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WEAK INTERACTION CORRECTIONS TO e^+e^- ANNIHILATION PROCESSES
AND TESTS OF UNIFIED THEORIES OF WEAK, ELECTROMAGNETIC
AND STRONG INTERACTIONS

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A B S T R A C T

We calculate weak interaction corrections to $R = \sigma(e^+e^- \rightarrow \text{all})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ at high energies, using a quark parton model. We show that the correction due to interference between γ and Z mediated annihilations (which would make itself manifest by a variation linear in the square of the centre-of-mass energy) is strongly model dependent. In particular the γ - Z interference contribution to R vanishes identically (to the order g_s^4 in the gluon coupling constant) in unified theories of weak, electromagnetic and string interactions provided the lepton and quark masses are neglected as compared with the energy, and if perturbation theory is valid.

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In this note we calculate the weak interaction corrections¹⁾ to e^+e^- annihilation cross-sections at high energies. Clearly, such a calculation has to be strongly model dependent and, indeed, we shall argue that the experimental precision at present (or in the near future) is such that measurement of these weak corrections may yield sensitive tests for some of the presently fashionable models. Our results are particularly sharp for the unified theories of weak, electromagnetic and strong interactions where we find that the weak correction to $\sigma(e^+e^- \rightarrow \text{all})$ due to interference between one-photon annihilation and one (or more) Z-annihilation diagrams vanishes identically. In general, this interference would give a departure, linear in the square of the c.m. energy, from the purely one-photon values.

We shall use the naïve quark-parton model when calculating $\sigma(e^+e^- \rightarrow \text{hadrons})$. This may be justified for colour-gluon models by invoking asymptotic freedom²⁾ plus the assumption that the asymptotic freedom argument may be applied in the time-like region³⁾: this will allow us to estimate the corrections to the naïve quark-parton calculation.

Calculations: Let f be any point-like fermion (quark or lepton). The piece of the Lagrangian containing the interaction of f with all neutral colourless vector fields may be written in the form⁴⁾

$$e \{ Q_f \bar{\psi}_f \gamma_\mu \psi_f A^\mu + \sum_i \bar{\psi}_f \gamma_\mu (v_{fi} - a_{fi} \gamma_5) \psi_f Z_i^\mu \}. \quad (1)$$

Here Q_f is the charge of the fermion f , in units of the proton charge e . A^μ is the photon field and the Z_i^μ massive vector fields; the v 's and a 's are the vector and axial vector couplings (again measured in units of e).

We calculate to lowest order in the fine structure constant, α , which for the energies we are considering should be a good approximation. Then, the scattering amplitude for $e^+e^- \rightarrow \bar{f}f$ (see Fig. 1) at energy $s^{1/2}$ is ^{*)}

$$T(e^+e^- \rightarrow \bar{f}f) = \frac{\alpha}{s} \{ Q_f \bar{e} \gamma_\mu e \bar{f} \gamma^\mu f + \sum_i \frac{s}{M_i^2 - s} \bar{e} \gamma_\mu (v_{ei} - a_{ei} \gamma_5) e \bar{f} \gamma^\mu (v_{fi} - a_{fi} \gamma_5) f \}, \quad (2)$$

*) In the particular case in which $\bar{f}f$ is e^+e^- , Eq. (2) takes into account only the contribution of the annihilation channel. We shall show later on how to correct for this.

where $e, \bar{e}, f$ and $\bar{f}$ are the appropriate spinors and $M_i = \text{mass of } Z_i$. We will define

$$R_f(s) = \frac{\sigma_{e^+e^- \rightarrow \bar{f}f}(s)}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}(s)}, \quad (3)$$

so, using Eq. (2), we can write

$$R_f(s) = R_f^{AA}(s) + R_f^{AZ}(s) + R_f^{ZZ}(s), \quad (4)$$

where R^{AA} is the pure electromagnetic term, R^{ZZ} the pure "weak" term, and R^{AZ} the interference term. Neglecting the mass of the quark or lepton f as compared with $s^{1/2}$, we have

$$R_f^{AA}(s) = Q_f^2, \quad (5a)$$

$$R_f^{AZ}(s) = 2 Q_f \sum_i \frac{s}{M_i^2 - s} v_{ei} v_{fi}, \quad (5b)$$

$$R_f^{ZZ}(s) = \sum_{ij} \frac{s^2}{(M_i^2 - s)(M_j^2 - s)} (v_{ei} v_{ej} + a_{ei} a_{ej})(v_{fi} v_{fj} + a_{fi} a_{fj}). \quad (5c)$$

