction generated by the (most general) chiral \( \pi \pi NN \)-contact vertex proportional to the second-order low-energy constants \( c_j \). The effects from explicit \( \Delta \)-excitation reappear in this description via resonance contributions to the low-energy constants \( c_{3,4} \). In both approaches we will separately discuss the isoscalar and isovector spin-orbit strengths.

Let us begin with writing down the explicit form of the spin-orbit coupling term in the nuclear energy density functional:

\[
E_{so}[\rho_p, \rho_n, \vec{J}_p, \vec{J}_n] = \bar{\nabla} \rho \cdot \vec{J} F_{so}(k_f) + \bar{\nabla} \rho_v \cdot \vec{J}_v G_{so}(k_f),
\]

where the sums \( \rho = \rho_p + \rho_n \), \( \vec{J} = \vec{J}_p + \vec{J}_n \) and differences \( \rho_v = \rho_p - \rho_n \), \( \vec{J}_v = \vec{J}_p - \vec{J}_n \) of proton and neutron quantities have been introduced.

\[
\rho_{p,n}(\vec{r}) = \frac{k_{p,n}^3(\vec{r})}{3\pi^2} = \sum_{\alpha \in \text{occ}} \Psi_{p,n}^{(\alpha)}(\vec{r}) \Psi_{p,n}^{(\alpha)}(\vec{r}),
\]

denote the local proton and neutron densities which we have rewritten in terms of the corresponding (local) proton and neutron Fermi-momenta \( k_{p,n}(\vec{r}) \) and expressed as sums over the occupied single-particle orbitals \( \Psi_{p,n}^{(\alpha)}(\vec{r}) \). The spin-orbit densities of the protons and neutrons are defined similarly:

\[
\vec{J}_{p,n}(\vec{r}) = \sum_{\alpha \in \text{occ}} \Psi_{p,n}^{(\alpha)}(\vec{r}) i \vec{\sigma} \times \bar{\nabla} \Psi_{p,n}^{(\alpha)}(\vec{r}).
\]

Furthermore, \( F_{so}(k_f) \) and \( G_{so}(k_f) \) in eq.(1) denote the density dependent isoscalar and isovector spin-orbit strength functions. In Skyrme parameterizations [9] these are just constants, \( F_{so}(k_f) = 3G_{so}(k_f) = 3W_0/4 \), whereas in our calculation their explicit density dependence originates from the finite range character of the two-pion exchange three-nucleon interaction.

The starting point for the construction of an explicit nuclear energy density functional \( E_{so}[] \) is the bilocal density-matrix as given by a sum over the occupied energy eigenfunctions:

\[
\sum_{\alpha \in \text{occ}} \Psi_{p,n}^{(\alpha)}(\vec{r} - \vec{a}/2) \Psi_{p,n}^{(\alpha)(\dagger)}(\vec{r} + \vec{a}/2).
\]

According to Negele and Vautherin [8] it can be expanded in relative and center-of-mass coordinates, \( \vec{a} \) and \( \vec{r} \), with expansion coefficients determined by purely local quantities (nucleon density, kinetic energy density and spin-orbit density). As outlined in Sec. 2 of ref.[10] the Fourier-transform of the (so expanded) density-matrix defines in momentum-space a medium-insertion \( \Gamma(\vec{p}, \vec{q}) \) for the inhomogeneous many-nucleon system. It is straightforward to generalize this construction to the isospin-asymmetric situation of different proton and neutron local densities \( \rho_{p,n}(\vec{r}) \) and \( \vec{J}_{p,n}(\vec{r}) \). We display here only that part of the medium-insertion \( \Gamma(\vec{p}, \vec{q}) \) which is actually relevant for the diagrammatic calculation of the isoscalar and isovector spin-orbit terms defined in eq.(1):

\[
\Gamma(\vec{p}, \vec{q}) = \int d^3r \, e^{-i\vec{q}\vec{r}} \left\{ \frac{1 + \tau_3}{2} \theta(k_p - |\vec{p}|) + \frac{1 - \tau_3}{2} \theta(k_n - |\vec{p}|) \right\}
\]
\[
+ \frac{\pi^2}{4k_f^4} \left[ \delta(k_f - |\vec{p}|) - k_f \delta'(k_f - |\vec{p}|) \right] (\vec{\sigma} \times \vec{p}) \cdot (\vec{J} + \tau_3 \vec{J}_v) \right\}.
\]

