Formulation of Vector Manifestation in Hot and/or Dense Matter *

Masayasu HARADA

Department of Physics, Nagoya University, Nagoya 464-8602, JAPAN

The vector manifestation (VM) was proposed as a novel manifestation of chiral symmetry in which the massless vector meson becomes the chiral partner of pion. In this write-up, I briefly summarize the following main ingredients to formulate the VM in hot and/or dense matter: Effective field theory (EFT) based on the hidden local symmetry; Wilsonian matching between the EFT and QCD; Intrinsic thermal and/or dense effects.

§1. Introduction

Spontaneous chiral symmetry breaking is one of the most important properties of QCD in low energy region. This chiral symmetry is expected to be restored in hot and/or dense QCD and properties of hadrons will be changed near the critical temperature of the chiral symmetry restoration. \(^{(1)-(3)}\) The CERN Super Proton Synchrotron (SPS) observed an enhancement of dielectron (\(e^+e^-\)) mass spectra below the \(\rho/\omega\) resonance. \(^{(4)}\) This can be explained by the dropping mass of the \(\rho\) meson (see, e.g., Refs. 2),3),5)) following the Brown-Rho scaling proposed in Ref. 6). Furthermore, the Relativistic Heavy Ion Collider (RHIC) has started to measure several physical processes in hot matter which include the dilepton energy spectra. Therefore it is interesting to study the temperature and/or density dependences of the vector meson mass which is one of the important quantities in the chiral phase transition.

In Ref. 7), it was shown how the vector manifestation (VM), \(^{(8)}\) in which the chiral symmetry is restored by the massless degenerate pseudoscalar meson denoted by \(\pi\) (pion and its flavor partners) and the vector meson denoted by \(\rho\) (\(\rho\) meson and its flavor partners) as the chiral partner, is formulated in hot matter using the model for \(\pi\) and \(\rho\) based on the hidden local symmetry (HLS). \(^{(9)}\) Furthermore, in Ref. 10), the formulation of the VM is done in the presence of dense matter by including the quasiquark into the HLS. In these formulations, the \textit{intrinsic temperature and/or density dependences} \(^{(7),10)}\) of the parameters of the HLS Lagrangian, which is introduced by applying the Wilsonian matching \(^{(11),12)}\) at non-zero temperature and/or non-zero density, played important roles to realize the chiral symmetry restoration consistently: In the framework of the HLS the equality between the axial-vector and vector current correlators at critical point can be satisfied only if the intrinsic thermal and/or density effects are included.

In this write-up, I show how the VM is formulated in hot and/or dense QCD

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following Refs. 7), 10), 13). I first show the difference between the VM and the conventional picture based on the linear sigma model in terms of the chiral representation of the low-lying mesons in section 2. I will introduce the model based on the HLS, and show the renormalization group equations for the parameters of the HLS Lagrangian in section 3. I will also briefly summarize the general idea of Wilsonian matching between the effective field theory and QCD. In section 4, the intrinsic thermal and/or density effects are briefly explained. Formulations of the VM in hot and dense matter are shown in sections 5 and 6. Finally, in section 7, I will give a brief summary and discussions.

§2. Vector Manifestation of Chiral Symmetry

In this section, following Ref. 8), 12), I briefly review the difference between the vector manifestation (VM) and the conventional manifestation of chiral symmetry restoration based on the linear sigma model in terms of the chiral representation of the mesons by extending the analyses done in Refs. 14), 15) for two flavor QCD.

The VM was first proposed in Ref. 8) as a novel manifestation of Wigner realization of chiral symmetry where the vector meson $\rho$ becomes massless at the chiral phase transition point. Accordingly, the (longitudinal) $\rho$ becomes the chiral partner of the Nambu-Goldstone (NG) boson $\pi$. The VM is characterized by

\[
(\text{VM}) \quad f_\pi^2 \to 0 \ , \quad m_\rho^2 \to m_\pi^2 = 0 \ , \quad f_\rho^2 / f_\pi^2 \to 1 \ , \quad (2.1)
\]

where $f_\rho$ is the decay constant of (longitudinal) $\rho$ at $\rho$ on-shell. This is completely different from the conventional picture based on the linear sigma model where the scalar meson $S$ becomes massless degenerate with $\pi$ as the chiral partner:

\[
(\text{GL}) \quad f_\pi^2 \to 0 \ , \quad m_S^2 \to m_\pi^2 = 0 \ . \quad (2.2)
\]

In Ref. 12) this was called GL manifestation after the effective theory of Ginzburg–Landau or Gell-Mann–Levy.