In asymptotically free theories, and assuming that $s^{1/2}$ is large compared with the quark masses the quantities

$$R_h^{XY}(s) = \sum_{f=\text{all quarks}} R_f^{XY}(s), \quad X, Y = A \text{ or } Z_i,$$

are corrected by multiplying their value as given by Eqs. (5) by the factor

$$1 + \frac{12}{33 - (2/3)N \log(s/m_o^2)}, \quad m_o \sim 1 \text{ GeV},$$

where N is the over-all number of quarks (counting also colour). This result is supposed to be accurate to

$$O[1/\log^2(s/m_o^2)].$$

These corrections are small if we have not too many quarks and s is large enough.

Models: Eqs. (4) and (5) are quite general. In the particular case of Weinberg-Salam-type models⁵⁾ (W-S models) the contribution to the interference term R^{AZ} from a left quark or lepton doublet $(\begin{smallmatrix} t \\ b \end{smallmatrix})_L$, with charge assignments $Q_t, Q_b = Q_t - 1$, and the appropriate right-handed singlets that need be introduced reads

$$\sum_{f=t,b} R_f^{AZ}(s) = \frac{s(1-4\sin^2\theta_W)[-1+4(Q_t^2+Q_b^2)\sin^2\theta_W]}{2(M_Z^2-s)\sin^2 2\theta_W}; \quad (6)$$

θ_W is the Weinberg angle. When summing over all leptons and hadrons we get

$$R^{AZ}(s) = \sum_{all f} R_f^{AZ}(s) = \frac{s(1-4\sin^2\theta_W)[-n_d^W+4(\sum_f Q_f^2)\sin^2\theta_W]}{2(M_Z^2-s)\sin^2 2\theta_W}. \quad (7)$$

Here n_d^W is the total number of doublets (including colour) of the theory. Next we consider vector-type models^{6,7)}. The contribution to the interference term due to a pair of fermions t, b (quarks or leptons) with charge assignments Q_t and $Q_b = Q_t - 1$ is

$$\sum_{f=t,b} R_f^{AZ}(s) = \frac{2s(1-2\sin^2\theta_V)[-1+2(Q_t^2+Q_b^2)\sin^2\theta_V]}{(M_Z^2-s)\sin^2 2\theta_V}, \quad (8)$$

where θ_V is the corresponding mixing angle. Summing over all fermions then gives

$$R^{AZ}(s) = \sum_{all f} R_f^{AZ}(s) = \frac{s(1-2\sin^2\theta_V)[-n_d^V+4(\sum_f Q_f^2)\sin^2\theta_V]}{(M_Z^2-s)\sin^2 2\theta_V}. \quad (9)$$

Here n_d^V is the total number of lepton and quark doublets (including colour) counting separately right handed and left handed. It is remarkable that both for W-S models and for V-models the interference term vanishes if

$$\sin^2 \theta_{W,V} = \frac{n_d^{W,V}}{4 \sum_f Q_f^2}, \quad (10)$$

which is precisely the value of the angle required to unify weak, e.m. and strong interactions^{8,7)} (grand unified schemes). Actually, these are just examples of a general result that we now state:

Theorem: To order g^4 in strong interactions (gluon corrections), and lowest order in weak and electromagnetic interactions^{*}) and neglecting fermion masses the interference term, viz.,

$$\sum_{\text{all fermions } f} R_f^{AZ}(s)$$

vanishes at large s in all grand unified schemes.

Proof: Suppose we have a grand unified scheme based on a group $S = S_L \times S_R$ where S_L and S_R are both simple (they may coincide), and assume that the left handed fermions belong to the representation α_L of S , the right handed ones to α_R and the gauge bosons to the representation ρ . The fermion-vector boson piece of the interaction Lagrangian may be written as

$$g \sum_{ijk} \{ C(\bar{\alpha}_L, \alpha_L | \rho)_{ijk} \bar{\psi}_i^L \gamma_\mu \psi_j^L W_k^\mu + C(\bar{\alpha}_R, \alpha_R | \rho)_{ijk} \bar{\psi}_i^R \gamma_\mu \psi_j^R W_k^\mu \}, \quad (11)$$

where g is the coupling constant and the C the appropriate Clebsch-Gordan coefficients and, as such, the components of an orthogonal matrix:

$$\sum_{ij} C(\bar{\alpha}_X, \alpha_X | \rho)_{ijk} C(\bar{\alpha}_X, \alpha_X | \rho)_{ijl} = \delta_{kl}, \quad X = L \text{ or } R. \quad (12)$$

As all the vector mesons are now in the same irreducible representation ρ of S , we may identify suitable orthonormal combinations of the neutral vector bosons, say $W_0, W_1, \dots, W_n$ with the physical fields $A, Z_1, \dots, Z_n$, A being the photon field, the Z massive. Equation (12) will still be valid when replacing the $W_0, \dots, W_n$ by the $A, Z_1, \dots, Z_n$ since it clearly is invariant under orthonormal transformations.