When working to quadratic order in deviations from isospin symmetry (i.e. proton-neutron differences) it is sufficient to use an average Fermi-momentum \( k_f \) in the prefactor of the spin-orbit density \( \vec{J} + \tau_3 \vec{J}_v \). The double-dash in the left picture of Fig.1 symbolizes the medium insertion
Γ(\vec{p}, \vec{q}) together with the assignment of the out- and in-going nucleon momenta \vec{p} \pm \vec{q}/2. The momentum transfer \vec{q} is provided by the Fourier-components of the inhomogeneous (matter) distributions \rho_{p,n}(\vec{r}) and \bar{J}_{p,n}(\vec{r}).

\[ \vec{p} + \vec{q}/2 \]
\[ -\Gamma(\vec{p}, \vec{q}) \]
\[ \vec{p} - \vec{q}/2 \]
\[ \vec{r} + \vec{a}/2 \]

**Fig. 1:** Left: The double-dash symbolizes the medium insertion \( \Gamma(\vec{p}, \vec{q}) \) defined by eq.(4). Next shown are the three-loop two-pion exchange Hartree and Fock diagrams involving one chiral \( \pi\pi NN \)-contact vertex (symbolized by the heavy dot). The combinatoric factors of these diagrams are 1/2 and 1, in the order shown.

\[ \vec{r} - \vec{a}/2 \]

**Fig. 2:** Two-pion exchange Hartree and Fock diagrams with (single) virtual \( \Delta \)-isobar excitation. The solid double-line denotes the \( \Delta \)-isobar and dashed and solid lines represent pions and nucleons, respectively. For isospin-symmetric nuclear matter the isospin factors of these diagrams are 8, 0, and 8. The combinatoric factor is 1 in each case.

Now we turn to the analytical evaluation of the two-pion exchange diagrams with (single) \( \Delta \)-isobar excitations shown in Fig. 2. We give for each diagram only the final result for the spin-orbit strengths \( F_{so}(k_f) \) and \( G_{so}(k_f) \) omitting all technical details related to extensive algebraic manipulations and solving elementary integrals. Putting a medium insertion at each of the three nucleon propagators of the Hartree diagram (left diagram in Fig. 2) we obtain the following contribution to the isoscalar spin-orbit strength:

\[
F_{so}(k_f)^{\Delta-\text{Hart}} = \frac{g_{A}^4 m_{\pi}}{8\pi^2 \Delta f_{\pi}^4} \left\{ \frac{u + 2u^3}{1 + 4u^2} - \frac{1}{4u} \ln(1 + 4u^2) \right\}, 
\]

where \( u = k_f/m_{\pi} \) denotes the ratio of the two small scales \( k_f \) and \( m_{\pi} \). The \( \Delta \)-propagator shows up in this expression merely via the (reciprocal) \( \Delta N \)-mass splitting, \( \Delta = 293 \text{ MeV} \). Additional corrections to the \( \Delta \)-propagator coming from differences of nucleon kinetic energies etc. will make a contribution to the spin-orbit strength \( F_{so}(k_f) \) at least one order higher in the small momentum expansion. In eq.(5) we have inserted the empirically well-satisfied relation \( g_{\pi N\Delta} = 3g_{\pi N}/\sqrt{2} \).
for the $\pi N\Delta$-coupling constant together with the Goldberger-Treiman relation $g_{\pi N} = g_A M/m_\pi$. Let us briefly sketch the main mechanism which generates the strength function $F_{so}(k_f)$. The exchanged pion-pair in the Hartree diagram transfers a momentum $q$ between the left and the right nucleon ring and this momentum $q$ enters also the pseudovector $\pi NN$-interaction vertices. The spin-orbit strength $F_{so}(k_f)$ arises from the spin-trace $\text{tr} [\vec{\sigma} \cdot (\vec{Q} + q/2) \vec{\sigma} \cdot (\vec{Q} - q/2) \vec{\sigma} \cdot (\vec{p} \times \vec{J})] = 2i(\vec{q} \times \vec{Q}) \cdot (\vec{p} \times \vec{J})$ where $i\vec{q}$ gets converted to $\vec{Q}k_f = (\pi^2/2k_f^2)\nabla \rho$ by Fourier transformation. The rest is a solvable integral over the product of three Fermi spheres. The second Fock diagram in Fig. 2 (with parallel pion lines) does also generate contributions from Fock-type diagrams (having two-body spin-orbit couplings. The latter are however genuine relativistic effects proportional to $(nucleon mass)$, $m$, $\pi^2/2k_f^2$. It is highly remarkable that the pertinent nine-dimensional integral over the product of two (different) pion-propagators and other momentum dependent factors can solved in terms of (a square of) elementary functions without the occurrence of any dilogarithm. The specific isospin structures of the $\pi NN$- and $\pi N\Delta$-vertices determine uniquely the ratio of isovector to isoscalar spin-orbit strength of each of the three diagrams. We find that the (left) Hartree diagram in Fig. 2 does not contribute to the isovector spin-orbit strength $G_{so}(k_f)$ while the combined result of both Fock diagrams in Fig. 2 reads:

$$F_{so}(k_f)^{\Delta-\text{Fock}} = \frac{g_A^4 M \pi^2}{\Delta (16 f_{\pi})^4} \left[ 8 u^2 - 12 + (3 u^2 + 4) \ln (1 + 4 u^2) \right]^2,$$

(6)

It is highly remarkable that the pertinent nine-dimensional integral over the product of two (different) pion-propagators and other momentum dependent factors can solved in terms of (a square of) elementary functions without the occurrence of any dilogarithm. The specific isospin structures of the $\pi NN$- and $\pi N\Delta$-vertices determine uniquely the ratio of isovector to isoscalar spin-orbit strength of each of the three diagrams. We find that the (left) Hartree diagram in Fig. 2 does not contribute to the isovector spin-orbit strength $G_{so}(k_f)$ while the combined result of both Fock diagrams in Fig. 2 reads:

$$G_{so}(k_f)^{\Delta-\text{Fock}} = \frac{7}{3} F_{so}(k_f)^{\Delta-\text{Fock}},$$

(7)

with a contribution of the second and third Fock diagram in the ratio $6 : 1$. It is important to note that the expressions in eqs.(5,6) are independent of the nucleon mass $M$ and therefore these $2\pi$-exchange three-body spin-orbit couplings are not relativistic effects. In fact the diagrams in Fig. 2 with two medium insertions (on non-neighboring nucleon propagators) do also generate two-body spin-orbit couplings. The latter are however genuine relativistic effects proportional to $1/M$ and therefore counted as one order higher in the small momentum expansion. Such two-body contributions to the spin-orbit strengths $F_{so}(k_f)$ and $G_{so}(k_f)$ can generally be expressed in terms of the spin-orbit amplitudes entering the T-matrix of elastic NN-scattering:

$$F_{so}(k_f)^{2-\text{body}} = -\frac{1}{6} \left\{ 3V_{SO}(0) + V_{SO}(2k_f) + 3W_{SO}(2k_f) \
+ \int_0^1 dx x \left[ V_{SO}(2xk_f) + 3W_{SO}(2xk_f) \right] \right\},$$

(8)

$$G_{so}(k_f)^{2-\text{body}} = \frac{1}{6} \left\{ W_{SO}(2k_f) - V_{SO}(2k_f) - 3W_{SO}(0) \
+ \int_0^1 dx x \left[ W_{SO}(2xk_f) + V_{SO}(2xk_f) \right] \right\}.$$

(9)

The terms $-V_{SO}(0)/2$ and $-W_{SO}(0)/2$ belong to Hartree-type diagrams (with two closed nucleon lines) while the remaining ones summarize the contributions from Fock-type diagrams (having just one closed nucleon line). Explicit expressions for the isoscalar and isovector spin-orbit NN-amplitudes $V_{SO}(q)$ and $W_{SO}(q)$ as they arise from $2\pi$-exchange with (single and double) $\Delta$-excitation can be found in the appendix of ref.[11] (modulo regularization dependent additive constants).