I first consider the representations of the following zero helicity ($\lambda = 0$) states under $\text{SU}(3)_L \times \text{SU}(3)_R$: the $\pi$, the (longitudinal) $\rho$, the (longitudinal) axial-vector meson denoted by $A_1$ ($a_1$ meson and its flavor partners) and the scalar meson denoted by $S$. The $\pi$ and the longitudinal $A_1$ are admixture of $(8, 1) \oplus (1, 8)$ and $(3, 3^*) \oplus (3^*, 3)$ since the symmetry is spontaneously broken:\cite{14, 15}

\[
|\pi \rangle = |(3, 3^*) \oplus (3^*, 3)\rangle \sin \psi + |(8, 1) \oplus (1, 8)\rangle \cos \psi \ , \\
|A_1(\lambda = 0)\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle \cos \psi - |(8, 1) \oplus (1, 8)\rangle \sin \psi \ , \quad (2.3)
\]

where the experimental value of the mixing angle $\psi$ is given by approximately $\psi = \pi/4$.\cite{14, 15} On the other hand, the longitudinal $\rho$ belongs to pure $(8, 1) \oplus (1, 8)$ and the scalar meson to pure $(3, 3^*) \oplus (3^*, 3)$:

\[
|\rho(\lambda = 0)\rangle = |(8, 1) \oplus (1, 8)\rangle , \\
|S\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle . \quad (2.4)
\]
When the chiral symmetry is restored at the phase transition point, it is natural to expect that the chiral representations coincide with the mass eigenstates: The representation mixing is dissolved. From Eq. (2.3) one can easily see that there are two ways to express the representations in the Wigner phase of the chiral symmetry: The conventional GL manifestation corresponds to the limit $\psi \rightarrow \pi/2i$ which $\pi$ is in the representation of pure $(3, \bar{3}) \oplus (\bar{3}^*, 3)$ together with the scalar meson, both being the chiral partners:

\[
\text{(GL)} \begin{cases}
|\pi\rangle, |S\rangle \rightarrow |(3, \bar{3}^*) \oplus (\bar{3}^*, 3)\rangle, \\
|\rho(\lambda = 0)\rangle, |A_1(\lambda = 0)\rangle \rightarrow |(8, 1) \oplus (1, 8)\rangle.
\end{cases}
\] (2.5)

On the other hand, the VM corresponds to the limit $\psi \rightarrow 0$ in which the $A_1$ goes to a pure $(3, \bar{3}^*) \oplus (\bar{3}^*, 3)$, now degenerate with the scalar meson $S$ in the same representation, but not with $\rho$ in $(8, 1) \oplus (1, 8)$:

\[
\text{(VM)} \begin{cases}
|\pi\rangle, |\rho(\lambda = 0)\rangle \rightarrow |(8, 1) \oplus (1, 8)\rangle, \\
|A_1(\lambda = 0)\rangle, |S\rangle \rightarrow |(3, \bar{3}^*) \oplus (\bar{3}^*, 3)\rangle.
\end{cases}
\] (2.6)

Namely, the degenerate massless $\pi$ and (longitudinal) $\rho$ at the phase transition point are the chiral partners in the representation of $(8, 1) \oplus (1, 8)$.

