Decay of $C/Z_n$: exact supergravity solutions
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We present a simple method to derive the general exact solution describing monopole scalar radiation coupled to gravity in 2 + 1 dimensions. The solution confirms the conjecture of Adams, Polchinski, and Silverstein regarding the late time behavior of the decay of the $C/Z_n$ orbifold in type II string theory.

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Introduction

Despite the impressive progress that has been made in the last several years in our understanding of open string tachyon condensation, the problem of closed string tachyon condensation remains deep and mysterious. Among the many advantages enjoyed by those studying open string tachyons, see Martinec for a nice review of open and closed string tachyon condensation, emphasizing the differences between the two cases. Perhaps the most fundamental is that, by taking the string coupling small, they can decouple the closed string degrees of freedom and work in a fixed gravitational background. Obviously, when studying closed string tachyons no such consistent truncation is possible, and one is forced to deal with the often complicated gravitational dynamics as an essential part of the condensation process. For example, since the energy density released during closed string tachyon condensation is proportional to $1/G_N$ (compared to $1/\sqrt{G_N}$ in the open string case), the gravitational back-reaction can never be neglected.

The situation is actually worse than this. Given that, generically, a string scale (times the above factor of $1/G_N$) energy density is being released, one would expect that massive string modes get excited during the decay, and that a fully stringy treatment is therefore necessary. There is no guarantee that the notion of a smooth geometry described by the massless fields continues to be valid; most dramatically, the spacetime itself could cataclysmically decay into nothing. However, if the tachyon locus has a transverse dimension for the energy to dissipate into (and if the decay doesn’t somehow open up a tear in spacetime), then it is plausible that at late times the massive string fields will dump their energy into the massless ones. In that case, as time goes on stringy effects will become less and less important, and eventually one will end up with a conventional gravity scenario in which energy propagates away from the tachyon locus at the speed of light, with a concomitant back-reaction of the geometry. This is essentially the picture proposed by Adams, Polchinski, and Silverstein Adams in their study of the decay of the $C/Z_n$ orbifold of type II string theory: within a few string times, H. Liu has argued that this process could in fact take a time inversely proportional to the string coupling. The energy released in the decay is converted into a circular dilaton wave that expands at the speed of light, leaving behind a bubble of flat space.

Typically in general relativity, even such “pedestrian” back-reaction problems are difficult or intractable, and one often resorts to an approximation in which the back-reaction of the geometry is treated as a perturbation on the equations of motion of the matter fields. The purpose of this note is to point out that, in the example of $C/Z_n$, there exists a simple method for solving the coupled dilaton-graviton equations of motion exactly and generally. Thus the back-reaction problem need not be an obstacle to understanding the decay process. In particular, the solution confirms the conjecture of APS regarding the late-time evolution of the system.

It is an oft-heard speculation that tachyon condensation processes are modelled by an RG flow of the worldsheet theory seeded by the tachyon vertex operator (which is relevant when stripped of its time dependence) Martinec–Minwalla. How the word “modelled” should be precisely understood in this context depends on who is speaking, and to understand the connection better it may be useful to have examples of decay processes in which both the time evolution and the RG flow are known explicitly. An exact solution to the RG equations for this system is already known Gutperle (this solution is valid only when the curvature scales are much larger than the string scale, just as for the time evolution solution considered in this paper). Thus, $C/Z_n$ can in principle now serve as a laboratory for testing ideas about the quantitative connection between these two types of evolution.
In Section 2 below, we set up the equations of motion for the system, namely a massless scalar minimally coupled to 2+1 gravity. We then show, using the rotational symmetry and making a judicious choice of coordinate system, that the equation of motion for the dilaton is the same as that for a dilaton in Minkowski space, i.e. is decoupled from the back-reaction of the geometry. This allows the equations to be solved in two steps: first, the dilaton equation of motion is straightforwardly solved, then the back-reaction of the geometry is found by solving a first-order equation for a metric coefficient that is sourced by the dilaton’s energy. Among other things, this method guarantees that under non-singular initial conditions the energy will indeed radiate to infinity and that no singularity can form.

In Section 3, we study solutions to the equations of motion. The quantitative behavior of the system at late times for generic initial conditions is easily found. One general result is that in the final state the dilaton returns to its initial value—the tachyon condensation does not lead the system to run either to strong or to weak coupling. A numerical example is presented to illustrate quantitatively how the dilaton wave carries the spatial curvature with it, leaving behind a bubble of flat space. In the limit of a spatially localized dilaton source, a formula is derived for the change in the deficit angle in terms of the source’s frequency spectrum. Finally, a thin-shell solution is derived for the limit in which the source is localized both in the space and in time.

This problem has previously been solved in the strong coupling limit by Gregory and Harvey Gregory. Their method is somewhat more involved than ours but their results are consistent with ours.

Equations of motion

We wish to study the decay of the unstable $C/Z_n$ orbifold of type II string theory as a classical time-dependent process. We can consistently truncate the dynamics to $2 + 1$ dimensions, namely the two of the orbifold plus time Adams. The reason is that the CFT describing the extra seven dimensions is coupled to the $2 + 1$ dimensional CFT neither in the initial configuration nor by the tachyon vertex operator, and therefore decouples entirely. In other words, the on-shell tachyon vertex operator must extend to an exactly marginal operator in the $2 + 1$ dimensional CFT alone. The extra seven dimensions may be in the form of $R^7$, or any other unitary $\hat{c} = 7$ CFT. The equation of motion for the $B$-field requires its field strength to be a constant multiple of the volume form on the unreduced $2 + 1$ dimensions. This constant vanishes in the initial orbifold configuration, and must therefore vanish everywhere. So the only active NS-NS fields are the metric and the dilaton $\Phi$, and their action is equation $S = 12\kappa_3^2 \int d^3 x \sqrt{-G} e^{-2\Phi} (R_3 + 4G^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi)$,