For the numerical evaluation of eqs.(5,6,7) we use the (physical) parameters: $M = 939$ MeV (nucleon mass), $m_\pi = 135$ MeV (neutral pion mass), $f_\pi = 92.4$ MeV (pion decay constant) and...
A = 1.3 (equivalent to a πNN-coupling constant of \( g_{\pi N} = g_A M/f_\pi = 13.2 \)). The full line in Fig. 3 shows the isoscalar spin-orbit strength \( F_{so}(k_f) \) generated by the two-pion exchange three-nucleon interaction involving virtual Δ-excitation as a function of the nucleon density \( \rho = 2k_f^3/3\pi^2 \). As it is typical for a three-body effect the spin-orbit strength \( F_{so}(k_f) \) starts from the value zero at zero density \( \rho = 0 \). The contribution of the Hartree diagram is by far the dominant one. At nuclear matter saturation density (where \( k_f \approx 2m_\pi \)) one finds for example \( F_{so}(2m_\pi)(\Delta-\text{Hart}) = 48.2 \text{ MeVfm}^5 \) to be compared with a Fock contribution of \( F_{so}(2m_\pi)(\Delta-\text{Fock}) = 1.2 \text{ MeVfm}^5 \). Clearly, this 2π-exchange three-body spin-orbit coupling is sizeable [4]. In the region around saturation density \( \rho_0 \approx 0.17 \text{ fm}^3 \) it amounts to about half of the ”empirical” value \( 3W_0/4 \approx 90 \text{ MeVfm}^5 \) deduced in the Skyrme phenomenology of nuclear structure [9]. The findings of refs.[4, 6] concerning spin-orbit splittings in light nuclei point of course in the same direction. Finally, the dashed line in Fig. 3 shows the isovector spin-orbit strength \( G_{so}(k_f) \) (magnified by a factor 10). In comparison to the isoscalar spin-orbit strength \( F_{so}(k_f) \) it is only a small 5% correction.

\[ \frac{i}{f_\pi^2} \left\{ 2\delta_{ab} \left[ c_3 \vec{q}_a \cdot \vec{q}_b - 2c_1 m_\pi^2 \right] + c_4 \epsilon_{abc} \vec{\tau}_c \cdot (\vec{q}_a \times \vec{q}_b) \right\}. \] (10)

Fig. 3: The spin-orbit strength generated by the two-pion exchange three-nucleon interaction involving virtual Δ-isobar excitation versus the nucleon density \( \rho = 2k_f^3/3\pi^2 \). The full curve shows the isoscalar spin-orbit strength \( F_{so}(k_f) \) and the dashed curve shows the isovector spin-orbit strength \( G_{so}(k_f) \) magnified by a factor 10.

Next, we turn to a more general derivation of the three-body spin-orbit coupling generated by two-pion exchange. Chiral symmetry determines the 2π-exchange three-nucleon interaction uniquely at leading order [7]. It follows from a tree-diagram involving the chiral ππNN-contact vertex proportional to the second-order low-energy constants \( c_j \):

\[ \frac{i}{f_\pi^2} \left\{ 2\delta_{ab} \left[ c_3 \vec{q}_a \cdot \vec{q}_b - 2c_1 m_\pi^2 \right] + c_4 \epsilon_{abc} \vec{\tau}_c \cdot (\vec{q}_a \times \vec{q}_b) \right\}. \] (10)
Here, $q_{a,b}$ denote out-going pion-momenta and we have already dropped the $c_0$-term proportional to the product of two pion-energies. In the present application these (off-shell) pion-energies become equal to differences of nucleon kinetic energies thus producing a higher order relativistic $1/M^2$-correction. A straightforward evaluation of the (three-loop) Hartree diagram in Fig. 1 with three medium insertions gives the following contribution to the isoscalar spin-orbit strength:

$$F_{so}(k_f)(c_j-\text{Hart}) = \frac{g_A^2 m_\pi}{4\pi^2 f_\pi^4} \left\{ \left( c_1 - c_3 \right) \frac{u - 2c_3 u^3}{1 + 4u^2} + \frac{c_3 - c_1}{4u} \ln(1 + 4u^2) \right\}, \quad (11)$$

where $u = k_f/m_\pi$. The contributions of the s-wave part and the p-wave part of the $\pi\pi NN$-contact vertex are distinguished by the two (isoscalar) low-energy constants $c_1$ and $c_3$. The isovectorial and spin-dependent $c_l$-vertex makes a non-vanishing contribution only through the Fock diagram. We find the following total result for the isoscalar spin-orbit strength from the three-loop Fock diagram in Fig. 1:

$$F_{so}(k_f)(c_j-\text{Fock}) = \frac{g_A^2 m_\pi u^3}{4\pi^2 (8f_\pi)^4} \left\{ -32c_1 \left[ 4u - u^{-1} \ln(1 + 4u^2) \right]^2 
- (c_3 + c_1) \left[ 8u^2 - 12 + (3u^{-2} + 4) \ln(1 + 4u^2) \right]^2 \right\}. \quad (12)$$