Next, I consider the helicity $\lambda = \pm 1$. Note that the transverse $\rho$ can belong to the representation different from the one for the longitudinal $\rho$ ($\lambda = 0$) and thus can have the different chiral partners. According to the analysis in Ref. 14), the transverse components of $\rho$ ($\lambda = \pm 1$) in the broken phase belong to almost pure $(3^*, 3)$ ($\lambda = +1$) and $(3, \bar{3}^*)$ ($\lambda = -1$) with tiny mixing with $(8, 1) \oplus (1, 8)$. Then, it is natural to consider in VM that they become pure $(3, \bar{3}^*)$ and $(3^*, 3)$ in the limit approaching the chiral restoration point:

\[
|\rho(\lambda = +1)\rangle \rightarrow |(3^*, 3)\rangle, \quad |\rho(\lambda = -1)\rangle \rightarrow |(3, \bar{3}^*)\rangle.
\] (2.7)

As a result, the chiral partners of the transverse components of $\rho$ in the VM will be themselves. Near the critical point the longitudinal $\rho$ becomes almost $\sigma$, namely the would-be NG boson $\sigma$ almost becomes a true NG boson and hence a different particle than the transverse $\rho$.

§3. Effective Field Theory

In this section I first show the effective field theory (EFT) in which the vector manifestation is formulated (subsection 3.1). Then, after showing the renormalization group equations (RGEs) for the parameters of the Lagrangian of the EFT (subsection 3.2), I briefly summarize a general idea of the Wilsonian matching between the EFT and QCD which determines the parameters of the Lagrangian of the EFT (subsection 3.3).

I should note that, as is stressed in Ref. 12), the VM can be formulated only as a limit by approaching it from the broken phase of chiral symmetry. Then, for the formulation of the VM, I need an EFT including $\rho$ and $\pi$ in the broken phase which is not necessarily applicable in the symmetric phase. One of such EFTs is the model
based on the hidden local symmetry (HLS)\(^9\) which includes \(\rho\) as the gauge boson of the HLS in addition to \(\pi\) as the Nambu-Goldstone (NG) boson associated with the chiral symmetry breaking in a manner fully consistent with the chiral symmetry of QCD. It should be noticed that, in the HLS, thanks to the gauge invariance one can perform the systematic chiral perturbation with including \(\rho\) in addition to \(\pi\).\(^{11},^{12},^{16}\)–\(^{18}\)

3.1. Hidden Local Symmetry

The HLS model is based on the \(G_{\text{global}} \times H_{\text{local}}\) symmetry, where \(G = SU(N_f)_L \times SU(N_f)_R\) is the chiral symmetry and \(H = SU(N_f)_V\) is the HLS. The basic quantities are the HLS gauge boson \(V_\mu\) and two matrix valued variables \(\xi_L(x)\) and \(\xi_R(x)\) which transform as

\[
\xi_{L,R}(x) \rightarrow \xi'_{L,R}(x) = h(x)\xi_{L,R}(x)g^\dagger_{L,R} ,
\]

where \(h(x) \in H_{\text{local}}\) and \(g_{L,R} \in [SU(N_f)_L,R]_{\text{global}}\). These variables are parameterized as

\[
\xi_{L,R}(x) = e^{i\sigma(x)/F_{\sigma}}e^{i\pi(x)/F_{\pi}} ,
\]

where \(\pi = \pi^aT_a\) denotes the pseudoscalar NG bosons associated with the spontaneous symmetry breaking of \(G_{\text{global}}\) chiral symmetry, and \(\sigma = \sigma^aT_a\) denotes the NG bosons associated with the spontaneous breaking of \(H_{\text{local}}\). This \(\sigma\) is absorbed into the HLS gauge boson through the Higgs mechanism. \(F_{\pi}\) and \(F_{\sigma}\) are the decay constants of the associated particles. The phenomenologically important parameter \(a\) is defined as

\[
a = \frac{F_{\sigma}^2}{F_{\pi}^2} .
\]

The covariant derivatives of \(\xi_{L,R}\) are given by

\[
D_\mu \xi_L = \partial_\mu \xi_L - iV_\mu \xi_L + i\xi_L D_\mu ,
\]
\[
D_\mu \xi_R = \partial_\mu \xi_R - iV_\mu \xi_R + i\xi_R D_\mu ,
\]

where \(V_\mu\) is the gauge field of \(H_{\text{local}}\), and \(D_\mu\) and \(R_\mu\) are the external gauge fields introduced by gauging \(G_{\text{global}}\) symmetry.