It is now easy to check that R_f^{AZ} summed over all f is given, in terms of the currents coupled to A and any Z_ℓ field by (see Fig. 2a)

$$\begin{aligned} R^{AZ_\ell} &\sim \sum_{ij} \langle 0 | J^A(0) | \bar{f}_i f_j \rangle \langle \bar{f}_i f_j | J^{Z_\ell}(0) | 0 \rangle \\ &\sim \sum_{X=L,R} \sum_{ij} C(\bar{\alpha}_X, \alpha_X | \rho)_{ijA} C(\bar{\alpha}_X, \alpha_X | \rho)_{ijZ_\ell} = 0, \end{aligned} \quad (13)$$

^{*}) Higher order corrections due to "weak" couplings are of order $g^2 \log M^2/\mu^2$, where M is the mass of the relevant vector boson. In models with superheavy bosons, one may have M so large that $g^2 \log M^2/\mu^2 \sim 1$, thus giving large corrections to lowest order calculations. In this case, our theorem no longer applies as indeed what happens is that perturbation theory breaks down. Another possible source of violations of the theorem is large Higgs couplings.

by virtue of Eq. (12): Q.E.D. The same arguments show that, in addition (see Fig. 2a-2c)

$$R^{Z_k Z_l} = 0, \quad k \neq l, \quad (14a)$$

$$\sum_{\text{all } f} |\langle 0 | J^A(0) | \bar{f}f \rangle|^2 = \sum_{\text{all } f} |\langle 0 | J^{Z_\ell}(0) | \bar{f}f \rangle|^2, \quad \ell = 1, \dots, n. \quad (14b)$$

The proof presented so far is valid only for pointlike fermions, and neglecting their masses, i.e., up to corrections

$$O\left(\sum \frac{m_f^2}{s}\right).$$

Equations (13) and (14) are broken by diagrams containing, besides the fermion loops, extra Z or W lines or Higgs particles. However, strong interaction corrections enter only to a very high order, namely, the gluon corrections to Eqs. (13) and (14) are of order g^6 or higher. This comes about because the A and Z are colour singlets; so, as colour symmetry is exactly preserved, the only gluon diagrams that would give a non-vanishing contribution to, e.g., the interference terms, would be those in which quarks with different flavours are attached to the J^A and J^Z currents (Fig. 2d). This indicates that the results may be much more accurate than could have been expected *a priori*, as (strong interaction) violations of Eqs. (13) and (14) involve "disconnected" quark diagrams which are strongly suppressed by the Zweig rule (or the asymptotic freedom version of it³).

In our proof we considered only $\bar{f}f$ pairs but not, for example, W^+W^- pairs. As we always sum over physical final state particles, W^+W^- pairs will only begin to contribute for $s > 4M^2(W)$, way above even PETRA energies. However, the theorem is also valid taking into account the coupling of the A, Z to the W; therefore, for energies $s \gg 4M^2(W)$, we shall still get Eqs. (13) and (14) valid with corrections that will be now

$$O\left(\sum \frac{m_f^2}{s}\right) + O\left(\sum \frac{M^2(W)}{s}\right).$$

This concludes our discussion of the Theorem.

Estimates of sizes of corrections: Generally speaking, R^{AZ} will not vanish in a theory not fully unified but will give a correction rising (or decreasing) linearly in s . We will now present the results that follow from our calculations, keeping

an open mind and thus letting free the mixing angles. Also, a few words have to be devoted to the contribution of the process $e^+e^- \rightarrow e^+e^-$ to the sum $\sum_f R_f$. As explained before, the trouble with $e^+e^- \rightarrow e^+e^-$ is that interference terms are also present in the exchange channel. Except in the near forward direction, this represents a small correction; but exact results can also be obtained replacing $e^+e^- \rightarrow e^+e^-$ by $e^+e^- \rightarrow \mu^+\mu^-$, which therefore gets summed twice. Hence, for comparison with experiment we may replace R as defined by Eq. (7) [or Eq. (9)] and Eqs. (3) and (4) by