Its analytical form is remarkably simple. Again there no contribution from the Hartree diagram in Fig. 1 to the isovector spin-orbit strength $G_{so}(k_f)$ while the Fock diagram in Fig. 1 leads to the combined result:

$$G_{so}(k_f)(c_j-\text{Fock}) = \frac{g_A^2 m_\pi u^{-3}}{4\pi^2 (8f_\pi)^4} \left\{ -32c_1 \left[ 4u - u^{-1} \ln(1 + 4u^2) \right]^2 
+ \left( \frac{c_4}{3} - c_3 \right) \left[ 8u^2 - 12 + (3u^{-2} + 4) \ln(1 + 4u^2) \right]^2 \right\}. \quad (13)$$

Here, the isospin structures of the $c_1$-$c_3$-vertex and the $c_4$-vertex shows up through relative factors $1$ and $-1/3$. We note aside that the previously calculated contributions from explicit $\Delta$-excitation (see eqs.(5,6,7)) reappear in the (general) expressions eqs.(11,12,13) through $\Delta$-resonance contributions to the low-energy constants: $-c_3^{(\Delta)} = 2c_4^{(\Delta)} = g_A^2/2\Delta$. This connection allows one also to trace back the origin of the factor $7/3$ in eq.(7). Finally, we consider the (leading-order) Weinberg-Tomozawa $\pi\pi NN$-contact vertex. It generates a three-body spin-orbit coupling first in form of a relativistic $1/M$-correction. Because of the isovector nature of the Weinberg-Tomozawa contact-vertex the Hartree diagram (in Fig. 1) can make a contribution only to the isovector spin-orbit strength:

$$G_{so}(k_f)(\text{WT-Hart}) = \frac{g_A^2 m_\pi}{96\pi^2 M f_\pi^4} \left[ 3 \arctan 2u - 2u - u^{-1} \ln(1 + 4u^2) \right]. \quad (14)$$

The Fock diagram (in Fig. 1) on the other hand generates isoscalar and isovector spin-orbit strengths in a fixed (relative) ratio:

$$F_{so}(k_f)(\text{WT-Fock}) = -3 G_{so}(k_f)(\text{WT-Fock})$$

$$= \frac{g_A^2 m_\pi u^{-2}}{4\pi^2 M (4f_\pi)^4} \left\{ 15u^2 \arctan 2u + \frac{3}{4u} - u - 39u^3 
- \frac{3 + 2u^2 + 10u^4}{8u^3} \ln(1 + 4u^2) + \frac{3 + 8u^2 - 64u^4}{64u^5} \ln^2(1 + 4u^2) 
+ \int_0^u dx x^{-2} \left[ 3(u^2 + u^4) + 6(u + u^3)(6x^2 - 1 - u^2) L 
+ \left[ 3(1 + u^2)^3 - (15 + 34u^2 + 19u^4)x^2 + (33 + 29u^2)x^4 - 13x^6 \right] L^2 \right] \right\}, \quad (15)$$
with the auxiliary function:

\[ L = \frac{1}{4x} \ln \frac{1 + (u + x)^2}{1 + (u - x)^2}. \]  

(16)

The power of the pion mass \( m_\pi \) in their prefactors indicates that all contributions written in eqs.(11-15) are of the same order in the small momentum expansion.

\[ \text{Fig. 4: The spin-orbit strength generated by the two-pion exchange three-nucleon interaction involving the general chiral } \pi\pi NN \text{-contact vertex versus the nucleon density } \rho = 2k_f^3/3\pi^2. \text{ The full curve shows the isoscalar spin-orbit strength } F_{so}(k_f) \text{ and the dashed curve shows the isovector spin-orbit strength } G_{so}(k_f) \text{ magnified by a factor } 10. \]