The HLS Lagrangian with lowest derivative terms is given by\(^9\)

\[
\mathcal{L}(2) = F_{\pi}^2 tr[\hat{\alpha}_{\perp \mu} \hat{\alpha}^\dagger_{\perp \mu}] + F_{\sigma}^2 tr[\hat{\alpha}_{\parallel \mu} \hat{\alpha}^\dagger_{\parallel \mu}] - \frac{1}{2g^2} tr[V_{\mu \nu}V^{\mu \nu} ] ,
\]

where \(g\) is the HLS gauge coupling, \(V_{\mu \nu}\) is the field strength of \(V_\mu\) and

\[
\hat{\alpha}_{\perp \parallel}^\mu = (D_\mu \xi_R \cdot \xi_R^\dagger \mp D_\mu \xi_L \cdot \xi_L^\dagger)/(2i) .
\]

3.2. Renormalization Group Equations

At one-loop level the Lagrangian (3.5) generates the \(\mathcal{O}(p^4)\) contributions including the divergent contributions which are renormalized by three leading order
parameters $F_\pi$, $a$ and $g$ (and parameters of $\mathcal{O}(p^4)$ Lagrangian). As was stressed in Ref. 12), it is important to include effects of quadratic divergences into the RGEs for studying the phase structure. The resultant RGEs for three leading order parameters are expressed as \cite{11,12,19}

$$
\mathcal{M} \frac{dF_\pi^2}{d\mathcal{M}} = C \left[ 3a^2 g^2 F_\pi^2 + 2(2 - a)\mathcal{M}^2 \right],
$$

$$
\mathcal{M} \frac{da}{d\mathcal{M}} = -C(a - 1) \left[ 3a(1 + a)g^2 - (3a - 1)\frac{\mathcal{M}^2}{F_\pi^2} \right],
$$

$$
\mathcal{M} \frac{dg^2}{d\mathcal{M}} = -C \frac{87 - a^2}{6} g^4,
$$

where $C = N_f / [2(4\pi)^2]$ and $\mathcal{M}$ is the renormalization point. It should be noted that the point $(g, a) = (0, 1)$ is the fixed point of the RGEs in Eq. (3.7) which plays an essential role to formulate the VM in the following analysis of the chiral symmetry restoration.

3.3. Wilsonian Matching

The basic concept of the EFT is that the effective Lagrangian, which has the most general form constructed from the chiral symmetry, give the same generating functional as that obtained from QCD:

$$
Z_{\text{EFT}}[J, F] = \int \mathcal{D} U e^{iS_{\text{eff}}[J, F]} \leftrightarrow \text{matching} \quad Z_{\text{QCD}}[J] = \int \mathcal{D} q \mathcal{D} \bar{q} \mathcal{D} G e^{iS_{\text{QCD}}[J]},
$$

where $J$ is a set of external source fields. In the EFT side $U$ denotes the relevant hadronic fields such as the pion fields, $S_{\text{eff}}$ is the action expressed in terms of these hadrons, $F$ a set of parameters included in the EFT. In QCD side $q$ ($\bar{q}$) denotes (anti) quark field, $G$ is gluon field and $S_{\text{QCD}}$ represents the action expressed in terms of the quarks and gluons.

In some matching schemes, the renormalized quantities of the EFT are determined from QCD. On the other hand, the matching in the Wilsonian sense is performed based on the following general idea: The bare Lagrangian of the EFT is defined at a suitable matching scale $\Lambda$ and the generating functional derived from the bare Lagrangian leads to the same Green's function as that derived in QCD at $\Lambda$:

$$
Z_{\text{EFT}}[J, F] \big|_{E=\Lambda} = e^{iS_{\text{eff}}[J, F_{\text{bare}}]} \leftrightarrow \text{matching} \quad Z_{\text{QCD}}[J] \big|_{E=\Lambda} = \int \mathcal{D} q \mathcal{D} \bar{q} \mathcal{D} G e^{iS_{\text{QCD}}[J]},
$$

where $F_{\text{bare}}$ denotes a set of bare parameters. Through the above matching, which is named Wilsonian matching in Ref. 11), the bare parameters of the EFT are determined. In other words, we obtain the bare Lagrangian of the EFT after integrating out the high energy modes, i.e., the quarks and gluons above $\Lambda$. Then the informations of the high energy modes are included in the parameters of the EFT.