$$R(s) = \frac{\sigma_{e^+e^- \rightarrow \text{hadrons}}(s) + \sigma_{e^+e^- \rightarrow \text{heavy leptons}}(s) + 2\sigma_{e^+e^- \rightarrow \mu^+\mu^-}(s)}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}(s)}, \quad (15a)$$

$$R(s) = R^{AA}(s) + R^{AZ}(s) + R^{ZZ}(s). \quad (15b)$$

We start with the Weinberg-Salam model with three lepton and three fractionally charged quark families, each in three colours⁵⁾. For the canonical value of the Z-mass, $M_Z = M_W \cos \theta_W$,

$$R^{AZ}(s) \simeq -3.6 \times 10^{-4} (1 - 4 \sin^2 \theta_W) (3 - 8 \sin^2 \theta_W) \frac{s}{(\text{GeV})^2}. \quad (16)$$

This quantity is very sensitive to the value of θ_W ; it is small for present energies and experimentally favoured values of θ_W , which are not far from the $\sin^2 \theta_W = 3/8$ required for grand unification. For example, at $s^{1/2} = 10$ GeV,

$$\begin{aligned} R^{AZ} &= -0.004, & \text{for } \sin^2 \theta_W &= 0.4, \\ R^{AZ} &= -0.04, & \text{for } \sin^2 \theta_W &= 0.1, \\ R^{AZ} &= -0.54, & \text{for } \sin^2 \theta_W &= 1. \end{aligned}$$

In the vector models the Z mass is a further parameter. We have used experimental information on $\bar{\nu}_\mu e$ scattering to eliminate this Z mass, assumed to be much larger than $s^{1/2}$. For the same number of quarks and leptons as for the W-S model we find

$$-0.3 \lesssim R^{AZ} \lesssim 0.4, \text{ at } s^{1/2} = 10 \text{ GeV.}$$

Thus the energy dependence of the quantity R^{AZ} may provide a good test of the grand unified schemes and perhaps yield some information on the nature of the underlying theory.

Finally, we turn to the purely "weak" correction as given by Eq. (5c). In the W-S model, the contribution of a pair of leptons or quarks, t, b is

$$\sum_{t,b} R_f^{ZZ}(s) = \left(\frac{s}{M_Z^2 - s} \right)^2 \frac{(1 - 2\sin^2\theta_W + 4\sin^4\theta_W) [1 - 2\sin^2\theta_W + 4(Q_t^2 + Q_b^2)\sin^4\theta_W]}{\sin^4 2\theta_W}, \quad (17)$$

while in the V-model

$$\sum_{t,b} R_f^{ZZ}(s) = 4 \left(\frac{s}{M_Z^2 - s} \right)^2 \frac{(1 - 2\sin^2\theta_V + 2\sin^4\theta_V) [1 - 2\sin^2\theta_V + 2(Q_t^2 + Q_b^2)\sin^4\theta_V]}{\sin^4 2\theta_V}. \quad (18)$$

In grand unified schemes we can use our relation (14b) to get at once

$$\sum_{all f} R_f^{ZZ}(s) = \left(\frac{s}{M_Z^2 - s} \right)^2 (v_e^2 + a_e^2) \sum_f Q_f^2, \quad (19)$$

where we have assumed that there is only one Z meson; otherwise the r.h.s. of Eq. (19) gets summed over all Z mesons. Using

$$v_e = \frac{1 - 4\sin^2\theta_W}{2\sin 2\theta_W}, \quad a_e = \frac{1}{2\sin 2\theta_W} \quad (\text{W-S model}), \quad (20)$$

$$v_e = \frac{1 - 2\sin^2\theta_V}{\sin 2\theta_V}, \quad a_e = 0 \quad (\text{V model}), \quad (21)$$

one gets for, e.g., the grand unified W-S model,

$$R^{ZZ}(s) = \left(\sum_f Q_f^2 \right) \left[1 + \frac{75}{(64)^2} \left(\frac{s^{1/2}}{37.4 \text{ GeV}} \right)^4 \right], \quad (22)$$

where the sum includes all leptons and all quarks. At present energies R^{ZZ} is minute, but would be of some 0.14 (for the six-quark model) at PETRA energies.

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Figure captions

Fig. 1 : e^+e^- -annihilation into $f\bar{f}$.

Fig. 2 : Contributions to R^{AZ} , R^{AA} and R^{ZZ} .

FIG.1

FIG.2