For the numerical evaluation we use the values \( c_1 = -0.64 \text{ GeV}^{-1}, c_3 = -3.90 \text{ GeV}^{-1} \) and \( c_4 = 2.25 \text{ GeV}^{-1} \) of the low-energy constants which have been determined (at tree-level) in ref.[12] from some low-energy \( \pi N \)-data. The full line in Fig.4 shows the resulting isoscalar spin-orbit strength \( F_{so}(k_f) \) as a function of the nucleon density \( \rho = 2k_f^3/3\pi^2. \) The contribution of the Hartree diagram involving the p-wave contact vertex proportional to \( c_3 \) is the absolutely dominant one. For example one finds at nuclear matter saturation density (where \( k_f \simeq 2m_\pi \)) the value \( F_{so}(2m_\pi)(c_3-\text{Hart}) = 65.3 \text{ MeVfm}^5. \) In comparison to this the s-wave Hartree contribution is very small, \( F_{so}(2m_\pi)(c_1-\text{Hart}) = 3.6 \text{ MeVfm}^5. \) Moreover, the even smaller Fock contributions have a tendency of cancelling each other. The somewhat smaller values of \( F_{so}(k_f) \) in Fig.3 compared to those in Fig.4 originate mainly from the fact that the \( \Delta \)-resonance saturates the low-energy constant \( c_3 \) to about only three quarters in magnitude: \( c_3^{(\Delta)} = -g_A^2/2\Delta \simeq -2.9 \text{ GeV}^{-1}. \) The dashed line in Fig.4 shows the isovector spin-orbit strength \( G_{so}(k_f) \) (magnified by a factor 10) as a function of the nucleon density \( \rho = 2k_f^3/3\pi^2. \) Again, in comparison to the isoscalar spin-orbit strength \( F_{so}(k_f) \) it is only a small 5% correction. The largest individual contribution comes here
from the Fock diagram involving the p-wave $c_3$-contact vertex which for example gives at nuclear matter saturation density: $G_{so}(2m_\pi)(c_3-F_{\text{Fock}}) = 3.2 \text{ MeVfm}^5$.

In summary we have calculated in this work the spin-orbit coupling generated by the two-pion exchange three-nucleon interaction. We have made use of the density-matrix expansion of Negele and Vautherin [8]. This method allows one to compute diagrammatically the nuclear energy density functional which contains the spin-orbit coupling term of interest. We have derived simple analytical expressions for the density-dependent isoscalar and isovector spin-orbit strengths $F_{so}(k_f)$ and $G_{so}(k_f)$. First, we have considered the two-pion exchange three-nucleon interaction of Fujita and Miyazawa [3] where one nucleon is excited to a p-wave $\Delta$-resonance. The corresponding three-loop Hartree and Fock diagrams generate spin-orbit couplings which are not relativistic effects but independent of the nucleon mass. The Hartree diagram and the isoscalar component $F_{so}(k_f)$ are by far dominant. At nuclear matter saturation density these $\Delta$-driven three-body mechanisms generate about half of the empirical isoscalar spin-orbit strength. The calculations of spin-orbit splittings in light nuclei [4, 6] point of course in the same direction. Secondly, have we derived more generally the three-body spin-orbit coupling generated by two-pion exchange on the basis of the chiral $\pi\pi NN$-contact vertex. In that framework we have obtained similar (numerical) results for the density dependent isoscalar and isovector spin-orbit strengths $F_{so}(k_f)$ and $G_{so}(k_f)$. The p-wave part of the chiral $\pi\pi NN$-contact interaction (proportional to the low-energy constant $c_3$) and the Hartree diagram give rise to the absolutely dominant contribution.

On the other hand it has been shown recently in ref.[10] that iterated one-pion exchange generates an isoscalar spin-orbit strength $F_{so}(k_f)$ that is sizeable but of the wrong negative sign. Combining those results [10] with the present ones, one may conclude that the net nuclear spin-orbit coupling generated by (multi) pion-exchange is rather small, at least for densities around nuclear matter saturation density. Lorentz scalar and vector mean-fields with their in-medium behavior governed by QCD sum rules could therefore be the appropriate dynamical framework for building up the strong (isoscalar) nuclear spin-orbit interaction. Indeed such a proposal has recently been successfully applied in ref.[13] to nuclear structure calculations.

References