In Ref. 11), 12), based on the above idea, the vector and axial-vector current correlators derived from the bare HLS theory are matched with those obtained by the operator product expansion in QCD. It was shown that the physical predictions are in remarkable agreement with experiments.
4. Intrinsic Thermal and/or Density Effects

In Refs. 7), 10) the Wilsonian matching, briefly explained in subsection 3.3, was applied to the analysis of QCD in hot and dense matter. As was discussed in subsection 3.3, the bare Lagrangian of the effective field theory (EFT) is obtained by integrating out high energy modes, i.e., quarks and gluons above the matching scale $\Lambda$, and as a result, the informations of the high energy modes are included into the parameters of the EFT. Thus when we integrate out high energy modes in hot and/or dense matter, the parameters are in general dependent on temperature and/or density. This is called the intrinsic temperature and/or density dependence,\(^{7),10}\) which is nothing but the signature that hadron has an internal structure constructed from the quarks and gluons. This is similar to the situation where the coupling constants among hadrons are replaced with the momentum-dependent form factor in high energy region. Thus the intrinsic thermal and/or dense effects play more important roles in higher temperature region, especially near the critical temperature.

Here, let me show an example of the Wilsonian matching condition to determine the bare decay constant of $\pi$ in hot and/or dense matter. This is obtained\(^{7),10}\) by putting possible temperature and/or density dependences on the gluonic and quark condensates in the Wilsonian matching condition at $T = \mu = 0$ *):

$$\frac{F_\pi^2(A; T, \mu)}{A^2} = \frac{1}{8\pi^2} \left[ 1 + \frac{\alpha_s}{\pi} + \frac{2\pi^2}{3} \frac{(\alpha_s G_{\mu\nu} G^{\mu\nu})_{T,\mu}}{A^4} + \frac{\pi^3}{27} \frac{1408 \alpha_s \langle \bar{q}q \rangle_T^2}{A^6} \right].$$  \hspace{1cm} (4.1)

Through this condition the temperature and/or density dependences of the quark and gluonic condensates determine the intrinsic temperature dependences of the bare parameter $F_\pi(A; T, \mu)$, which is then converted into those of the on-shell parameter $F_\pi(\mu = 0; T, \mu)$ through the Wilsonian renormalization group equations.

5. Vector Manifestation in Hot Matter

In this section, I show how the vector manifestation (VM) is formulated in hot QCD following Refs. 7), 13).

Let me start from the axial-vector and the vector current correlators derived from the bare hidden local symmetry (HLS):

$$G_A^{(\text{HLS})}(Q^2) = \frac{F_\pi^2(A; T)}{Q^2} - 2z_2(A; T),$$
$$G_V^{(\text{HLS})}(Q^2) = \frac{F_\sigma^2(A; T)[1 - 2g^2(A; T)z_3(A; T)]}{M_\rho^2(A; T) + Q^2} - 2z_1(A; T).$$  \hspace{1cm} (5.1)

* It should be noticed that there is no longer Lorentz symmetry in hot and/or dense matter, and the Lorentz non-scalar operators such as $\bar{q}_\nu D_\mu q$ may exist in the form of the current correlators derived by the operator product expansion.\(^{20}\) This leads to, e.g., a difference between the temporal and spatial bare pion decay constants. However, I neglect the contributions from these operators here since they give a small correction compared with the main term $1 + \frac{\alpha_s}{\pi}$. This implies that the Lorentz symmetry breaking effect in the bare pion decay constant is small, $F_{\pi, \text{bare}}^t \simeq F_{\pi, \text{bare}}^s$.\(^{21}\) The Wilsonian matching with including the effect of such Lorentz non-scalar operators was recently done in Ref. 22), which shows that the difference between $F_{\pi, \text{bare}}^t$ and $F_{\pi, \text{bare}}^s$ is actually small.
It should be noticed that the bare parameters have the intrinsic temperature dependences as I explained in the previous section.

At the critical temperature, the axial-vector and vector current correlators derived in the operator product expansion (OPE) agree with each other for any value of $Q^2$. Thus I require that these current correlators in the HLS are equal at the critical temperature for any value of $Q^2$ around $\Lambda^2$. By taking account of the fact $F_\pi^2(A;T_c) \neq 0$ derived by applying the Wilsonian matching condition in Eq. (4.1) at $T = T_c$, the requirement $G_A^{(\text{HLS})} = G_V^{(\text{HLS})}$ is satisfied only if the following conditions are met:

$$g(A;T) \to 1 \quad , \quad a(A;T) \to 0 \quad , \quad z_1(A;T) - z_2(A;T) \to 0 \quad .$$

These conditions ("VM conditions in hot matter") for the bare parameters are converted into the conditions for the on-shell parameters through the Wilsonian renormalization group equations (RGEs). Since $g = 0$ and $a = 1$ are separately the fixed points of the RGEs for $g$ and $a$, the on-shell parameters also satisfy $(g,a) = (0,1)$, and thus the parametric $\rho$ mass satisfies $M_\rho = 0$.

Now, let me include the hadronic thermal effects to obtain the $\rho$ pole mass near the critical temperature. As I explained above, the intrinsic temperature dependences imply that $M_\rho/T \to 0$ for $T \to T_c$, so that the $\rho$ pole mass near the critical temperature is expressed as

$$m_\rho^2(T) = M_\rho^2 + g^2 N_f \frac{15 - a^2}{144} T^2 \quad .$$

Since $a \simeq 1$ near the restoration point, the second term is positive. Then the $\rho$ pole mass $m_\rho$ is bigger than the parametric $M_\rho$ due to the hadronic thermal corrections. Nevertheless, the intrinsic temperature dependence determined by the Wilsonian matching requires that the $\rho$ becomes massless at the critical temperature:

$$m_\rho^2(T) \to 0 \quad \text{for} \quad T \to T_c \quad ,$$

since the first term in Eq. (5.3) vanishes as $M_\rho \to 0$, and the second term also vanishes since $g \to 0$ for $T \to T_c$. This implies that the vector manifestation (VM) actually occurs at the critical temperature.\(^7\)

§6. Vector Manifestation in Dense Matter

In this section, I briefly summarize how the vector manifestation (VM) is formulated in dense QCD following Ref. 10).

In Ref. 10), following the picture shown in Ref. 23), the quasiquark degree of freedom is added into the Lagrangian of the hidden local symmetry (HLS) near the critical chemical potential with assuming that its mass $m_q$ becomes small ($m_q \to 0$). The Lagrangian introduced in Ref. 10) for including one quasiquark field $\psi$ and one anti-quasiquark field $\bar{\psi}$ is counted as $O(p)$ and given by

$$\mathcal{L}_Q = \bar{\psi} \left( i D_\mu \gamma^\mu + \mu \gamma^0 - m_q \right) \psi + \bar{\psi} \left( \kappa \gamma^\mu \hat{a}_{\parallel \mu} + \lambda \gamma_5 \gamma^\mu \hat{a}_{\perp \mu} \right) \psi \quad .$$

\(^{19}\)
where $\mu$ is the chemical potential, $D_{\mu}\psi = (\partial_{\mu} - ig\rho_{\mu})\psi$ and $\kappa$ and $\lambda$ are constants to be specified later.

Inclusion of the quasiquark changes the renormalization group equations (RGEs) for $F_{\pi}, a$ and $g$. Furthermore, the RGE for the quasiquark mass $m_q$ should be considered simultaneously. The explicit forms of the RGEs are shown in Eq. (7) of Ref. 10), which show that, although $g = 0$ and $a = 1$ are not separately the fixed points of the RGEs for $g$ and $a$, $(g, a, m_q) = (0, 1, 0)$ is a fixed point of the coupled RGEs for $g$, $a$ and $m_q$.

Let me consider the intrinsic density dependences of the bare parameters of the HLS Lagrangian. Similarly to the intrinsic temperature dependences in hot QCD, the intrinsic density dependences are introduced through the Wilsonian matching. Noting that the quasiquark does not contribute to the current correlators at bare level, one arrives at the following “VM conditions in dense matter” similar to the one in hot matter in Eq. (5.2) near the critical chemical potential $\mu_c$:

$$g(A; \mu) \xrightarrow{\mu \to \mu_c} 0, \quad a(A; \mu) \xrightarrow{\mu \to \mu_c} 1, \quad z_1(A; \mu) - z_2(A; \mu) \xrightarrow{\mu \to \mu_c} 0. \quad (6.2)$$

These conditions are converted into the conditions for the on-shell parameters through the RGEs. Since $(g, a, m_q) = (0, 1, 0)$ is a fixed point of the coupled RGEs for $g$, $a$ and $m_q$, the above conditions together with the assumption $m_q \to 0$ for $\mu \to \mu_c$ imply that the on-shell parameters behave as $g \to 0$ and $a \to 1$, and thus the parametric $\rho$ mass vanishes for $\mu \to \mu_c$: $M_\rho \to 0$.

Now, let me study the $\rho$ pole mass near $\mu_c$ by including the hadronic dense-loop correction from the quasiquark. By taking $(g, a, m_q) \to (0, 1, 0)$ in the quasiquark loop contribution, the $\rho$ pole mass is expressed as

$$m_{\rho}^2(\mu) = M_{\rho}^2(\mu) + g^2 \frac{(1 - \kappa)^2}{6\pi^2} \mu^2. \quad (6.3)$$

Since $M_\rho(\mu) \to 0$ and $g \to 0$ for $\mu \to \mu_c$ due to the intrinsic density dependence, the above expression implies that the $\rho$ pole mass vanishes at the critical chemical potential, i.e., the VM is realized in dense matter:

$$m_{\rho}(\mu) \to 0 \text{ for } \mu \to \mu_c. \quad (6.4)$$

§7. Summary and Discussions

In this write-up I summarized how the vector manifestation (VM) is formulated in hot and/or dense matter based on the hidden local symmetry (HLS). One of the most important ingredients to formulate the VM in hot and/or dense matter is the intrinsic temperature and/or density dependence of the bare parameters of the bare HLS Lagrangian derived through the Wilsonian matching. For satisfying the equality between the axial-vector and vector current correlators at the chiral restoration point ($T = T_c$ and/or $\mu = \mu_c$), which is needed for consistency with the chiral symmetry restoration, the intrinsic temperature and/or density dependence leads to the VM conditions in Eq. (5.2) and/or Eq. (6.2). These conditions are
protected by the VM fixed point of the renormalization group equations, and play crucial role to formulate the VM in hot and/or dense matter.

In this write-up, I explained only the formulation of the VM, and did not introduce the following several predictions of the VM in hot matter done so far: The vector and axial-vector susceptibilities are predicted to be equal;\textsuperscript{21)} The pion velocity becomes the speed of light when we neglect the small Lorentz violating effects in the bare HLS Lagrangian;\textsuperscript{21)} the vector dominance of the electromagnetic form factor of pion is largely violated.\textsuperscript{13)} Recently in Ref. 24), Sasaki included the Lorentz breaking effects into the bare HLS Lagrangian, and showed that the pion velocity at the critical temperature receives neither quantum nor hadronic thermal corrections. In Ref. 22), based on this “non-renormalization theorem” and the Lorentz non-invariant version of the Wilsonian matching, the pion velocity near the critical temperature was shown to be close to the speed of light. This result is drastically different from the result obtained in the standard chiral theory\textsuperscript{25)} for the two-flavor QCD which predicts that the pion velocity should go to zero at the critical point.

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